*SOCIETY. INTEGRATION. EDUCATION*
*Proceedings of the International Scientific Conference. Volume VI, May 25th -26th, 2018. 42-52*

# APPLICATION OF THE THEORY OF LATENT VARIABLES TO PERSONNEL MANAGEMENT METHODS

**Tatiana Averina**

**Sergey Barkalov**

**Sergey Moiseev**

Voronezh State Technical University, Russian Federation

***Abstract.*** *One of the main tasks of the human-resources (HR) department is the qualitative management of personnel, which is the basis for effective work of the organization. The paper presents models of personnel management based on the theory of latent variables. Research objective is to use the Rasch model for assessing latent variables to evaluate the competence of employees and their suitability for certain positions and also quality control of their work.*
*Three problems of personnel management are considered.*
*Organization of qualitative selection of candidates for vacancies by objective assessment of the degree of professional suitability of candidates with a view to selecting the best employees.*
*Multilateral monitoring as the quality of performance of duties of each employee on the one hand, and the efficiency of the work of the whole team on the other. A dynamic comparison is made between the quality of performance of their duties in different periods of time.*
*Effective appointments of specific employees to certain positions. Based on the criteria of the employee's suitability for certain positions, the integral degree of the employee's suitability for each vacancy is calculated. In addition, the model makes it possible to assess the degree of influence of the criteria on the evaluation of employees.*
*The models proposed in the paper allow obtaining estimates on a linear interval scale, which can be translated into any other scale, for example, probabilistic. Estimates do not depend on a set of criteria and the set of evaluated employees.*
***Keywords:*** *competence assessment, latent variables, mathematical model, personnel management, personnel monitoring, the Rasch method, vacancy selection.*

## Introduction

At present, no one doubts that the most important resource of any company is its employees. On how effective the work of employees will be, the success of any company depends. The task of managers is to make the best use of the capabilities of the staff. And this can happen only if the personnel policy of the organization is successfully developed. All this confirms the relevance of research


*http://dx.doi.org/10.17770/sie2018vol1.3121*

in the field of labour management, the importance of developing new methods of personnel management.

To solve the problem successfully, it is necessary to evaluate the competence of employees, their suitability for certain positions and constantly monitor the quality of work. However, traditional methods of assessing employees are subjective and depend on the opinion of managers. Research objective is the development of methods for an objective assessment of the basic indicators of the employees' work quality.

Research objective is to use the Rasch model for assessing latent variables to evaluate the competence of employees and their suitability for certain positions and also quality control of their work. The choice of this approach is due to the fact that many indicators of personnel policy are latent, they cannot be directly measured, but can be estimated using criteria. As will be shown below, methods based on the theory of latent variables have several advantages over traditional methods.

To obtain estimates of latent parameters, a criterial approach was used. Estimation of indicators based on the Rasch model allows to obtain estimates on a linear scale, the estimates do not depend on a set of criteria and on the set of evaluated employees.

The following tasks of labor management are solved in the paper:

1. Organization of qualitative selection of candidates for vacancies by objective assessment of the degree of professional suitability of candidates for vacancies in order to select the best employees.
2. Multilateral monitoring as the quality of the performance of duties of each employee on the one hand, and the efficiency of the work of the entire workforce on the other.
3. Effective appointments of specific employees to certain positions.

## Model of selection of candidates for vacant positions

The task of qualitative selection of personnel for vacant positions is becoming especially relevant at the present time, in view of the new professional standards that have entered into force and the approval of the rules for conducting an independent evaluation of employees. However, there are no clear recommendations on the methodology for assessing the quantitative indicators of professional suitability, standard methods of expert evaluation are usually used for these purposes. As practice shows, the evaluation of competencies for which ballroom behaviour scales are currently used is of a qualitative nature and depends, in many respects, on the experience and qualification of the evaluator (Averina et al., 2011).

From this, it follows that the main difficulty in assessing the professional suitability of candidates for vacancies is that the concept of a complex assessment of professional suitability is, from the point of view of mathematics, a latent variable, that is, a qualitative and subjective indicator (Barkalov et al., 2014).

