



TITLE:

Numerical evaluation of vortex-induced vibration amplitude of a box girder bridge using forced oscillation method

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1 **Numerical evaluation of vortex-induced vibration amplitude of a box girder bridge using forced**
2 **oscillation method**

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16

17

18 **Abstract**

19 The evaluation of the amplitude of the vortex-induced vibration (VIV) of a long-span bridge is necessary
20 to implement a wind-resistant design. The development of high-performance computing has led to the
21 use of computational fluid dynamics (CFD) in this domain, but the evaluation of VIV amplitude using
22 the free vibration method in CFD incurs a high computational cost because of the small negative
23 aerodynamic damping in the wind speed region of VIV. In this study, the use of flutter derivatives based
24 on the forced oscillation method with a large eddy simulation is proposed for evaluating the VIV
25 amplitude to reduce computational cost. The heaving VIV amplitude of a box girder was evaluated using
26 simulated flutter derivatives and the results were validated by corresponding free vibration wind tunnel
27 tests. Because the aerodynamic damping obtained by the flutter derivatives showed a clear dependence
28 on the oscillation amplitude, the VIV amplitude can be evaluated using the proposed method. The effects
29 of the spanwise domain size and Reynolds number were also significant.

30

31 **Keywords**

32 Large Eddy Simulation, Flutter derivatives, Vortex-induced vibration, Spanwise domain size, Reynolds
33 number.

34

35 1. Introduction

36

37 Recent developments in high-performance computing have led to greater use of computational fluid
38 dynamics (CFD) in engineering (Noguchi et al., 2017; Ishihara and Oka, 2018; Okaze et al., 2018; Zhou
39 et al., 2018). In practice, however, CFD has not been adequately employed to achieve the wind-resistant
40 design of a long-span bridge because aerodynamic stability as well as aerostatic stability must first be
41 investigated. Some challenges persist in evaluating vortex-induced vibration (VIV) because calculating
42 its amplitude and wind speed of occurrence is significantly important, which is different from the
43 evaluation of flutter and galloping.

44 The VIV amplitude of a body is usually evaluated by the spring-supported free-vibration method in
45 wind tunnel tests (Washizu et al., 1978; Miyata et al., 1983), and some researches have obtained this
46 amplitude using the numerical free vibration method (Sun et al., 2008; Shimada and Ishihara, 2012;
47 Nguyen et al., 2018; Álvarez et al., 2019). However, because the VIV amplitude increases (or decreases)
48 gradually with time because of small aerodynamic damping, which makes it difficult to determine
49 whether a steady-state response has been obtained, the free vibration method in CFD still incurs high
50 computational costs. Thus, an alternative method is needed to evaluate the VIV amplitude using CFD
51 for practical use.

52 Flutter derivatives have frequently been calculated based on the forced oscillation method (Sarpkaya,
53 1978; Staubli, 1983; Morse and Williamson, 2009) using CFD. This method can obtain aerodynamic
54 forces in the periodic state in less time than the free vibration method, which reduces computational cost.
55 For example, Šarkić et al. (2012) conducted unsteady Reynolds-Averaged Navier–Stokes (URANS)
56 simulations for a box girder to obtain flutter derivatives. Using large eddy simulations (LES), Maruoka
57 and Hirano (2000) focused on a thin plate, and Sarwar et al. (2008) and Ito and Graham (2016, 2017)
58 conducted simulations of box girders to successfully calculate flutter derivatives with good accuracy.
59 Because flutter derivatives can be used to evaluate aerodynamic damping, the VIV amplitude can also
60 be evaluated using flutter derivatives by considering its dependence on the oscillation amplitude.
61 However, because these authors focused on regions of relatively large wind speed to evaluate flutter
62 instability, the possibility of evaluating the VIV amplitude using the forced oscillation method was not
63 discussed. Sarwar and Ishihara (2010) evaluated the VIV using the forced oscillation method and
64 obtained the wind speed region of VIV, but employed the free vibration method to evaluate the VIV
65 amplitude. Thus, the forced oscillation method has not been often applied to calculate the VIV amplitude.

66 In light of the above, this study discusses the possibility of evaluating the VIV amplitude based on
67 the forced oscillation method using LES. Because the inertia force does not act on the structure in
68 simulations performed in CFD, unlike in wind tunnel tests, the forced oscillation method can calculate
69 the aerodynamic forces accurately even in regions of low wind speed where the VIV appears. Moreover,
70 because aerodynamic damping derived from the forced oscillation method does not depend on structural
71 variables such as mass and damping, the simulated results can be used to calculate the VIV amplitude
72 of a system with any Scruton number. The dependence of the aerodynamic damping term of the flutter

73 derivatives on the amplitude of oscillation is first investigated in the wind speed region of VIV. Then,
74 the VIV amplitude calculated by the aerodynamic damping based on the forced oscillation method using
75 LES is compared with corresponding spring-supported free vibration wind tunnel tests to discuss its
76 accuracy and validity. Further, the factors influencing aerodynamic damping and the VIV amplitude in
77 the proposed method are investigated and the corresponding mechanisms are clarified.

78

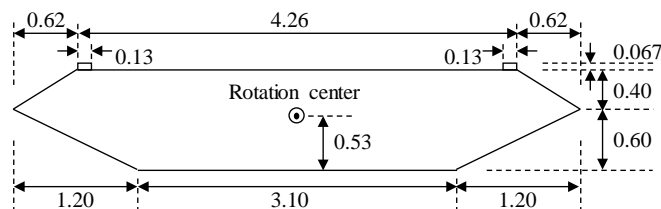
79 2. Experimental setup and numerical method

80

81 Ito and Graham (2016, 2017) focused on the cross-section of a box girder bridge (Šarkić et al., 2012;
82 Nieto et al., 2015) and accurately calculated the flutter derivatives using LES in a region of large reduced
83 wind speed. In this study, the VIV amplitude was calculated using the same section on which the
84 protuberances were additionally installed to excite aerodynamic vibrations for both the wind tunnel tests
85 and LES (Nagao et al., 1997; Guan et al., 2012). Fig. 1 shows the cross-section of the target bridge in
86 this study, the VIV amplitude of which with vertical one-degree-of-freedom (1DOF) was evaluated by
87 wind tunnel tests and LES. Protuberances were installed upstream and downstream on the upper surface.
88 The side ratio of the bridge section was $B/D = 5.5$, where B = deck width and D = deck height excluding
89 protuberances.

90 In this section, the experimental setup and the employed numerical method are first explained. Then,
91 the definitions of the aerostatic and aerodynamic forces are provided. The aerostatic forces were
92 calculated by both wind tunnel tests and numerical simulations to validate the numerical method
93 employed in this study.

94



95

96 **Fig. 1.** Bridge section used for wind tunnel tests and LES (non-dimensionalized by deck height, D).

97

98 2.1 Experimental setup

99

100 The wind tunnel used was of the closed-circuit type located at Shimizu Corporation, and had a working
101 section of 1.1 m in width and 0.9 m in height. The height (D), width (B), and length (L) of the section
102 model used in the experiment were 30 mm, 165 mm, and 1,000 mm, respectively. All tests in this study
103 were conducted under smooth flow conditions using this wind tunnel. The turbulence intensities of flow
104 at 5 m/s and 10 m/s were $\sim 1\%$, which can still be regarded as smooth in accordance with the Honshu–
105 Shikoku Bridge Authority (2001). The spring-supported free vibration method was employed to obtain
106 the VIV amplitude in the vertical 1DOF. The model was suspended using eight coil springs in the

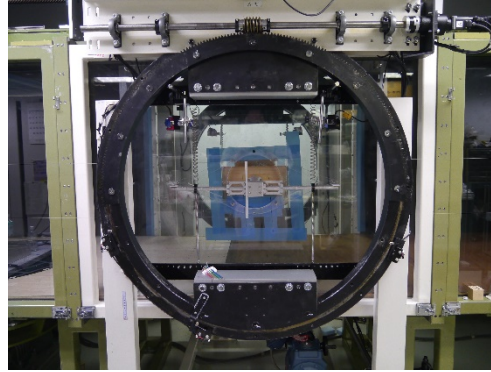
107 working section of the wind tunnel. The movements along the torsional and swaying directions were
108 fixed to realize the vertical 1DOF condition. The experimental setup is shown in Fig. 2.

