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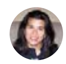

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DESIGN TOOL TO PREDICT THE OPEN-HOLE FAILURE STRENGTH OF COMPOSITE LAMINATES SUBJECTED TO IN-PLANE LOADS

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Abstract

A design tool based on Whitney-Nuismer model is proposed to predict the failure strength of open-hole composite plates subjected to in-plane loads, both tensile and compressive. One of the limitations of this model is the use of characteristic distances, which have to be estimated, and which are mostly considered constant material properties. In this work, it is demonstrated that these characteristic distances are a function of the plate geometry and are not a material property. Different analytical expressions are proposed to estimate the characteristic distances by using the results of only three experimental tests. This approach enables an accurate and fast estimation of the failure strength when the parameters of the plate are modified without using numerical simulations that require a longer time to be carried out.

Keywords: open hole; notched compression; notched tension; non-dimensional model.

Nomenclature

PSC:	Point Stress Criterion
ASC:	Average Stress Criterion
W-N model	Whitney-Nuismer model
$\boldsymbol{d_o}$:	Characteristic distance for the Point Stress Criterion
$\boldsymbol{a_o}$:	Characteristic distance for the Average Stress Criterion
$\boldsymbol{W}$:	Width of the laminate plate
$\boldsymbol{R}$:	Radius of the open-hole

$\boldsymbol{\sigma_N}$:	Uniaxial in-plain load applied to the plate
$\boldsymbol{K}$:	Stress concentration factor
ξ:	Dimensionless geometric ratio
$\boldsymbol{\eta}$:	Dimensionless geometric ratio
$\boldsymbol{K_{max}^{\infty}}$:	Stress concentration factor at the edge of the hole for an infinite orthotropic plate
$\mathbf{f_w}$:	Finite-width correction factor
$\mathbf{A_{ij}}$:	Components of the in-plane stiffness matrix **[A]**
M:	Magnification factor
$\hat{\boldsymbol{d}}_{\boldsymbol{o}}$:	Dimensionless characteristic distance for the Point Stress Criterion
$\hat{\boldsymbol{a}}_{\mathbf{0}}$:	Dimensionless characteristic distance for the Average Stress Criterion
K_e:	Effective stress concentration factor
$\boldsymbol{\sigma_0}$:	Failure strength of the unnotched laminate
OHT:	Open-hole tensile tests
$\sigma_N^{failure}$:	Laminate failure strength
OHC:	Open-hole compressive tests
$\boldsymbol{E_1}$:	Longitudinal elastic modulus
$\boldsymbol{E_2}$:	Transversal elastic modulus
$\boldsymbol{G_{12}}$:	In-plane shear modulus
$\boldsymbol{\nu_{12}}$:	Poisson coefficient
$\boldsymbol{t_k}$:	Ply thickness

1. Introduction

The presence of irregularities in the geometry of a structural element, such as holes, notches or changes in cross-section causes a significant change in the stress distribution, known as stress concentration. During the manufacturing of a laminate structure, open holes are made, for example, for joining several elements or for access (doors, windows, vents, etc.) [1].

The stress concentration related to an open hole is an essential topic in the design of composite structures, and therefore, several studies have been conducted to investigate its effect on the strength of the structure [2-4]. A reduction in the strength of the notched laminate is observed compared to that of the unnotched laminate [2-7]. Hence, there is a need for a model that can predict the stress distribution around the hole, as well as the strength [8-11].

Many authors study the mechanical behaviour of open-hole composite laminates carrying out experimental tests [12-15]. Such tests are costly, time-consuming and are valid only for the configuration tested (plate width and hole diameter) since a change in a single parameter requires the entire new configuration to be tested.

Numerical models have been successfully applied for modelling notched plates subjected to in-plane loads. There are several models capable of predicting the open hole strength and even predict the damage evolution on the plate [12-18]

In the pre-design of a structure containing holes, it is of interest to have a tool that estimates the failure strength of a plate containing holes, and that allows quick estimation of that failure when modifications on the geometry are introduced. If the numerical models mentioned above are used, the time needed to obtain results is longer.

Analytical models can be useful in providing a sufficiently accurate solution with a lower computational cost than that of the numerical methods [8, 9, 16, 18-21]. Many analytical models seek to predict the stress distribution around several discontinuities in anisotropic materials, among the best known being the models of Camanho [15,18], Waddoups [16], Mar and Lin [19], Eriksson and Aronsson [20] or Whitney and Nuismer [8, 9].

Whitney and Nuismer [8, 9] developed two simple criteria to estimate the open-hole strength of composite laminates with a centred open hole under uniaxial tension: the Point Stress Criterion (PSC) and the Average Stress Criterion (ASC). Both criteria assume that the damage of an open-centred hole laminate appears when the normal stress (for PSC criterion) or the average stress (for the ASC criterion) reaches the failure strength of the unnotched laminate over some characteristic distances $\boldsymbol{d_o}$ and $\boldsymbol{a_o}$ away from the edge of the hole. Although Whitney and Nuismer model was proposed in 1974, it is easy to implement and apply for the prediction of open-hole laminate strength. This model has been applied in several scientific studies in the last decade as well as nowadays [7, 22-25]. To determine the failure strength of notched laminates using Whitney and Nuismer model, it is necessary to know the value of the characteristic distances. Whitney and Nuismer [8, 9], Barbero [22], Mallick [23] and Tan [7], assume that the characteristic distance $\boldsymbol{d_o}$ and $\boldsymbol{a_o}$ are constant for each laminate regardless of the size of the hole. Frequently, values for $\boldsymbol{d_o}$ between 1 and 2 mm [20] and $\boldsymbol{a_o}$ between 2.5 and 5 mm [8, 9, 26, 27] are useful in the design for several E-glass/epoxy and carbon/epoxy laminates [22]. However, other authors [10, 28-32] affirm that $\boldsymbol{d_o}$ and $\boldsymbol{a_o}$ depend on the size of the hole and therefore cannot be considered material properties. These distances are estimated by fitting an expression to experimental data obtained from tensile tests. Several expressions have been proposed. For example, Karlak [29] established that the characteristic distance $\boldsymbol{d_o}$ is proportional to the square root or the hole radius. Pipes et al., [30] suggested an expression with three parameters, being one of those an exponential function of the hole radius. These authors do not consider the influence of the specimen width on the failure strength of the laminate. However, it has been observed experimentally that laminates with the same hole radius and different width, show different notched strength [33]. Kim et al. [33] proposed an exponential function related to the width and hole radius, for which, only two experimental tests carried out in two notched specimens are needed to fit the parameters. Nevertheless, the error of the expression and the value of the coefficient of determination are not showed.

