

The Functional Composition of Sense*

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Abstract

A central dispute in understanding Frege's philosophy concerns how the sense of a complex expression relates to the senses of its component expressions. According to one reading, the sense of a complex expression is a whole built from the senses of the component expressions. On this interpretation, Frege is an early proponent of structured propositions. A rival reading says that senses compose by functional application: the sense of a complex expression is the value of the function denoted by its functional component for the arguments denoted by its remaining components. I argue that two non-negotiable Fregean theses entail that senses compose by functional application. One thesis is that referents compose by functional application. The other thesis is that an expression in an indirect context refers to its customary sense.

Two aspects of Frege's mature semantic theory have each been heralded as central advances in the development of a rigorous semantics. One aspect—introduced in *Function and Concept*—is that reference is not merely compositional, but functionally compositional. According to Frege, each expression has a *Bedeutung*, or referent. Some expressions refer to functions. Others expressions refer to arguments for these functions. A complex expression refers to the value of the function referred to by one component for the arguments referred to by the remaining components. Letting $\llbracket \cdot \rrbracket_R$ take an expression to its referent, Frege's model can be stated as follows.¹

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¹The operation $\llbracket \cdot \rrbracket_R$ will be type-sorted since the referent of an incomplete expression is of a different type from the referent of a complete expression. This point is emphasized in (Wright 2004).

FUNCTIONAL COMPOSITION OF REFERENCE: If α is a complex expression composed of a functional expression β applied to n argument expressions $\gamma_1 \dots \gamma_n$, then $\llbracket \alpha \rrbracket_R = \llbracket \beta \rrbracket_R(\llbracket \gamma_1 \rrbracket_R, \dots, \llbracket \gamma_n \rrbracket_R)$.

In addition to its core role in Frege's later philosophy, this model of semantic composition has been extraordinarily fruitful as a "leading idea" in contemporary formal semantic theories (Heim and Kratzer 1998, p. 13).²

In the same work, Frege observes that reference does not explain all of the relevant properties of an expression.³ Two terms such as 'the capital of Texas' and 'Austin' refer to the same city. Yet, one can understand both expressions without knowing that they co-refer. To explain this cognitive difference, Frege posits that expressions have *senses* in addition to referents. The sense of an expression is grasped by everyone who understands it. Although the expressions 'the capital of Texas' and 'Austin' co-refer, they differ in sense.

In *On Sense and Reference*, Frege appeals to the distinction between sense and reference to explain why certain co-referential expressions do not substitute in all contexts without change of truth-value. Although 'Austin' and 'the capital of Texas' co-refer, the attitude ascriptions (1) and (2) differ in truth-value.

(1) Ben believes that Austin is hot.

(2) Ben believes that the capital of Texas is hot.

²The expression *functional compositionality* comes from (Cresswell 2002). In this use, *functional compositionality* is a more specific thesis than compositionality, which says that the meaning of a complex is uniquely determined by the meaning of its parts and their arrangement. Functional compositionality demands that the meaning of the complex expression is determined by a specific semantic composition rule, functional application. Cresswell's use of the expression 'functional compositionality' differs subtly from another standard use of the expression (Pagin and Westerståhl 2010a: 254), on which the semantic value of a complex expression is the result of an application of a function (determined by the syntactic composition rule) to the semantic values of the constituent expressions.

³(Frege 1891/1997, p. 138).

Frege's explanation is that the referent of an expression in an opaque context shifts to its customary sense.

REFERENCE SHIFT SEMANTICS: In a singly embedded opaque context such as a propositional attitude ascription, the referent of an expression α is its ordinary sense.

By appeal to the reference shift semantics, Frege preserves his insight that the referent of a complex expression is a function of the referents of its parts.

The FUNCTIONAL COMPOSITION OF REFERENCE and the REFERENCE SHIFT SEMANTICS are uncontroversially Fregean theses. I argue that they together entail the controversial thesis that senses functionally compose, just as referents do. That is, the sense of a complex expression is the result of functionally applying the sense expressed by its functional component to the senses of its argument components. Letting $\llbracket \cdot \rrbracket_S$ map an expression to its sense, the thesis can be expressed as follows:

FUNCTIONAL COMPOSITION OF SENSE: If α is a complex expression composed of a functional expression β applied to n argument expressions $\gamma_1 \dots \gamma_n$, then $\llbracket \alpha \rrbracket_S = \llbracket \beta \rrbracket_S(\llbracket \gamma_1 \rrbracket_S, \dots, \llbracket \gamma_n \rrbracket_S)$.

The functional composition of sense entails that the sense of a predicate is a sense function, a function that maps senses of terms to the senses of sentences. Although historically popular,⁴ this view has recently been criticized in favor of the rival view that the sense of a complex expression is a *structured complex* containing the senses of its components.⁵ If the sense of a complex expression is a structured whole, the components of which correspond to the components of the sentence, then two sentences differing structurally cannot express the same thought. But, if the sense of a sentence is the *value* of the function denoted by

⁴See (Church 1951) and (Geach 1976). See (Klement 2002, 66, footnote 9) for a more comprehensive list.

⁵Sources of this criticism include (Dummett 1981a, chapter 6), (Sullivan 1992), (Klement 2002), and (Heck and May 2011).

its functional component for the arguments denoted by the remaining components, then structurally different sentences may express the same thought.

The choice between these interpretations of the composition of sense is of central importance for understanding Frege's core aim, *logicism*—the thesis that mathematics reduces to logic. Frege defends logicism by regimenting ordinary mathematical claims into the language of *Begriffsschrift* and showing that the regimented sentences follow from purely logical principles. Proponents of the two interpretations often differ as to whether two ordinary sentences with different structures or a sentence and its regimentation into *Begriffsschrift* express the same thought.⁶ This makes a difference to whether Frege's regimentations replace or interpret ordinary mathematical sentences. More centrally, Frege also defends some of the axioms of *Begriffsschrift* by suggesting that distinct sentences of this formal language express the same thought.⁷ Proponents of structured conceptions of sense such as Klement (2002, p. 87) and Heck and May (2011, p. 155) see this as a mistake or "slip" on Frege's part.⁸

The rival interpretations echo in contemporary debates about propositions, the contemporary analogues of Frege's thoughts. According to some contemporary theories, propositions lack a part-whole structure.⁹ On such views, the meanings of the parts of a sentence may determine the proposition expressed by the sentence, but they are not constituents of this proposition. So sentences with different structures can express the same proposition. Structured propositionalists say that a proposition is a structured whole.

⁶For the former issue, see (Dummett 1981a, pp. 228-9). For the latter issue, Weiner (1999, pp. 67-8) and Makin (2000, chapter 7) hold that the regimentations replace ordinary mathematical sentences. (Blanchette 2012) argues that the natural language sentences and their *Begriffsschrift* regimentation express the same thought.

⁷See (Frege 1891/1997, 136).

⁸A major source of this objection is (Dummett 1991, chapter 14).

⁹Propositions may be identified with *intensions*, functions from possibilities to truth-values, or with more fine-grained entities. See Cresswell (2002) for intensionalism and Keller (2013, 2019) and Merricks (2015) for more-fine grained views.

Many say that its parts are identical to the semantic values of the components of the sentence that expresses it. They thereby deny functional compositionality.¹⁰ According to such views, sentences with different structures never express the same proposition.¹¹ The FUNCTIONAL COMPOSITION OF REFERENCE and the REFERENCE SHIFT SEMANTICS have been deeply instrumental in the development of formal semantics. Respecting these principles or their descendants in contemporary semantics imposes constraints on any conception of propositions on offer.¹²

1 The Functional composition of Reference

Starting with *Function and Concept*, the FUNCTIONAL COMPOSITION OF REFERENCE takes a central place in Frege's philosophy. Frege arrives at this conception of semantic composition by generalizing from the case of complex terms in mathematical languages. In this section, I explain his analysis of these expressions and then discuss the principles by which he generalizes this analysis to other expressions.

1.1 The Mathematical Model

Frege's core model for complex expressions comes from mathematics.¹³ According to the Fregean analysis, a complex expression of mathematics such as '7-1' is partly composed of "complete" expressions such as '7' and '1'. The expression '7-1' also leaves an incomplete remainder when its complete components are removed, which Frege writes as ' $\xi-\zeta$ '. Frege says that this incomplete expression refers to a function that is similarly incomplete. The

¹⁰See Salmon (1986), Soames (1987), and King (1995, 2007).