109

110 (a)



(b)



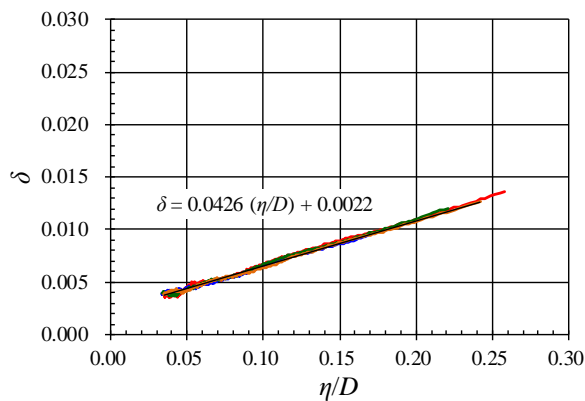
111

112 **Fig. 2.** Experimental setup. (a) front view, (b) side view.

113

114 The heaving natural frequency and equivalent mass were 3.104 Hz and 2.016 kg, respectively. Fig.
115 3 shows the relationship between the logarithmic decrement in structural damping (δ) and non-
116 dimensional amplitude (η/D), where η = half amplitude of response, obtained from vibrations in calm
117 conditions. Four similar trials were conducted, and the results of each are shown in Fig. 3. The
118 logarithmic decrement was $\delta = 0.0065$ (Scruton number $S_c = 2m\delta/(\rho B^2) = 0.80$) at $\eta = 3.0$ mm ($\eta/D =$
119 0.10), where m = mass of the system.

120



121

122 **Fig. 3.** Relationship between logarithmic decrement (δ) and amplitude (η/D).

123

124 Moreover, the aerostatic force coefficients (mean wind force coefficients) were measured to validate
125 the numerical simulation. Two three-component force balance devices (LMC-3501A-50N, Nissho
126 Electric Works, Japan) were installed at the ends of the section model. The drag, lift, and pitching
127 moment coefficients were calculated for every 2° of the angle of attack from -10° to 10° .

128

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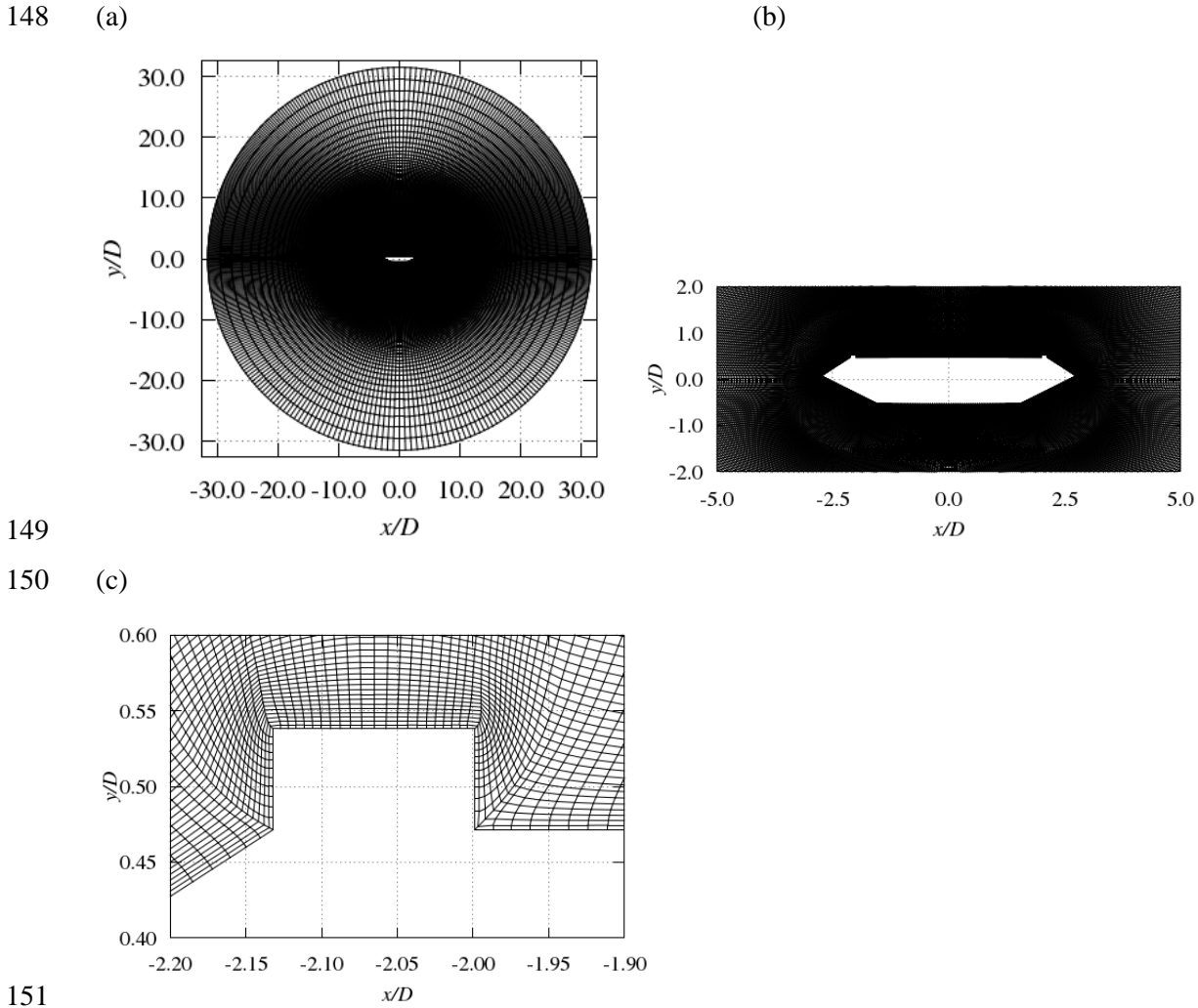
130 2.2 Numerical method

131

132 The numerical method used to calculate the flutter derivatives also followed Ito and Graham (2016,
133 2017), where the incompressible Navier–Stokes equation in generalized coordinates under the moving
134 grid condition was solved using the equation of continuity (Kajishima, 2014). A summary of the
135 numerical method is provided below. The fractional method was used for time advancement, the second-
136 order Adams–Bashforth scheme for the convection term, and the Crank–Nicolson scheme for the
137 diffusion term. The improved Morinishi method (Morinishi, 1995) was used for the discretization of the
138 convection term and the second-order central difference scheme for the other terms. A small amount of
139 numerical viscosity was added to the convection term to stabilize the simulation. The sub-grid scale
140 model was the standard Smagorinsky model ($C_s = 0.12$). The van Driest damping function was applied
141 around the wall surfaces.

142 Fig. 4 shows the computational domain. The structured O-type grid was used. The diameter of the
143 domain was $63.0D$ and the spanwise domain size was $1.0D$. The vertical size of the wall-adjacent grids
144 was $D/400$ and the spanwise grid size was $D/20$. The spanwise size of the grids used in this study was
145 sufficiently small according to Tamura et al. (1998). The number of grids in the circumferential, radial,
146 and spanwise directions were, respectively, 519, 264, and 21.

147



152 **Fig. 4.** Computational domain. (a) overall view, (b) enlarged view around the bridge section, (c) enlarged
153 view around the protuberance.

154

155 The Reynolds number ($Re = UD/\nu$) was 20,000, and the time increment was 3.0×10^{-4} – 8.0×10^{-4}
156 depending on the reduced wind speed (U/fD) and the nondimensional oscillation amplitude (η_0/D),
157 where U = wind speed, ν = kinematic viscosity, and f = oscillation frequency. All the simulations were
158 conducted under smooth flow conditions as with the wind tunnel tests. The flutter derivatives were
159 calculated using 15-cycle oscillations after a preliminary calculation of 5–15 cycle oscillations. U/fD
160 was set to 6.0, 8.0, 9.0, 10.0, 11.0, 12.0, and 14.0, and η_0/D was set between 0.025 and 0.300 at intervals
161 of 0.025. Therefore, 84 cases with different combinations of U/fD and η_0/D were obtained. The case of
162 $U/fD = 10.0$ and $\eta_0/D = 0.075$ yielded a nondimensional wall distance (y^+) between 0.9 and 1.5. This
163 means that the wall-adjacent grids were within the laminar sub-layer and no special wall treatment was
164 necessary. The maximum Courant number was 0.46, and guaranteed a stable simulation (Ferziger and
165 Perić, 2002). Moreover, the aerostatic force coefficients were calculated for every 2° angle of attack
166 from -10° to 10° , using 800 non-dimensional time after a preliminary calculation of 100 non-
167 dimensional time, for comparison with the results of the wind tunnel test for validation.