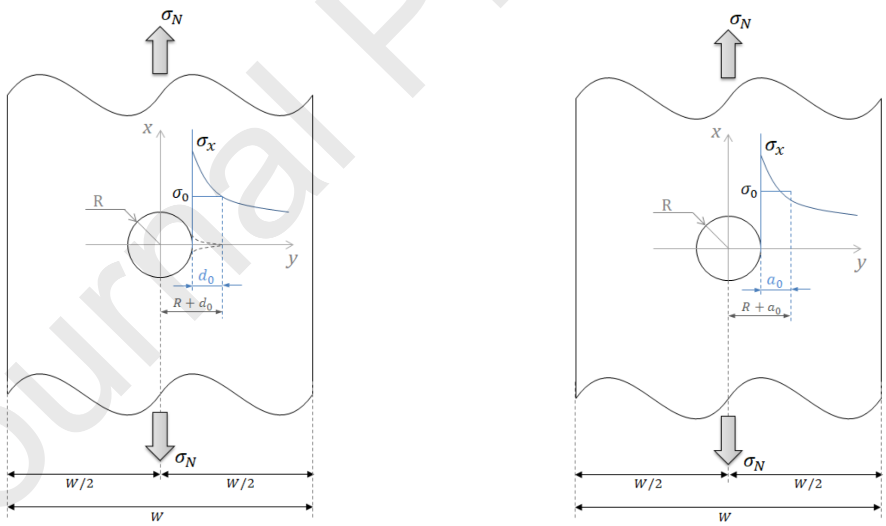
No more expressions to fit the experimental data have been found in the literature. Also, depth analysis of the number of tests which is required to fit the expression has not been carried out. Additionally, many authors use the same experimental data to fit the expression and to estimate the precision of the method. Other authors, as Govindan et al. [34] and Kim et al. [33] presented expressions to determine the values of $\boldsymbol{d_o}$ and $\boldsymbol{a_o}$ applying concepts of fracture mechanics of isotropic materials to the Whitney and Nuismer model. Nevertheless, these models required also experimental test to fit the parameters.

Although Whiney and Nuismer model was developed for in-plane tensile loads, the equations could also be used in compression, assuring that the failure is not controlled by the global buckling of the plate [35, 36]. However, not enough information has been found in the scientific literature which evaluates the applicability of Whitney and Nuismer model in notched laminates under compressive loads.

In the present work, a predictive design tool is proposed to estimate the failure strength of notched laminates subjected to tensile and compressive in-plane loads. Whitney and Nuismer model in a dimensionless formulation is used. A methodology to estimate the value of the characteristic distances is defined, by using several fitting expressions as a function of the laminate width and hole radius, and for both criteria (PSC and ASC). A comparison of the precision for each fitting equation is shown. Experimental data from the literature for different materials, laminates, and geometries are used. For each material, some of the experimental results were selected to fit the parameters of the expressions, while the rest of the results (with different geometries) were used to estimate the precision of the method. Additionally, a graphical representation of the proposed method is presented and can be used as a design curve to estimate the effective stress concentration factor.

2. Model description

The model proposed by S.G. Lekhnitskii [37] is used in this work to estimate the stresses close to the edge of the hole of a composite-laminate plate subjected to uniaxial in-plane loads. Considering an orthotropic plate of width $\boldsymbol{W}$**,** with a centred circular hole of radius $\boldsymbol{R}$, and subjected to uniaxial in-plane load $\boldsymbol{\sigma_N}$ parallel to the $\boldsymbol{x}$-axis (Fig.1), the stress increment close to the edge of the hole is represented by the stress concentration factor $\boldsymbol{K}$, which is written in dimensionless variables as shown in Eq. 1.



(a) (b)

Figure 1. Stress distribution of $\boldsymbol{\sigma_x}$ in the vicinity of the hole of a laminate subjected to uniaxial loading. Definition of the characteristic distances for: a) PSC criteria and b) ASC criteria.

$$k(\xi) = \frac{1}{2} \cdot f_w \cdot \left\{ 2 + \left(\frac{\eta}{\eta + 2\xi}\right)^2 + 3\left(\frac{\eta}{\eta + 2\xi}\right)^4 - \left(K_{max}^{\infty} - 3\right) \cdot \left[5\left(\frac{\eta}{\eta + 2\xi}\right)^6 - 7\left(\frac{\eta}{\eta + 2\xi}\right)^8 \right] \right\} \quad (1)$$

Where ξ and $\boldsymbol{\eta}$ are two dimensionless geometric ratios which have been defined using the width of the plate $\boldsymbol{W}$, as shown in Eq. 2.

$$\eta = \frac{2R}{W} \;,\; \xi = \frac{y}{W} \quad (2)$$

In Eq.1, $\boldsymbol{K_{max}^{\infty}}$ denotes the stress concentration factor at the edge of the hole for an infinite orthotropic plate (Eq.3), and $\boldsymbol{f_w}$ is a finite-width correction factor [7], defined by Eq.4.

$$K_{max}^{\infty} = 1 + \sqrt{\frac{2}{A_{22}}\left(\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2\,A_{66}}\right)} \quad (3)$$

$$f_w = \left(\frac{3(1-\eta)}{2 + (1-\eta)^3} + \frac{1}{2}(\eta \cdot M)^6 \cdot \left(K_{max}^{\infty} - 3\right)\left[1 - (\eta \cdot M)^2\right]\right)^{-1} \quad (4)$$

being $\mathbf{A_{ij}}$ the components of the in-plane stiffness matrix **[A]** calculated by the Classical Laminate Theory [22]**.** **M** is a magnification factor defined by Eq. 5. [7].

$$M = \sqrt{\frac{\sqrt{1 - 8 \cdot \left[\frac{3(1-\eta)}{2 + (1-\eta)^3} - 1\right]} - 1}{2 \cdot \eta^2}} \quad (5)$$

The above equations apply only to balanced symmetric laminates with uniaxial in-plane loading [22]. The expression for the finite width correction factor $\boldsymbol{f_w}$ (Eq. 4) provides good results even with a $\boldsymbol{\eta = 2R/W}$ ratio of over 90% [7].

Whitney and Nuismer [8] developed two stress criteria for the notched strength prediction of open-hole laminated composites under uniaxial tension; the Point Stress Criterion (PSC) and the Average Stress Criterion (ASC).

The point-stress criterion assumes that failure occurs when the stress $\boldsymbol{\sigma_x}$, at a distance $\boldsymbol{d_o}$ ($\hat{\boldsymbol{d}}_{\boldsymbol{o}} = \frac{d_0}{W}$ in dimensionless variables) from the edge of the hole, reaches the strength of the unnotched laminate $\boldsymbol{\sigma_0}$. The effective stress concentration factor $\boldsymbol{K_e}$ can be defined as the one which causes laminate failure, Eq. 6.

$$K_e^{PSC} = \left[1 + \frac{1}{2}\left(\frac{\eta}{\eta + 2\hat{d}_o}\right)^2 + \frac{3}{2}\left(\frac{\eta}{\eta + 2\hat{d}_o}\right)^4 - \frac{(K_{max}^{\infty} - 3)}{2} \cdot \left(5\left(\frac{\eta}{\eta + 2\hat{d}_o}\right)^6 - 7\left(\frac{\eta}{\eta + 2\hat{d}_o}\right)^8\right)\right] \cdot f_w \quad (6)$$

The second criterion is the Average Stress Criterion, *ASC* [8]. This criterion assumes that failure occurs when the average stress $\boldsymbol{\sigma_x}$ at a distance $\boldsymbol{a_o}$ ($\boldsymbol{\hat{a}_0} = \frac{a_0}{W}$ in dimensionless variables) from the edge of the hole, reaches the failure strength of the unnotched laminate $\boldsymbol{\sigma_0}$, defined in Eq. 7:

$$\frac{1}{a_0}\int_o^{a_0} \sigma_x(y)\, dy = \sigma_0 \quad (7)$$

The parameter $\boldsymbol{a_o}$ corresponds to the distance at which the stress average measured from the edge of the hole is located, as shown in Fig. 1. In this case, the stress concentration factor is defined as:

$$K_e^{ASC} = \left[\frac{2 - \left(\frac{\eta}{\eta + 2\hat{a}_o}\right)^2 - \left(\frac{\eta}{\eta + 2\hat{a}_o}\right)^4 + (K_{max}^{\infty} - 3) \cdot \left(\left(\frac{\eta}{\eta + 2\hat{a}_o}\right)^6 - \left(\frac{\eta}{\eta + 2\hat{a}_o}\right)^8\right)}{2\,(1 - \left(\frac{\eta}{\eta + 2\hat{a}_o}\right))}\right] \cdot f_w \quad (8)$$

One of the limitations of W-N model is the necessity of determining the characteristic distances $\boldsymbol{d_o}$ and $\boldsymbol{a_o}$, which requires experimental tests for each laminate lay-up and plate geometry. To use the criteria as a predicting tool that can be useful in design purposes, it is necessary to have a model able to predict the value of these characteristic distances as a function of the geometry.

