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# A Bayesian analysis of complete multiple breaks in a panel autoregressive (CMB-PAR(1)) time series model

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## ABSTRACT

Most economic time series, such as GDP, real exchange rate and banking series are irregular by nature as they may be affected by a variety of discrepancies, including political changes, policy reforms, import-export market instability, etc. When such changes entail serious consequences for time series modelling, various researchers manage this problem by applying a structural break. Thus, the aim of this paper is to develop a generalised structural break time series model. The paper discusses a panel autoregressive model with multiple breaks present in all parameters, i.e. in the autoregressive coefficient and mean and error variance, which is a generalisation of various sub-models. The Bayesian approach is applied to estimate the model parameters and to obtain the highest posterior density interval. Strong evidence is observed to support the Bayes estimator and then it is compared with the maximum likelihood estimator. A simulation experiment is conducted and an empirical application on the SARRC association's GDP per capita time series is used to illustrate the performance of the proposed model. This model is also extended to a temporary shift model.

**Key words:** panel autoregressive model, structural break, MCMC, posterior probability.

## 1. Introduction

When modelling any time series, one may identify characteristics of series such as stationarity, seasonality, outliers, linear trend, structural breaks, etc., and then produce a good forecast for making a better conclusion. If there is an unexpected shift in time series, then this may occur due to outlier(s) or structural break(s). In the structural break, mainly any or all model parameters are affected for a particular time interval, which may have different inferences. These break points may split time series into two or multiple parts. If at multiple time points, which are identified in terms of change on

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model parameters, the series changes temporarily or permanently, then the model must be analysed in such a way that it gives better explanation and prediction. Handling of such time series received importance by several researchers, who made inference about the break and show its impact in real applications. The problem of estimation and testing of change points in the linear model was proposed by Bai and Perron (1998) and then extended into multiple breaks in a multiple regression model. Altissimo and Corradi (2003) considered a nonlinear process which has dependent and heterogeneous observations and contained a break in the mean component. They proposed an estimator for the detection and estimation of the number of breaks and applied for weekly Eurodollar interest rate. Jin *et al.* (2013) addressed the problem of multiple breaks in piecewise stationary AR process and detected the breaks by the penalized model selection approach. Topal *et al.* (2016) compared various detection techniques of multiple break points in artificially modified time series and applied to vine sprout length data as well as mercury injection capillary pressure curve. Jibrin *et al.* (2015) modelled an AR fractionally integrated moving average process and used Bayes information criterion to study the structural breaks in crude oil prices of Brent and WTI series.

The consequences of the structural break under Bayesian approach is studied by several researchers, see Albert and Chib (1993), Bai (2010), Kumar et al. (2012), Eo (2012) and Maheu and Song (2018). Further on, Chin *et al.* (2016) combined both robust-jump volatility estimator and a structural break heterogeneous autoregressive (HAR) model to battle the structural break in stock market volatility modelling and added the empirical literature of high-frequency volatility analysis by using modified HAR models and robust-jump volatility estimators. Yamamoto (2016) considered a simple modification in EM confidence set proposed by Elliott and Muller (2007) in a linear regression model having a single structural break and achieved a shorter confidence set than the EM method. Baltagi *et al.* (2016) considered both cross sectional dependence and a structural break in Pesaran (2006) heterogeneous panels and applied least square and common correlated effects estimators to estimate the change points. Pestova and Pesta (2017) constructed an estimator for a break in panel mean without a boundary condition, which was also consistent in no break situation and demonstrated in non-life insurance application. Meligkotsiduo *et al.* (2017) suggested a Bayesian approach to detect stationarity from AR(p) model with multiple breaks in mean, variance and autoregressive coefficients. To determine the marginal likelihood and posterior probability for comparing models, filtering recursions algorithm is used in the structural break model. Hwang and Shin (2017) proposed a sequential test for detecting mean breaks that allow long memory errors. The proposed test is consistent with asymptotic normal distribution and produced an unbiased break estimate as compared with Bai and Perron (1998) biased estimates.

Many studies have also been carried out on a structural break in the panel data model in reference to testing for unit root hypothesis, break point detection, estimation, etc. Karavias and Tzavalis (2017) studied the asymptotic properties of least squares based fixed-T panel unit root tests of panel AR(1) model considering a structural break in the deterministic components and obtained the limiting distribution which is dependent on the break date and time. Chen and Huang (2018) considered a non-parametric method to analyse the consistence of changing parameters and developed two types of consistent tests to check the stability of model parameter in time varying interaction panel model. Okui and Wang (2018) established a new model which allows a common structural change in the coefficients, while the number of breaks, break points, and the size of breaks are different across groups. They also obtained a hybrid estimation procedure under grouped fixed effects and an adaptive group, fused in panel data model with heterogeneous structural breaks. Bardwell *et al.* (2019) developed an approach to detect the change point in panel data model that pools the information across time series and come up with the most recent break points in multiple series at the same time point.

This paper is an extension of Agiwal *et al.* (2018), which discussed the panel autoregressive time series model of order one (PAR(1)) with a break in mean and error variance. This model does not allow a change on autoregressive parameter. However, it may also have multiple breaks so a PAR(1) time series model with multiple breaks is explored in the present study that considers a break in autoregressive coefficient also. As this allows breaks on all parameters of the model including coefficients, mean and error variance. Therefore, this is termed as a complete multiple breaks panel autoregressive time series model of order one (CMB-PAR(1)). A Bayesian analysis of the proposed model has been carried out to estimate the parameters under both symmetric and asymmetric loss functions and then compared with MLE through both simulation and empirical study. This paper has also discussed the temporary shift model, where a change occurs in the parameter for a short time interval, then it comes to the original structure. This model is a particular form of CMB-PAR(1) model with two break points.