168 2.3 Definition of aerostatic force coefficients and flutter derivatives

169

170 The time-averaged drag force (D_f), lift force (L_f), and pitching moment (M_f) were defined using the
171 aerostatic force coefficients (drag force (C_D), lift force (C_L), and pitching moment (C_M)) as below:

$$D_f = 1/2 \rho U^2 C_D B L, \quad L_f = 1/2 \rho U^2 C_L B L, \quad M_f = 1/2 \rho U^2 C_M B^2 L \quad (1)$$

172 where ρ = air density, B = deck width, and L = deck length. The directions of the wind forces were set
173 as downstream-side positive for the drag force, upside positive for the lift force, and upstream-side up
174 positive for the pitching moment.

175 The unsteady lift force (L_{ae}) (downside positive) in the vertical 1 DOF was defined using two flutter
176 derivatives as below (Scanlan and Tomko, 1971);

$$L_{ae} = 1/2 \rho U^2 B L \left[K H_1^* \dot{\eta}/U + K^2 H_4^* \eta/B \right] \quad (2)$$

177 where K = reduced frequency ($B\omega/U$), ω = circular frequency, and η = heaving displacement (downside
178 positive). H_1^* and H_4^* are the flutter derivatives used in this study to define the unsteady lift force. H_1^*
179 describes aerodynamic damping because it is included in the heaving velocity term, which is defined as
180 below:

$$H_1^* = - \frac{L_\eta \sin \Psi_L}{B/D \times \tilde{\omega}^2 \times \eta_0/D} \quad (3)$$

181 where L_η = nondimensional amplitude of the unsteady lift force, Ψ_L = phase lag between heaving
182 displacement (at the downward maximum) and unsteady lift force (at the downward maximum), and $\tilde{\omega}$
183 = nondimensional circular frequency ($=\omega D/U$).

184

185 **3. Experimental and numerical results**

186

187 In this section, first, the numerical results of aerostatic force coefficients are compared with the
188 experimental results to validate the numerical method. Second, the results of spring-supported wind
189 tunnel tests are described and the characteristics of the observed aerodynamic vibrations are noted. Third,
190 the aerodynamic damping obtained from the forced oscillation method is described with various
191 combinations of reduced wind speed and oscillation amplitude. Finally, the dependence of the
192 aerodynamic damping on the oscillation amplitude is explained to discuss the possibility of calculating
193 the VIV amplitude.

194

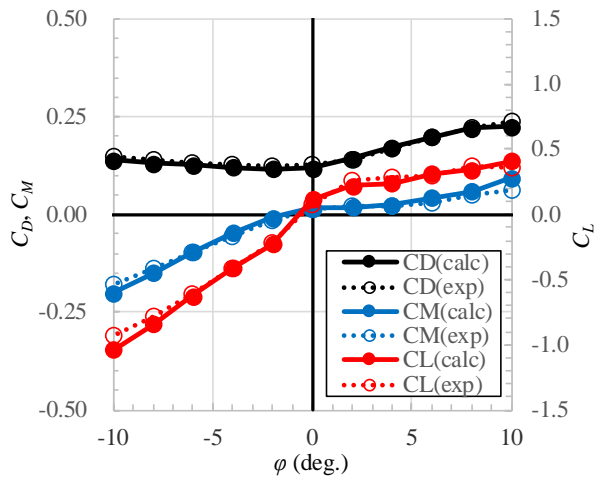
195 3.1 Aerostatic force coefficients

196

197 Fig. 5 shows the aerostatic force coefficients obtained by the wind tunnel tests and numerical simulations.
198 The simulated values were in good agreement with the experimental results. Therefore, the numerical
199 method used in this study is sufficiently accurate to describe the basic characteristics of the wind

200 resistance of the bridge section examined.

201



202

203 **Fig. 5.** Comparison of the aerostatic force coefficients obtained by the wind tunnel tests and numerical
204 simulations.

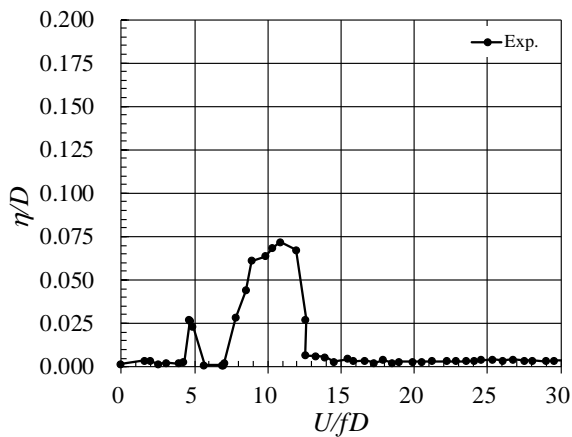
205

206 *3.2 VIV obtained by the spring-supported free vibration wind tunnel tests*

207

208 Fig. 6 shows the experimental results of the spring-supported free vibration tests in the wind tunnel.
209 VIVs were observed at $U/fD = 5.0$ and 8.0 – 12.5 ; the maximum amplitude was $\eta/D = 0.072$ at $U/fD =$
210 10.9 . According to Shiraishi and Matsumoto (1983), the critical reduced wind speed of the motion-
211 induced-type VIV in the heaving mode was $U/fD = 1.67 \times (1/N) \times B/D$, where $N =$ natural number.
212 Because this equation yields $U/fD = 4.6$ ($N = 2$) and 9.2 ($N = 1$) for the bridge section studied, and were
213 in good agreement with the results of the wind tunnel test, the aerodynamic vibrations shown in Fig. 6
214 are assumed to be of the motion-induced-type VIV.

215 By numerical simulations using the forced oscillation method with LES, the authors reproduced the
216 VIV at $U/fD = 8.0$ – 12.5 that yielded the maximum amplitude.



217

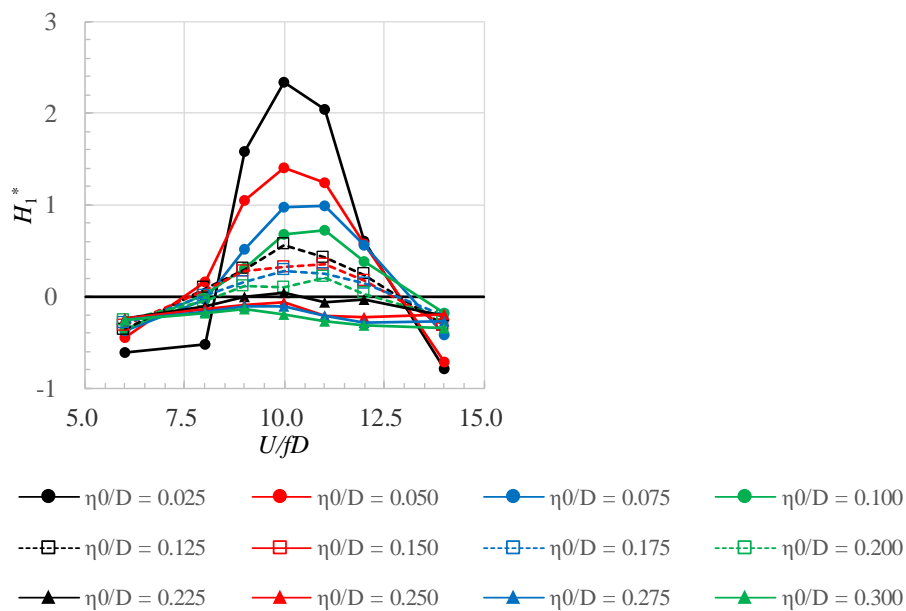
218 **Fig. 6.** Velocity–amplitude diagram obtained by the wind tunnel tests.

