

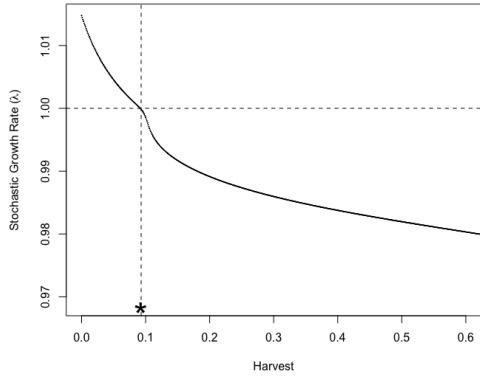
Supplementary Information for Plant and disperser co-harvest model

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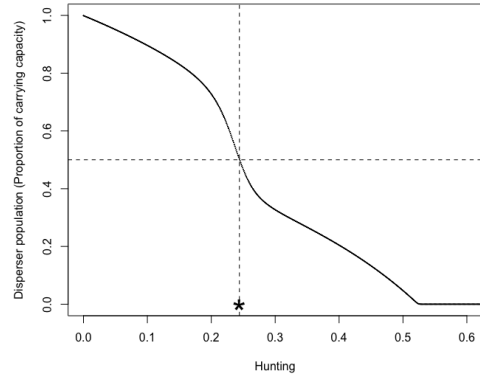
December 4, 2020

1 Supplemental Analysis

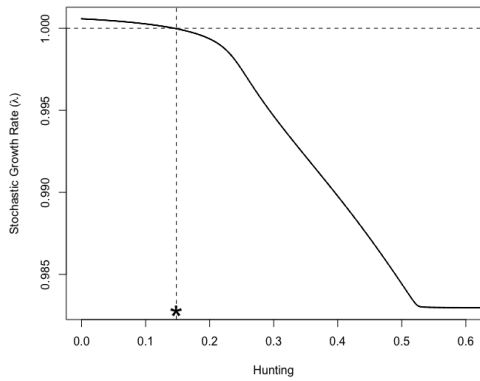
The figures below presented extended results supporting the analysis presented in the manuscript.



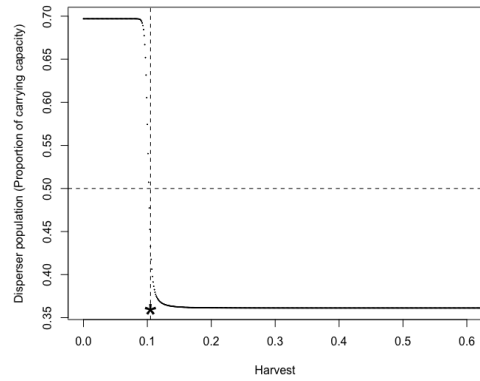
(a)



(b)



(c)



(d)

Figure 6: The sustainable thresholds for both Brazil nut and agouti under varying harvest and hunting level regimes. Note that the other parameters were held at these values: $\delta_{d \rightarrow p} = 1$, $\delta_{p \rightarrow d} = 0.5$, $G_t = 0.85$, $S_t = 0.9$.