## 2. Model and Assumptions

Let $\{y_{it}\}$ be a PAR(1) time series model having multiple structural breaks and break points in each panel that are assumed to be same and known. Due to multiple breaks, the structure of PAR(1) model may be shifted temporarily or permanently depending on the situation. If all parameters are instable permanently for assumed time intervals, at that time structure of the series also shifted permanently. Let there be $B$ break points, then permanently shifted PAR(1) model (PS-PAR(1)) is

$$y_{it} = \begin{cases} \rho_1 y_{i,t-1} + (1-\rho_1)\mu_{i1} + \sigma_1 \varepsilon_{it} & T_0 < t \le T_1 \\ \vdots & \\ \rho_j y_{i,t-1} + (1-\rho_j)\mu_{ij} + \sigma_j \varepsilon_{it} & T_{j-1} < t \le T_j \\ \vdots & \\ \rho_{B+1} y_{i,t-1} + (1-\rho_{B+1})\mu_{i,B+1} + \sigma_{B+1} \varepsilon_{it} & T_B < t \le T_{B+1} \end{cases} \tag{1}$$

There are several practical situations where a change occurs on a model for a temporary period, i.e. a change in the series only for a particular time interval and later on it comes back to the original model/process. Such a model is called a temporary shift (TS) model. So, this type of series contains only two breaks to observe the short term changes in the model parameters. In that situation, temporary shift PAR(1) model (TS-PAR(1)) is expressed as

$$y_{it} = \begin{cases} \rho_1 y_{i,t-1} + (1-\rho_1)\mu_{i1} + \sigma_1 \varepsilon_{it} & 1 \le t \le T_1 \\ \rho_2 y_{i,t-1} + (1-\rho_2)\mu_{i2} + \sigma_2 \varepsilon_{it} & T_1 < t \le T_2 \\ \rho_1 y_{i,t-1} + (1-\rho_1)\mu_{i1} + \sigma_1 \varepsilon_{it} & T_2 < t \le T \end{cases} \tag{2}$$

where $\{y_{it}, t=1,2,\ldots,T; i=1,2\ldots,n\}$ is a sequence of observations which contains $n$ cross-sectional units recorded at $T$ time period between $T_0=0$ to $T_{B+1}=T$. The error term $\varepsilon_{it}$ is a sequence of an independently distributed normal random variable with mean zero and variance $\sigma_j^2$ for $j^{th}$ break point. Models (1) and (2) are complete multiple structural breaks PAR(1) models (CMB-PAR(1)), which contain breaks in autoregressive coefficient, mean as well as error variance. The likelihood function for the observed data under model (1) is

$$L(\Theta \mid y) = (2\pi)^{-\frac{nT}{2}} \prod_{j=1}^{B+1} \left(\sigma_j^{-n(T_j - T_{j-1})}\right) \exp\left[-\frac{1}{2}\sum_{j=1}^{B+1}\left\{\frac{1}{\sigma_j^2}\sum_{i=1}^{n}\sum_{t=T_{j-1}}^{T_j}\left(y_{it} - \rho_j y_{i,t-1} - (1-\rho_j)\mu_{ij}\right)^2\right\}\right] \tag{3}$$

where $\Theta = \left\{\left(\mu_{ij}, \sigma_j^2, \rho_j\right), \forall\, i = 1,2,\ldots,n;\; j = 1,2,\ldots,B\right\}$.

Similarly for model (2), the likelihood function is

$$L(\mu_{i1}, \mu_{i2}, \sigma_1^2, \sigma_2^2, \rho_1, \rho_2 \mid y) = (2\pi)^{-\frac{nT}{2}} \sigma_1^{-n(T+T_1-T_2)} \sigma_2^{-n(T_2-T_1)} \exp\left[-\frac{1}{2\sigma_1^2}\sum_{i=1}^{n}\sum_{t \ne T_1+1}^{T_2}\left(y_{it} - \rho_1 y_{i,t-1} - (1-\rho_1)\mu_{i1}\right)^2 - \frac{1}{2\sigma_2^2}\sum_{i=1}^{n}\sum_{t=T_1+1}^{T_2}\left(y_{it} - \rho_2 y_{i,t-1} - (1-\rho_2)\mu_{i2}\right)^2\right] \tag{4}$$

## 3. Bayesian Analysis

In Bayesian inference, the current sample information is incorporated within the available prior information because the prior distribution gives additional information about the unknown parameters that are useful to improve further inference. For Bayesian estimation, prior distribution is required to obtain the estimator for unknown parameters. If enough information about the parameter is available then it is better to incorporate the informative prior, otherwise non-informative prior is considered. In general, normal and inverse gamma distributions are the most often used conjugate priors for intercept ($\mu_{ij}$) and error variance ( $\sigma_j^2$ ) parameters in various time series model (see Meligkotsidou et al. (2017)). For autoregressive coefficient, non-informative prior as a uniform distribution is considered that provides little information related to the proposed model. Therefore, we assume $\mu_{ij}$ parameter is conditionally independent and other parameters are mutually independent, having the form as

$$\pi\left(\mu_{ij} \mid \sigma_j^2\right)=\frac{1}{\sqrt{2\pi}\sigma_j}\exp\left[-\frac{1}{2\sigma_j^2}\left(\mu_{ij}-\gamma_{ij}\right)^2\right];\quad \mu_{ij},\gamma_{ij}\in\Re,\sigma_j>0$$

$$\pi\left(\sigma_j^2\right)=\frac{d_j^{c_j}}{\Gamma c_j}\left(\sigma_j^2\right)^{-c_j-1}\exp\left[-\frac{d_j}{\sigma_j^2}\right];\quad c_j,d_j>0$$

$$\pi\left(\rho_j\right)=\frac{1}{1-l_j};\quad l_j>-1,-1<\rho_j<1$$

Then, the joint prior distribution for $\Theta=\left\{\left(\mu_{ij},\sigma_j^2,\rho_j\right),\ \forall\ i=1,2,\dots,n;\ j=1,2,\dots,B\right\}$ is given as

$$\pi(\Theta)=(2\pi)^{-\frac{n(B+1)}{2}}\prod_{j=1}^{B+1}\left(\frac{d_j^{c_j}}{\Gamma c_j\left(1-l_j\right)}\left(\sigma_j^2\right)^{-c_j-1-\frac{n}{2}}\right)\exp\left[-\sum_{j=1}^{B+1}\frac{1}{\sigma_j^2}\left\{d_j+\frac{1}{2}\sum_{i=1}^{n}\left(\mu_{ij}-\gamma_{ij}\right)^2\right\}\right] \quad (5)$$

Without loss of generality, one may know that prior distributions accurately describe the nature of the parameter and assist correctly to find the best estimator. The joint posterior distribution of PS-PAR(1) model obtained from the likelihood function given in equation (3) with incorporating the joint prior distribution given in equation (5) is expressed as

$$\pi(\Theta\mid y)=K_P(2\pi)^{-\frac{n(T+B+1)}{2}}\prod_{j=1}^{B+1}\left(\frac{d_j^{c_j}}{\Gamma\left(c_j\right)\left(1-l_j\right)}\left(\sigma_j^2\right)^{-\left[\frac{n\left(T_j-T_{j-1}+1\right)}{2}+c_j+1\right]}\right)\exp\left[-\frac{1}{2}\sum_{j=1}^{B+1}\left\{\frac{1}{\sigma_j^2}\left(\sum_{i=1}^{n}\sum_{t=T_{j-1}}^{T_j}\left(y_{it}-\rho_j y_{i,t-1}-\left(1-\rho_j\right)\mu_{ij}\right)^2+\sum_{i=1}^{n}\left(\mu_{ij}-\gamma_{ij}\right)^2+2d_j\right)\right\}\right] \quad (6)$$