219 3.3 Flutter derivatives obtained by the forced oscillation method using LES

220

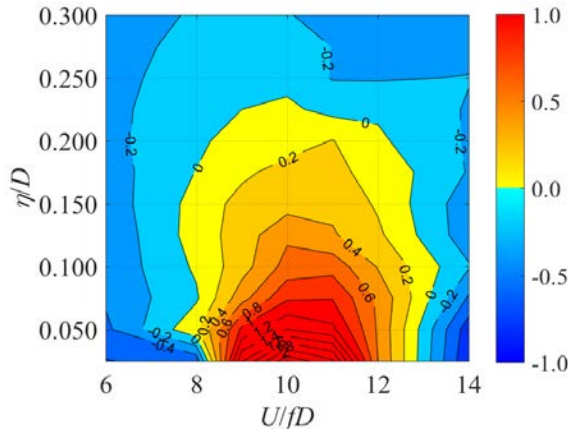
221 Fig. 7 shows the flutter derivatives of heaving aerodynamic damping (H_1^*) simulated by LES using the
 222 forced oscillation method. Ito and Graham (2016, 2017) have shown that flutter derivatives simulated
 223 in the same way as in this study are in good agreement with those obtained by wind tunnel tests. Thus,
 224 a discussion based on this should be sufficiently reliable. Fig. 8 shows the results as a contour figure of
 225 H_1^* based on Fig. 7 to investigate the relationship among reduced wind speed, nondimensional
 226 oscillation amplitude, and heaving aerodynamic damping. A positive H_1^* means negative aerodynamic
 227 damping and a negative H_1^* indicates positive aerodynamic damping. H_1^* in Fig. 8 shows a clear
 228 dependence on the oscillation amplitude, especially at approximately $U/fD = 8-13$, which corresponded
 229 with the wind speed region of VIV by wind tunnel tests. Moreover, H_1^* , in an area of Fig. 8, was positive,
 230 which suggests the occurrence of VIV.

231



233

234 **Fig. 7.** Flutter derivatives of heaving aerodynamic damping (H_1^*) for each nondimensional oscillation
 235 half amplitude (η_0/D) as a function of reduced wind speed (U/fD).
 236



237
238 **Fig. 8.** Contour figure of aerodynamic damping (H_1^*) as a function of reduced wind speed (U/fD) and
239 nondimensional oscillation half amplitude (η/D).

240
241 Thus, the possibility of calculating the VIV amplitude using the proposed method has been shown,
242 and incurs a lower computational cost than the free vibration method. In the next section, the VIV
243 amplitude is calculated on the basis of aerodynamic damping as shown in Fig. 8 and compared with the
244 results of the free vibration wind tunnel tests. Then, factors influencing aerodynamic damping are
245 investigated to accurately evaluate the VIV amplitude.

246
247 **4. Discussion**

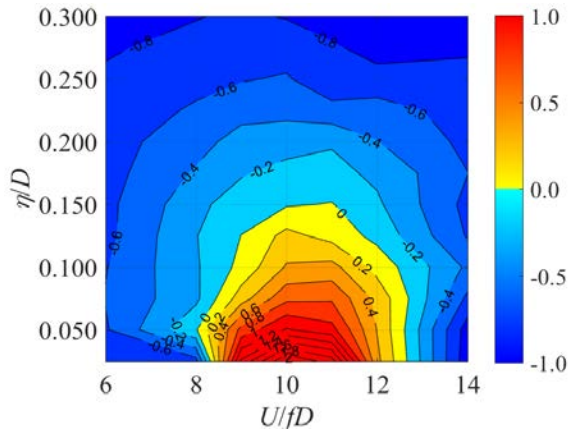
248
249 As mentioned in the previous section, aerodynamic damping based on the forced oscillation method
250 using LES showed a clear dependence on the oscillation amplitude. In this section, first, the VIV
251 amplitude is calculated using aerodynamic damping and compared with the results of free vibration
252 wind tunnel tests to discuss the accuracy and validity of the proposed method. Following this, the effects
253 of spanwise domain size and the Reynolds number on aerodynamic damping and the VIV amplitude are
254 investigated, and their mechanisms are discussed in terms of unsteady pressure characteristics.

255
256 *4.1 VIV amplitude based on forced oscillation method using LES*

257
258 While aerodynamic damping was calculated as shown in Fig. 8, the damping of the bridge deck was
259 determined on the basis of structural damping as well as the aerodynamic damping. Fig. 9 describes the
260 damping of the system ($H_1^* - 2m\delta/(\pi\rho B^2)$), where m = mass of the system and δ = logarithmic
261 decrement in structural damping. The logarithmic decrement is given by the relationship shown in Fig.
262 3. A positive $H_1^* - 2m\delta/(\pi\rho B^2)$ describes the excitation of the vibration and a negative sign the
263 attenuation. $H_1^* - 2m\delta/(\pi\rho B^2)$ still exhibited a clear dependence on the oscillation amplitude, such
264 as H_1^* , although structural damping rendered the area with positive values smaller than that in Fig. 8.
265 Further, because $H_1^* - 2m\delta/(\pi\rho B^2)$ was positive at approximately $U/fD = 8-13$ and became negative
266 at larger oscillation amplitude, the simulated results indicate the occurrence of the VIV. The simulated

267 region of wind speed where the VIV appeared was in good agreement with that in the experiments. Thus,
268 the forced oscillation method using CFD with LES can be used to calculate the wind speed region of
269 VIV with adequate accuracy.

270



271

272 **Fig. 9.** Contour figure of damping of the system ($H_1^* - 2m\delta/(\pi\rho B^2)$) as a function of reduced wind
273 speed (U/fD) and nondimensional oscillation half amplitude (η_0/D).

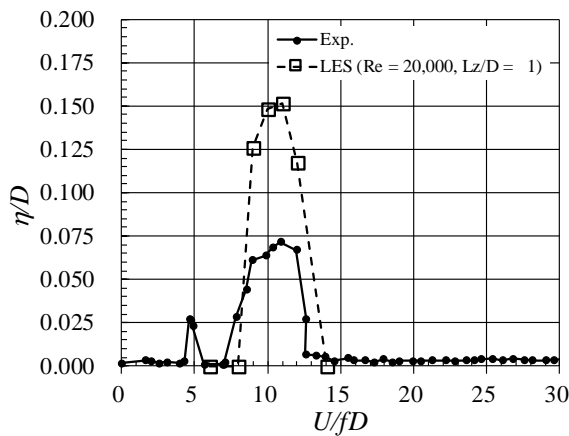
274

275 Because the relationship among reduced wind speed, oscillation amplitude, and damping of the
276 system was obtained as shown in Fig. 9, the amplitude of the steady-state response can be easily obtained
277 by calculating the wind speed and amplitude that satisfy $H_1^* - 2m\delta/(\pi\rho B^2) = 0$. The response
278 amplitudes obtained are shown in Fig. 10 together with those from the free vibration wind tunnel tests.
279 The VIV response was successfully evaluated, and the wind speed that yielded the maximum amplitude
280 was comparable to that in the wind tunnel tests. Thus, the VIV amplitude can be determined by the
281 forced oscillation method using LES as well as the wind speed region. This study is the first to adopt
282 the forced oscillation method using CFD for calculating the VIV amplitude. Although the authors focus
283 only on the motion-induced-type VIV of a box girder bridge, the proposed method can be widely
284 employed to any kind of VIV because the method here follows those in previous research (Sarpkaya,
285 1978; Staubli, 1983; Morse and Williamson, 2009) that have employed experimental forced oscillation.
286 Moreover, the damping of the system ($H_1^* - 2m\delta/(\pi\rho B^2)$) can be written as $H_1^* - S_c/\pi$, where S_c
287 is the Scruton number. This means that aerodynamic damping derived from the forced oscillation method
288 (H_1^*) can be used for calculating the VIV amplitude with any S_c . Thus, the calculation of the VIV
289 amplitude using the forced oscillation method has an advantage compared with the free vibration method.

290 However, the VIV amplitude simulated by LES was overestimated more than that by the
291 experimental free vibration method because of very large negative aerodynamic damping. Thus, for the
292 appropriate wind-resistant design of a long-span bridge, the causes of the discrepancy between the
293 results of the wind tunnel tests and the numerical simulations should be clarified.

294 In the following sections, first, the effects of the size of the nondimensional spanwise domain (L_z/D)
295 are investigated. The characteristics of a flow field are different along the span due to three-
296 dimensionality. Therefore, the dependence of aerodynamic damping on L_z/D should be examined.

297 Second, the effects of the Reynolds number (Re) are interesting to investigate. In wind tunnel tests
298 and CFD analyses, Re is usually set to a smaller value, such as several tens of thousands, than that of a
299 real bridge because the separation points are fixed at the corners of the bridge and, thus, the difference
300 in Re between a prototype and an experimental or numerical model is expected to not affect the flow
301 field. However, the Re of a wind speed region of VIV in wind tunnel tests is still small. Thus, the effects
302 of Re on the VIV amplitude should be investigated to determine the validity of the numerical simulation
303 used in this study, and to discuss the suitability of the VIV amplitude obtained from wind tunnel tests
304 for the wind-resistant design.
305



306

307 **Fig. 10.** Velocity–amplitude diagram obtained by the wind tunnel tests and LES.