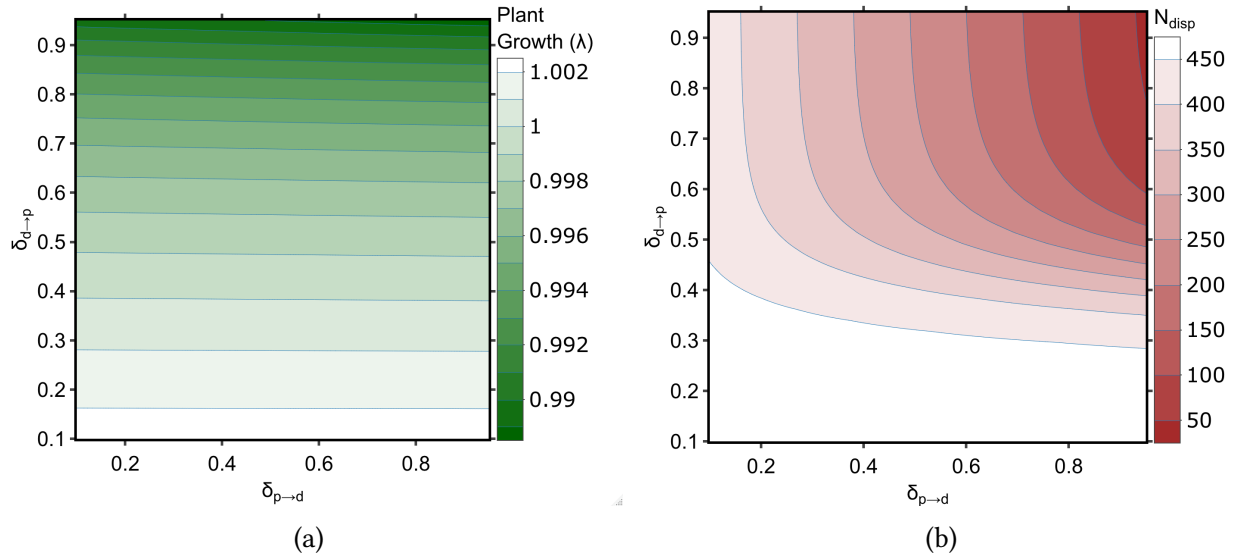


Figure 7: Varying strengths of coupling - Low harvest and high hunting ($G_t = 0.95$, $S_t = 0.92$ and $\hat{R}_t = 0.5$)