¹¹See (King 2013). (Båve 2019) offers similar considerations to criticize act-based accounts of propositional attitudes developed in Moltmann (2003), Soames (2010), and Hanks (2011).

¹²For views of structured propositions that do attempt to respect the developments of functional compositionality, see Elbourne (2011) and Pickel (2019, forthcoming).

¹³See (Frege 1891/1997, p. 131). See also the letter to Russell dated 20.10.1902 (p. 149) in (Frege 1980).

function term needs an argument to form a complete expression. The function itself needs an argument to return a complete entity.

[O]ne can always speak of the name of a function as having empty places, since what fills them does not, strictly speaking, belong to them. Accordingly I call the function unsaturated, or in need of supplementation, because its name has first to be completed with the sign of an argument if we are to obtain a [referent] that is complete in itself. I call such a [referent] an object and, in this case, the value of the function for the argument that effects the supplementing or saturating. (Frege 1892/1972, p. 118)

The last sentence expresses the FUNCTIONAL COMPOSITION OF REFERENCE for expressions which result from saturating a functional expression with expressions referring to arguments of the right type. Frege next generalizes this picture from the expressions of mathematical language to all expressions of a well-constructed language.

1.2 Extending the Model

Frege (1891/1997, p. 137) observes that the notion of a function has been extended in two directions. One extension expands the range of expressions that contain functional expressions. Frege (1891/1997, pp. 139-40) argues that expressions such as ‘the capital of England’, ‘the capital of the German Empire’, and ‘the capital of Texas’ take the same syntactic positions as proper names. These expressions can also be split up into parts. One part of each—‘England’, ‘the German Empire’, and ‘Texas’, respectively—is saturated. The remainder (‘the capital of ...’) is unsaturated; it has a gap. Entering a complete name into the expression ‘the capital of ...’ outputs a complex term: a name for London, for Berlin, and for Austin, respectively. Saturating ‘the capital of ...’ with co-referential

names such as 'Texas' and 'the Lone Star State' always results in co-referential names. Based on this correspondence with mathematical expressions, Frege says the incomplete expression 'the capital of . . . ' refers to a function. Saturating this incomplete expression with a complete expression refers to the value of the function when it takes the referent of the complete expression as argument. The overall argument is not that Frege regards these *natural language expressions* as functional expressions but rather that the *Begriffsschrift* regimentations of these natural language expressions are functional expressions.

The other extension expands the concept of a function. A function takes any objects as arguments and output any objects as values. The question now arises whether Frege can delimit the range of possible objects and functions. Frege answers that each thing is either an object or a function.

When we have thus admitted objects without restriction as arguments and values of functions, the question arises what it is that we are here calling an object. [. . .] an object is anything that is not a function, so that an expression for it does not contain any empty place. (Frege 1891/1997, p. 140)

Complete expressions refer to objects. Frege is clearly committed to the thesis that the expression for a function must be incomplete. But I believe that he is likewise committed to the converse as well: an incomplete expression always refers to a function. As Frege says, "We recognize the function in the expression by imagining the latter as split up" (Frege 1891/1997, p. 134, cf. Frege 1879/1970, p. 13). This commitment explains why Frege immediately concludes that the incomplete component 'the capital of . . . ' expresses a function from the fact that (the *Begriffsschrift* regimentation of) 'the capital of the German Empire' can be divided into a complete and an incomplete component.

Indeed, Frege moves further. Since sentences lack gaps, they are complete expressions. Sentences therefore refer to objects and not to functions.

A statement contains no empty place, and therefore we must take its *Bedeutung* as an object. (Frege 1891/1997, p. 140)

A sentence such as ‘Caesar conquered Gaul’ contains complete components: ‘Caesar’ and ‘Gaul’. Just as the result of removing a complete expression from a complex term yields an expression with a gap, the result of removing a complete expression from a sentence yields an expression with a gap, ‘ξ conquered Gaul’. Therefore, the result of removing a name from a sentence denotes a function. This argument *presupposes* that if a complete expression can be divided into a complete component and an incomplete component, then the remainder refers to a function. Predicates such as ‘... conquered Gaul’ refer to functions, which Frege calls *concepts*. These functions take as input the objects referred to by complete expressions and output the objects referred to by sentences.¹⁴

2 Sense and Indirect Reference

Sentences that differ by the substitution of co-referential constituents always agree in reference. But they may “tell us quite different things, express quite different thoughts” (Frege 1891/1997, p. 138). A thought is the *sense* of the sentence that expresses it.¹⁵ Individual words also have senses. The thought expressed by a sentence is determined by the senses of the words that make it up.

[A] proposition [that is, sentence] consists of parts which must somehow contribute to the expression of the sense of the proposition, so they themselves must have a sense. (Frege to Jourdain, undated, Frege 1980, p. 79)

¹⁴Frege equates the “predicative nature” of the referent of a predicate with the incompleteness or “unsaturatedness” of a function (Frege 1892/1972, p. 119).

¹⁵I believe that Frege’s early formulation of the reason for positing sense in terms of what the sentences “tell us” strongly supports the views of (Noonan 2001, p. 187) and (Makin 2010, pp. 153-4)—as against (Dummett 1981a, p. 293) and (Geach 1961, p. 162)—that the theory of indirect reference is not merely some “optional extra which can be grafted on to the distinction of sense and reference” (Noonan 2001, p. 187).

Frege posits senses because substituting co-referential terms in a sentence does not always preserve the thought expressed. The sentences 'Austin is hot' and 'the capital of Texas is hot' express different thoughts, even though 'Austin' and 'the capital of Texas' co-refer.¹⁶

2.1 Indirect Reference

A belief ascription contains an embedded sentence and ascribes the thought the embedded sentence is customarily used to express. Thus, (1) ascribes to Ben the thought that Austin is hot and (3) ascribes to him the thought that Seattle is rainy.

(1) Ben believes that Austin is hot.

(3) Ben believes that Seattle is rainy.

Although the embedded sentences—'Austin is hot' and 'Seattle is rainy'—are both true, (1) and (3) may differ in truth-value. Frege concludes from such cases that the referent of a sentence in a belief ascription is not its truth-value, but rather the thought it expresses, its customary sense.

Sentences are not the only expressions that acquire a different referent in an attitude ascription. Component expressions also refer to their customary senses in attitude ascriptions.

In reported speech one talks about the sense, e.g., of another person's remarks.

It is quite clear that in this way of speaking words do not have their customary reference but designate what is usually their sense. [...] In reported speech, words are used indirectly or have their indirect reference. We distinguish accordingly the customary from the indirect reference of a word; and its

¹⁶See (Frege 1892/1970, p. 62) and (Frege to Russell 13.11.1904, Frege 1980, p. 164). This marks a change from Frege's earlier position according to which a sentence has a *judgeable content* as its single semantic value. See discussion in (Currie 1984) and (Weiner 1999, pp. 38-40; pp. 78-9).

customary sense from its indirect sense. The indirect reference of a word is accordingly its customary sense. (Frege 1892/1970, p. 59)

This requires making explicit an assumption that Frege often leaves implicit. A sentence has the same syntactic structure inside and outside of an attitude ascription.¹⁷

EMBEDDED STRUCTURE: A sentence has the same syntactic and semantic structure both inside and outside of an attitude ascription. This principle can be broken into two sub-theses:

If a sentence ϕ results from the application of a functional expression π to argument expressions $\alpha_1, \dots, \alpha_n$ outside of an attitude ascription, then it has this same syntactic derivation inside of an attitude ascription and *vice-versa*.

All expressions that have referents outside of an attitude ascription have referents inside of an attitude ascription.

EMBEDDED STRUCTURE says that a sentence inside an attitude ascription isn't a *syntactic atom*. It has components. It isn't a *semantic atom* either. Its components have *some reference or other*. But EMBEDDED STRUCTURE does not tell us what those referents are.