308

309 4.2 Effects of spanwise domain size

310

311 4.2.1 Characteristics of unsteady lift force

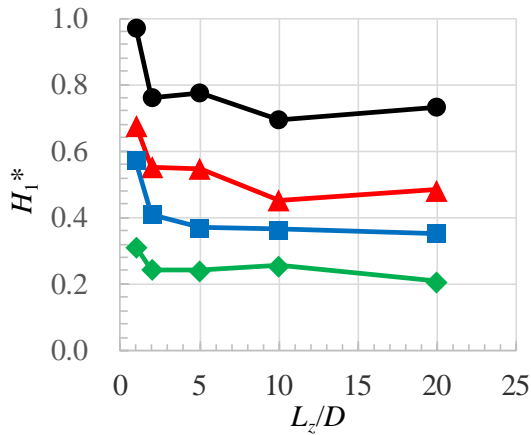
312 The effects of the spanwise domain size (L_z/D) on the flutter derivatives for heaving aerodynamic
313 damping (H_1^*) were investigated. The nondimensional spanwise domain size was set to $L_z/D = 1.0, 2.0,$
314 $5.0, 10.0,$ and $20.0,$ respectively. The spanwise grid size was maintained as $D/20$ for all cases; therefore,
315 the numbers of grids in the spanwise direction were 21, 41, 101, 201, and 401. The oscillation amplitude
316 was set to $\eta_0/D = 0.075, 0.100, 0.125,$ and $0.150,$ and the reduced wind speed was fixed at $U/fD = 10.0.$
317 The other conditions were the same as in the previous section.

318 As shown in Fig. 11(a), H_1^* tended to decrease with increasing L_z/D . H_1^* also exhibited an
319 asymptotic trend against L_z/D . Ito and Graham (2016, 2017) observed a similar trend for aerostatic force
320 coefficients at $L_z/D = 10$ – 20 because of the 3-D flow field. The same phenomenon was observed in this
321 study as well. Thus, L_z/D affects aerodynamic damping and, consequently, the VIV amplitude. The
322 amplitude for $U/fD = 10.0$ was $\eta/D = 0.133$ in case of $L_z/D = 20.0$ while $\eta/D = 0.148$ in case of $L_z/D =$
323 1.0 . To explore the mechanisms, the effects of L_z/D on the amplitude (L_η) and phase lag (Ψ_L) of the
324 unsteady lift force, which directly influenced H_1^* (Eq. (3)), were also investigated as shown in Figs.
325 11(b) and 11(c). The amplitude of the unsteady lift force (L_η) tended to decrease with increasing L_z/D ,

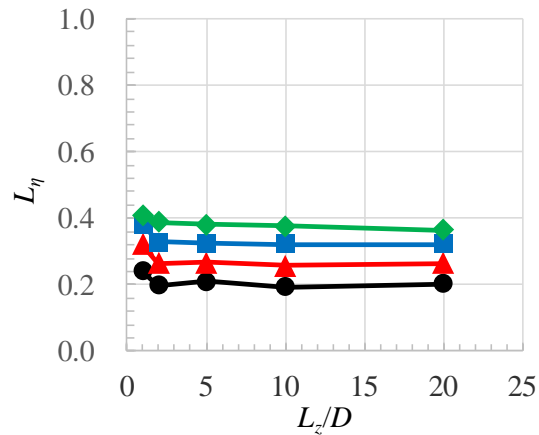
326 especially in the range $L_z/D = 1.0\text{--}2.0$. The phase lag (Ψ_L) shows negative values for all cases, which
 327 indicates the excitation of the vibration. The absolute value of Ψ_L exhibited a clear tendency to decrease
 328 with increasing L_z/D . Because a small amplitude and phase lag of the unsteady lift force reduce the
 329 aerodynamic damping as well, H_1^* also decreased with an increase in L_z/D . Thus, a sufficiently large
 330 spanwise domain size is necessary to accurately evaluate the VIV amplitude.

331

332 (a)

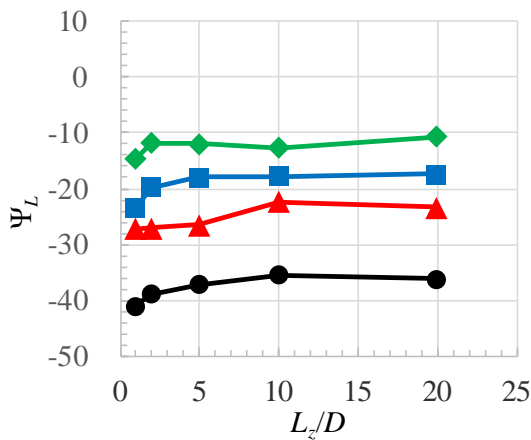


(b)



333

334 (c)



335

336 ● $\eta_0/D = 0.075$ ▲ $\eta_0/D = 0.100$ ■ $\eta_0/D = 0.125$ ◆ $\eta_0/D = 0.150$

337 **Fig. 11.** Effects of spanwise domain size on unsteady lift force. (a) aerodynamic damping (H_1^*), (b)
 338 amplitude (L_η) of unsteady lift force, (c) phase lag (Ψ_L) of unsteady lift force.

339

340 4.2.2 Characteristics of unsteady pressure

341 The mechanisms of the effect of spanwise domain size (L_z/D) on the characteristics of unsteady lift force
 342 were examined in detail. Because the unsteady lift force was calculated by integrating pressure around
 343 the bridge section, discussing the characteristics of unsteady pressure is important and suggestive.

344 The forced oscillation method was employed under $L_z/D = 1.0$ or 10.0 , $\eta_0/D = 0.075$, and $U/fD =$
 345 10.0 . $L_z/D = 10.0$ was employed because H_1^* converged to a certain value at $L_z/D = 10\text{--}20$ as mentioned

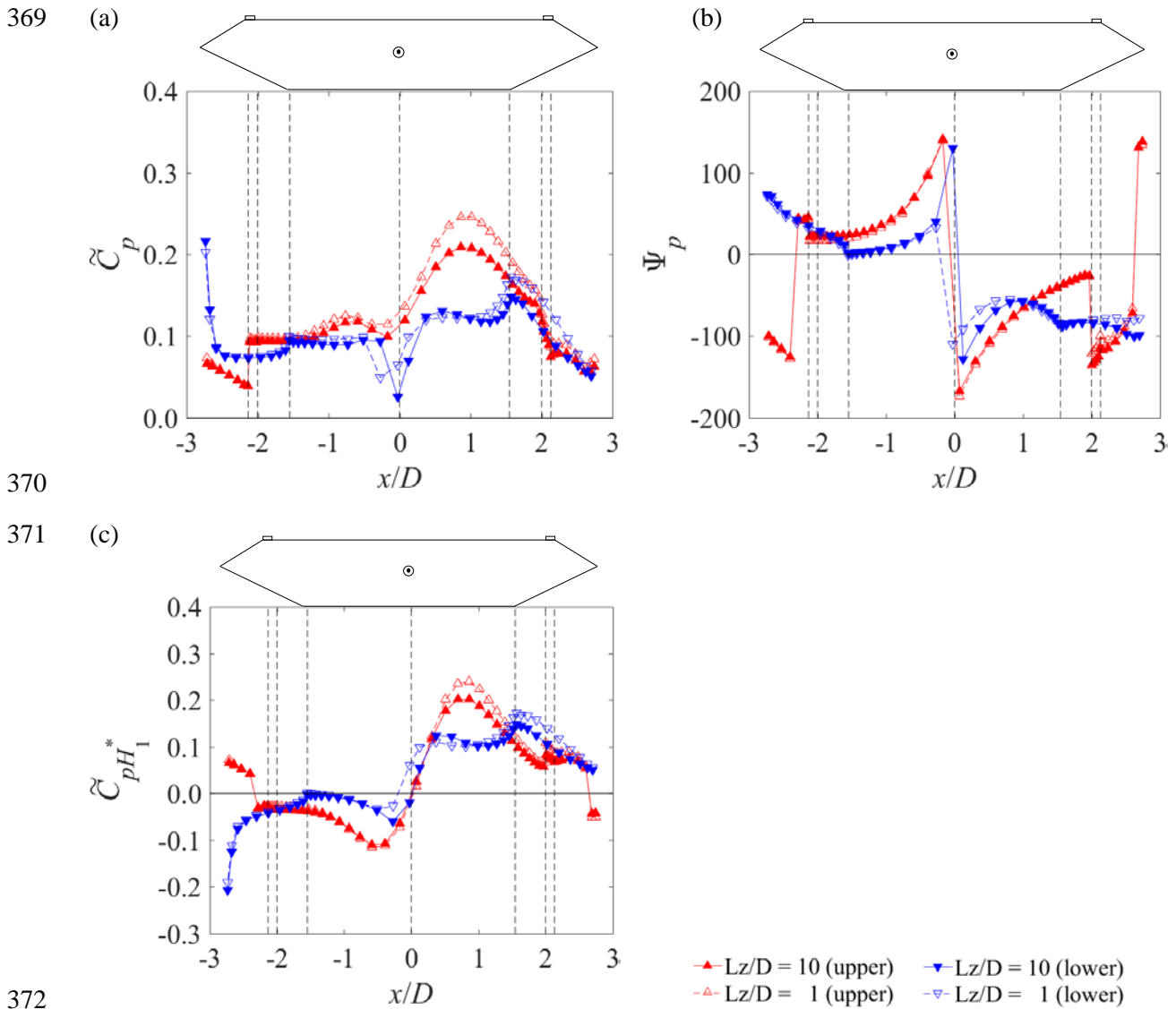
346 above. Spanwise-averaged unsteady wall pressures at every five grids along the circumferential
347 direction were obtained. Then, the nondimensional half amplitude of unsteady pressure (\tilde{C}_p) and phase
348 lag (Ψ_p) at each grid were evaluated. The phase lag was defined as a difference in phase between the
349 heaving displacement (at the downward maximum) and the unsteady pressure (with the maximum
350 positive value on the upper surface and the maximum negative value on the lower surface, both of which
351 resulted in a downward lift force). Finally, aerodynamic damping in the heaving mode at each grid
352 ($\tilde{C}_{pH_1^*}$), and the relationship between $\tilde{C}_{pH_1^*}$ and H_1^* can be written as below (Matsumoto et al., 1996).