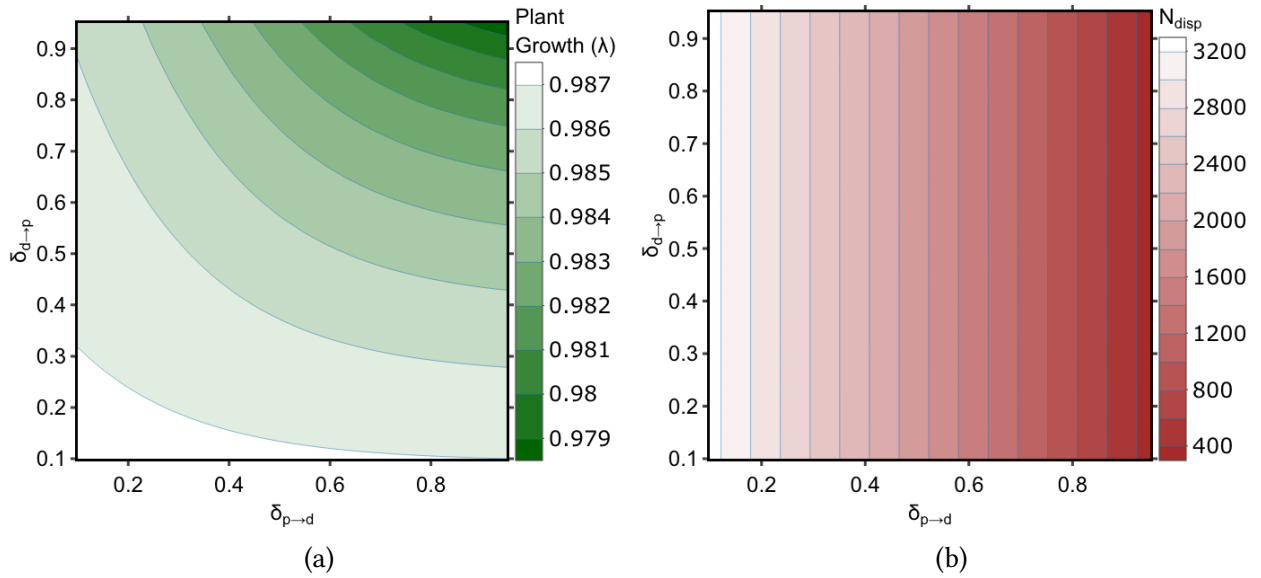
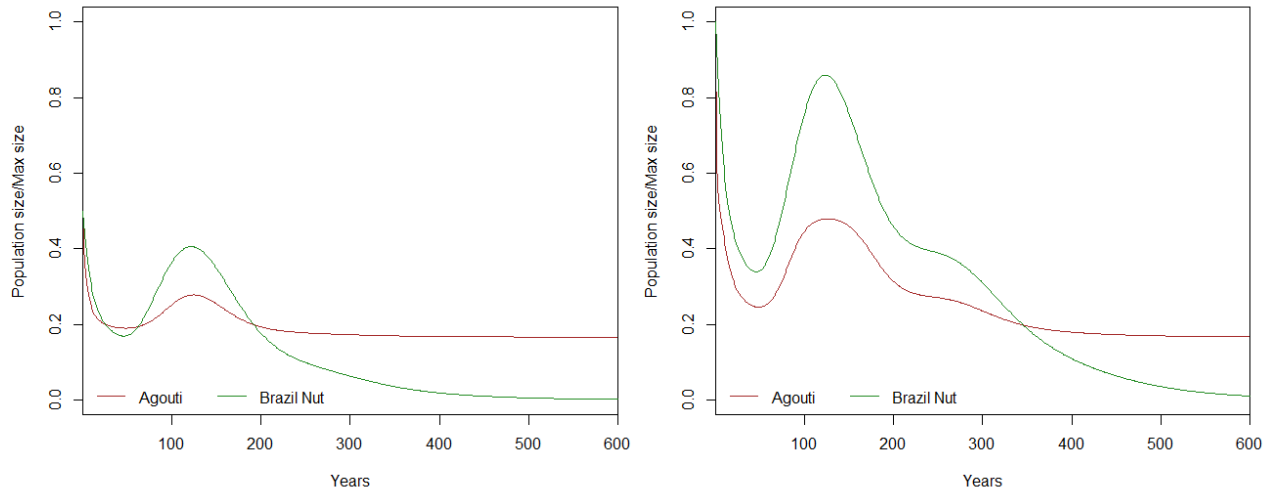
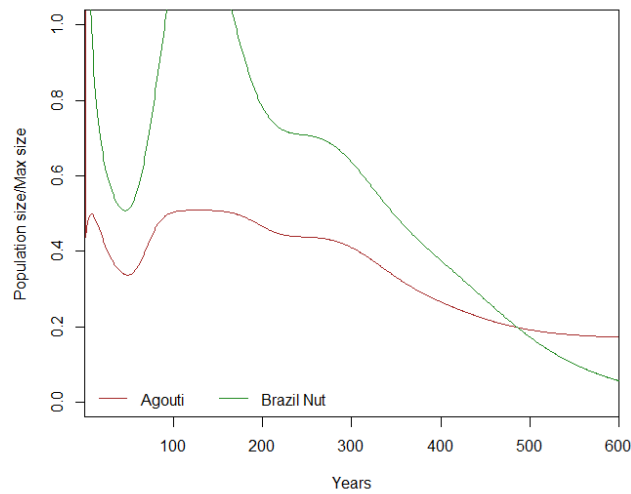