3. Analysis of characteristic distances of W-N model

3.1. Experimental data

In this work, three different materials from four different authors of the scientific literature are used: AS4/3501-6 [13], AS4/3502 [7, 38], and AS4/PEEK [38] (see Table 1). For AS4/3502, different properties have been used according to each author [7] and [38]. These materials were selected due to the broad range of experimental failure-strength results for several laminate lay-ups and geometries.

Table 1. Mechanical Properties of the materials selected.

Material	$E_1\ [GPa]$	$E_2\ [GPa]$	$G_{12}\ [GPa]$	ν_{12}	$t_k\ [mm]$
AS4/3501-6 ***[13]***	142.0	10.3	7.2	0.270	0.1300
AS4/3502 ***[7]***	143.9	11.9	6.7	0.326	0.1308
AS4/3502 ***[38]***	127.6	11.3	6.0	0.300	0.1360
AS4/PEEK ***[38]***	133.8	8.9	5.1	0.38	0.136

Laminates with different stacking sequences were analysed, as shown in Table 2. Unnotched strength of these laminates in tension and compression are shown in Table 3. Both open-hole tensile tests (OHT) and open-hole compressive tests (OHC) with several geometries were selected (Table 3). Experimental results for open-hole laminates subjected to in-plane compressive loads, in which buckling do not exist, were chosen. In this latter case, for each notched laminate a minimum of four geometries was used, as shown in Table 3.

Table 2. Materials and stacking sequences analysed.

Laminate	Material	Stacking Sequence
L1	AS4/3501-6 *[13]*	$[45/0/-45/90]_{2s}$
L2	AS4/3502 *[7]*	$[0/90/45/-45]_s$
L3	AS4/3502 *[7]*	$[0_2/45/-45]_{2s}$
L4	AS4/3502 *[7]*	$[\pm 60_2]_s$
L5	AS4/3502 *[38]*	$[(\pm 45)_2/0_4/90/\pm 45/0_2/90]_s$
L6	AS4/PEEK *[38]*	$[(\pm 45)_2/0_4/90/\pm 45/0_2/90]_s$

Table 3. Unnotched and notched failure strength of the laminates analyzed. Plate geometry of each sample and type of test (OHT: Open-Hole Tension and OHC: Open-Hole Compression) [7, 13, 38].

Laminate	Samples Nomenclature	W $[mm]$	R $[mm]$	$\eta = \frac{2R}{W}$	Test Type	$\sigma_N^{failure}$ $[MPa]$
L1	L1- S1(*)	38.10	-	-	OHT	702.90
	L1- S2	38.10	1.000	0.052		558.00
	L1- S3	38.10	1.905	0.100		494.90
	L1- S4	38.10	3.175	0.167		472.40

	L1- S5	38.10	4.775	0.251		447.90
	L2- S1(*)	12.70	-	-		695.16
	L2- S2	12.70	0.229	0.036		648.90
	L2- S3	12.70	1.270	0.200		435.69
L2	L2- S4	25.40	3.175	0.250	OHT	349.04
	L2- S5	25.40	3.810	0.300		325.00
	L2- S6	34.80	5.200	0.299		311.88
	L2- S7	47.50	7.750	0.326		271.32
	L3- S1(*)	25.40	-	-		909.00
	L3- S2	25.40	1.270	0.100		577.00
L3	L3- S3	38.10	2.415	0.127	OHC	512.00
	L3- S4	38.10	3.685	0.193		443.00
	L3- S5	37.85	7.620	0.403		345.00
	L3- S6	25.40	7.745	0.610		250.00
	L4- S1(*)	12.39	-	-		186.00
	L4- S2	12.39	0.585	0.094		179.00
L4	L4- S3	12.39	1.255	0.203	OHC	151.00
	L4- S4	24.99	3.785	0.303		112.00
	L4- S5	31.83	5.170	0.325		115.00
	L4- S6	46.20	7.745	0.335		115.00
	L5- S1(*)	76.20	-	-		536.40
	L5- S2	76.20	3.950	0.104		418.40
	L5- S3	76.20	6.350	0.167		345.60
L5	L5- S4	76.20	9.550	0.251	OHC	394.30
	L5- S5	76.20	12.700	0.333		316.30
	L5- S6	76.20	19.050	0.500		251.50
	L5- S7	76.20	25.400	0.667		165.40
	L6- S1(*)	76.20	-	-		532.70
	L6- S2	76.20	3.950	0.104		379.90
	L6- S3	76.20	6.350	0.167		329.60
L6	L6- S4	76.20	9.550	0.251	OHC	288.50
	L6- S5	76.20	12.700	0.333		240.90
	L6- S6	76.20	19.050	0.500		192.40
	L6- S7	76.20	25.400	0.667		132.00

(*) Unnotched strength values

3.2. Estimation of characteristic distances

The characteristic distances of the W-N model were estimated applying Eq. 6 and 8 to the experimental results shown in Table 3. As an example, in Fig. 2, it is shown the procedure used to estimate the characteristic distances for laminates L2-S3 and L2-S4. Then, the values of $\hat{do}$ and $\hat{ao}$ are obtained by minimizing the error ($err = Ke - Kexp$) using the least square adjusting method. In all the laminates shown in Table 3, the same procedure was applied to estimate the characteristic distances.

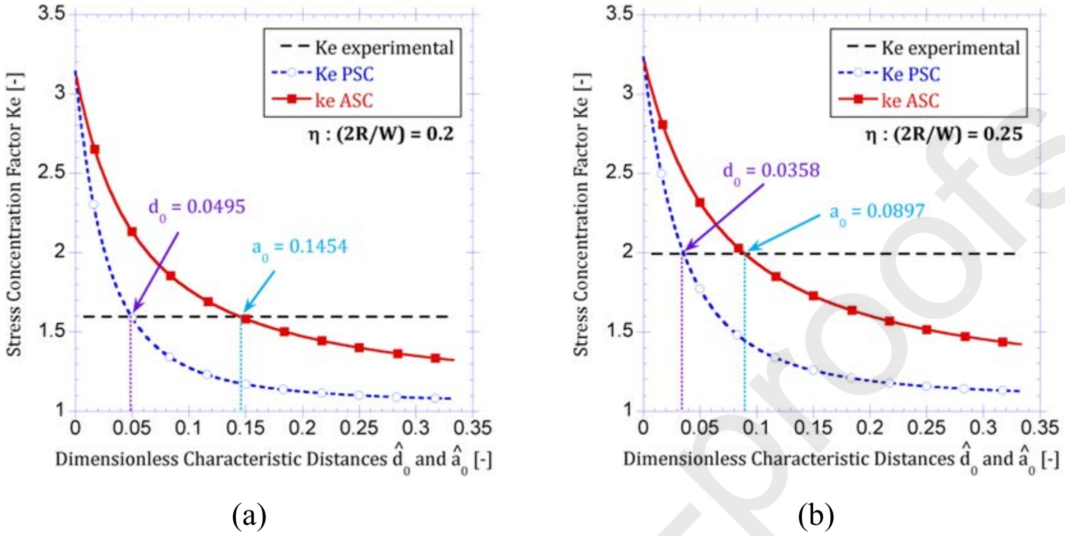


Figure 2. Estimation of $\hat{do}$ and $\hat{ao}$ for laminates: (a) L2- S3 and (b) L2- S4.

