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Forecasting the amplitude of high-intensity chaotic laser pulses

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ABSTRACT

Forecasting the dynamics of chaotic systems from the analysis of their output signals is a challenging problem with applications in most fields of modern science. In this work, we use a laser model to compare the performance of several machine learning algorithms for forecasting the amplitude of upcoming emitted chaotic pulses. We simulate the dynamics of an optically injected semiconductor laser that presents a rich variety of dynamical regimes when changing the parameters. We focus on a particular regime where the intensity shows a chaotic pulsing dynamics, and occasionally an ultra-high pulse, reminiscent of a rogue wave, is emitted. Our goal is to predict the amplitude (height) of the next pulse, knowing the amplitude of the three preceding pulses. We compare the performance of several machine learning methods, namely neural networks, support vector machine, nearest neighbors and reservoir computing. We analyze how their performance depends on the length of the time-series used for training.

Keywords: semiconductor lasers, optical injection, injection locking, optical chaos, optical rogue waves, machine learning, reservoir computing

1. INTRODUCTION

Complex dynamical systems often exhibit extreme or rare events. Examples in nature include earthquakes, hurricanes, financial crises, and epileptic attacks, to name just a few.[1] In recent years the generation of extreme events in optical systems has attracted attention,[2,3] as such systems serve as experimental platforms for testing the physics of extreme event generation in a controlled environment, where parameters can be tuned with high precision.

In particular, the chaotic dynamics of continuous-wave (cw) optically injected semiconductor lasers has attracted attention, because, under appropriated conditions, the laser can emit excitable pulses,[4,5] or rare giant pulses[6] that have been referred to as optical rogue waves. Thus, the cw optically injected laser has been used for testing methods either to suppress[7] or to generate "on demand"[8,9] high optical pulses. In addition, in contrast to what can be achieved in other fields, optics laser systems allow to record long datasets containing large numbers of extreme events. Such optical "big data" has been used for testing data analysis tools for extreme event prediction.[10–14]

Predicting the dynamical evolution of complex systems is an extremely challenging problem with important practical applications.[15] With unprecedented advances in computer science and artificial intelligence, many algorithms[16–19] are nowadays available for time series forecasting.

Here we simulate the dynamics of an injected laser using a well-known rate equation model,[20–22] and use the chaotic regime to compare the performance of several machine learning algorithms (neural networks, support vector machine, nearest neighbors and reservoir computing) for forecasting the amplitude of the next intensity pulse. Within the chaotic regime, we consider two different situations: the intensity pulses display occasional extreme values, or the intensity pulses are chaotic but do not display extreme fluctuations. In the first case, the

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probability distribution function of pulse amplitudes is long tailed, while in the second case, it has a well-defined cut off. Our goal is to predict the amplitude of the next optical pulse, knowing the amplitude of the three preceding pulses. We find that, while both regimes can be forecasted, the existence of extreme events bounds the prediction accuracy.

2. MODEL

We use the following rate equations to describe the dynamics of a semiconductor laser with optical injection. In the Appendix we present the derivation of these equations for the deterministic free-running laser.

$$\frac{dE}{dt} = \kappa(1+i\alpha)(N-1)E + i\Delta\omega E + \sqrt{P_{inj}} + \sqrt{D}\xi(t), \quad (1)$$

$$\frac{dN}{dt} = \Gamma_N(\mu - N - N|E|^2). \quad (2)$$

Here E is the slow envelope of the complex optical field, $S = |E|^2$ is the intensity, N is the carrier density, κ is the field decay rate, α is the line-width enhancement factor, and Γ_N is the carrier decay rate. $\Delta\omega = \omega_s - \omega_m$ is the angular frequency detuning between the slave laser and the master laser, P_{inj} is the injection strength and μ is the injection current parameter (normalized such that the threshold of the free-running laser is at $\mu_{th} = 1$). $\xi(t)$ is a complex uncorrelated Gaussian noise of zero mean and unity variance that represents spontaneous emission and D is the noise strength.