Figure 8: Varying strengths of coupling - High harvest and low hunting ($G_t = 0.5$, $S_t = 0.5$ and $\hat{R}_t = 0.25$)



(a)

(b)



(c)

Figure 9: Agouti and Brazil Nut Tree population simulated throughout 600 years under differing initial conditions (0.5x, 1.0x, and 1.5x baseline of 5200 agoutis and 100 adult trees, respectively).

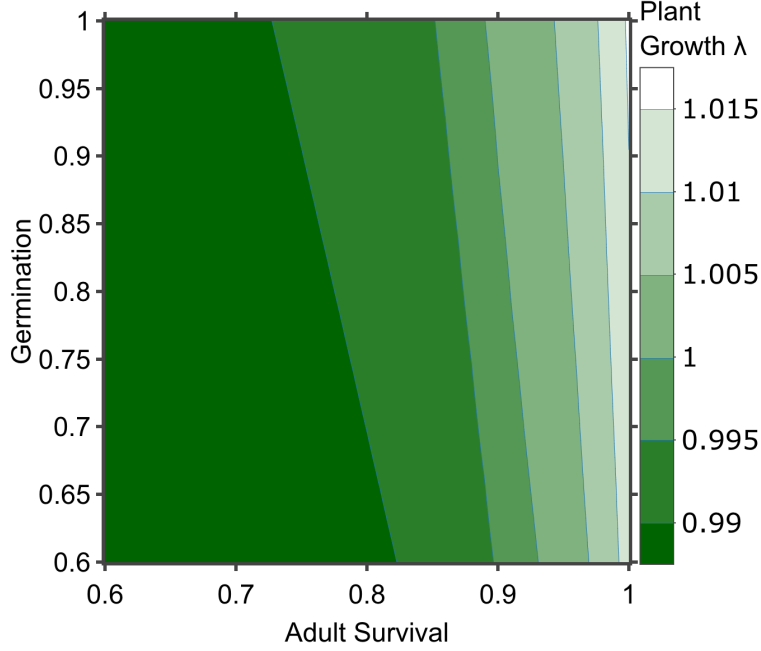


Figure 10: Brazil nut growth rate under varying levels of G_t and S_t . Other parameters were held at the following: $\delta_{d \rightarrow p} = 1$, $\delta_{p \rightarrow d} = 0.5$, and $\hat{R} = 0.25$.

1.1 Incorporating Plant Density Dependence

We extend our existing modeling framework to include plant density dependence. We introduce a threshold value of plant abundance beyond which the disperser will not continue to experience population growth benefits associated with the plant abundance. This model case applies specifically to plant-disperser systems that are not subjected to strong harvesting pressure and can survive to reach or exceed their carrying capacity. We incorporated the Allen (1998) density dependence Leslie matrix model into our plant-disperser harvest model.

The matrix model has the form $V_{t+1} = P_t(N_t)V_t$, where $P_t(N_t) = P(N_t)u^{-1}$, P is the plant transition matrix, V_t represents the plant population stage distribution and u is the density dependence term defined below:

$$u(v_{a,t}) = \frac{K_a + (\lambda_o - 1)v_{a,t}}{K_a}$$

λ_o is the dominant eigenvalue of P , $v_{a,t}$ is the plant adult population size at time t and K_a represents the adult plant carrying capacity. We only consider the adult plant population as it directly provides growth benefits to the seed disperser.

1.1.1 Analysis

In our main text, we present all analyses based on plant-disperser model without plant density dependence. We now present analyses from our extended model framework. Incorporating plant density dependence reduced the parameter space needed for the growth of Brazil nut trees (Figure 2,

Figure 11), and decreased the hunting rate (less than 0.4) that can support more than 5000 agoutis (Figure 11b). This suggests that plant density dependence reduced the sustainable level of hunting.

Figure 12 represents the sustainable hunting and harvest threshold needed for the persistence of the Brazil nut trees and the agouti. Plant density dependence appears to contribute to no change in the sustainable threshold level needed for both the plant-disperser pair.

Figure 13 shows a wider parameter space for both coupling interaction terms for the disperser population compared to Figure 4, allowing increased persistence of the agouti.

When conducting Sobol sensitivity analysis for the plant density dependence model (Figure 14), we observe $\delta_{p \rightarrow d}$ and \hat{R}_t as the important factors. However, in the total-order indices we see all of the parameters having a substantial effect on both the plant and animal abundance, reflecting the increased complexity associated with a density-dependent model for the Brazil nut tree and agouti.