As it is shown in Fig. 2 and Table 4, the values of $\hat{d}_o$ and $\hat{a}_o$ change with the geometric ratio η for the same laminate, and therefore cannot be considered material properties as some authors concluded [7-9, 22, 26, 27]. For example, when the geometric ratio of laminate L4 is reduced by 53%, the characteristic distance a_o almost doubles. This dependence with the geometry ratio suggests fitting a function with ratio η to estimate the characteristic distances.

Table 4. Estimated characteristic distances for all laminates analyzed.

Sample	$\eta = \frac{2R}{W}$	W $[mm]$	f_w	$\hat{d}_0$	$d_0\ [mm]$	$\hat{a}_0$	$a_0\ [mm]$
L1- S2	0.052	38.10	1.003	0.0235	0.8954	0.0904	3.4442
L1- S3	0.100	38.10	1.011	0.0312	1.1887	0.1006	3.8329
L1- S4	0.167	38.10	1.031	0.0482	1.8364	0.1505	5.7341
L1- S5	0.251	38.10	1.077	0.0704	2.6822	0.2172	8.2753
L2- S2	0.036	12.70	1.001	0.0373	0.4737	0.2494	3.1674
L2- S3	0.200	12.70	1.047	0.0495	0.6287	0.1454	1.8466
L2- S4	0.250	12.70	1.076	0.0358	0.9093	0.0897	2.2784
L2- S5	0.300	25.40	1.116	0.0386	0.9804	0.0945	2.4003
L2- S6	0.299	25.40	1.115	0.0336	1.1693	0.0803	2.7944

L2- S7	0.326	34.80	1.141	0.0240	1.1400	0.0540	2.5650
L3- S2	0.100	25.40	1.011	0.0215	0.5461	0.0672	1.7069
L3- S3	0.127	25.40	1.018	0.0200	0.7620	0.0571	2.1755
L3- S4	0.193	38.10	1.043	0.0220	0.8382	0.0579	2.2060
L3- S5	0.403	38.10	1.228	0.0358	1.3550	0.0891	3.3724
L3- S6	0.610	37.85	1.719	0.0551	1.3995	0.1378	3.5001
L4- S2	0.094	12.39	1.010	0.1632	2.0220	1.5868	19.6605
L4- S3	0.203	12.39	1.048	0.1254	1.5537	0.5310	6.5791
L4- S4	0.303	12.39	1.122	0.0933	2.3316	0.2448	6.1176
L4- S5	0.325	24.99	1.144	0.1128	3.5904	0.3177	10.1124
L4- S6	0.335	31.83	1.156	0.1196	5.5255	0.3425	15.8235
L5- S2	0.104	76.20	1.012	0.0429	3.2690	0.1701	12.9616
L5- S3	0.167	76.20	1.031	0.0400	3.0480	0.1277	9.7307
L5- S4	0.251	76.20	1.076	0.1048	7.9858	0.4172	31.7906
L5- S5	0.333	76.20	1.146	0.0839	6.3932	0.2725	20.7645
L5- S6	0.500	76.20	1.401	0.1155	8.8011	0.3646	27.7825
L5- S7	0.667	76.20	1.959	0.1224	9.3269	0.3601	27.4396
L6- S2	0.104	76.20	1.012	0.0316	2.4079	0.1108	8.4430
L6- S3	0.167	76.20	1.031	0.0355	2.7051	0.1098	8.3668
L6- S4	0.251	76.20	1.076	0.0416	3.1699	0.1197	9.1211
L6- S5	0.333	76.20	1.146	0.0401	3.0556	0.1063	8.1001
L6- S6	0.500	76.20	1.400	0.0559	4.2596	0.1458	11.1100
L6- S7	0.667	76.20	1.954	0.0662	5.0444	0.1681	12.8092

In laminate L4, for the geometry L4-S2, the diameter of the hole is small and the width of the plate too. The ratio η is also one of the smallest analysed. The fact that $a_o > W$ suggests that the W-N model predicts that this plate will fail with a load equal to the failure strength without hole. Experimentally, however, the value of the failure strength is slightly lower.

4. Design Tool Proposal

In this section, the methodology to estimate the values of $\boldsymbol{a_o}$ and $\boldsymbol{d_o}$ is presented as a function of the geometric ratio $\boldsymbol{\eta}$. Using these characteristic distances and applying the W-N model (both for PSC and ASC criteria), an estimation of the open-hole strength is carried out for each of the laminates studied. Finally, an alternative to estimate the effective stress-concentration factor through carpet plots is presented.

4.1. Methodology to estimate a_o and d_o

In this section, phenomenological equations to estimate characteristic distances $\hat{d}_o$ *and* $\hat{a}_o$ as a function of the geometric ratio η are presented. To estimate the coefficients of these equations, the least-squares method was applied to three experimental results of the same laminate (selected from Table 4), with different values of η. The criteria used for selecting a combination of three tests out of 4 to 6 tests presented in Table 4 for each laminate, was done in such a way that at least the extremes of the range of the geometric parameter (η) and a middle value between them were included. Following this criterion all the values of η are covered.

Four approaches are proposed to estimate the characteristic distances $\hat{d}_o$ *and* $\hat{a}_o$: the average value of three experimental values used as a reference (FIT1, Eq. 9a), a linear variation of η (FIT2, Eq. 9b), a parabolic variation of η (FIT3, Eq. 9c), and a function of the square root of the geometric ratio η (FIT4, Eq. 9d). All these expressions depend on different coefficients, $m_i^{(j)}$ or $n_i^{(j)}$, where 'i' can go from 1 to 6 and corresponds to the laminate considered, and 'j' can go from 1 to 7, being the number of coefficients. Therefore, there will be 14 different coefficients, 7 for the four approaches of $\hat{d}_o$ and 7 for the four approaches of $\hat{a}_o$.

FIT1	$\hat{d}_o$ *or* $\hat{a}_o$ = **Average of experimental for laminates of table 5**	9.a
FIT2	$\hat{a}_o = m_i^{(1)}\cdot + m_i^{(2)}\cdot\eta$ $\hat{d}_o = n_i^{(1)}\cdot + n_i^{(2)}\cdot\eta$	9.b
FIT3	$\hat{a}_o = m_i^{(3)} + m_i^{(4)}\cdot\eta + m_i^{(5)}\cdot\eta^2$ $\hat{d}_o = n_i^{(3)} + n_i^{(4)}\cdot\eta + n_i^{(5)}\cdot\eta^2$	9.c
FIT4	$\hat{a}_o = m_i^{(6)} + m_i^{(7)}\cdot\eta^{1/2}$ $\hat{d}_o = n_i^{(6)} + n_i^{(7)}\cdot\eta^{1/2}$	9.d

Table 5. Experimental data used to estimate the characteristic distances.