The model equations were simulated with a second order Runge-Kutta method with a time step of 1 ps. The parameter values used are listed in Table 1. We consider two values of the pump current parameter: for $\mu = 2.2$ the intensity time series occasionally displays an extreme pulse (as shown in Fig. 1), while for $\mu = 2.45$ the intensity time series displays chaotic oscillations, but without extreme pulses.

Table 1. Model parameters.

Name	Symbol	Value
Field decay rate	κ	300 ns^{-1}
Carrier decay rate	Γ_N	1 ns^{-1}
Linewidth enhancement factor	α	3
Frequency detuning	$\Delta\omega = \omega_s - \omega_m$	$2\pi\times0.49$ GHz
Injection strength	P_{inj}	60ns^{-2}
Noise strength	D	0 or 10^{-4} ns^{-1}
Pump current parameter	μ	2.2 or 2.45

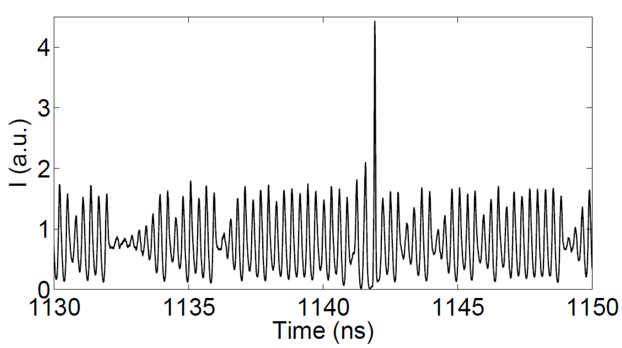

Figure 1. Example of a simulated intensity time series showing chaotic pulses and an ultra-high pulse.

3. FORECAST ALGORITHMS

The machine learning algorithms used to predict the amplitude of the next optical pulse, knowing the amplitude of the three preceding pulses are:

- K-Nearest neighbors (KNN)
- Support vector machine
  1. Linear (Linear SVM)
  2. Gaussian (Gaussian SVM)
- Artificial neural networks
  1. Shallow neural network (Shallow NN)
  2. Deep neural network (Deep NN)
  3. Reservoir computing (RC)

These are well-known algorithms that tackle the problem of function approximation, and have been described in.[23] Except for reservoir computing (that has an internal state with memory of the history of the inputs), all other algorithms are memory-less.

3.1 Quantification of the performance of the algorithms

Let y_i be the ith intensity peak amplitude, and $\widetilde{y}_i$ the predicted value. We use the mean absolute relative error (MARE) to quantify the algorithms' performance:

$$MARE = \left\langle \frac{|y_i - \widetilde{y}_i|}{y_i} \right\rangle, \tag{3}$$

where $\langle \dots \rangle$ denotes the temporal average.

4. RESULTS

Figure 2 displays the scatter plots of the real intensity peak amplitudes, y_i and the predicted values, $\widetilde{y}_i$, for the different algorithms. An algorithm with good prediction accuracy aligns the points to the diagonal line. We see that the deep NN, KNN, and RC algorithms give good precision, while the Shallow NN and the Gaussian SVM algorithms tend to underestimate the amplitude of medium to large pulses. The linear SVM algorithm completely fails to capture the complexity of the dynamics.

Next, we evaluate the number of training points needed to have accurate results. The results are presented in Fig. 3. For the noise-free simulations with $\mu = 2.2$ or $\mu = 2.45$ (Fig. 3, top panels), when there are extreme pulses ($\mu = 2.2$, top left panel) the value of the MARE is at least two orders of magnitude worse than for $\mu = 2.45$ (for which there are no extreme events, top right panel). This is due to the added complexity of the extreme events that occasionally occur for $\mu = 2.2$, which deteriorates the performance of all forecasting methods. We note that that the KNN, Deep NN, and RC methods, in this order, yield the most accurate predictions. These methods, together with the shallow NN, yield the lowest MARE for $\mu = 2.45$ (top right panel). We also note that the performance of the RC algorithm improves when the number of training data points increases. Overall, the prediction of the amplitude of the upcoming pulse for $\mu = 2.45$ (for which there are no extreme pulses) requires less training points than for $\mu = 2.2$ (for which there are occasional extreme pulses). These results suggest that the intensity dynamics with extreme pulses is harder to predict.