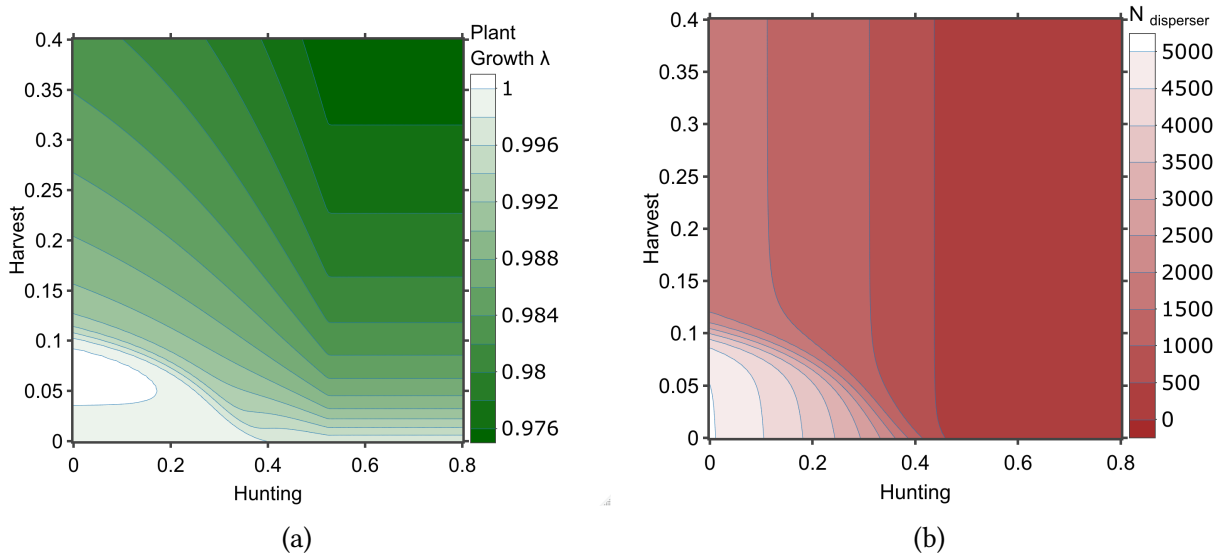


Figure 11: Agouti population size ($N_{\text{disperser}}$) and the average growth rate of Brazil nut (plant; λ) under varying levels of harvesting and hunting. The simulation was run with the parameters in Table 1 as well as $\delta_{d \rightarrow p} = 1$ and $\delta_{p \rightarrow d} = 0.7$.

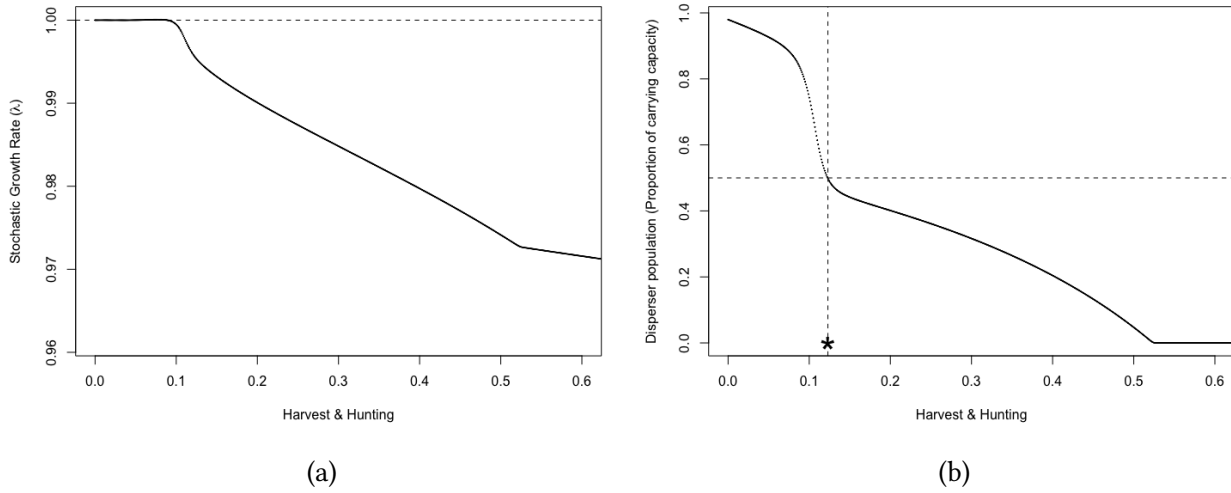


Figure 12: The impact of harvesting regimes (plant harvest and disperser hunting) on Brazil nut and agouti populations. Note that in these simulations, $\delta_{d \rightarrow p} = 1$, $\delta_{p \rightarrow d} = 0.5$, $G_t = 0.85$, $S_t = 0.9$. * represents the sustainable harvesting regime corresponding to a persistent population.