The REFERENCE SHIFT SEMANTICS does tell us what the referent of an expression in an attitude ascription is: its customary sense. Because expressions that are customarily co-referential do not substitute *salva veritate* when embedded in attitude ascriptions, these expressions cannot have their customary referents when embedded in these ascriptions. For example, (1) may be true while (2) is false even though 'Austin' and 'the capital of Texas' co-refer in direct contexts.

¹⁷This assumption is deeply reflected in Frege's practice. For instance, in a letter to Russell (28.12.1902, Frege 1980, p. 153), Frege typographically differentiates expressions embedded under a single attitude operator from those that are twice embedded. This would be unnecessary if the embedded sentence had no internal structure. And it would be mysterious if the embedded sentence had a different structure from an unembedded occurrence of the same sentence.

- (1) Ben believes that Austin is hot.
- (2) Ben believes that the capital of Texas is hot.

Frege concludes that the terms ‘Austin’ and ‘the capital of Texas’ do not refer to their customary referents in (1) and (2), respectively. Rather, these expressions refer to their customary senses. The customary senses of ‘Austin’ and ‘the capital of Texas’ differ. Therefore, they also differ in reference when embedded in belief ascriptions, allowing for the possibility that (1) and (2) may differ in truth-value.

2.2 Regimenting Indirect Reference

The REFERENCE SHIFT SEMANTICS introduces complications to the representation of the semantic theory. A single expression has different referents (and potentially different senses) in different contexts. To handle this, one might relativize the reference function $\llbracket \cdot \rrbracket_R$ to a context so that $\llbracket \cdot \rrbracket_R^C$ represents the customary referent of an expression and $\llbracket \cdot \rrbracket_R^I$ represents the indirect referent. The referent of a whole would be determined by the referents of its parts in their respective contexts. This approach has been called *the method of indirect discourse* (Kaplan 1964: pp. 22-23). It has been developed to a considerable degree of technical sophistication. It has even been argued that it paves the way to an “occurrence based” alternative to the standard “expression based” semantics.¹⁸

Frege nowhere articulates anything this sophisticated. Geach (1961, p. 162) even suggests that “Frege never worked out this theory far enough to have to consider how it should be symbolically expressed.” Yet, in a letter to Russell, Frege does develop his approach, which has subsequently called been *the method of direct discourse* (Kaplan 1964: pp. 22-23). Frege treats the fact that a word has different referents in different contexts as a kind of *ambiguity*. One expression occurs unembedded and a different expression occurs

¹⁸See (Pagin and Westerståhl 2010b) and (Pickel and Rabern forthcoming).

embedded. In representing this formally, one expression is used in ordinary discourse and a different expression is used embedded.

To avoid ambiguity, we ought really to have special signs in indirect speech, though their connection with the corresponding signs in direct speech should be easy to recognize. (Frege to Russell 28.12.1902, Frege 1980, p. 153)

Frege singly underlines expressions when they are used to refer to their customary senses and doubly underlines expressions when they are used to refer to their indirect senses.¹⁹

For my purposes, there is no difference between the two approaches. I follow Frege in deploying the method of direct discourse. In regimenting an attitude ascription such as (1) ‘Ben believes that Austin is hot’, I underline the expressions occurring in the ‘that’-clause.

(1a) Ben believes that Austin is hot.

An underlined expression refers to the customary sense of the expression: $\llbracket \text{Austin} \rrbracket_S = \llbracket \underline{\text{Austin}} \rrbracket_R$.²⁰ By EMBEDDED STRUCTURE, every expression occurring in the derivation of a sentence has an underlined correlate occurring in the derivation of the underlined correlate of the sentence.

3 The Composition of Sense

The sense of a whole sentence such as ‘Austin is hot’ is somehow determined by the senses of its components, ‘Austin’ and ‘ξ is hot’, and their organization. This raises two

¹⁹The translation in (Frege 1980) uses italics for singly embedded contexts and capitalized expressions for doubly embedded. Either approach makes the relation between the expression referring to the customary referent and the expression referring to the customary sense is “easy to recognize”. Kripke (2011a, p. 263) rightly questions what it means to say that the correspondence should be “easy to recognize”.

²⁰This provision handles unembedded attitude ascriptions. Cases of embedded attitude ascriptions are contentious.

questions. First, how do the senses of the parts go together to determine the sense of the whole? Second, what is the sense of the function term? I argue that senses functionally compose just as referents do. The sense of a functional expression is a *sense function*, a function from senses to senses. The sense of a composite expression is the value of the sense of its functional component for the senses of its saturated components as arguments. Thus, just as the referent of ‘the capital of Texas’ is the value of the referent of ‘the capital of ξ ’ when applied to the referent of ‘Texas’, the sense of ‘the capital of Texas’ will be the value of the sense of ‘the capital of ξ ’ when applied to the sense of ‘Texas’.

Historically, this interpretation was defended by pointing to the fact that Frege repeatedly writes that the sense of a functional expression such as a predicate or a logical operator is unsaturated.²¹ For instance, Frege says that the sense of a negated sentence such as ‘Texas is not hot’ results from somehow combining the unsaturated sense of ‘not’ with the saturated sense of the negated sentence ‘Texas is hot’.

The thought does not, by its make-up, stand in need of any completion; it is self-sufficient. Negation on the other hand needs to be completed by the thought. (Frege 1918-9/1984a, p. 386)

Frege refers to this unsaturated or incomplete sense of ‘not’ using an unsaturated expression: ‘the negation of. . .’. The thought that Texas is not hot is the negation of the thought that Texas is hot.

The fact that the sense of a functional expression is unsaturated suggests that this sense is itself a function for two reasons. First, functions are Frege’s core model of unsaturated entities. Frege does not provide any alternative model of unsaturated entities.²² Second, as we have seen, Frege believes that the division between objects and functions is total.

²¹Emphasized by (Geach 1976).

²²This is a crucial point of (Geach 1976, p. 445).

An argument is anything that is not a function, so that an expression for it does not contain any empty place. (Frege 1891/1997, p. 140)

Since the senses of functional expressions are unsaturated, they cannot be objects. So, they must be functions. Any rival interpretation must provide some alternative model for the incompleteness of senses.

This argument is suggestive, but not conclusive. Opponents of the FUNCTIONAL COMPOSITION OF SENSE have suggested that the senses and referents of incomplete expressions are unsaturated in different ways. So, even if they are—in some sense—unsaturated, the senses of incomplete expressions might nevertheless be objects.

To move beyond the impasse, I will argue that the reference shift semantics provides a direct route from the FUNCTIONAL COMPOSITION OF REFERENCE to the FUNCTIONAL COMPOSITION OF SENSE. This argument tells decisively in favor of the functional composition of sense, because it does not rely on the premise that there must be an *analogy* between senses and reference, but rather directly connects them by way of the REFERENCE SHIFT SEMANTICS.

4 The Simple Argument

I turn now to the argument for the FUNCTIONAL COMPOSITION OF SENSE. For concreteness, consider the sentence ‘Austin is hot’ composed by saturating the functional expression ‘ ξ is hot’ with the complete expression ‘Austin’. In this case, the FUNCTIONAL COMPOSITION OF SENSE says that the sense of this sentence is the result of applying the sense expressed by ‘ ξ is hot’ to the sense expressed by ‘Austin’. That is, $\llbracket \text{Austin is hot} \rrbracket_S = \llbracket \xi \text{ is hot} \rrbracket_S(\llbracket \text{Austin} \rrbracket_S)$. This can be shown by considering (1).

- (1) Ben believes that Austin is hot.

According to the REFERENCE SHIFT SEMANTICS, the expressions in the sentence ‘Austin is hot’ refer to their customary senses in the context of (1). This is implemented in the semantics by regimenting the attitude ascription so that every embedded expression is replaced by an expression referring to a sense.

(1a) Ben believes that Austin is hot.

The sentence ‘Austin is hot’ refers to the sense of the original sentence ‘Austin is hot’. That is, $\llbracket \underline{\text{Austin is hot}} \rrbracket_R = \llbracket \text{Austin is hot} \rrbracket_S$. EMBEDDED STRUCTURE guarantees that the structure of the resulting embedded sentence mirrors the structure of the original: ‘Austin is hot’ syntactically derives from ‘Austin’ and ‘ξ is hot’. The REFERENCE SHIFT SEMANTICS tells us that the referents of these expressions are the senses of their unembedded counterparts: $\llbracket \underline{\text{Austin}} \rrbracket_R = \llbracket \text{Austin} \rrbracket_S$ and $\llbracket \underline{\xi \text{ is hot}} \rrbracket_R = \llbracket \xi \text{ is hot} \rrbracket_S$.