$$H_1^* = \frac{1}{B/D \times \tilde{\omega}^2 \times \eta_0/D} \int -\tilde{C}_p \sin \Psi_p dx = \frac{1}{B/D \times \tilde{\omega}^2 \times \eta_0/D} \int \tilde{C}_{pH_1^*} dx \quad (4)$$

353 These definitions of the parameters prompt a similar discussion with the characteristics of unsteady
354 lift force of L_η , Ψ_L , and H_1^* . A positive Ψ_p results in a negative $\tilde{C}_{pH_1^*}$ (positive aerodynamic damping =
355 attenuation of vibration), and a negative Ψ_p leads to a positive $\tilde{C}_{pH_1^*}$ (negative aerodynamic damping =
356 excitation of vibration). Considering these parameters at each grid on the girder, the difference in the
357 pressure characteristics and, thus, the flow field caused by L_z/D can be discussed in detail.

358 Fig. 12 shows the characteristics of unsteady pressure obtained, the lateral axis of which is the
359 nondimensional horizontal coordinate (x/D). The values of both the upper surface (\blacktriangle : $L_z/D = 10.0$, \triangle :
360 $L_z/D = 1.0$) and the lower surface (\blacktriangledown : $L_z/D = 10.0$, \triangledown : $L_z/D = 1.0$) are plotted in each figure. The
361 amplitude of pressure (\tilde{C}_p) in $L_z/D = 1.0$ tended to be higher in both the upper and the lower surfaces.
362 Whereas the phase lag (Ψ_p) of the upper surface is almost identical for the two cases, that of the lower
363 surface in $L_z/D = 1.0$ has a slightly larger area with negative values around $x/D = 0$. The characteristics
364 of \tilde{C}_p and Ψ_p resulted in those of $\tilde{C}_{pH_1^*}$, showing larger values of $L_z/D = 1.0$ in both the upper and lower
365 surfaces, especially in the downstream side and, thus, exhibited greater instability. In other words, L_z/D
366 affected the flow field around the bridge deck, especially downstream. These characteristics of the
367 unsteady pressure explain those of the unsteady lift force as shown in Fig. 11 well.

368



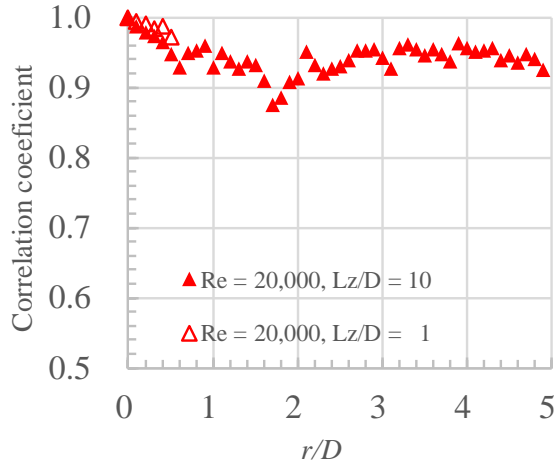
373 **Fig. 12.** Effects of spanwise domain size on unsteady pressure at each location. (a) amplitude (\tilde{C}_p) of
374 unsteady pressure, (b) phase lag (Ψ_p) of unsteady pressure, (c) aerodynamic damping ($\tilde{C}_{pH_1^*}$).

375

376 Moreover, the correlation coefficient of the oscillation frequency of pressure along the span at x/D
377 $= 0$ was investigated in case of $L_z/D = 1.0$ and 10.0 . Pressure in this discussion is defined as the difference
378 between pressure at the upper and lower surfaces. The correlation of the unsteady lift forces exhibited
379 the same relations as that of the unsteady pressures. Fig. 13 shows the correlation coefficient of pressure
380 at $z/D = 0$ and $z/D = r/D$, where $r =$ distance from $z/D = 0$. The correlation coefficient was large because
381 of the oscillation. On the contrary, the correlation coefficient tended to become smaller with increasing
382 r/D , which indicates a significant three-dimensionality of the flow field. This also indicates that a
383 sufficiently large L_z/D is necessary to consider the 3-D flow field. The overestimated correlation of flows
384 in the spanwise direction in $L_z/D = 1.0$ is considered to have affected the flow field around the bridge
385 section, as shown in Fig. 11, which resulted in a larger VIV amplitude.

386 Thus, a sufficiently large size of the spanwise domain is required to consider the 3-D flow field for

387 a proper evaluation of aerodynamic damping and the VIV amplitude. Although a larger spanwise domain
388 increases computational cost, the free vibration method also requires this size to accurately evaluate the
389 phenomena. This is because L_z/D depends on the target flow field and not the methodology of simulation.
390 Thus, the forced oscillation method still has an advantage in terms of computational cost against the free
391 vibration method.



392
393 **Fig. 13.** The correlation coefficient of pressure along the span at $x/D = 0$.

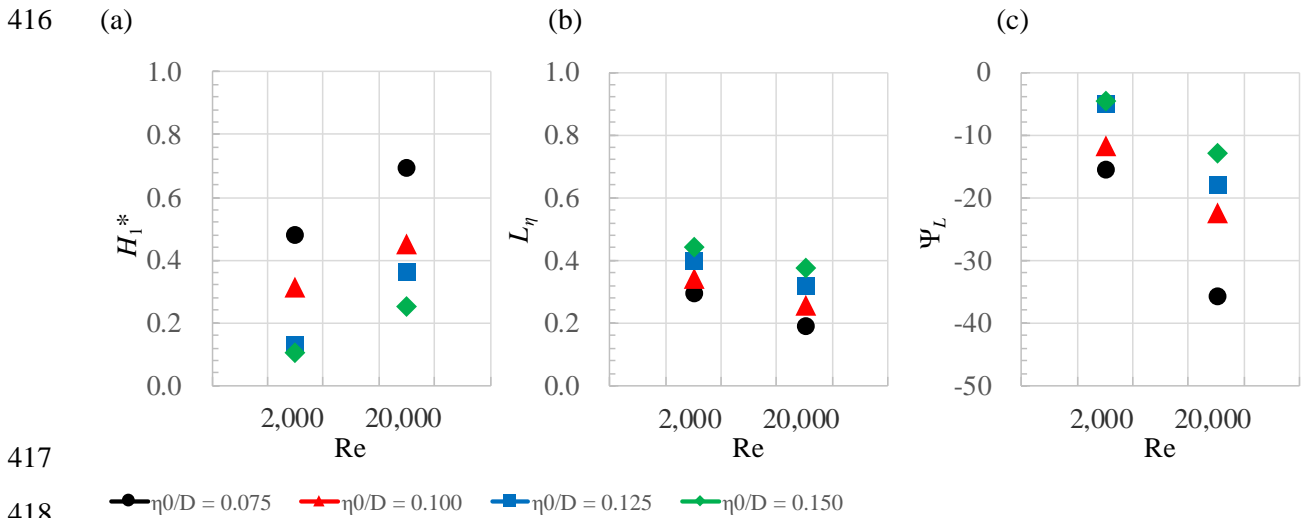
394
395 *4.3 Effects of Reynolds number*

396
397 The previous section clarified that a sufficiently large domain is necessary to implement the three-
398 dimensionality of the flow field. The section below discusses the effects of Re on the VIV amplitude in
399 terms of the characteristics of unsteady lift force and unsteady pressure.