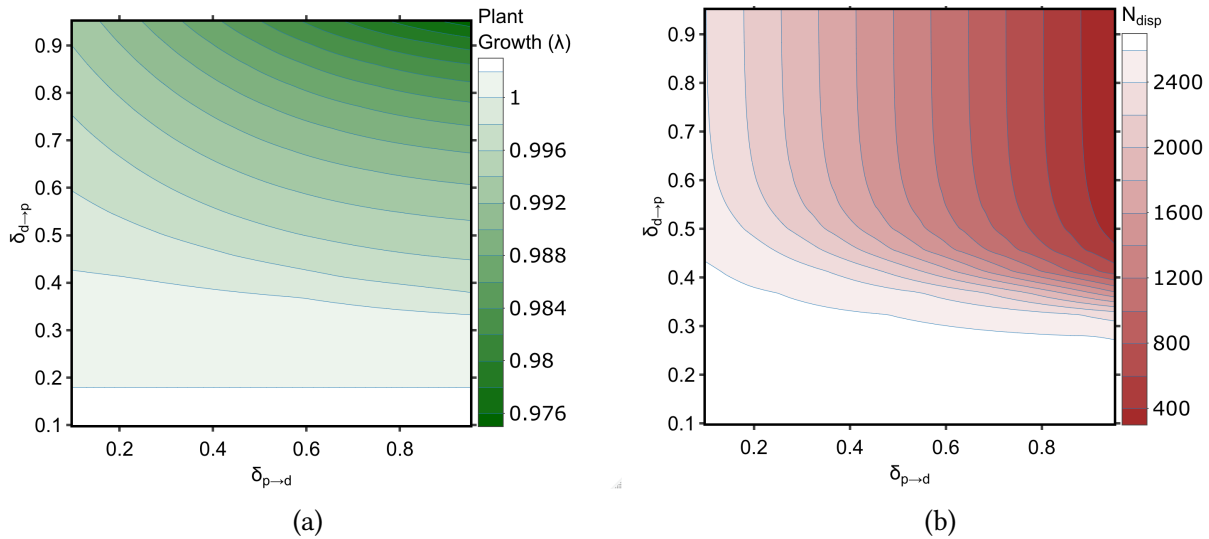
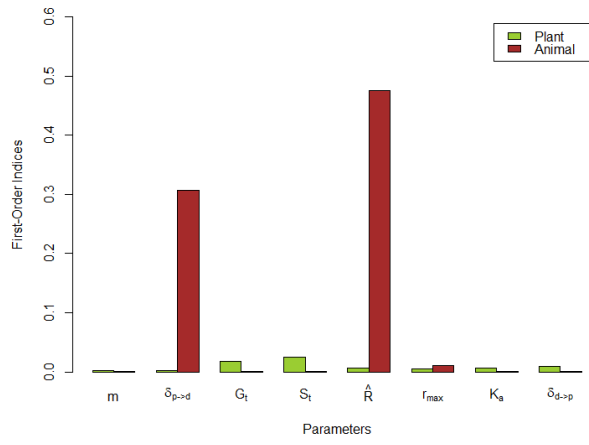
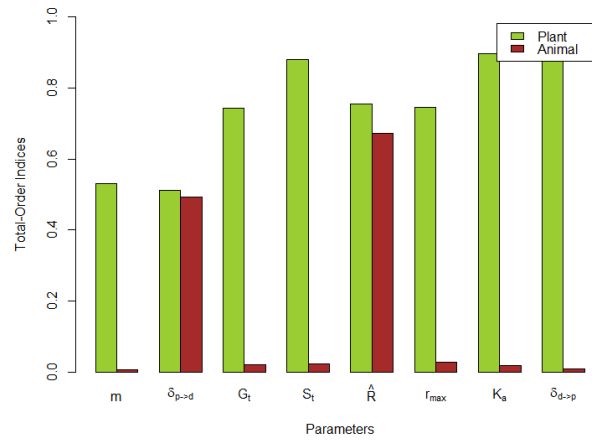


Figure 13: Average growth rate of Brazil nut (λ) and Agouti abundance (N_{disp}) under varying strengths of coupling (represented by $\delta_{p \rightarrow d}$, reliance of Brazil nut on agouti, and $\delta_{d \rightarrow p}$, the impact of the Brazil nut on the agouti). The simulation was run with the following harvest regime: $G_t = 0.85$, $S_t = 0.9$ and $\hat{R}_t = 0.35$.



(a)



(b)

Figure 14: Sobol indices for the density-dependent plant model.