With stochastic simulations (Fig. 3, bottom panels), as expected, we see that the MARE value increases with respect to deterministic simulations. The presence of noise has a strong influence on the forecast of the chaotic dynamics for $\mu = 2.45$, causing an increase of two orders of magnitude in the MARE value.

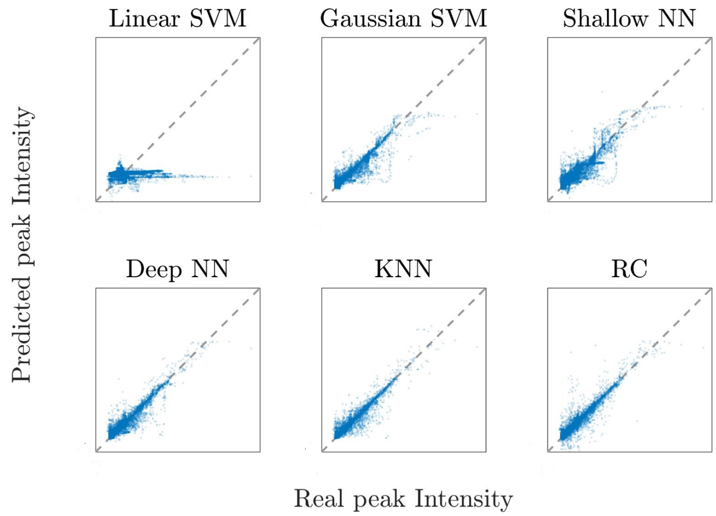

Figure 2. Scatter plots of the real intensity peak amplitudes, y_i and the predicted values, $\widetilde{y}_i$ (arb. units) obtained with the different algorithms. The parameters are $\mu = 2.2$ and $D = 10^{-4}$ ns^{-1}. 65534 data points were used for training.

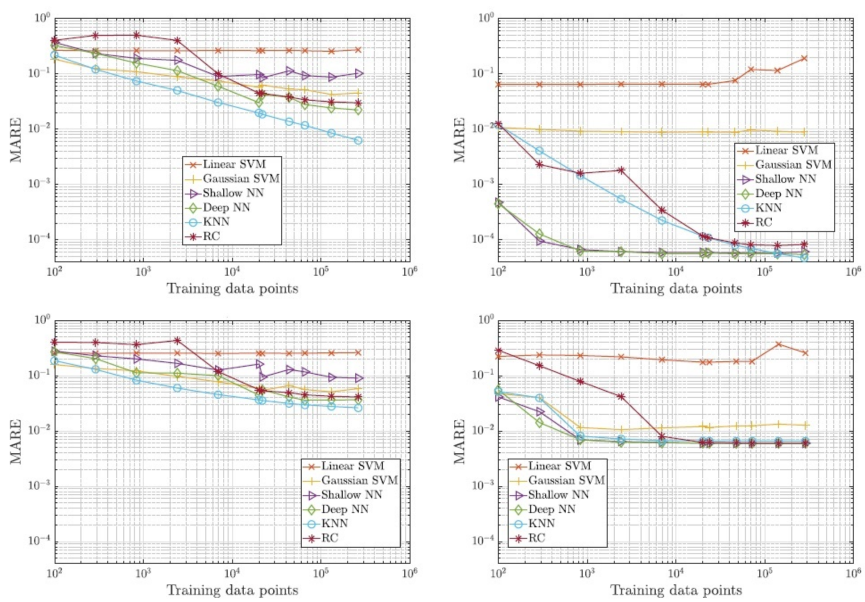

Figure 3. Mean absolute relative error (MARE) obtained with the different forecast algorithms, as a function of the number of training data points. The parameters are $\mu = 2.2$ (left column) and $\mu = 2.45$ (right column); $D = 0$ (top row) and $D = 10^{-4}$ ns^{-1} (bottom row).