where $K_P$ is the normalizing constant. Using equation (6), the Bayesian estimator is obtained but due to complexity in expression under different loss functions, a numerical technique is used to solve the posterior distribution. So, we use MCMC sampler technique to generate posterior samples. For this, we obtain the form of conditional posterior distributions for PS-PAR(1) model as given by (see Gilks *et al.* (1995), page 75–76)

$$\pi\left(\mu_{ij} \mid \rho_j, \sigma_j^2, y\right) \sim N\left(\frac{B_{ij}}{A_j}, \frac{\sigma_j^2}{A_j}\right) \tag{7}$$

$$\pi\left(\sigma_j^2 \mid \rho_j, \mu_{ij}, y\right) \sim IG\left(\frac{n\left(T_j - T_{j-1}\right) + n}{2} + c_j, C_j\right) \tag{8}$$

$$\pi\left(\rho_j \mid \mu_{ij}, \sigma_j^2, y\right) \sim TN\left(\frac{E_j}{D_j}, \frac{\sigma_j^2}{D_j}, l_j, 1\right) \tag{9}$$

where

$$A_j = \left(1 - \rho_j\right)^2 \left(T_j - T_{j-1}\right) + 1$$

$$B_{ij} = \left(1 - \rho_j\right) \sum_{t=T_{j-1}+1}^{T_j} \left(y_{it} - \rho_j y_{i,t-1}\right) + \gamma_{ij}$$

$$C_j = d_j + \frac{1}{2} \sum_{i=1}^{n} \left( \sum_{t=T_{j-1}}^{T_j} \left(y_{it} - \rho_j y_{i,t-1} - \left(1 - \rho_j\right)\mu_{ij}\right)^2 + \left(\mu_{ij} - \gamma_{ij}\right)^2 \right)$$

$$D_j = \sum_{i=1}^{n} \sum_{t=T_{j-1}+1}^{T_j} \left(y_{it-1} - \mu_{ij}\right)^2$$

$$E_j = \sum_{i=1}^{n} \sum_{t=T_{j-1}+1}^{T_j} \left(y_{it} - \mu_{ij}\right)\left(y_{i,t-1} - \mu_{ij}\right)$$

A temporary shifted model contains only two break points so that joint prior distribution has parameters $\Theta = \left\{\left(\mu_{i1}, \mu_{i2}, \sigma_1^2, \sigma_2^2, \rho_1, \rho_2\right), \forall\ i = 1,2,\ldots,n\right\}$. Then, posterior distribution for the given likelihood function is obtained as

$$\pi(\Theta \mid y) = K_T \frac{(2\pi)^{-\frac{n(T+2)}{2}} d_1^{c_1} d_2^{c_2}}{\Gamma(c_1)\Gamma(c_2)(1-l_1)(1-l_2)} \left(\sigma_1^2\right)^{-\left[\frac{n(T+T_1-T_2+1)}{2}+c_1+1\right]} \left(\sigma_2^2\right)^{-\left[\frac{n(T_2-T_1+1)}{2}+c_2+1\right]}$$
$$\exp\left[-\frac{1}{2\sigma_1^2}\left\{\sum_{i=1}^{n}\sum_{t\neq T_1+1}^{T_2}\left(y_{it}-\rho_1 y_{i,t-1}-(1-\rho_1)\mu_{i1}\right)^2+\sum_{i=1}^{n}(\mu_{i1}-\gamma_{i1})^2+2d_1\right\}\right]$$
$$\exp\left[-\frac{1}{2\sigma_2^2}\left\{\sum_{i=1}^{n}\sum_{t=T_1+1}^{T_2}\left(y_{it}-\rho_2 y_{i,t-1}-(1-\rho_2)\mu_{i2}\right)^2+\sum_{i=1}^{n}(\mu_{i2}-\gamma_{i2})^2+2d_2\right\}\right] \tag{10}$$

where $K_T$ is the normalizing constant. For computing the conditional posterior distribution, one may integrate equation (10) with respect to other parameters and get the expression. The expressions of conditional posterior distribution for various parameters are (see Gilks *et al.* (1995), page 75-76)

$$\pi\left(\mu_{i1} \mid \rho_1, \sigma_1^2, y\right) \sim N\left(\frac{(1-\rho_1)\sum_{t\neq T_1+1}^{T_2}\left(y_{it}-\rho_1 y_{i,t-1}\right)+\gamma_{i1}}{(1-\rho_1)^2(T+T_1-T_2)+1}, \frac{\sigma_1^2}{(1-\rho_1)^2(T+T_1-T_2)+1}\right) \tag{11}$$

$$\pi\left(\sigma_1^2 \mid \rho_1, \mu_{i1}, y\right) \sim IG\left(\frac{n(T+T_1-T_2+1)}{2}+c_1, d_1+\frac{1}{2}\sum_{i=1}^{n}\left(\begin{array}{l}\sum_{t\neq T_1+1}^{T_2}\left(y_{it}-\rho_1 y_{i,t-1}-(1-\rho_1)\mu_{i1}\right)^2\\ +(\mu_{i1}-\gamma_{i1})^2\end{array}\right)\right) \tag{12}$$

$$\pi\left(\rho_1 \mid \mu_{i1}, \sigma_1^2, y\right) \sim TN\left(\frac{\sum_{i=1}^{n}\sum_{t\neq T_1+1}^{T_2}(y_{it}-\mu_{i1})(y_{i,t-1}-\mu_{i1})}{\sum_{i=1}^{n}\sum_{t\neq T_1+1}^{T_2}(y_{it-1}-\mu_{i1})^2}, \frac{\sigma_1^2}{\sum_{i=1}^{n}\sum_{t\neq T_1+1}^{T_2}(y_{it-1}-\mu_{i1})^2}, l_1, 1\right) \tag{13}$$

$$\pi\left(\mu_{i2} \mid \rho_2, \sigma_2^2, y\right) \sim N\left(\frac{(1-\rho_2)\sum_{t=T_1+1}^{T_2}\left(y_{it}-\rho_2 y_{i,t-1}\right)+\gamma_{i2}}{(1-\rho_2)^2(T_2-T_1)+1}, \frac{\sigma_2^2}{(1-\rho_2)^2(T_2-T_1)+1}\right) \tag{14}$$

$$\pi\left(\sigma_2^2 \mid \rho_2, \mu_{i2}, y\right) \sim IG\left(\frac{n(T_2-T_1+1)}{2}+c_2, d_2+\frac{1}{2}\sum_{i=1}^{n}\left(\sum_{t=T_1+1}^{T_2}\left(y_{it}-\rho_2 y_{i,t-1}-(1-\rho_2)\mu_{i2}\right)^2 + \left(\mu_{i2}-\gamma_{i2}\right)^2\right)\right) \tag{15}$$

$$\pi\left(\rho_2 \mid \mu_{i2}, \sigma_2^2, y\right) \sim TN\left(\frac{\sum_{i=1}^{n}\sum_{t=T_1+1}^{T_2}\left(y_{it}-\mu_{i2}\right)\left(y_{i,t-1}-\mu_{i2}\right)}{\sum_{i=1}^{n}\sum_{t=T_1+1}^{T_2}\left(y_{it-1}-\mu_{i2}\right)^2}, \frac{\sigma_2^2}{\sum_{i=1}^{n}\sum_{t=T_1+1}^{T_2}\left(y_{it-1}-\mu_{i2}\right)^2}, l_2, 1\right) \tag{16}$$