Laminate	Samples used to fit the model
L1	L1- S2

	L1- S4
	L1- S5
L2	L2- S2
	L2- S3
	L2- S7
L3	L3- S2
	L3- S5
	L3- S6
L4	L4- S2
	L4- S3
	L4- S6
L5	L5- S2
	L5- S5
	L5- S7
L6	L6- S2
	L6- S5
	L6- S7

For each of the laminates studied, all the coefficients which define the expressions shown from eq. 9.b to 9.d are calculated using only the samples shown in Table 5. The results are shown in Table 6.

Table 6. Curve fit equations for the characteristic distance $\hat{d}_o$ *and* $\hat{a}_o$ for the laminates used in this study.

Fitting equation	**Laminate**	$\hat{d}_o$	R^2	$\hat{a}_o$	R^2
FIT1	L1	$\hat{d}_o = 0.04740$	-	$\hat{a}_o = 0.15270$	-
	L2	$\hat{d}_o = 0.03693$	-	$\hat{a}_o = 0.14960$	-
	L3	$\hat{d}_o = 0.03747$	-	$\hat{a}_o = 0.09803$	-
	L4	$\hat{d}_o = 0.13610$	-	$\hat{a}_o = 0.82000$	-
	L5	$\hat{d}_o = 0.08307$	-	$\hat{a}_o = 0.26757$	-
	L6		-		-

		$\hat{d}_o = 0.04597$		$\hat{a}_o = 0.12840$	
FIT2	L1	$\hat{d}_o = 0.2355 \cdot \eta + 0.0105$	0.9968	$\hat{a}_o = 0.6333 \cdot \eta + 0.0535$	0.9862
	L2	$\hat{d}_o = -0.04 \cdot \eta + 0.0444$	0.2081	$\hat{a}_o = -0.6712 \cdot \eta + 0.2754$	0.9986
	L3	$\hat{d}_o = 0.0645 \cdot \eta + 0.0135$	0.9631	$\hat{a}_o = 0.1336 \cdot \eta + 0.0485$	0.8990
	L4	$\hat{d}_o = -0.1759 \cdot \eta + 0.1731$	0.8032	$\hat{a}_o = -5.0253 \cdot \eta + 1.8792$	0.8175
	L5	$\hat{d}_o = 0.1394 \cdot \eta + 0.0318$	0.9847	$\hat{a}_o = 0.3321 \cdot \eta + 0.1454$	0.9774
	L6	$\hat{d}_o = 0.0627 \cdot \eta + 0.0229$	0.9680	$\hat{a}_o = 0.1078 \cdot \eta + 0.0887$	0.7843
FIT3	L1	$\hat{d}_o = 0.2421 \cdot \eta^2 + 0.1633 \cdot \eta + 0.0143$	1	$\hat{a}_o = 1.3512 \cdot \eta^2 + 0.2303 \cdot \eta + 0.0746$	1
	L2	$\hat{d}_o = -0.9516 \cdot \eta^2 + 0.299 \cdot \eta + 0.0278$	1	$\hat{a}_o = -0.3081 \cdot \eta^2 - 0.5614 \cdot \eta + 0.27$	1
	L3	$\hat{d}_o = 0.09 \cdot \eta^2 + 0.7266 \cdot \eta + 0.2205$	1	$\hat{a}_o = 0.3191 \cdot \eta^2 - 0.088 \cdot \eta + 0.0728$	1
	L4	$\hat{d}_o = 1.2697 \cdot \eta^2 - 0.1759 \cdot \eta + 0.1731$	1	$\hat{a}_o = 34.623 \cdot \eta^2 - 20.043 \cdot \eta + 3.1704$	1
	L5	$\hat{d}_o = -0.1119 \cdot \eta^2 + 0.2274 \cdot \eta + 0.0205$	1	$\hat{a}_o = -0.3252 \cdot \eta^2 + 0.588 \cdot \eta + 0.1126$	1
	L6	$\hat{d}_o = 0.0733 \cdot \eta^2 + 0.005 \cdot \eta + 0.0303$	1	$\hat{a}_o = 0.3641 \cdot \eta^2 - 0.1787 \cdot \eta + 0.1254$	1
FIT 4	L1	$\hat{d}_o = 0.16799 \cdot \eta^{1/2} - 0.016357$	0.9556	$\hat{a}_o = 0.4491 \cdot \eta^{1/2} - 0.017663$	0.9556
	L2	$\hat{d}_o = -0.02245 \cdot \eta^{1/2} + 0.045965$	0.1171	$\hat{a}_o = -0.49581 \cdot \eta^{1/2} + 0.34928$	0.9740
	L3	$\hat{d}_o = 0.067881 \cdot \eta^{1/2} - 0.0017163$	0.9148	$\hat{a}_o = 0.1385 \cdot \eta^{1/2} + 0.01809$	0.8291
	L4	$\hat{d}_o = -0.16231 \cdot \eta^{1/2} + 0.20837$	0.8683	$\hat{a}_o = -4.6281 \cdot \eta^{1/2} + 2.8817$	0.8804
	L5	$\hat{d}_o = 0.16076 \cdot \eta^{1/2} - 0.0088804$	1	$\hat{a}_o = 0.38441 \cdot \eta^{1/2} + 0.047707$	0.9993
	L6	$\hat{d}_o = 0.069554 \cdot \eta^{1/2} + 0.0061856$	0.9101	$\hat{a}_o = 0.11437 \cdot \eta^{1/2} + 0.062989$	0.6738

4.2. Estimation of the open-hole strength

To validate the model and determine the accuracy of each fit in the estimation of the characteristic lengths $\hat{\boldsymbol{d}}_{\boldsymbol{o}}$ *and* $\hat{\boldsymbol{a}}_{\boldsymbol{o}}$, the remaining samples (shown in Table 7) are used.

Table 7. Experimental data used to validate the model.

Laminate	Samples used to validate the model
L1	L1- S3
L2	L2- S4 L2- S5 L2- S6
L3	L3- S3 L3- S4
L4	L4- S4 L4- S5
L5	L5- S3 L5- S4 L5- S6
L6	L6- S3 L6- S4 L6- S6

Once the different fit curves for estimating the values of the dimensionless characteristic distances are obtained, the failure strength (for both criteria, PSC and ASC) can be estimated applying the W-N model.

As an example, Fig. 3 shows the open-hole failure strength of laminates L2 and L6 calculated using the PSC and ASC criteria of the W-N model, and applying the four different fittings for $\hat{d}_o$ *and* $\hat{a}_o$, (according to Eq. 9a to 9d). In this figure, the experimental results of the samples which were not used to fit the tool are represented to validate and determine the accuracy of each fit. A good approximation with experimental results is observed. The difference (percentual error) between the estimated failure strength and the experimental data for all six laminates analysed in this study, for the samples used to validate the tool, using both criteria, are shown in Table 8.