400
401 *4.3.1 Characteristics of unsteady lift force*

402 The effects of Re on H_1^* were investigated at $L_z/D = 10.0$, $\eta_0/D = 0.075, 0.100, 0.125, \text{ and } 0.150$, and
403 $U/fD = 10.0$. Re was set to Re = 20,000 and 2,000. Re = 20,000 was the same value as in the former
404 sections, and Re = 2,000 was selected because it corresponded with the occurrence wind speed of the
405 VIV in the free vibration wind tunnel tests.

406 As shown in Fig. 14(a), Re affected H_1^* and, consequently, the VIV amplitude; the amplitude for
407 $U/fD = 10.0$ and $L_z/D = 10.0$ was $\eta/D = 0.107$ in case of Re = 2,000, and $\eta/D = 0.136$ in case of Re =
408 20,000. To explore the mechanisms, effects of Re on amplitude (L_η) and phase lag (Ψ_L) of the unsteady
409 lift force were examined as shown in Figs. 14(b) and 14(c). Values of L_η at Re = 2,000 were larger than
410 those at Re = 20,000. On the contrary, although Ψ_L showed negative values for all cases, the absolute
411 values of Ψ_L in Re = 2,000 were clearly smaller. As indicated by Eq. (3), a large amplitude and a large
412 absolute value of negative phase lag result in a large negative aerodynamic damping (i.e., positive H_1^*).
413 Thus, although L_η and Ψ_L in Re = 2,000 had opposite roles, the aerodynamic damping of each oscillation
414 amplitude in Re = 2,000 was smaller because the effect of Ψ_L seemed to be dominant. Thus, considering
415 the effect of Re was very important to reproduce the VIV amplitude obtained from wind tunnel tests.



417

418

419 **Fig. 14.** Effects of Reynolds number on unsteady lift force. (a) aerodynamic damping (H_1^*), (b) amplitude
420 (L_η) of unsteady lift force, (c) phase lag (Ψ_L) of unsteady lift force,

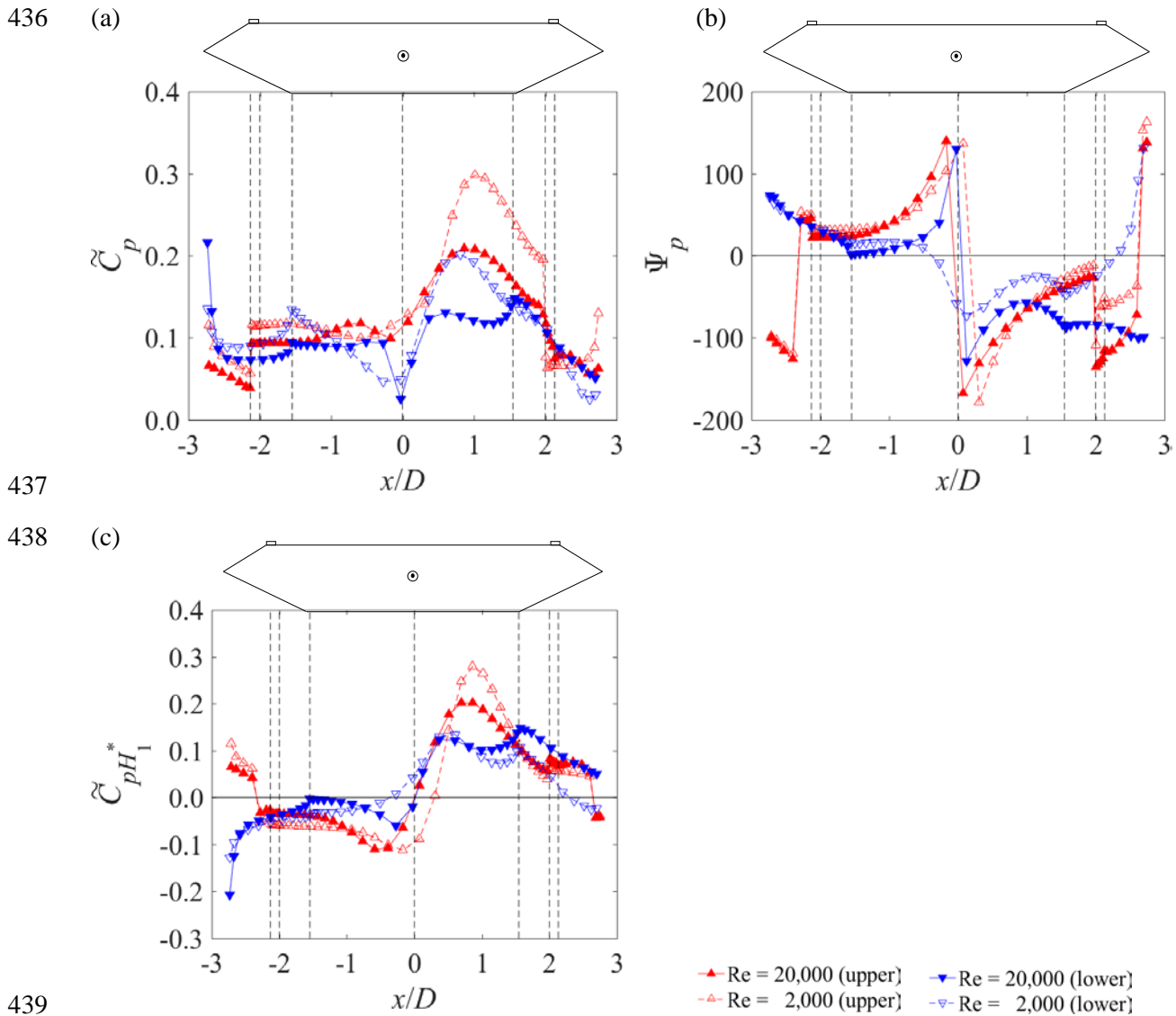
421

422 4.3.2 Characteristics of unsteady pressure

423 The characteristics of unsteady pressure were also investigated to discuss the mechanisms of the effects
424 of Re on the characteristics of unsteady lift force.

425 Fig. 15 shows the obtained characteristics of unsteady pressure. The values of both the upper surface
426 (\blacktriangle : Re = 20,000, \triangle : Re = 2,000) and the lower surface (\blacktriangledown : Re = 20,000, \triangledown : $L_c/D = \text{Re} = 2,000$) are
427 plotted. First, in the upper surface, \tilde{C}_p for Re = 20,000 tended to be smaller, and Ψ_p had a slightly larger
428 area with negative values for Re = 20,000. Therefore, although the peak value of $\tilde{C}_{pH_1^*}$ for Re = 2,000
429 was larger, Re = 20,000 gave a larger area of $\tilde{C}_{pH_1^*}$ with positive values. Second, in the lower surface,
430 \tilde{C}_p for Re = 20,000 also tended to be smaller. On the contrary, Ψ_p in the downstream side for Re =
431 20,000 indicated significantly larger negative values than those for Re = 2,000. Therefore, $\tilde{C}_{pH_1^*}$ for Re
432 = 20,000 had larger values over the downstream side, which appeared to result in larger H_1^* in Re =
433 20,000. These characteristics of unsteady pressure explain well the difference in the characteristics of
434 unsteady lift force between Reynolds numbers.

435



440 **Fig. 15.** Effects of Reynolds number on unsteady pressure. (a) amplitude (\tilde{C}_p) of unsteady pressure, (b)
441 phase lag (Ψ_p) of unsteady pressure, (c) aerodynamic damping ($\tilde{C}_{pH_1^*}$).