For getting better estimator form the conditional posterior distribution, a suitable loss function is generally adopted. The commonly used loss function is squared error (symmetric) loss function (SELF) that takes equal magnitude due to over and under-estimation and another one is entropy (asymmetric) loss function (ELF). The Bayes estimator and its posterior risk for both loss functions are described below:

| Loss Function | Bayes Estimator | Posterior Risk |
|---|---|---|
| $\text{SELF} = \left(\theta - \hat{\theta}\right)^2$ | $E(\theta \mid y)$ | $Var(\theta \mid y)$ |
| $\text{ELF} = \left[\frac{\hat{\theta}}{\theta} - \ln\frac{\hat{\theta}}{\theta} - 1\right]$ | $\left(E\left(\theta^{-1} \mid y\right)\right)^{-1}$ | $E(\ln\theta \mid x) - \ln\left(E\left(\theta^{-1} \mid x\right)\right)$ |

It is obvious that the form of the posterior distribution will not be tractable and the computation of its respective Bayes estimator under different loss functions will not be analytically obtained. Consequently, one can choose stochastic simulation procedures, namely, the Gibbs and Metropolis samplers (Gilks *et al.*, 1995) to generate samples from the posterior distributions. Then, compute Bayes estimates of the parameters and their corresponding interval. This study utilizes the following steps to obtain the posterior samples using Gibbs sampling algorithm:

1. Starting with initial values $\rho_j^{(0)}, \mu_{ij}^{(0)}, \left(\sigma_j^2\right)^{(0)}$ and set k=1
2. Generate $\mu_{ij}^{(k)}$ from conditional posterior density $\pi\left(\mu_{ij}^{(k)} \mid \rho^{(k-1)}, \left(\sigma_j^2\right)^{(k-1)}, \underline{y}\right)$.
3. Generate $\left(\sigma_j^2\right)^{(k)}$ from conditional posterior density $\pi\left(\left(\sigma_j^2\right)^{(k)} \mid \rho^{(k-1)}, \mu_{ij}^{(k-1)}, \underline{y}\right)$.
4. Generate $\rho_j^{(k)}$ from conditional posterior density $\pi\left(\rho_j^{(k)} \mid \mu_{ij}^{(k-1)}, \left(\sigma_j^2\right)^{(k-1)}, \underline{y}\right)$.
5. Set k=k+1
6. Repeat steps 3-6, P times and record the sequence of observations of parameters.
7. Obtained Bayes estimate under different loss functions.

## 4. Simulation Study

To investigate and compare the performance of the various proposed estimators, a simulation study is conducted to observe the behaviour of the proposed models for various values of true parameters. For generating a series of sample size 1000, consider the following series size T =200 with different break points combination {(T/4, T/2); (T/2, 3T/4); (T/4, 3T/4)} for a set of true value: $(y_{10}, y_{20}, y_{30}) = (20, 40, 60)$; $(\rho_1, \rho_2, \rho_3) = (0.8, 0.85, 0.9)$; $(\sigma_1^2, \sigma_2^2, \sigma_3^2) = (2, 3, 4)$; $(\mu_{11}, \mu_{12}, \mu_{13}) = (10, 35, 65)$; $(\mu_{21}, \mu_{22}, \mu_{23}) = (15, 40, 70)$ and $(\mu_{31}, \mu_{32}, \mu_{33}) = (20, 45, 75)$. For numerical purpose, hyper parameters are to be known in normal and inverse gamma prior. We have taken $c_j = 0.01$, $d_j = 1$ for all break points and normal prior mean is equal to average of the generated series at $(T_{j-1}, T_j)$ break interval with parallel variance given in disturbances term. For simulation experiment, each pair of break point series is generated based on 10,000 replications. The generated samples are obtained using an iterative procedure of Gibbs sampling algorithm and get the estimates. We mainly compare the performances of the Bayes estimator with the maximum likelihood estimator (MLE) by calculating average absolute biases (AB) and mean squared error (MSE). A confidence interval (CI) of MLE and highest posterior density (HPD) interval of the Bayes estimator are also computed. Tables 1-6 report the MSE, AB and confidence/HPD interval of all parameters present in both permanent and temporary shifted models.