Combined with the FUNCTIONAL COMPOSITION OF REFERENCE, these results immediately entail the FUNCTIONAL COMPOSITION OF SENSE. The functional composition of reference demands that $\llbracket \underline{\text{Austin is hot}} \rrbracket_R = \llbracket \underline{\xi \text{ is hot}} \rrbracket_R(\llbracket \underline{\text{Austin}} \rrbracket_R)$. Substituting identicals on both sides results in the desired conclusion: $\llbracket \underline{\text{Austin is hot}} \rrbracket_S = \llbracket \underline{\xi \text{ is hot}} \rrbracket_S(\llbracket \underline{\text{Austin}} \rrbracket_S)$.

Generalizing this argument, EMBEDDED STRUCTURE says that a sentence is embedded in an attitude ascription retains its syntactic structure. So if the sentence ϕ is derived from the application of a functional expression π to n argument expressions, $\alpha_1, \dots, \alpha_n$, then its embedded correlate $\underline{\phi}$ is derived by applying a correlated functional expression $\underline{\pi}$ to n correlated argument expressions $\underline{\alpha_1}, \dots, \underline{\alpha_n}$. The REFERENCE SHIFT SEMANTICS says that the referents of $\underline{\phi}$, $\underline{\pi}$, and $\underline{\alpha_1}, \dots, \underline{\alpha_n}$ are the senses of ϕ , π and $\alpha_1, \dots, \alpha_n$, respectively. Finally, the FUNCTIONAL COMPOSITION OF REFERENCE says that the referent of $\underline{\phi}$ is the result of applying the referent of $\underline{\pi}$ to the referents of $\underline{\alpha_1}, \dots, \underline{\alpha_n}$. It follows immediately by the substitution of identicals that the sense of ϕ is the result of applying the sense of π to the senses of $\alpha_1, \dots, \alpha_n$.

This argument reinforces the traditional argument for the FUNCTIONAL COMPOSITION OF SENSE from the fact that the senses of functional expressions are unsaturated. When Frege describes how the thought expressed by a negated sentence is “made up” from its component parts—negation and the negated thought—he introduces a device for referring to each component. He *refers* to the sense of the word ‘not’ using the incomplete expression ‘the negation of . . . ’.

To bring out in language the need for completion, we may write ‘the negation of . . . ’ where a blank is left after ‘of’ indicates where the completed expression is to be inserted. For the relation of completing, in the realm of thoughts and their parts, has something similar to it in the realm of sentences and their parts. (Frege 1918-9/1984a, p. 386)

The negated thought is referred to using the construction ‘the thought . . . ’. Frege describes the construction of a thought as follows.

The thought

$(21/20)^{100}$ is not equal to $\sqrt[10]{10^{21}}$

is the negation of the thought:

$(21/20)^{100}$ is equal to $\sqrt[10]{10^{21}}$ (Frege 1918-9/1984a, 386).

Notice that here, Frege *refers* to the sense of ‘ $(21/20)^{100}$ is not equal to $\sqrt[10]{10^{21}}$ ’ using the complex term (4).

(4) the negation of the thought that $(21/20)^{100}$ is equal to $\sqrt[10]{10^{21}}$

A complex term such as (4) should be analyzed like any other complex term. The referent of (4) is the result of applying the referent of ‘the negation of . . . ’ to the referent of ‘the

thought that $(21/20)^{100}$ is equal to $\sqrt[10]{10^{21}}$. But this implies that the sense of $'(21/20)^{100}$ is not equal to $\sqrt[10]{10^{21}}$ ' is the result of applying the sense of 'not' to the sense of $'(21/20)^{100}$ is equal to $\sqrt[10]{10^{21}}$ '.

5 Responses to the Simple Argument

The FUNCTIONAL COMPOSITION OF REFERENCE and the REFERENCE SHIFT SEMANTICS straightforwardly entail the FUNCTIONAL COMPOSITION OF SENSE. Yet influential works on Frege disavow the conclusion. Few address this argument in detail. But the issue does bubble up to the surface periodically. I now consider Dummett's (1981a) rejection of the REFERENCE SHIFT SEMANTICS and Klement's (2002) rejection of the FUNCTIONAL COMPOSITION OF REFERENCE.

5.1 Rejecting the reference shift

Dummett believes that the sense of a sentence is a structured complex composed of the senses of its components. He rejects the identification of the sense of a function term with a sense function. Yet, Dummett squarely faces the argument I developed above, focussing on the sentence 'Plato believes that Socrates was wise'.

The whole clause, 'Socrates was wise' stands . . . for the thought which it expresses, and therefore functions as a complex proper name. A complex proper name cannot be composed merely out of other proper names—expressions standing for objects: it must contain functional expressions also. So it appears that, in this sentence, ' ξ is wise' cannot stand for an object, but must stand for the function whose value for the sense of 'Socrates' as argument is the thought that Socrates was wise . . . Since we said that in an opaque context ' ξ was wise'

stands for its sense, it will follow that the sense of 'ξ was wise' is this function.
(Dummett 1981a, p. 292)

To respond to the argument, Dummett proposes a change to Frege's view.

... Frege's doctrine of indirect reference requires one further emendation. The sense of a predicate is indeed to be considered as an object—the referent of the expression 'the sense of the predicate'; but its referent in opaque contexts is not this sense, but the associated function, which maps the sense of a name on to the thought expressed by the sentence formed by attaching the predicate to that name. The referent of an incomplete expression, whether direct or indirect, must always be incomplete. (Dummett 1981a, p. 294)

According to Dummett, the referent of a simple proper name such as 'Texas' in an indirect context is its customary sense. However, the incomplete expression 'the capital of ...' in indirect contexts does not refer to its customary sense, but rather to a function which maps the sense of 'Texas' to the sense of 'the capital of Texas'. Sense functions play a role in the semantics of opaque contexts, but they are not the senses of incomplete expressions.

Observe first that Dummett proposes an *emendation* of Frege and not an interpretation. Dummett simply concedes that the functional composition of sense follows from Frege's own explicit views. But we also saw that Frege refers to the senses of incomplete expressions using incomplete expressions even in direct discourse. If the functional expression 'the negation of ...' refers to the sense of the incomplete expression 'it is not the case that ...', then the same argument would apply in direct discourse. Dummett cannot plausibly extend his emendation of Frege to these contexts because, in the context that Frege is writing, he is illustrating how the sense of a composite is made up of the senses of its components. It is not plausible for Dummett to reject this passage without also

rejecting a considerable portion of the evidence that the senses of composite expressions are composed of the senses of the constituent expressions.

5.2 Rejecting the functional composition of reference

In *Frege and the Logic of Sense and Reference*, Klement defends an alternative picture of structured senses. For Klement, the senses of incomplete expressions constitute a new category of unsaturated entity:²³

[Senses] are neither functions nor objects, but a particular type of unsaturated entity in the realm of [sense]. (Klement 2002, p. 74)

The sense of an incomplete expression such as ‘ ξ is hot’ is incomplete, but it is not a function. It composes with the sense of a saturated expression such as the sense of ‘Austin’ to yield a thought, the thought that Austin is hot.

Klement says that the sense of a composite expression is the result of some operation suitably like functional application applied to the senses of its component expressions. I will indicate this operation using corner brackets ‘ $\langle \dots \rangle$ ’. In the simple case, sense composition works as follows:

SENSE COMPOSITION: If α is an n -adic functional expression and β_1, \dots, β_n are n expressions of the right type, then $\llbracket \alpha(\beta_1, \dots, \beta_n) \rrbracket_s = \llbracket \alpha \rrbracket_s \langle \llbracket \beta_1 \rrbracket_s, \dots, \llbracket \beta_n \rrbracket_s \rangle$.

While the reference of a whole is the result of functionally applying the function referred to by its functional component to the arguments referred to by the remaining components, sense composition proceeds by applying an operation parallel to—but distinct from—functional application to senses of these sentential components.