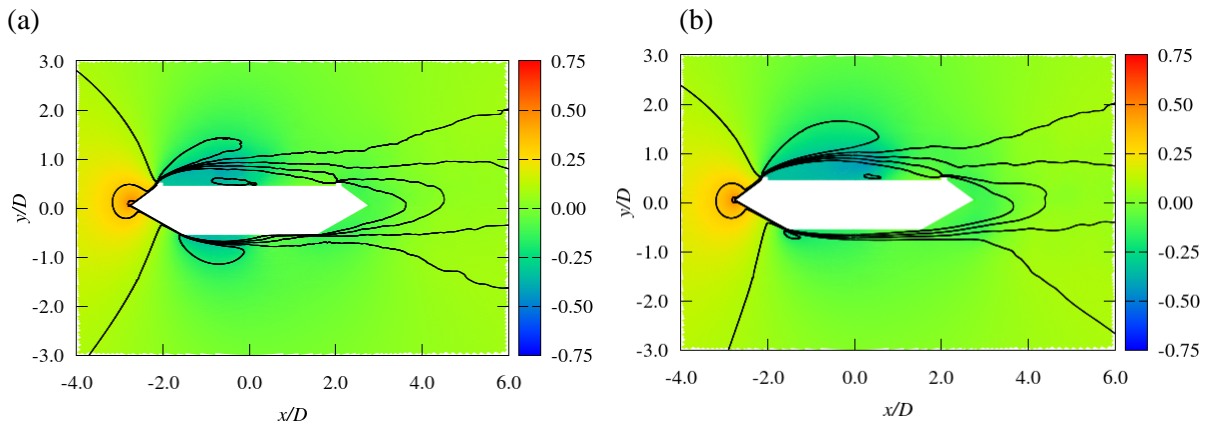
442

443 Fig. 16 shows the time-averaged pressure contours and velocity magnitude isograms for $Re = 20,000$
444 and $Re = 2,000$, respectively. The difference in the flow field between values of Re at the upper surface
445 was small because the separation point was fixed on the protuberance and, thus, the flow field on the
446 upper surface was insensitive to Re . On the contrary, the difference on the lower side was relatively
447 large, including at the reattachment point because the lower side had a streamlined form, because of
448 which a slight change in the separation point had a large influence on the flow field. This is also obtained
449 because flows at $Re = 20,000$ featured more turbulence than those at $Re = 2,000$, which caused earlier
450 reattachment (Laneville et al., 1975). Thus, the numerical results reproduced the qualitative
451 characteristics well, although discussing the discrepancy between the wind tunnel tests and LES is
452 necessary. These characteristics of the flow field correspond with those of unsteady pressure shown in
453 Fig. 15, in which the effects of Re are larger at the lower surface than at the upper surface. As a result,

454 the effects of the Reynolds number on the flow field were significant, and thus the unsteady lift force
455 and VIV amplitude were significantly different between $Re = 20,000$ and $Re = 2,000$.

456

457



458

459 **Fig. 16.** Time-averaged pressure contours and velocity magnitude isograms (colormap = mean pressure,
460 lines = mean velocity magnitude isograms). (a) $Re = 20,000$, (b) $Re = 2,000$.

461

462 As discussed above, the flow fields around the bridge section at $Re = 20,000$ and $2,000$ were different,
463 especially downstream, and this resulted in a difference in the unsteady lift force, aerodynamic damping,
464 and the VIV amplitude. Thus, the Reynolds number should be attended to, especially the reproduction
465 of the VIV amplitude obtained from wind tunnel tests in case of a small Reynolds number. In other
466 words, the considered case in this study implies as a possibility that the VIV amplitude calculated by
467 wind tunnel tests may be affected by the Reynolds number and, thus, underestimated compared with
468 that obtained from the prototype. This is because a small Reynolds number sometimes has to be
469 employed for the wind-resistant design because of the limitation of experimental facilities. Thus, the
470 experimental results of the VIV should be carefully interpreted. However, because the dependence of
471 the VIV amplitude on the Reynolds number can be affected by the shape of the bridge section, including
472 the existence of such attachments as handrails, further investigations are needed for a comprehensive
473 understanding of the numerical performance and accuracy of the wind tunnel tests, especially at low
474 Reynolds numbers.

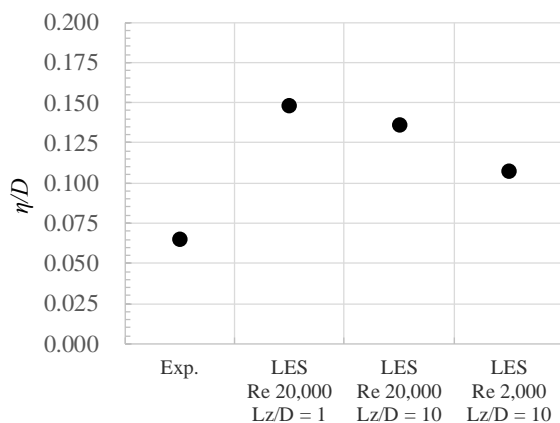
475

476 4.4 Comparison of VIV amplitude obtained by wind tunnel tests and LES

477

478 Finally, the VIV amplitude obtained by wind tunnel tests and numerical simulations using LES were
479 compared as shown in Fig. 17. The reduced wind speed was set to $U/fD = 10.0$. As mentioned above,
480 the size of the spanwise domain and Re should be appropriately determined to better calculate the VIV
481 amplitude. In particular, the effect of Re is more significant than that of the size of the spanwise domain.
482 However, even if the effects of Re and domain size are considered, the VIV amplitude obtained by the
483 forced oscillation method using LES is still larger than that obtained from the free vibration wind tunnel
484 tests.

485 The spanwise size of the grids used in this study was determined according to Tamura et al. (1998).
 486 However, as mentioned by Bruno et al. (2012), the spanwise discretization may play a great role in the
 487 accuracy of the simulations. Therefore, the effects of the spanwise discretization should be investigated
 488 in future work. Moreover, the small turbulence in the approaching flow of the wind tunnel tests might
 489 have reduced amplitude. For example, Wardlaw et al. (1983) noted that the VIV amplitude of a bridge
 490 deck decreased dramatically under turbulent flow. In addition, the accuracy of the wind tunnel tests was
 491 not necessarily adequate because the measurements were carried out in regions of low wind speed.
 492 Additional numerical simulations using LES are thus necessary in future work to further investigate the
 493 effects of turbulent flow on the VIV amplitude.
 494



495
 496 **Fig. 17.** VIV amplitude obtained by wind tunnel tests and numerical simulations using LES.

497
 498 **5. Concluding remarks**

499
 500 In this study, the amplitude of vortex-induced vibration (VIV) was calculated using the forced oscillation
 501 method with a large eddy simulation (LES) to avoid the large computational cost involved in the free
 502 vibration method, and to realize the practical use of a numerical simulation for the wind-resistant design
 503 of a long-span bridge. The conclusions are as follows:

- 504 · Flutter derivatives for aerodynamic damping obtained from the forced oscillation method using LES
- 505 showed a clear dependence on the oscillation amplitude. Thus, the proposed method can calculate
- 506 the VIV amplitude with a smaller computational cost than the free vibration method.
- 507 · A sufficiently large spanwise domain is required to consider the 3-D flow field for the proper
- 508 evaluation of the aerodynamic damping and, consequently, the VIV amplitude.
- 509 · The Reynolds number changes the flow field around the bridge section to affect the aerodynamic
- 510 damping and, thus, the VIV amplitude. The effect of Reynolds number is important, especially for
- 511 reproducing the VIV amplitude obtained from wind tunnel tests with a small value of this number.
- 512 · The considered case in this study implies as a possibility that the VIV amplitude calculated by wind
- 513 tunnel tests may be underestimated compared with that obtained from the prototype, because of a low
- 514 Reynolds number resulting from the limitation of experimental facilities. In future work, investigations

515 are necessary to explore the effects of the shape of the bridge section, including attachments such as
516 handrails, on the dependence of the VIV amplitude on the Reynolds number.
517 · Even if the effects of the Reynolds number and the size of the spanwise domain were considered, the
518 VIV amplitude obtained by the forced oscillation method using LES was still larger than that of the
519 free vibration wind tunnel tests. The cause of this discrepancy should be investigated further to
520 improve accuracy, for example, from the viewpoint of the effects of spanwise discretization and
521 turbulent flow.

522

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524 The supercomputer of ACCMS, Kyoto University, was used in this study for the flow simulation.

525

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