**Table-1**. MSE, AB and CI/HPD of $\mu$ parameter under PS-PAR(1) model

| $T_B$ | Estimator | $\mu_{11}$ | | | $\mu_{12}$ | | | $\mu_{13}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 1.0097 | 0.7992 | (8.3520,11.6731) | 2.8189 | 1.3439 | (32.1162,37.6330) | 4.1938 | 1.6321 | (61.4577,68.2851) |
| | SELF | 0.4467 | 0.5310 | (8.8963,11.1026) | 0.7323 | 0.6838 | (33.5560,36.3717) | 1.0261 | 0.8037 | (63.2851,66.6140) |
| | ELF | 0.4587 | 0.5385 | (8.8201,11.0390) | 0.7386 | 0.6865 | (33.5144,36.3355) | 1.0315 | 0.8057 | (63.2497,66.5852) |
| (T/4,3T/4) | MLE | 1.0214 | 0.8077 | (8.3261,11.6969) | 1.3548 | 0.9268 | (33.0093,36.8435) | 6.2002 | 2.4134 | (59.7821,69.6432) |
| | SELF | 0.4480 | 0.5349 | (8.8875,11.1057) | 0.6333 | 0.6338 | (34.6719,36.3143) | 0.9074 | 0.7529 | (63.3898,66.4942) |
| | ELF | 0.4619 | 0.5440 | (8.8113,11.0439) | 0.6345 | 0.6340 | (34.6440,36.2917) | 0.9193 | 0.7584 | (63.3398,66.4532) |
| (T/2, 3T/4) | MLE | 0.5090 | 0.5702 | (8.7913,11.1464) | 2.8941 | 1.3561 | (32.0412,.37.5400) | 6.0916 | 2.4069 | (59.7758,69.7308) |
| | SELF | 0.3249 | 0.4553 | (9.0410,10.9053) | 0.7501 | 0.6878 | (33.5291,36.3639) | 0.8998 | 0.7517 | (63.4293,66.5347) |
| | ELF | 0.3303 | 0.4589 | (8.9923,10.8737) | 0.7589 | 0.6916 | (33.4820,36.3237) | 0.9088 | 0.7561 | (63.3835,66.4915) |
| $T_B$ | Estimator | $\mu_{21}$ | | | $\mu_{22}$ | | | $\mu_{23}$ | | |
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 1.0579 | 0.8203 | (13.3809,16.7410) | 2.9550 | 1.3646 | (37.0322,42.7015) | 4.2959 | 1.6558 | (66.4581,73.2309) |
| | SELF | 0.4524 | 0.5361 | (13.9168,16.1238) | 0.7642 | 0.6949 | (38.5207,41.3918) | 1.0474 | 0.8145 | (68.2761,71.6419) |
| | ELF | 0.4560 | 0.5387 | (13.8672,16.0811) | 0.7689 | 0.6967 | (38.4815,41.3583) | 1.0519 | 0.8162 | (68.2468,71.6148) |
| (T/4,3T/4) | MLE | 1.0466 | 0.8161 | (13.3834,16.7071) | 1.4273 | 0.9550 | (37.9952,41.9061) | 6.1377 | 2.4031 | (64.7961,74.6504) |
| | SELF | 0.4472 | 0.5327 | (13.9129,16.1081) | 0.6651 | 0.6507 | (38.6354,41.3231) | 0.8987 | 0.7514 | (68.3781,71.5061) |
| | ELF | 0.4525 | 0.5356 | (13.8627,16.0628) | 0.6670 | 0.6517 | (38.6153,41.2989) | 0.9076 | 0.7550 | (68.3328,71.4683) |
| (T/2, 3T/4) | MLE | 0.5177 | 0.5759 | (13.8422,16.2097) | 2.8491 | 1.3429 | (37.0584,42.6770) | 9.3331 | 2.4443 | (64.6742,74.5916) |
| | SELF | 0.3274 | 0.4569 | (14.0637,15.9513) | 0.7371 | 0.6835 | (38.5278,41.3831) | 0.9208 | 0.7632 | (68.3538,71.4903) |
| | ELF | 0.3291 | 0.4575 | (14.0380,15.9231) | 0.7416 | 0.6849 | (38.4910,41.3515) | 0.9317 | 0.7677 | (68.3074,71.4531) |
| $T_B$ | Estimator | $\mu_{31}$ | | | $\mu_{32}$ | | | $\mu_{33}$ | | |
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 1.1202 | 0.8484 | (18.3352,21.8126) | 2.8274 | 1.3391 | (42.1176,47.5853) | 4.1250 | 1.6185 | (71.4899,78.1837) |
| | SELF | 0.4531 | 0.5383 | (18.9068,21.1262) | 0.7377 | 0.6808 | (43.5558,46.3537) | 1.0014 | 0.7949 | (73.2988,76.5888) |
| | ELF | 0.4555 | 0.5395 | (18.8665,21.0915) | 0.7431 | 0.6831 | (43.5204,46.3205) | 1.0052 | 0.7963 | (73.2724,76.5637) |
| (T/4,3T/4) | MLE | 1.1247 | 0.8477 | (18.3371,21.8030) | 1.4071 | 0.9446 | (43.0046,46.8866) | 9.4117 | 2.4297 | (69.6114,79.7753) |
| | SELF | 0.4522 | 0.5374 | (18.9187,21.1211) | 0.6536 | 0.6443 | (43.6595,46.3211) | 0.9259 | 0.7596 | (73.3484,76.5294) |
| | ELF | 0.4546 | 0.5394 | (18.8757,21.0888) | 0.6544 | 0.6443 | (43.6383,46.2961) | 0.9353 | 0.7637 | (73.3094,76.4956) |
| (T/2, 3T/4) | MLE | 0.5293 | 0.5790 | (18.8400,21.2299) | 2.8747 | 1.3513 | (42.0962,47.6055) | 9.0290 | 2.3882 | (69.6876,79.5348) |
| | SELF | 0.3290 | 0.4569 | (19.0554,20.9419) | 0.7446 | 0.6863 | (43.5311,46.5311) | 0.8870 | 0.7422 | (73.3785,76.4407) |
| | ELF | 0.3310 | 0.4585 | (19.0332,20.9224) | 0.7500 | 0.6889 | (43.4978,46.3552) | 0.8973 | 0.7468 | (73.3380,76.4061) |

**Table-2**. MSE, AB and CI/HPD of $\rho$ parameter under PS-PAR(1) model

| $T_B$ | Estimator | $\rho_1$ | | | $\rho_2$ | | | $\rho_3$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 3.86E-04 | 0.0155 | (0.7617,0.8231) | 6.05E-04 | 0.0191 | (0.8009,0.8735) | 3.45E-04 | 0.0144 | (0.8623,0.9156) |
| | SELF | 3.08E-04 | 0.0140 | (0.7775, 0.8152) | 3.85E-04 | 0.0156 | (0.8226,0.8665) | 2.37E-04 | 0.0120 | (0.8897,0.9088) |
| | ELF | 3.11E-04 | 0.0141 | (0.7710,0.8149) | 3.91E-04 | 0.0157 | (0.8220,0.8662) | 2.40E-04 | 0.0121 | (0.8893,0.9086) |
| (T/4,3T/4) | MLE | 3.70E-04 | 0.0153 | (0.7635,0.8238) | 4.40E-04 | 0.0165 | (0.8018,0.8718) | 5.46E-04 | 0.0183 | (0.8537,0.9293) |
| | SELF | 3.01E-04 | 0.0139 | (0.7786,0.8159) | 3.42E-04 | 0.0147 | (0.8250,0.8653) | 2.72E-04 | 0.0130 | (0.8776,0.9099) |
| | ELF | 3.04E-04 | 0.0140 | (0.7782,0.8156) | 3.46E-04 | 0.0147 | (0.8245,0.8650) | 2.76E-04 | 0.0131 | (0.8772,0.9097) |
| (T/2, 3T/4) | MLE | 3.12E-04 | 0.0140 | (0.7660,0.8211) | 6.39E-04 | 0.0197 | (0.8000,0.8741) | 5.55E-04 | 0.0182 | (0.8527,0.9188) |
| | SELF | 2.75E-04 | 0.0132 | (0.7801,0.8139) | 4.06E-04 | 0.0160 | (0.8221,0.8669) | 2.73E-04 | 0.0129 | (0.8775,0.9198) |
| | ELF | 2.77E-04 | 0.0133 | (0.7796,0.8136) | 4.12E-04 | 0.0162 | (0.8215,0.8665) | 2.77E-04 | 0.0130 | (0.8771,0.9196) |