²³Klement leaves ‘Sinn’, ‘Bedeutung’, and ‘Gedanke’ (and their correlates) untranslated in his writing. In quotation from him, I will replace these with ‘sense’, ‘reference’, and ‘thought’, respectively, to clarify how Klement’s view is connected to my own discussion.

But the reference shift semantics ties sense composition to reference composition. Indeed, Klement cites as an advantage of his proposal that—unlike Dummett—he may preserve the reference shift semantics.

This complication is avoided on my view, since I can take the indirect [referents] of function expressions to be the incomplete entities they normally express as [senses], and it is entirely natural how such expressions would come together with other expressions to form expressions for full [thoughts]. (Klement 2002, p. 75)

Consider a specific case: the sentence 'Austin is hot' embedded in (1a). According to Klement, the component expressions 'Austin' and 'ξ is hot' refer to the customary senses of 'Austin' and 'ξ is hot', respectively. The *referent* of the composite expression 'Austin is hot' is *not* the result of functionally applying the referent of the functional expression to the referents of its components. Rather, the referent of this expression is the result of applying the operation $\dots\langle\dots\rangle$ to the senses of 'ξ is hot' and 'Austin'. In symbols: $\llbracket \text{Austin is hot} \rrbracket_R = \llbracket \xi \text{ is hot} \rrbracket_R \langle \llbracket \text{Austin} \rrbracket_R \rangle \neq \llbracket \xi \text{ is hot} \rrbracket_R (\llbracket \text{Austin} \rrbracket_R)$. So Klement's proposal demands rejecting the FUNCTIONAL COMPOSITION OF REFERENCE. Indeed, Klement develops an interesting system containing expressions specifically typed to refer to senses which do not functionally compose. However, in my view, the FUNCTIONAL COMPOSITION OF REFERENCE is a non-negotiable component of Frege's view after *Function and Concept*.

Klement himself observes that his proposal conflicts with another claim of Frege's: everything is either an object or function. The senses of incomplete expressions, in particular, are neither objects nor functions.

Perhaps the biggest drawback to this view is that it forces us to revise Frege's explicit claims that everything is either a function or an object. However, it is

worth noting that at the spots in which Frege explicitly makes this claim . . . , he is speaking of the language of the *Begriffsschrift*. (Klement 2002, p. 74)

For Klement, Frege's explicit claim that everything is either an object or a function should be interpreted as applying only to expressions already named in the language of the *Begriffsschrift*. Moreover, since the senses of incomplete expressions are still incomplete in some sense, the expressions standing for them must be incomplete as well. These expressions standing for incomplete senses would be written with argument places just as normal function expressions.

Incomplete [senses] are similar to functions in being somehow unsaturated and, thus, if signs were introduced standing for them in the *Begriffsschrift*, they too would have to be written with argument-places such that when the argument-place is filled, the whole stands for an object. (Klement 2002, p. 75)

Klement concludes that it was natural for Frege to overlook the senses of incomplete expressions when making his *explicit claim*.

However, I disagree about the role of Frege's claim that everything is either an object or function. As I read the passage, Frege uses the explicit claim to decide how expressions of natural language should be regimented into *Begriffsschrift*. Consider again the relevant passage from *Function and Concept*. Immediately prior to claiming that everything is either an object or function, Frege argued that the functional composition of reference should generalize from mathematical expressions such as '7+5' to the *Begriffsschrift* regimentations of natural language descriptions such as 'the capital of the German Empire' (Frege 1891/1997, p. 147). The referent of 'the capital of the German Empire' is the result of applying the function referred to by 'the capital of ξ ' to the argument referred to by 'the German Empire'. Frege then asks how far this generalization of the notions of object and

function extends. As we have seen, his answer is that an object is anything that can be the referent of a complete expression. Frege immediately concludes that since a statement—a sentence—lacks empty places, it refers to an object. This argument would not work if Frege implicitly restricts his criterion to expressions already in *Begriffsschrift* rather than using it to decide how to regiment new expressions into *Begriffsschrift*. If Klement is right to deny that the referent of a sentence in an attitude ascription is the value of the function referred to by one component for the argument referred to by the other component, then Frege’s argument that complex referring expressions and sentences of natural language should be treated similarly is threatened. So Klement’s proposal rejects not only the letter of Frege’s explicit claim, but undermines the arguments which rely on the spirit of the claim.²⁴

6 Against Structured Senses

Proponents of the FUNCTIONAL COMPOSITION OF SENSE as an interpretation of Frege must address Frege’s repeated use of part-whole language to describe the relationship between the sense of a composite expression and the senses of its components. For instance, Frege says that many thoughts are composed of an incomplete component and other complete components.

²⁴I have suppressed some complexity in Klement’s view. According to Klement (2002, pp. 127-8), ‘Austin is hot’ has two relevant syntactic derivations when it is embedded in an attitude ascription. It can be arrived at by applying the expression ‘ ξ is hot’ to the name ‘Austin’. Semantically, Klement agrees with Dummett that ‘ ξ is hot’ denotes a sense function and not its own customary sense. In doing so, I take him to reject the letter of the REFERENCE SHIFT SEMANTICS. However, ‘Austin is hot’ can also be derived from applying the functional expression ‘ δ is hot’ to the name ‘Austin’. Here ‘ δ ’ is a mark of incompleteness allowing only terms denoting the senses of saturated names. The referent of ‘Austin is hot’ is not computed by functionally applying the referent ‘ δ is hot’ to the referent of ‘Austin’, but by determining a structured whole composed of these referents. Since Klement (2002, p. 128) is explicit that ‘ δ is hot’ is an “unsaturated” expression, he must reject the letter of the FUNCTIONAL COMPOSITION OF REFERENCE. Finally, since ‘Austin is hot’ unembedded cannot be syntactically derived from applying the functional expression ‘ δ is hot’ to the name ‘Austin’, Klement abandons EMBEDDED STRUCTURE. A sentence has a syntactic derivation when embedded under an attitude ascription that it lacks outside of the attitude ascription. (Klement 2010) discusses these issues extensively.

[A] thought is made up out of parts that are not themselves thoughts. The simplest case of this kind is where one of the two parts is in need of supplementation and is completed by the other part, which is saturated: that is to say, it is not in need of supplementation. The former part corresponds to a concept, the latter to an object (Frege 1915/1979b, p. 254)

The problem is that neither a function nor its argument are—in general—components of the value of the function for that argument. To take one example from (Frege 1915/1979b), there is a function that takes states to their capitals. This function takes Sweden to Stockholm, but neither the function nor the country Sweden are components of the city Stockholm.

Insofar as Frege endorses the part-whole metaphor, it is tempting to say that he must also reject FUNCTIONAL COMPOSITION OF SENSE.

Where our intuitions invite us to think in terms of parts combined in whole they thereby resist the functional model, since it would in general be absurd to suppose that the value of a function for some argument is a whole in which the argument and function are combined. (Sullivan 1992, 91-2, cf. Klement 2002, p. 67)

While there is nothing incoherent about a function or argument being a constituent of its value for that argument,²⁵ these cases would seem to be exceptional. So those who interpret Fregean senses as functionally composing must provide some story about Frege's use of the language of parts and wholes to describe them.

²⁵Levine (2002) offers a reading according to which Frege accepts both the functional composition of reference and part-whole composition. There is an additional worry arising from the fact that it is now customary to model functions as sets, and sets are treated as well-founded. In standard treatments, a function cannot be a constituent of its value without violating the well-foundedness of set theory. Frege does not identify his correlates of sets, value-ranges, as sets. Even if he were to model functions using value-ranges, Frege's own theory of value-ranges is (infamously) non-well-founded.

Geach (1976, pp. 444-5) suggested that this part-whole language is misleading. Frege himself sometimes wrote of the referent of an expression as composed of the referents of its components, but later came to view this language as untenable because the referent of 'the capital of Sweden' does not contain the referent of 'Sweden' as a part. Geach would suggest similarly discounting Frege's part-whole language at the level of sense.

Sullivan has rightly pointed out that Frege is willing to retain part-whole language at the level of sense even while disavowing it at the level of reference.