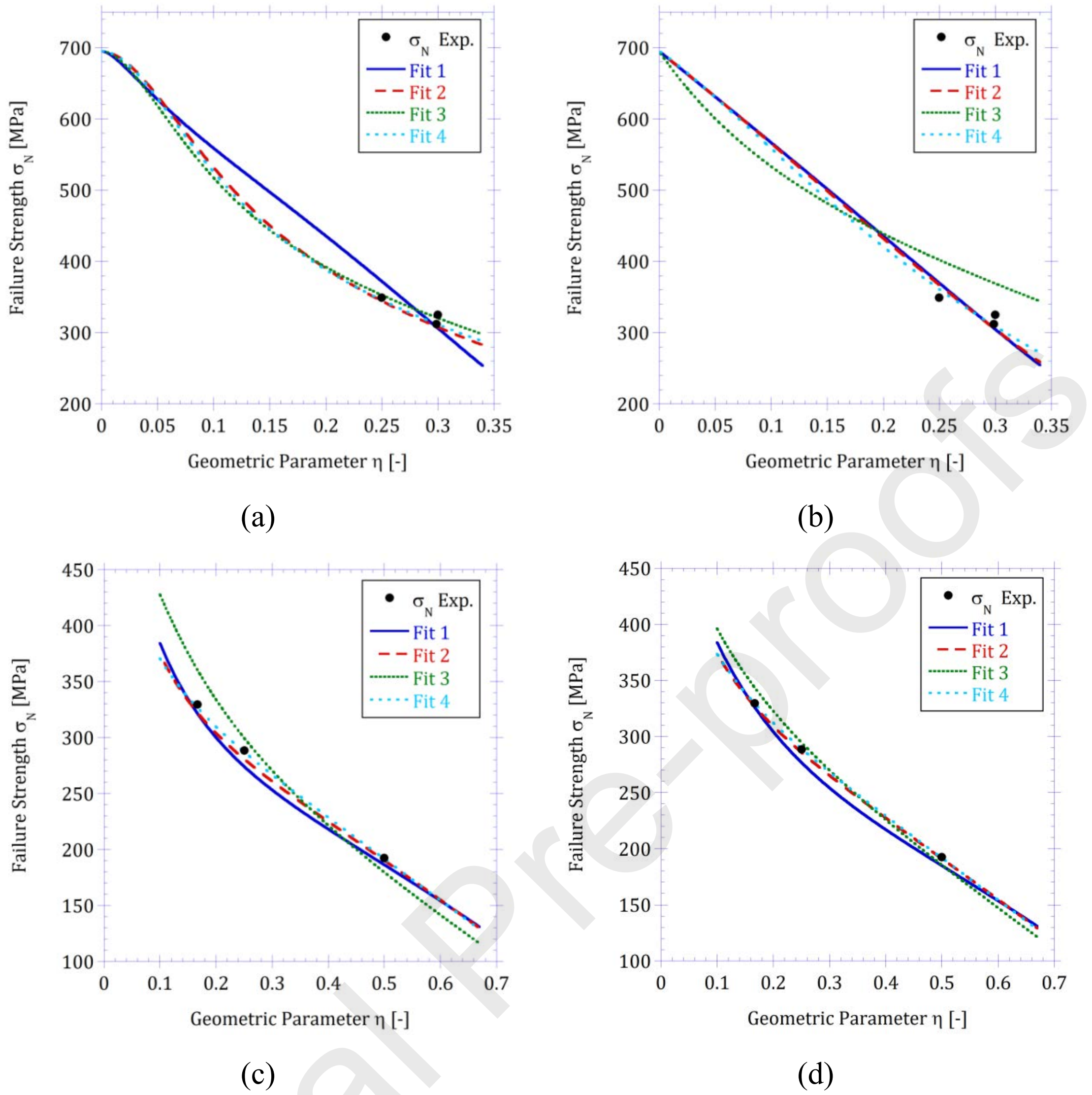


Figure 3. Failure strength as a function of $\boldsymbol{\eta}$ estimated by W-N model (for both criteria, PSC and ASC) using $\hat{\boldsymbol{d}}_{\boldsymbol{o}}$ *and* $\hat{\boldsymbol{a}}_{\boldsymbol{o}}$, calculated with FIT1, FIT2, FIT3 and FIT4: (a) Laminate L2 PSC criteria, (b) Laminate L2 ASC criteria, (c) Laminate L6 PSC criteria, and (d) Laminate L6 ASC criteria.

Table 8. Percentual error of the failure strength estimation for the samples used to validate the model (using PSC and ASC criteria), and applying the fitting curves defined in Eq.9a to Eq.9d to calculate $\hat{\boldsymbol{d}}_{\boldsymbol{o}}$ *and* $\hat{\boldsymbol{a}}_{\boldsymbol{o}}$.

Sample	**FIT 1**		**FIT 2**		**FIT 3**		**FIT 4**	
	PSC %	***ASC*** %	***PSC*** %	***ASC*** %	***PSC*** %	***ASC*** %	***PSC*** %	***ASC*** %
L1- S3	13.68	13.68	2.96	3.47	1.95	2.32	5.54	4.89
L2- S4	85.01	15.21	1.30	5.17	6.62	5.92	0.97	3.44

L2- S5	91.69	13.52	5.29	6.19	5.77	6.26	4.17	5.03
L2- S6	99.94	18.53	1.06	1.80	1.31	1.84	0.09	0.67
L3- S3	24.73	16.21	2.94	3.99	3.68	4.59	4.28	4.87
L3- S4	21.48	16.62	6.16	7.60	3.37	4.70	9.33	9.54
L4- S4	14.89	26.57	9.93	9.28	8.95	2.98	9.66	7.71
L4- S5	7.11	19.69	1.06	6.01	1.87	0.35	1.02	6.29
L5- S3	50.47	18.26	11.17	11.31	11.33	11.63	12.22	12.00
L5- S4	26.37	8.84	12.93	12.47	11.33	11.20	10.86	11.14
L5- S6	52.42	8.16	4.50	4.65	2.92	3.02	3.40	3.49
L6- S3	9.39	4.28	2.20	0.78	2.40	1.02	0.92	0.02
L6- S4	3.72	2.04	2.72	0.97	5.04	4.13	0.54	0.14
L6- S6	6.56	3.59	1.08	0.64	3.11	3.87	0.36	0.38

In all the laminates analysed, the failure strength estimated by using PSC criterion with an average fit of the characteristic distance $\boldsymbol{d_o}$ (FIT 1), presents a high error level than the one using ASC criterion, in some cases reaching almost 100% (for laminate L2). On the other hand, by using the rest of fits (linear, parabolic, and square root as a function of $\boldsymbol{\eta}$), the error level is small. Only in laminate L5, the error is around 13% for two of the geometries. In some other laminates and geometries the error is even less than 1%. None of the fits for characteristic distance $\boldsymbol{d_o}$ provides the most accurate prediction of failure strength in all cases studied. Nevertheless, the estimation of $\boldsymbol{d_o}$ using FIT4 provides the best estimate failure strength in 8 of the 14 geometries analysed.

The results for ASC criteria show similar trends. In all cases except one (for the L5 laminate) the FIT1 is the worst. Except in the L5 laminate, the rest of the fits give percentual errors in the estimation of the failure strength below 10%, and in the case of laminate L6, the error is below 1%. In half of the cases studied, the best fit is FIT4.

In view of the results obtained, FIT 4 is recommended to estimate the characteristic distances that will be used in the W-N model to estimate failure strength. In the worst-case scenario, the maximum expected error would be around 12%.

Therefore, it can be assumed that the proposed methodology for estimating characteristic distances from a small number of experimental data is valid, which allows, using the W-N model, to calculate accurately the failure strength.

4.3. Carpet plots for K_e estimation

An alternative to the equations of the model (Eq.6 and 8) to estimate the effective stress-concentration factor which causes the failure of the plate is the use of carpet plots, so that these values can be calculated for a particular laminate of width $\boldsymbol{W}$ and hole radius $\boldsymbol{R}$. In Fig. 4 and 5, the evolution of effective stress concentration factor $\boldsymbol{K_e}$ as a function of the hole radius $\boldsymbol{R}$ and the width $\boldsymbol{W}$ of the plate for laminate L2 both for PSC and ASC criteria respectively. In these plots, the characteristic distance ($\hat{\boldsymbol{d}}_{\boldsymbol{o}}$ and $\hat{\boldsymbol{a}}_{\boldsymbol{o}}$) were estimated using the FIT4 showed in table 7.