5. CONCLUSIONS

We have used the chaotic dynamics of the intensity of an optically injected laser as a benchmark to test the performance of several machine learning algorithms for forecasting the amplitude of the next intensity pulse. This laser system is described by a simple rate-equation model that, with a small change of parameters, produces time series which have extreme events in the form of high peak intensities, resembling the dynamics of much more complex systems. We have shown that good prediction accuracy can be achieved with some of the proposed methods, namely, the KNN, Deep NN, and RC methods. Although we do not exclude that even more complex methods (e.g., a neural network with additional hidden layers) can outperform the presented algorithms, the results in this contribution illustrate the power of machine learning methods for the analysis of dynamical systems.

APPENDIX A. DERIVATION OF THE MODEL EQUATIONS

Here present the derivation of Eqs. (1) and (2). Without noise or optical injection, the model equations of a free-running diode laser are (see Eq.(9) in[20]):

$$\frac{dE_0}{dt} = \frac{1}{2}\xi(1+i\alpha)(N-N_{thr})E_0, \tag{4}$$

$$\frac{dN}{dt} = J-\Gamma_N N-[\Gamma_E+\xi(N-N_{thr})]|E_0|^2. \tag{5}$$

With N_0 and ξ being the transparency carrier density and the gain coefficient respectively, and $\Gamma_E = 2\kappa$. The threshold carrier density satisfies the threshold condition, $\xi(N_{thr}-N_0) = \Gamma_E$.

Defining the dimension-less carrier variable as $N' = (N-N_0)/(N_{thr}-N_0)$ and using the threshold condition we have $N' = \xi(N-N_0)/\Gamma_E$.

Using the threshold condition, the term $\Gamma_E + \xi(N-N_{thr})$ in Eq. (5) can be rewritten as

$$\Gamma_E + \xi(N-N_{thr}) = \xi(N_{thr}-N_0) + \xi(N-N_{thr}) = \xi(N-N_0) = \Gamma_E N'. \tag{6}$$

Substituting in Eq. (5) we have

$$\frac{dN}{dt} = (N_{thr}-N_0)\frac{dN'}{dt} = \frac{\Gamma_E}{\xi}\frac{dN'}{dt} = J-\Gamma_N\left(N_0+\frac{\Gamma_E}{\xi}N'\right)-\Gamma_E N'|E_0|^2. \tag{7}$$

Defining the pump current parameter as

$$\mu = \frac{\xi}{\Gamma_E\Gamma_N}\left(J-\Gamma_N N_0\right), \tag{8}$$

and the dimensionless complex optical field as $E' = E_0\sqrt{\xi/\Gamma_N}$ we obtain Eq. (2):

$$\frac{dN'}{dt} = \Gamma_N(\mu - N' - N'|E'|^2). \tag{9}$$

Rewritting Eq. (4) as

$$\frac{dE'}{dt} = \frac{1}{2}\xi(1+i\alpha)(N-N_{thr})E', \tag{10}$$

and using that $\xi(N-N_{thr}) = \Gamma_E N' - \Gamma_E$ [Eq. (6)] we obtain Eq. (1) with $P_{inj} = D = 0$ (note that $\Gamma_E = 2\kappa$):

$$\frac{dE'}{dt} = \frac{\Gamma_E}{2}(1+i\alpha)(N'-1)E'. \tag{11}$$

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REFERENCES

[1] Albeverio, S., Jentsch, V. and Kantz, H. (Eds.) [Extreme Events in Nature and Society], Springer, Berlin/Heidelberg, Germany (2006).

[2] Solli, D. R., Ropers, C., Koonath, P., Jalali, B., "Optical rogue waves," Nature 450, 1054–1057 (2007).

[3] Akhmediev, N., Kibler, B., Baronio, F., Belic, M., Zhong, W.P., Zhang, Y., Chang, W., Soto-Crespo, J. M., Vouzas, P., Grelu, P. et al. "Roadmap on optical rogue waves and extreme events," J. Opt. 18, 063001 (2016).