Frege constantly reiterates the part-whole model of thoughts . . . even in the same breath as he denies its appropriateness for the complexity of reference[.] So Geach's proposal that we adopt a function-argument model of the complexity of sense, must be assessed as emendation rather than as exegesis. (Sullivan 1992, p. 101; cf. Klement 2002, p. 69)

According to Sullivan, Frege abandons the use of part-whole language at the level of reference because he recognizes that it is misleading. An argument of a function doesn't usually stand in anything like the relation of parthood to the value of the function for that argument. But Frege continues to describe the relationship between the sense of a sentence and the senses of its constituents in terms of the part-whole relation. This suggests—at very least—that Frege's reasons for regarding the part-whole relation as misleading at the level of reference do not extend to the level of sense.

However, Frege himself also warns about part-whole language as applied to thoughts. Frege says that this language "may lead to our looking at it the wrong way", largely because of the incompleteness of the senses of the functional expressions (Frege 1918-9/1984a, p. 386). He also calls this part-whole language figurative and analogical.

[W]e really talk figuratively when we transfer the relation of whole and part

to thoughts; yet the analogy is so ready to hand and so generally appropriate that we are hardly even bothered by the hitches which occur from time to time.
(Frege 1923-26/1984, p. 390)

While Frege's use of part-whole language is suggestive, it strikes me as hardly decisive. The important question is whether proponents of functional composition can explain the appropriateness, if not the literal truth, of part-whole language in describing senses.

6.1 The Legitimacy of the Part-Whole Metaphor

If Frege's part-whole language as applied to senses is metaphorical, why is this metaphor apt in the case of senses but not in the case of referents? One of Frege's most explicit statements distinguishing the use of part-whole language at the levels of sense and reference occurs in "Notes for Ludwig Darmstaedter". This passage is frequently cited to criticize the FUNCTIONAL COMPOSITION OF SENSE.²⁶ I will briefly examine what Frege says in this passage in order to argue that it is consistent with the FUNCTIONAL COMPOSITION OF SENSE. I then examine whether the passage creates an explanatory burden that functional composition cannot meet.

In the passage, Frege says that the part-whole structure of a thought "maps" the part-whole structure of a sentence that expresses it.

[C]orresponding to the part-whole relation of a thought and its parts we have, by and large, the same relation for the sentence to its parts. (Frege 1915/1979b, p. 255)

The part-whole metaphor suggests that there is a close correspondence between the syntactic derivation of a sentence and the thought it expresses. For each expression occurring

²⁶Dummett (1981b, pp. 262-3), Klement (2002, p. 69), and Heck and May (2011, p. 128). (Sullivan 1992, p. 101) cites also (Frege to Russell 20.10.1902, Frege 1980, p. 149) and (Frege 1979, pp. 191-2 and 225) to support his reading given above.

in a sentence, a corresponding sense occurs in the thought it expresses. Sentences that differ in structure or in corresponding constituents express senses that differ in their parts. This contrasts with the case of reference. For example, although the sense of ‘the capital of Sweden’ will contain the sense of ‘Sweden’ as a part, its *referent* does not contain the referent of ‘Sweden’ as a part: “We cannot say that Sweden is part of the capital of Sweden” (1915/1979b, p. 255). In the remainder of this section, I spell out what I take to be this passage’s face-value commitments about the individuation of senses, showing that the functional composition of sense is compatible with these face-value commitments. In the next section, I examine the charge that Frege’s account of sense composition must explain these face-value commitments and that an account based on the FUNCTIONAL COMPOSITION OF SENSE cannot offer such an explanation.

One face-value commitment of the mapping metaphor is that two sentences in the language of *Begriffsschrift* cannot express (and therefore map) the same thought without having the same structure.

SAME STRUCTURE: If $\llbracket \alpha \rrbracket_S = \llbracket \beta \rrbracket_S$ and α results from syntactic operation O on expressions $\alpha_1, \dots, \alpha_n$, then β results from syntactic operation O on expressions β_1, \dots, β_n and each of the α_i and β_i are of the same syntactic type.

By way of contrast, expressions of different structures can have the same referent.

Another commitment is that substituting expressions with different senses guarantees to result in composite expressions with different senses.

If in a sentence or part of a sentence one constituent is replaced by another with a different [referent], the different sentence or part that results does not have to have a different [referent] from the original; on the other hand, it always has a different sense. (Frege 1915/1979b, p. 255)

The idea here is that if two sentences with the same structure differ only by the substitution of constituent expressions with different senses, then the sentences themselves have different senses. Taken at face value, Frege seems committed to the view that sentences ‘Austin is hot’ and ‘the capital of Texas is hot’ will have different senses if ‘Austin’ and ‘the capital of Texas’ have different senses. Similarly, ‘Sam is a cordate’ and ‘Sam is a renate’ will express different thoughts if ‘cordate’ and ‘renate’ have different senses (Quine 1970/1986, pp. 8-10). More generally, if two expressions with the same sense each result from the application of a function term to n -arguments, then the function terms and all of the arguments agree in sense.

SAME CORRESPONDING CONSTITUENTS: If α results from applying n -ary functional expression π_α to n -argument expressions, $\alpha_1, \dots, \alpha_n$ and β results from applying n -ary functional expression π_β to n -argument expressions, β_1, \dots, β_n and $\llbracket \alpha \rrbracket_S = \llbracket \beta \rrbracket_S$, then $\llbracket \pi_\alpha \rrbracket_S = \llbracket \pi_\beta \rrbracket_S$ and $\llbracket \alpha_i \rrbracket_S = \llbracket \beta_i \rrbracket_S$, for each i .²⁷

SAME CORRESPONDING CONSTITUENTS is equivalent to saying that the result of substituting one constituent for another with different sense always results in an expression with a different sense.²⁸

Together SAME STRUCTURE and SAME CORRESPONDING CONSTITUENTS impose strong constraints on the relationship between the sense of a composite expressions and the senses of its constituents. Two composite expressions express the same sense only if they have the same structure and their corresponding constituents have the same sense. This requirement is a natural interpretation of the claim that a sentence maps the thought it expresses.

²⁷Compare the notion of inverse compositionality in (Pagin 2003, 295).

²⁸Frege’s criterion of sense identity would therefore very closely match Carnap’s (1947/1988, p. 59, 14-1(b)) notion of intensional isomorphism or Church’s Alternative (0), as is suggested by (Dummett 1981a, pp. 227-8, 379). Klement (2002, pp.102-5) discusses the issue extensively and argues that Frege should endorse a principle that is slightly weaker than SAME CORRESPONDING CONSTITUENTS.

However, there is no conflict between this observation and THE FUNCTIONAL COMPOSITION OF SENSE. If Frege is simply making the point that sentences with different syntactic structures or sentences whose corresponding constituents differ in sense have different senses, then the proponent of functional composition can readily agree. Effectively, this is just a strengthening of the point that expressions with the same referent may differ in sense. Indeed, one of the most prominent developments of the FUNCTIONAL COMPOSITION OF SENSE, Church's Alternative (0), nearly approximates these constraints on semantic values (Church 1951). Pagin (2003) offers a similar argument that most of what is wanted out of the part-whole metaphor can be extracted by requiring principles analogous to SAME STRUCTURE and SAME CORRESPONDING CONSTITUENTS.

6.2 Explanation

Some proponents of structured senses would argue that while the FUNCTIONAL COMPOSITION OF SENSE is compatible with these strong constraints, it does not explain them. These constraints are immediately predicted by the structured view.²⁹ According to Heck and May (2011, pp. 144-5; cf. Dummett 1981a, p. 152): "... [We] can earn a right to [SAME CORRESPONDING CONSTITUENTS] if we regard compositions of senses in a more structural light."

Why exactly does the structural conception of senses entail SAME STRUCTURE and SAME CORRESPONDING CONSTITUENTS? Recall, that this is the view that there is a composition operation which I denoted as ' $\dots \langle \dots \rangle$ ' so that:

SENSE COMPOSITION: If α is an n -adic functional expression and β_1, \dots, β_n are n ex-

²⁹Klement (2002, p. 77) and Anderson (1980: pp. 223-4) say that Frege only endorses a more limited principle. This more limited principle says that sentences differing by the substitution of *saturated* names with different senses always have different senses, but the same does not hold for sentences that differ by the substitution of *unsaturated* senses. I am doubtful that this more limited principle can motivate the part-whole conception of semantic composition since it is no longer clear that the sense of a function term is a component of the sense of a sentence that contains it.

pressions of the right type, then $\llbracket \alpha(\beta_1, \dots, \beta_n) \rrbracket_S = \llbracket \alpha \rrbracket_S \langle \llbracket \beta_1 \rrbracket_S, \dots, \llbracket \beta_n \rrbracket_S \rangle$.