**Table-3**. MSE, AB and CI/HPD of $\sigma^2$ parameter under PS-PAR(1) model

| $T_B$ | Estimator | $\sigma_1^2$ | | | $\sigma_2^2$ | | | $\sigma_3^2$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 0.0553 | 0.1871 | (1.6443,2.4148) | 0.1313 | 0.2881 | (2.4492,3.6385) | 0.1075 | 0.2624 | (3.4804,4.5571) |
| | SELF | 0.0526 | 0.1829 | (1.8313,2.2798) | 0.1257 | 0.2830 | (2.6303,3.3918) | 0.1052 | 0.2598 | (3.7656,4.3302) |
| | ELF | 0.0524 | 0.1824 | (1.9096,2.2477) | 0.1249 | 0.2825 | (2.6976,3.3445) | 0.1052 | 0.2600 | (3.7420,4.3005) |
| (T/4,3T/4) | MLE | 0.0561 | 0.1885 | (1.6373,2.4258) | 0.0623 | 0.2001 | (2.6040,3.4266) | 0.2228 | 0.3725 | (3.2728,4.8284) |
| | SELF | 0.0534 | 0.1846 | (1.7258,2.2931) | 0.0609 | 0.1982 | (2.7886,3.3007) | 0.2150 | 0.3676 | (3.4610,4.6897) |
| | ELF | 0.0530 | 0.1848 | (1.7034,2.2607) | 0.0612 | 0.1988 | (2.7713,3.2783) | 0.2125 | 0.3677 | (3.4182,4.6233) |
| (T/2, 3T/4) | MLE | 0.0278 | 0.1317 | (1.7418,2.2914) | 0.1246 | 0.2800 | (2.4502,3.6277) | 0.2198 | 0.3721 | (3.2543,4.7904) |
| | SELF | 0.0269 | 0.1299 | (1.8322,2.1748) | 0.1190 | 0.2738 | (2.6383,3.3839) | 0.2114 | 0.3654 | (3.5519,4.5576) |
| | ELF | 0.0269 | 0.1300 | (1.8209,2.1597) | 0.1182 | 0.2739 | (2.6046,3.3360) | 0.2098 | 0.3652 | (3.5085,4.4927) |

**Table-4**. MSE, AB and CI/HPD of $\mu$ parameter under TS-PAR(1) model

| $T_B$ | Estimator | $\mu_{11}$ | | | $\mu_{12}$ | | |
|---|---|---|---|---|---|---|---|
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 0.3484 | 0.4701 | (9.0470,10.9694) | 9.6882 | 2.4927 | (59.7725,70.0058) |
| | SELF | 0.2544 | 0.4018 | (9.1776,10.6238) | 0.8789 | 0.7509 | (63.4696,66.5473) |
| | ELF | 0.2569 | 0.4041 | (9.1451,10.7981) | 0.8860 | 0.7538 | (63.4210,66.5083) |
| (T/4,3T/4) | /.MLE | 0.5264 | 0.5769 | (8.8041,11.1993) | 4.3461 | 1.6701 | (61.4640,68.2793) |
| | SELF | 0.3320 | 0.4578 | (9.0383,10.9339) | 1.0267 | 0.8107 | (63.2910,66.6277) |
| | ELF | 0.3380 | 0.4618 | (8.9952,10.9010) | 1.0311 | 0.8126 | (63.2588,66.6000) |
| (T/2, 3T/4) | MLE | 0.3564 | 0.4763 | (9.0041,10.9704) | 9.5505 | 2.4672 | (59.5605,69.8348) |
| | SELF | 0.2601 | 0.4070 | (9.1457,10.8226) | 0.8662 | 0.7403 | (63.3814,66.5010) |
| | ELF | 0.2627 | 0.4092 | (9.1124,10.7953) | 0.8746 | 0.7442 | (63.3389,66.4576) |
| $T_B$ | Estimator | $\mu_{21}$ | | | $\mu_{22}$ | | |
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 0.3389 | 0.4640 | (14.0701,15.9820) | 9.0765 | 2.4004 | (64.7812,74.5734) |
| | SELF | 0.2460 | 0.3951 | (14.2963,15.6310) | 0.8259 | 0.7231 | (68.4579,71.4092) |
| | ELF | 0.2470 | 0.3958 | (14.2751,15.6134) | 0.8342 | 0.7265 | (68.4169,71.3705) |
| (T/4,3T/4) | MLE | 0.5396 | 0.5827 | (13.8021,16.2317) | 4.2236 | 1.6387 | (66.5093,73.2619) |
| | SELF | 0.3371 | 0.4609 | (14.0440,15.9567) | 1.0094 | 0.7992 | (68.2958,71.6303) |
| | ELF | 0.3397 | 0.4627 | (14.0114,15.9304) | 1.0123 | 0.8003 | (68.2650,71.5994) |
| (T/2, 3T/4) | MLE | 0.3463 | 0.4674 | (14.0265,15.9676) | 8.9987 | 2.3973 | (64.9211,74.7977) |
| | SELF | 0.2516 | 0.3983 | (14.1652,15.8195) | 0.8139 | 0.7184 | (68.5092,71.4686) |
| | ELF | 0.2528 | 0.3989 | (14.1437,15.8033) | 0.8190 | 0.7211 | (68.4693,71.4312) |
| $T_B$ | Estimator | $\mu_{31}$ | | | $\mu_{32}$ | | |
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 0.3410 | 0.4622 | (19.0455,20.9890) | 9.1295 | 2.4025 | (69.7670,79.6721) |
| | SELF | 0.2465 | 0.3930 | (19.1830,20.7309) | 0.8263 | 0.7213 | (73.4572,76.4584) |
| | ELF | 0.2470 | 0.3934 | (19.1673,20.7180) | 0.8335 | 0.7245 | (73.4118,76.4259) |
| (T/4,3T/4) | MLE | 0.5579 | 0.5957 | (18.7864,21.2446) | 4.1963 | 1.6336 | (71.5202,78.2553) |
| | SELF | 0.3452 | 0.4683 | (19.0392,20.9707) | 0.9911 | 0.7936 | (73.3388,76.6031) |
| | ELF | 0.3464 | 0.4691 | (19.0184,20.9520) | 0.9944 | 0.7948 | (73.3101,76.5762) |
| (T/2, 3T/4) | MLE | 0.3559 | 0.4773 | (19.0131,20.9763) | 9.7078 | 2.4736 | (69.6701,79.8414) |
| | SELF | 0.2579 | 0.4063 | (19.1466,20.8161) | 0.8784 | 0.7447 | (73.4009,76.4541) |
| | ELF | 0.2590 | 0.4071 | (19.1316,20.8015) | 0.8865 | 0.7481 | (73.3617,76.4218) |