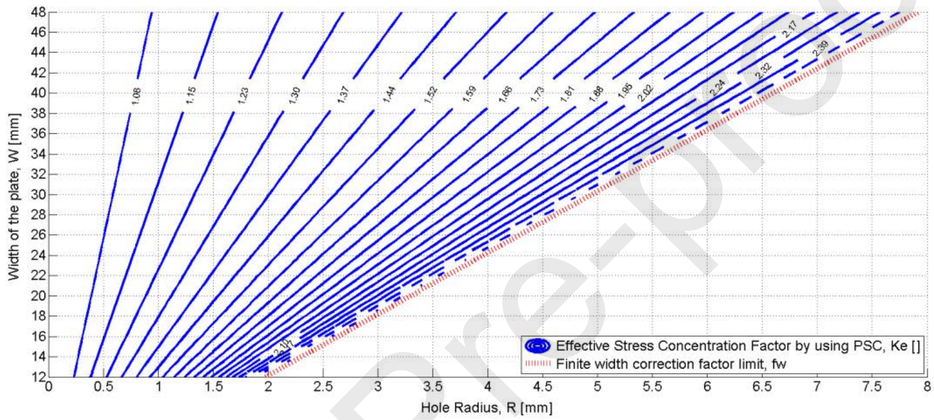


Figure 4. Carpet plot for the effective stress concentration factor $\boldsymbol{K_e}$ using PSC for laminate L2.

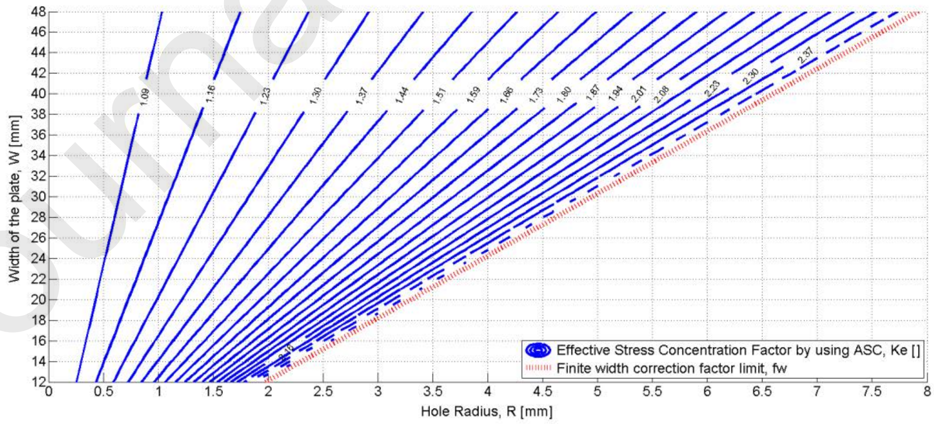


Figure 5. Carpet plot for the effective stress-concentration factor $\boldsymbol{K_e}$ using ASC for laminate L2.

5. Conclusions

In this work, Whitney and Nuismer model is applied as a design tool to estimate the open-hole failure strength of laminates subjected to in-plane loads. The model has been expressed in dimensionless variables to determine the parameters that control the stresses around a hole and the failure load. The failure load is controlled by the relationship between the hole diameter and the plate width.

Whitney and Nuismer model can be used to predict the tensile or compressive strength of a laminate and the stress-concentration factor employing fitting equations for the characteristic distances, using only three experimental results with three different geometries to estimate the coefficients of these equations. Four different phenomenological equations were proposed to estimate the characteristic distances $\hat{\boldsymbol{d}}_{\boldsymbol{o}}$ and $\hat{\boldsymbol{a}}_{\boldsymbol{o}}$ as a function of the geometric ratio $\boldsymbol{\eta}$; an average value, a linear variation, a parabolic variation and a function of the square root. The estimation of the open-hole strength in laminated plates by using the average of the characteristic distances leads to high errors in all the cases analysed, especially under tension, in this case, the error is almost 100%. Under compression, the maximum error is around 50%. Therefore, this fitting equation is not recommended.

Although there is no fitting expression that provides the best estimation of the failure strength for all laminates and geometries analysed, the fit which depends on the square root of the ratio between the hole diameter and width of the plate provides a good estimation in a high percentage of the cases analysed. With this fitting, the maximum error is around 5% in tension and 12% in compression.

Alternatively, carpet plots are proposed to estimate graphically the failure strength of open-hole laminates as a function of the width of the plate and the hole radius of a particular laminate.

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Bibliography

[1] G. Mingming, C. Xiaoquan, Z. Qian, Compressive test and numerical simulation of center-notched composite laminates with different crack configurations, *Polym. Compos. 38 (2017) 2631-2641.*

[2] A.B. De Morais, Open-hole strength of quasi-isotropic laminates, *Compos. Sci. Technol*. 60 (2000) 1997-2004.

[3] C. Soutis, N.A. Fleck, P.A. Smith, Failure prediction technique for compression loaded carbon fibre-epoxy laminate with open holes, *J. Compos. Mater*. 25 (1991) 1476-1498.

[4] S.R. Hallett, M.Wisnom, Experimental investigation of progressive damage and the effect of layup in notched tensile test, *J. Compos. Mater*. 40 (2006)119-141.

[5] R.M. O'Higgins, G.S. Padhi, M.A. McCarthy, C.T. McCarthy, Experimental and numerical study of the open-hole tensile strength of carbon/epoxy composites, *Mech. Compos. Mater*. 40 (2004) 269-278.

[6] D.S. Pandita, K. Nishiyabu, I. Verpoest, Strain concentrations in woven fabric composites with holes. *Compos. Struct*. 59 (2003) 361-368.

[7] S.C. Tan, Stress concentrations in laminated composites, *Technomic Publishig*, Lancaster, 1994.

[8] J.M. Whitney JM, R.J. Nuismer, Stress fracture criteria for laminated composites containing stress concentrations, *J. Compos. Mater*. 8 (1974) 253-265.

[9] R.J. Nuismer, J.M. Whitney, Uniaxial Failure of Composite Laminates Containing Stress Concentrations, *ASTM Special Technical Publication*, 593 (1974) 117-142.

[10] P.P. Camanho, G.H. Erçin,, G. Catalanotti, S. Mahdi, P. Linde, A finite fracture mechanics model for the prediction of the open-hole strength of composite laminates. *Compos. Part. A-Appl. S*. 43 (2012) 1219.1225.

[11] A.Yudhantoa, N. Watanabea, Y. Iwahorib, H. Hoshia, The effects of stitch orientation on the tensile and open hole tension properties of carbon/epoxy plain weave laminates, *Mater. Des*. 35 (2012) 563-571.

[12] S.R. Hallett, B.G. Green, W.G. Jiang, M Wisnom, An experimental and numerical investigation into the damage mechanisms in notched composites, *Compos. Part. A-Appl. S*. 40 (2009) 613-624.

[13] J Wang, P.J. Callus, M.K. Bannister, Experimental and numerical investigation of the tension and compression strength of un-notched and notched quasiisotropic laminates, *Compos. Struct*. 64 (2004) 297-306.