Assume that this composition operation is injective. That is: if $f \langle a_1 \dots, a_n \rangle = g \langle b_1 \dots, b_n \rangle$, then $f = g$ and $a_i = b_i$, for each i . If being injective is built into the very concept of the relevant composition operation, then the conception of senses as structured explains SAME CORRESPONDING CONSTITUENTS. Of course, not all composition operations are injective. Summation, for instance, is not since one object may be the sum of its top half and bottom half or of its left half and right half. Nonetheless, if we assume that the composition operation is what unites entities into a *structured* whole, then there is reason to accept this reversibility principle.³⁰

If Frege is indeed committed to SAME STRUCTURE and SAME CORRESPONDING CONSTITUENTS, then there is some pressure on him to accept that senses are structured. To repeat, the advantage is that this conception explains these two theses. They do not come out true by accident. But it is worth investigating the depth of Frege's commitment to these principles. In the next two subsections, I will suggest that neither SAME STRUCTURE NOR SAME CORRESPONDING CONSTITUENTS should fall immediately out of Frege's view of sense composition. In particular, both principles raise tension with Frege's other commitments.

6.3 Against Same Structure

Frege's commitment to SAME STRUCTURE does not seem very deep. Even in the passage of "Notes for Ludwig Darmstaedter" which motivates SAME STRUCTURE and SAME CORRESPONDING CONSTITUENTS, Frege says that the sentence only "by and large" maps its sense. That is, it is only "by and large" true that the parts of the sentence correspond to the parts of the sense. But if semantic composition is governed by a principle such as structured

³⁰The alternative is to view the composition relation as somehow "plastic" in the sense of (MacBride 2003, p. 127). Cf. (Bronzo 2017).

senses, and the composition relation is injective, then it is hard to see how there could ever be exceptions. I believe that this concession reflects the fact that Frege commits to many instances of sentences with different structures expressing the same sense.

As is well known, Frege repeatedly suggests exceptions to SAME STRUCTURE. These exceptions include three kinds of case. Sometimes, Frege finds cases where two natural language sentences with different structure express the same sense. Elsewhere, Frege argues that a natural language sentence and its differently structured *Begriffsschrift* regimentation express the same thought. Most importantly, Frege offers cases where the structure of two sentences of *Begriffsschrift* express the same thought. Consider, for instance, the following commitments:

- ‘Sea-water is salt’ and ‘it is true that sea-water is salt’ have the same sense.³¹
- ‘A and B’ and ‘B and A’ have the same sense. In *Begriffsschrift* notation: ‘ $\overline{\overline{B}}$ ’ and ‘ $\overline{\overline{A}}$ ’ have the same sense.³²
- ‘A’ and ‘A and A’ have the same sense. In *Begriffsschrift* notation: ‘A’ and ‘ $\overline{\overline{A}}$ ’ have the same sense.³³
- ‘A’ and ‘it is not the case that it is not the case that A’ have the same sense. In *Begriffsschrift* notation: ‘A’ and ‘ $\overline{\overline{\overline{A}}}$ ’ have the same sense.³⁴
- ‘for any x , $f(x) = g(x)$ ’ and ‘the value range of $f(\xi) =$ the value range of $g(\xi)$ ’ have

³¹This example is from (Frege 1915/1979a, 251). cf (Frege 1892/1970, p. 64) and (Frege 1918-9/1984b, p. 354).

³²(Frege 1923-26/1984, p. 393).

³³(Frege 1923-26/1984, p. 393, footnote 21).

³⁴(Frege 1923-26/1984, p. 399).

the same sense. In *Begriffsschrift* notation: ' $\ulcorner f(a) = g(a) \urcorner$ ' and ' $\ulcorner \epsilon f(\epsilon) = \epsilon g(\alpha) \urcorner$ ' have the same sense.³⁵

Treating sense as functionally compositional allows for these failures of SAME STRUCTURE. On the other hand, many versions of the view that senses are structured have difficulty making sense of these passages. They often cite these very examples as a “slip” on Frege’s part.³⁶ Insofar as they have to reject Frege’s explicit pronouncements (which are often important components of his logicism), this seems to be again a cost of the view. But even if we take Frege as merely oscillating in his opinion about SAME STRUCTURE, it seems unfair to think that the principle is a direct consequence of his conception of sense composition. It would be as though he is repeatedly forgetting his view of sense composition. A more reasonable interpretation is that Frege is simply undecided about precisely how fine grained senses are.

6.4 Against Same Corresponding Constituents

SAME CORRESPONDING CONSTITUENTS says that if α and β differ only by the substitution of one constituent for another with different sense, then α and β differ in sense. We should not accept this principle because it conflicts with Frege’s syntactic commitments as developed in Basic Laws §30. Although (Hodes 1982) and (Klement 2002 pp. 85-6, 147-8) have offered a similar argument, they focus on different ways of *breaking up* a single name into a functional expression and its arguments. I look instead at different ways in which Frege thinks we may *syntactically derive* a single name. My aim is also more limited in that I am trying to show only that the argument for structured senses offered by Heck and May fails.

³⁵(Frege 1891/1997, p. 136)

³⁶A major source of this interpretation is (Dummett 1991, chapter 14).

Names (which include function names and sentences) of the language of *Begriffsschrift* are derived using two formation rules.

RULE 1: If $\pi(\xi_1, \dots, \xi_n)$ is an n -ary function name and $\gamma_1, \dots, \gamma_n$ are n well formed names of appropriate type, then $\pi(\gamma_1, \dots, \gamma_n)$ is a well formed name.

RULE 2: If ϕ is a complete name containing occurrences of γ , then the result of replacing γ in ϕ by a mark of incompleteness ξ is a function name.

RULE 1 allows one to form a name by saturating the argument places of a function name by arguments of the appropriate type. RULE 2 allows one to form a function name by removing a component of a saturated name.

One consequence of RULE 2 is that the same name may be derived in multiple ways. For instance, there are multiple syntactic derivations of the self-identity claim $\gamma = \gamma$. The language of *Begriffsschrift* has a primitive dyadic relation term $\xi = \zeta$. One may sequentially saturate the argument places of this relation term with γ to yield $\xi = \gamma$ and then $\gamma = \gamma$. However, the function name $\xi = \xi$ can also be syntactically derived using RULE 2 by saturating both positions of $\xi = \zeta$ with a name and then removing it. We can then fill the argument position of $\xi = \xi$ with γ to yield $\gamma = \gamma$. So the name $\gamma = \gamma$ results (A) from the application of $\xi = \gamma$ to γ and also (B) from the application of $\xi = \xi$ to γ . The semantic rules assign the same reference and (presumably) the same sense to $\gamma = \gamma$ under either derivation.³⁷

³⁷Klement (2002, p. 46) reconstructs Frege's syntax using a restricted version of Rule 1 which only permits (atomic) Roman letter function names. He also introduces a separate rule for quantified formulae which does not require RULE 2. As reconstruction, this is obviously fine. But as an exegesis of Frege's own syntax, it is strictly inaccurate, since Frege derives $\underbrace{\xi}_{\alpha} = \alpha$ from a combination of both rules. This means that the derived function name $\xi = \xi$ can be fed into the quantifier $\underbrace{\phi}_{\alpha}(\alpha)$ using the first rule. Frege introduces the rule allowing one to fill the arguments of a first-level function name with a saturated name and the rule allowing one to fill the argument of a second-level function name with a first-level function name in the same sentence. So, just as he explicitly allows a non-Roman function name to fill the quantifier, we should also interpret him as allowing a name to fill a non-Roman function name. Elsewhere, Klement (2010, p. 177) is clear about the

But this creates a problem with SAME CORRESPONDING CONSTITUENTS. We obviously have that $\gamma = \gamma$ has the same sense as itself. But we also have that it is formed from the application of $\xi = \gamma$ to γ and also from the application of $\xi = \xi$ to γ . But $\xi = \gamma$ and $\xi = \xi$ differ in reference and—therefore—in sense. This result is inconsistent with SAME CORRESPONDING CONSTITUENTS. So Frege can accept SAME CORRESPONDING CONSTITUENTS, only by rejecting his explicit syntax.