**Table-5**. MSE, AB and CI/HPD of $\rho$ parameter under TS-PAR(1) model

| $T_B$ | Estimator | $\rho_1$ | | | $\rho_2$ | | |
|---|---|---|---|---|---|---|---|
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 6.84E-05 | 0.0066 | (0.7852,0.8120) | 1.44E-04 | 0.0095 | (0.8765,0.9140) |
| | SELF | 6.50E-05 | 0.0064 | (0.7863,0.8127) | 8.61E-05 | 0.0074 | (0.8828,0.9031) |
| | ELF | 6.51E-05 | 0.0064 | (0.7863,0.8126) | 8.66E-05 | 0.0074 | (0.8827,0.9030) |
| (T/4,3T/4) | MLE | 7.53E-05 | 0.0068 | (0.7842,0.8125) | 1.06E-04 | 0.0081 | (0.8795,0.9115) |
| | SELF | 6.94E-05 | 0.0066 | (0.7865,0.8109) | 8.22E-05 | 0.0072 | (0.8828,0.9109) |
| | ELF | 6.96E-05 | 0.0066 | (0.7864.0.8108) | 8.27E-05 | 0.0072 | (0.8827,0.9100) |
| (T/2, 3T/4) | MLE | 6.78E-05 | 0.0065 | (0.7851,0.8122) | 1.46E-04 | 0.0094 | (0.8758,0.9138) |
| | SELF | 6.48E-05 | 0.0064 | (0.7861,0.8130) | 8.51E-05 | 0.0072 | (0.8831,0.9128) |
| | ELF | 6.49E-05 | 0.0064 | (0.7860,0.8129) | 8.56E-05 | 0.0072 | (0.8830,0.9128) |

**Table-6**. MSE, AB and CI/HPD of $\sigma^2$ parameter under TS-PAR(1) model

| $T_B$ | Estimator | $\sigma_1^2$ | | | $\sigma_2^2$ | | |
|---|---|---|---|---|---|---|---|
| | | MSE | AB | CI/HPD | MSE | AB | CI/HPD |
| (T/4,T/2) | MLE | 0.0179 | 0.1060 | (1.7948,2.2324) | 0.2226 | 0.3779 | (3.2909,4.8390) |
| | SELF | 0.0113 | 0.0810 | (1.8216,2.1787) | 0.1358 | 0.2834 | (3.5064,4.5448) |
| | ELF | 0.0113 | 0.0811 | (1.9124,2.1674) | 0.1355 | 0.2842 | (4.5515,4.4714) |
| (T/4,3T/4) | MLE | 0.0266 | 0.1292 | (1.7484,2.2860) | 0.1069 | 0.2602 | (3.4820,4.5781) |
| | SELF | 0.0163 | 0.0978 | (1.8903,2.2176) | 0.0673 | 0.1976 | (3.6646,4.3453) |
| | ELF | 0.0165 | 0.0987 | (1.8756,2.1985) | 0.0678 | 0.1988 | (3.6351,4.3069) |
| (T/2, 3T/4) | MLE | 0.0174 | 0.1051 | (1.7963,2.2275) | 0.2146 | 0.3666 | (3.2875,4.8103) |
| | SELF | 0.0111 | 0.0804 | (1.8269,2.1748) | 0.1319 | 0.2756 | (3.4099,4.6227) |
| | ELF | 0.0112 | 0.0809 | (1.8176,2.1635) | 0.1330 | 0.2794 | (3.3519,4.5439) |

For the simulation study, we observed that both PS-PAR(1) and TS-PAR(1) models are having minimum AB and average MSE when estimated through the Bayesian estimator as compared to MLE. It is also observed that there is a considerable difference in AB and MSE in respective sets of break points on both models with complete and temporary shifts. We observe the same performance of the Bayes estimates under both symmetric and asymmetric loss functions and approximately same magnitude in terms of their MSE and AB.

## 5. Real Data Analysis

An empirical application is the way of analysis of real data to get the applicability of the proposed model. There are sufficient studies that show a change on economic series due to a change on economic policy, trade strategy, market fluctuation, etc. For example, present scenario of India is making several policies specially demonetization, good and service tax (GST), which may be improving the economic condition in the future. For analysis purpose, we have taken annual series of gross domestic product (GDP) per capita of South Asian Association for Regional Cooperation (SAARC) countries over the period from 1981 to 2016. Due to restrictions in data availability, it was not possible to include the economy series of Afghanistan as it is available since

2002. GDP per capita determines the growth of the economy of a country and compares it with its trading participant countries as well as applies it in better economic analysis and policy-making in the future. Over the world, SAARC association has a common cultural background and shared political experience and decides five areas namely agriculture, rural development, telecommunications, meteorology, health and population activities, where economic prosperity is the best achieved. The purpose is to investigate whether the presence of break point(s) in GDP per capita series may be varying due to a change in all model parameters or not and then find the estimates of the parameter for the best fitted model. For better understanding, we require a strongly balanced panel that has multiple breaks at the same time point. For this, it is natural to determine the number and location of structural breaks, which is developed by Zeileis *et al.* (2002). The most preferred break point(s) and its location for GDP per capita series for all countries are summarized in Table 7.

**Table-7.** Number of breaks and its location for GDP series of SAARC countries

| Country | Number of Breaks | $T_1$ | $T_2$ |
|---|---|---|---|
| Bangladesh | 1 | 2008 | - |
| Bhutan | 2 | 1997 | 2008 |
| India | 2 | 1997 | 2008 |
| Maldives | 2 | 1997 | 2008 |
| Nepal | 1 | 2008 | - |
| Pakistan | 2 | 1992 | 2008 |
| Sri Lanka | 2 | 1994 | 2008 |

Results reported in Table 7 indicate that the break arises mostly in 1997 and 2008. These break points occur when Asian financial crisis and Global financial crisis happened. These financial crises were analysed by various researchers from both theoretical and application point of view. To study the PS-PAR(1) model, assembly Bhutan, India and Maldives as a panel, which has similar break points $T_B$ = (1997, 2008) and compute the estimated values of the proposed model. To check the validity of the proposed PS-PAR(1) model to the other change point models which have a break in lesser number of parameter(s), i.e. incomplete multiple breaks PAR(1) models. For GDP per capita series, we verify the applicability of PS-PAR(1) model using Akaike information criterion (AIC) and Bayesian information criterion (BIC). The AIC and BIC values are based on the likelihood function, which needs to be determined by Bayesian estimators. The mathematical formula for the calculation of AIC and BIC is

$$AIC = -2\log L\left(\hat{\Theta} \mid y\right) + 2K$$

$$BIC = -2\log L\left(\hat{\Theta} \mid y\right) + K\log\left(nT\right)$$

where $L\left(\hat{\Theta} \mid y\right)$ is the likelihood of the PS-PAR(1) model given the data when it is evaluated at the Bayesian estimator of Θ for 1000 iterations and K is the number of estimated parameters in the proposed model. The results are obtained by taking the average of all values of AIC and BIC.