[14] A.R. Abu Talib, A.A. Ramadhan, A.S. Mohd Rafie, R. Zahari, Influence of cut-out hole on multi-layer Kevlar-29/epoxy composite laminated plates, *Mater. Des*. 43 (2013) 89-98.

[15] P.P. Camanho, P. Maimí, C.G. Dávila, Prediction of size effects in notched laminates using continuum damage mechanics, *Compos. Sci. Technol*. 67 (2007) 2715-2727

[16] M.E. Waddoups, J.R. Eisenmann, B.E. Kaminski, Macroscopic fracture mechanics of advanced composite materials, *J. Compos. Mater*. 5 (1971) 446-454.

[17] M.M. Moure, F. Otero, S.K. García-Castillo, S. Sánchez-Sáez, E. Barbero, E.J. Barbero, Damage evolution in open-hole laminated composite plates subjected to in-plane loads, *Compos. Struct*. 133 (2015) 1048-1057.

[18] A. Arteiro, G. Catalanotti, J. Xavier, P. P. Camanho. Notched response of non-crimp fabric thin-ply laminates: analysis methods. Composites Science and Technology, 88 (2013) 165-171.

[19] L.W. Mar, K.Y. Lin, Fracture mechanics correlation for tensile failure of filamentary composites with holes, *J. Aircraf*. 14 (1977) 703-704.

[20] I. Eriksson, C.G. Aronsson, Strength of tensile loaded graphite/epoxy laminates containing cracks, open and filled holes, *J. Compos. Mater*. 24 (1990) 456-482.

[21] J. Lindhagen, Berglund L, Notch sensitivity and damage mechanisms of glass mat reinforced polypropylene, *Polym. Compos.* 18 (1997) 40-47.

[22] E.J. Barbero, Introduction to Composite Materials Design. Third edition. CRC Press, Boca Raton, 2017.

[23] P.K. Mallick, Fiber-Reinforced Composites. CRC Press Taylor & Francis Group, Boca Raton (2008).

[24] K.-H. Tsai, C.-L. Hwan, M.-J. Lin and Y. S. Huang. Finite Element Based Point Stress Criterion for Predicting the Notched Strengths of Composite Plates. Journal of Mechanics. 28(3) (2012) 401-406.

[25] C. Wallner, S. F. M. Almeida, C. Kassapoglou, Modified Whitney-Nuismer criteria for prediction of notched strength of composite laminates, 31st Congress of International Council of the Aeronautical Sciences, Belo Horizonte, Brazil, September 09-14, 2018.

[26] C.S. Hong, J.H. Crews, Stress-Concentration factors for finite orthotropic laminates with a circular hole and uniaxial loading, NASA Technical Paper 1469, Langley Research Center Hampton, Virginia

[27] I. Varellis, T.L Norman, Failure of unnotched and notched composites with adhesive strips, *Compos. Sci. Technol*. 51 (1994) 367-376.

[28] C.J. Liu, A.H.J. Nijhof, L.J. Ernst and R. Marissen, A new ultimate strength model of notched composite laminates-Including the effects of matrix failure, *J. Compos. Mater.* 44 (2010) 1335-1349.

[29] R.F. Karlak, Hole effects in a related series of symmetrical laminates, Proceedings of Failures Modes in Composties IV, *Metallurgical Society of Aime* (1977) 105-117.

[30] R.B. Pipes, J.W. Gillespie, R.C. Wetherhold, Notched Strength of Composite Materials, *J. Compos. Mater.* 13 (1979) 148-160.

[31] S.-Y. Kim, J.-M. Koo, D. Kim, C.-S. Seok. Prediction of the static fracture strength of hole notched plain weave CFRP Composites. Composites Science and Technology. 71 (2011) 1671–1676.

[32] C.-L. Hwan, K.-H. Tsai, W.-L. Chen, C.-H. Chiu and C.M. Wu. Strength prediction of braided composite plates with a center hole. Journal of Composite Materials 45(19) (2011):1991-2002

[33] J.K. Kim, D.S. Kim, N. Takeda, Notched Strength and Fracture Criterion in Fabric Composite Plates Containing a Circular Hole, *J. Compos. Mater.* 29 (1995) 982-998.

[34] P.K. Govindan, B. Nageswara, V.K. Srivastava, Notched strength of carbon fibre/epoxy composite laminates with a circular hole, Forschung. Im. Ingenieurwesen. 65 (2000) 295-300.

[35] H.S. Sultan Aljibori, W.P. Chong, T.M.I. Mahlia, W.T. Chong, P.Edi, H. Al-qrimli, I. Anjum, R. Zahari, Load–displacement behavior of glass fiber/epoxy composite plates with circular cut-outs subjected to compressive load, *Mater. Des*. 31 (2010) 466-474.

[36] C. Wallner, S.F. Muller Almeida, C. Kassapoglou. Novel criteria for strength predictions of open-hole composite laminates for preliminary design. Composite Structures 229 (2019) 111-409

[37] S.G. Lekhnitskii, Anisotropic Plates, Gordon and Breach Science Publishers, New York, 1968.

[38] D.C.Jegley, Compression behavior of graphite-thermoplastic and graphite-epoxy panels with circular holes or impact damage, NASA technical paper 3071 (1991).

FIGURE CAPTION

Figure 1. Stress distribution of σ_x in the vicinity of the hole of a laminate subjected to uniaxial loading. Definition of the characteristic distances for: a) PSC criteria and b) ASC criteria.

Figure 2. Estimation of $\hat{\boldsymbol{d}}\boldsymbol{o}$ and $\hat{\boldsymbol{a}}\boldsymbol{o}$ for laminates: (a) L2- S3 and (b) L2- S4.

Figure 3. Failure strength as a function of $\boldsymbol{\eta}$ estimated by W-N model (for both criteria, PSC and ASC) using $\hat{\boldsymbol{d}}_{\boldsymbol{o}}\ and\ \hat{\boldsymbol{a}}_{\boldsymbol{o}}$, calculated with FIT1, FIT2, FIT3 and FIT4: (a) Laminate L2 PSC criteria, (b) Laminate L2 ASC criteria, (c) Laminate L6 PSC criteria, and (d) Laminate L6 ASC criteria.

Figure 4. Carpet plot for the effective stress concentration factor $\boldsymbol{K}_{\boldsymbol{e}}$ using PSC for laminate L2.

Figure 5. Carpet plot for the effective stress-concentration factor $\boldsymbol{K}_{\boldsymbol{e}}$ using ASC for laminate L2.

TABLE CAPTION

Table 1. Mechanical Properties of the materials selected.

Table 2. Materials and stacking sequences analysed.

Table 3. Unnotched and notched failure strength of the laminates analyzed. Plate geometry of each sample and type of test (OHT: Open-Hole Tension and OHC: Open-Hole Compression) [7, 13, 38].

Table 4. Estimated characteristic distances for all laminates analyzed.

Table 5. Experimental data used to estimate the characteristic distances

Table 6. Curve fit equations for the characteristic distance $\hat{\boldsymbol{d}}_{\boldsymbol{o}}\ and\ \hat{\boldsymbol{a}}_{\boldsymbol{o}}$ for the laminates used in this study.

Table 7. Experimental data used to validate the model.

Table 8. Percentual error of the failure strength estimation for the samples used to validate the model (using PSC and ASC criteria), and applying the fitting curves defined in Eq.9a to Eq.9d to calculate $\hat{\boldsymbol{d}}_{\boldsymbol{o}}\ and\ \hat{\boldsymbol{a}}_{\boldsymbol{o}}$.