One might attempt to save SAME CORRESPONDING CONSTITUENTS by appealing to the distinction in (Dummett 1981b) between analysis and decomposition. A simple way to develop this approach—as for instance in (Sullivan 2010)—would have it that $\gamma = \gamma$ contains the constituents that show up in a privileged syntactic derivation, say (A), but not (B). So the advocate of structured senses can say that the sense of $\xi = \xi$ is not a constituent of the sense of $\gamma = \gamma$. But given that the function name $\gamma = \gamma$ results from the application of $\xi = \xi$ to γ , we still need a semantic composition principle that does not require the sentence that results of syntactically combining a function name with an argument name to express a thought which contains the senses of the function and argument name as constituents. The functional composition of sense can deliver this. Dummett’s own approach is more radical: incomplete expressions such as ‘ $\xi = \xi$ ’ do not occur in the syntactic derivation of $\gamma = \gamma$. Rather, $\gamma = \gamma$ is formed by combining two instances of γ with the *simple* predicate ‘=’. Because Dummett (1981a, p. 30) acknowledges that Frege “tacitly assimilated simple predicates to complex ones”, it is reasonable to question whether this accurately presents Frege’s own syntax. Indeed, the approach seems to conflict with the syntax explicitly offered in *Basic Laws* §30. A full accounting of Dummett’s proposal as a positive position

Fregean syntax. Landini (1996) also thinks that Fregean senses have unique analysis into parts extractable from their syntactic presentation. This might suggest that names of *Begriffsschrift* in Landini’s account have a unique syntactic derivation. However, Landini’s account leans heavily on a notion of “mutual satisfaction” (p. 131), which I do not find in Frege.

is called for, but is beyond the scope of this paper.³⁸

Klement (2002, pp. 85-6, 147-8) offers a similar argument to the conclusion that the thesis that I am calling *SAME CORRESPONDING CONSTITUENTS* must be weakened. According to Klement, replacing a saturated name by another with a different sense *must* result in a new expression with a different sense. But the same cannot hold of functional expressions. In making this claim, Klement weakens the case that the passage in “Notes for Ludwig Darmstaedter” supports structured conceptions of sense rather than the *FUNCTIONAL COMPOSITION OF SENSE*. Even a proponent of structured senses such as Klement must offer a restricted reading of the passage. Klement must either reject the view that the sense of a composite is the structured whole containing the senses of its immediate constituents or he must allow that a single sense may have multiple structures. Either concession, in my view, undermines the motivation offered by Heck and May for abandoning the *FUNCTIONAL COMPOSITION OF SENSE*.

7 Conclusion

The *FUNCTIONAL COMPOSITION OF SENSE* is entailed by two central Fregean commitments: the *FUNCTIONAL COMPOSITION OF REFERENCE* and the *REFERENCE SHIFT SEMANTICS*. Whether Frege spotted the entailment or not, it is immediate. Opponents of the *FUNCTIONAL COMPOSITION OF SENSE* wind up rejecting not only the letter, but also the spirit, of the two underlying Fregean commitments. Moreover, we have seen that the strongest arguments against taking sense to be functionally compositional—Frege’s use of part-whole language and his flirtation with *SAME STRUCTURE* and *SAME CORRESPONDING CONSTITUENTS*—are hardly decisive considerations. Indeed, core arguments in Frege’s development of his logicism go unexplained, if we take *SAME STRUCTURE* and *SAME CORRESPONDING CONSTITUENTS* to simply

³⁸(Sullivan 2010) is a nice discussion.

fall out of his view of sense composition. Together these theses amount to the claim that the sense of a composite expression encodes the senses of the component expressions and their arrangement. Frege's vacillation regarding SAME STRUCTURE and SAME CORRESPONDING CONSTITUENTS is completely explained, on the other hand, if he endorses the FUNCTIONAL COMPOSITION OF SENSE.

The core arguments of this paper have repercussions for other debates in Frege exegesis. One salient application arises because Frege applies his reference shift semantics not only to expressions embedded in belief ascriptions, but also to expressions embedded in quotation.

If words are used in the ordinary way, what one intends to speak of is their reference. It can also happen, however, that one wishes to talk about the words themselves or their sense. This happens, for instance, when the words of another are quoted. One's own words then first designate the words of the other speaker, and only the latter have their usual reference. In writing, the words are in this case enclosed in quotation marks. Accordingly, a word standing between quotation marks must not be taken as having its ordinary reference. (Frege 1892/1970, 58-9)

In a quotation context, an expression denotes itself. This means that when the sentence 'Austin is hot' is embedded under quotation, it denotes itself, the word 'Austin' refers to itself, and the predicate 'ξ is hot' refers to itself. Because a sentence is a composite expression, its referent must be the result of applying the function referred to by 'ξ is hot' to the argument referred to by 'Austin'. But since 'ξ is hot' refers to itself in a quotation context, it must be a function from a name to a sentence. The result is another controversial thesis in Frege exegesis known as the *Functional Conception of Sentential Complexity*.³⁹

³⁹The thesis is defended explicitly in (Geach 1976) and (Sullivan 1992) and criticized in (Oliver 2010).

While this paper does not examine the *Functional Conception of Sentential Complexity* in the detail it deserves, similar issues arise regarding Frege's use of part-whole terminology and inverse compositionality.⁴⁰ Indeed, there are passages where Frege seems to explicitly endorse the view that the predicates are objects and not functions.⁴¹ Yet the commitment to the FUNCTIONAL COMPOSITION OF REFERENCE and the REFERENCE SHIFT SEMANTICS, as applied to quotation, seem to entail the functional conception of sentential complexity. It is possible, again, that Frege himself was undecided or didn't draw the connection. But the entailment is straightforward.

Another point that will need further examination is Frege's notion of understanding or grasp.⁴² In a famous passage, Frege appeals to the part-whole metaphor to explain how language users can understand infinitely many novel thoughts on the basis of finite understanding.

It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even if a thought has been grasped by an inhabitant of the Earth for the very first time, a form of words can be found in which it will be understood by someone else to whom it is entirely new. This would not be possible, if we could not distinguish parts in the thought corresponding to parts of a sentence, so that the picture of the sentence can serve as a picture of the structure of the thought. (Frege 1923-26/1984, p. 390)

At first glance, this passage—and its near duplicate (Frege 1979, 225)—strongly supports a literal part-whole reading. Yet, it is immediately following this passage—and in the

⁴⁰In particular, we might ask whether the result of combining a monadic first-level functional expression α with a saturated expression β may also be the result of applying a distinct functional expression γ to a distinct argument expression δ .

⁴¹For instance, (Frege to Russell 29.06.1902, Frege 1980, pp. 135-6). Thanks to an anonymous referee for pointing me to the passage.

⁴²Thanks to an anonymous referee for suggesting the need to explicitly address this passage.

same paragraph—that Frege (1923-26/1984, p. 390) says we that “talk figuratively” in transferring the part-whole language to thoughts. This strongly suggest that Frege himself does not think that the metaphor needs to be taken literally to account for the ability of speakers to express or understand novel thoughts. Perhaps our ability to express and grasp novel thoughts can be fully accounted for by principles such as SAME STRUCTURE and SAME CORRESPONDING CONSTITUENTS. However, this paper has made no attempt to work out a robust theory of understanding or grasping a thought, either based on internal considerations of Frege’s text or on independent considerations. Further work is required to see whether a reasonable conception of understanding or grasping a thought is fully compatible with the FUNCTIONAL COMPOSITION OF SENSE.⁴³

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⁴³(Dummett 1981a, 251-2)—in particular—argues that to grasp a thought, it is not sufficient that one understands the thought merely as the *output* of some function. One must grasp it by its constituents. Although I admit that I feel the pull of this idea, I do not see conclusive considerations for this view as a textual interpretation of Frege, especially because Frege explicitly says that the part-whole language is metaphorical.

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