Table 8 records the AIC and BIC values for each model as per the break presence in the parameters. From Table 8, one can observe that the PS-PAR(1) model with a break present in autoregressive coefficient, mean and error variance having minimum AIC and BIC value with other permanent shifted models at breaks (1997, 2008). Hence, the PS-PAR(1) model is well fitted for the GDP series. We also verify the result based on the Bayes factor. The Bayes factor is the ratio of posterior probability under null and alternative hypothesis. Higher values of the Bayes factor lead to rejection of null hypothesis. This shows that series is well fitted from the alternative model, i.e. proposed model. Hence, Table 9 records the value of Bayes factor (BF) to take decision about the best fitted model. This table shows that there is a strong evidence to support the presence of breaks in all parameters as Bayes factor is so much high to reject the null hypothesis. Overall, we conclude that PS-PAR(1) model is well fitted for the GDP series at breaks (1997, 2008).

**Table-8**. Selection the parameter(s) shifting in PS-PAR(1) model using information criterion

| Model | Break in Parameter(s) | -logL | AIC | BIC |
|---|---|---|---|---|
| PAR($\rho_j$, $\mu_{ij}$, $\sigma_j$) | AR coefficient, mean & error variance | 205.8028 | 441.6056 | 481.8375 |
| PAR($\rho_j$, $\mu_{ij}$, $\sigma$) | AR coefficient & mean | 250.9187 | 527.8375 | 562.7052 |
| PAR($\rho_j$, $\mu_i$, $\sigma_j$) | AR coefficient & error variance | 221.6518 | 461.3035 | 485.4427 |
| PAR($\rho$, $\mu_{ij}$, $\sigma_j$) | Mean & error variance | 471.7729 | 969.5459 | 1004.414 |
| PAR($\rho_j$, $\mu_i$, $\sigma$) | AR coefficient | 232.5329 | 479.0658 | 497.8407 |
| PAR($\rho$, $\mu_{ij}$, $\sigma$) | Mean | 8.35E+28 | 1.67E+29 | 1.67E+29 |
| PAR($\rho$, $\mu_i$, $\sigma_j$) | Error variance | 2.15E+30 | 4.30E+30 | 4.30E+30 |
| PAR($\rho$, $\mu_i$, $\sigma$) | - | 7.19E+30 | 1.44E+31 | 1.44E+31 |

**Table-9**. Model selection using Bayes factor when alternative hypothesis ($H_1$) considers multiple breaks in all parameters

| Model | Null hypothesis ($H_0$ ) consider breaks in | BF | Evidence against $H_0$ |
|---|---|---|---|
| PAR($\rho_j$, $\mu_{ij}$, $\sigma$) | AR coefficient & mean | 1.13E+34 | Very Strong |
| PAR($\rho_j$, $\mu_i$, $\sigma_j$) | AR coefficient & error variance | 6.26E+13 | Very Strong |
| PAR($\rho$, $\mu_{ij}$, $\sigma_j$) | Mean & error variance | 1.20E+11 | Very Strong |
| PAR($\rho_j$, $\mu_i$, $\sigma$) | AR coefficient | 1.00E+38 | Very Strong |
| PAR($\rho$, $\mu_{ij}$, $\sigma$) | Mean | 9.69E+29 | Very Strong |
| PAR($\rho$, $\mu_i$, $\sigma_j$) | Error variance | 3.09E+20 | Very Strong |
| PAR($\rho$, $\mu_i$, $\sigma$) | - | 4.60E+30 | Very Strong |

After identifying the best suitable model, the estimated value of the maximum likelihood and Bayesian estimators of PS-PAR(1) model parameters are summarized in Table 10.

**Table-10**. MLE and Bayes estimates based on GDP series using PS-PAR(1) model

| Parameter | MLE | SELF | ELF |
|---|---|---|---|
| $\rho_1$ | 9.54E-01 | 9.97E-01 | 9.97E-01 |
| $\rho_2$ | 9.61E-01 | 9.66E-01 | 9.66E-01 |
| $\rho_3$ | 9.78E-01 | 9.29E-01 | 9.29E-01 |
| $\mu_{11}$ | 3.15E+02 | 4.65E+02 | 4.38E+02 |
| $\mu_{21}$ | 2.80E+02 | 3.41E+02 | 2.85E+02 |
| $\mu_{31}$ | 2.97E+02 | 1.17E+03 | 1.16E+03 |
| $\mu_{12}$ | 5.54E+02 | 1.01E+03 | 1.80E+03 |
| $\mu_{22}$ | 2.94E+02 | 6.33E+02 | 4.98E+02 |
| $\mu_{32}$ | 1.93E+03 | 3.84E+03 | 3.81E+03 |
| $\mu_{13}$ | 8.07E+03 | 2.34E+03 | 2.27E+03 |
| $\mu_{23}$ | 4.72E+03 | 1.50E+03 | 1.33E+03 |
| $\mu_{33}$ | 3.80E+04 | 9.70E+03 | 1.00E+04 |
| $\sigma_1^2$ | 2.45E+03 | 7.42E+04 | 7.84E+04 |
| $\sigma_2^2$ | 1.03E+04 | 1.24E+05 | 2.21E+05 |
| $\sigma_3^2$ | 9.95E+03 | 1.16E+05 | 1.78E+05 |

## 6. Conclusion

There is a sufficient literature on the time series model with a structural break, which allows a break on mean and variance, but the present paper has extended the frontier of knowledge in a PAR(1) model, which allows a break on all parameters of the model at multiple time points, and carried out the Bayesian analysis. Sometimes, changes on parameters are temporary, so the model with a temporary shift is also discussed. It recorded better results in a simulation study. An empirical application on GDP per capita time series of SARRC association is applied to PS-PAR(1) model and it is observed that both Asian and World financial crises have affected the GDP series of SAARC countries due to a break in all parameters permanently and the same may be applied in other areas like insurance, agriculture, administrative, crime, etc. The result may be extended for other structural break models with non-normal error and time trend.

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