[4] Goulding, D., Hegarty, S. P., Rasskazov, O., Melnik, S., Hartnett, M., Greene, G., McInerney, J. G., Rachinskii, D. and Huyet, G.,"Excitability in a quantum dot semiconductor laser with optical injection. Phys. Rev. Lett. 98, 153903 (2007).

[5] Turconi, M., Garbin, B., Feyereisen, M., Giudici, M. and Barland, S., "Control of excitable pulses in an injection-locked semiconductor laser," Phys. Rev. E 88, 022923 (2013).

[6] Bonatto, C., Feyereisen, M., Barland, S., Giudici, M., Masoller, C., Leite, J. R. and Tredicce, J. R., "Deterministic optical rogue waves," Phys. Rev. Lett. 107, 053901 (2011).

[7] Zamora-Munt, J., Garbin, B., Barland, S., Giudici, M., Leite, J. R., Masoller, C. and Tredicce, J. R., "Rogue waves in optically injected lasers: Origin, predictability, and suppression," Phys. Rev. A 87, 035802 (2013).

[8] Jin, T., Siyu, C. and Masoller, C., "Generation of extreme pulses on demand in semiconductor lasers with optical injection," Opt. Express 25, 031326 (2017).

[9] Torre, M. S. and Masoller, C., "Exploiting the nonlinear dynamics of optically injected semiconductor lasers for optical sensing," Photonics 6, 45 (2019).

[10] Akhmediev, N., Ankiewicz, A., Soto-Crespo, J. M., and Dudley, J. M.,"Rogue wave early warning through spectral measurements?," Phys. Lett. A 375, 541-544 (2011).

[11] Birkholz, S., Bree, C., Demircan, A. and Steinmeyer, G., "Predictability of rogue events," Phys. Rev. Lett. 114, 213901 (2015).

[12] Martinez Alvarez, N., Borkar, S. and Masoller, C., "Predictability of extreme intensity pulses in optically injected semiconductor lasers," Eur. Phys. J. Spec. Top. 226, 1971–1977 (2017).

[13] Colet, M. and Aragoneses, A.,"Forecasting events in the complex dynamics of a semiconductor laser with optical feedback," Sci. Rep. 8, 10741 (2018).

[14] Narhi, M., Salmela, L., Toivonen, J., Billet, C., Dudley, J. M. and Genty, G., "Machine learning analysis of extreme events in optical fibre modulation instability," Nat. Comm. 6, 4923 (2018).

[15] Jaeger, H. and Haas, H., "Harnessing nonlinearity: Predicting chaotic systems and saving energy in wireless communication," Science 304, 78-80 (2004).

[16] Hinton, G., Deng, L., Yu, D., Dahl, G. E., Mohamed, A. R., et al., "Deep neural networks for acoustic modeling in speech recognition," IEEE Signal Process. Mag. 29, 82-97 (2012).

[17] Jordan, M. I. and Mitchell, T. M., "Machine learning: Trends, perspectives, and prospects," Science 349, 255-260 (2015).

[18] LeCun, Y., Bengio, Y. and Hinton, G.,"Deep learning," Nature 521, 436-444 (2015).

[19] Pathak, J., Hunt, B., Girvan, M., Lu, Z. X. and Ott, E., "Model-free prediction of large spatiotemporally chaotic systems from data: a reservoir computing approach," Phys. Rev. Lett. 120, 024102 (2018).

[20] Wieczorek, S., Krauskopf, B., Simpson, T. B. and Lenstra, D., "The dynamical complexity of optically injected semiconductor lasers," Phys. Rep. 416, 1–128 (2005).

[21] Kovanis, V., Gavrielides, A. and Gallas, J. A. C., "Labyrinth bifurcations in optically injected diode lasers," Eur. Phys. J. D 58, 181–186 (2010).

[22] Ohtsubo, J. [Semiconductor Lasers: Stability, Instability and Chaos], Springer, Berlin/Heidelberg, Germany (2012).

[23] Amil, P., Soriano, M. C. and Masoller, C., "Machine learning algorithms for predicting the amplitude of chaotic laser pulses," Chaos 29, 113111 (2019).