In this part, we propose an approach to the organization of qualitative selection of candidates for vacancies by evaluating the generalized assessment of the professional suitability of workers based on the Rasch method of estimating latent variables (Andrich, 1988; Engelhard, 2013).

Let us consider a mathematical model for solving the problem.

Let there be $N$ candidates for vacant positions: $A_1, A_2, \ldots, A_N$, from which the best $n$ are to be selected. To solve this problem, it is necessary to quantify the professional suitability of all candidates. For estimation $M$ criteria are defined: $K_1$, $K_2, \ldots K_M$.

The choice of criteria depends on the kind of professional activity of the team, in which the selection of candidates is made. Criteria can be arbitrary, they can be changed or supplemented over time, candidate scores obtained for each criterion can be measured on different scales.

In the future, we will give the solution of the problem in general form, without reference to specific criteria.

We denote by $U_{ij}$ - evaluation of a candidate of $i$-th by $j$-th criterion, $i=1,2,\ldots, N$; $j=1,2,\ldots, M$. These estimates can be of different nature and have different dimensions. To bring the estimates to a single scale, it is necessary to carry out the procedure for their normalization (Barkalov et al., 2015), as a result of which all normalized estimates of employees according to the criteria $u_{ij}$ will take values from the interval (0;1).

Let us now turn directly to the Rasch model of estimating latent variables. In accordance with (Moiseev & Zenin, 2015), we introduce latent variables:

$\theta_i$ - an integral indicator of the degree of professional suitability of the candidate for the vacant position of $A_i$ , the higher this indicator, the more attractive the candidate for the vacant position is;

$\beta_j$ – the degree of non-feasibility of the evaluation criterion $K_j$ for the entire group of candidates for vacant seats, the lower this indicator, the more the entire set of candidates in the aggregate satisfies the criterion.

Then, according to the Rasch model, the probability that the candidate $A_i$ satisfies the employer by the criterion $K_j$ is described by the logistic function:

$$P_{ij} = \frac{e^{\theta_i - \beta_j}}{1 + e^{\theta_i - \beta_j}}. \qquad (1)$$

These probabilities can be interpreted as normalized complex assessments of candidates according to the criteria $u_{ij}$.

To apply expression (1) in practice, it is necessary to find estimates of the following latent $\theta_i$ and $\beta_j$. These estimates are calculated on the basis of known estimates of these candidates by private evaluation criteria $u_{ij}$, which are obtained empirically by means of a questionnaire or expert evaluation.

In view of the fact that the estimates $u_{ij}$ are measured in the general case on a continuous scale from the interval [0; 1], for this purpose it is necessary to use the Rasch model based on the method of least squares (Moiseev, 2015): latent variables $\theta_i$ and $\beta_j$ of model (1) are chosen so that the sum of squares of deviations of empirical data $u_{ij}$ from theoretical probabilities 1) was the smallest. Mathematically, this reduces to solving an optimization problem of the form:

$$\sum_{i=1}^{N}\sum_{j=1}^{M}(u_{ij}-P_{ij})^2=\sum_{i=1}^{N}\sum_{j=1}^{M}\left(u_{ij}-\frac{e^{\theta_i-\beta_j}}{1+e^{\theta_i-\beta_j}}\right)^2\to\min. \tag{2}$$

Estimates of latent variables $\theta_i$ and $\beta_j$, calculated from the solution of problem (2), will be measured on interval and linear scales, but the origin in them will be indeterminate. The initial level of the scale can be chosen so that the values of all the estimates are nonnegative. Then the objective function (2) will be supplemented by the conditions:

$$\theta_i\geq 0;\ \beta_j\geq 0;\ i=1,2,...,N;\ \ j=1,2,...,M \tag{3}$$

For the integral estimates obtained, it is also possible to normalize them, for example, they can be normalized to a scale at which the value of the generalized indicator of a candidate's professional fitness will be equal to the fraction in the unit sum of all estimates. The normalized estimates $\tilde{\theta}_i$ are obtained from the obtained $\theta_i$ by the formula:

$$\tilde{\theta}_i=\frac{\theta_i}{\sum_{i=1}^{N}\theta_i}. \tag{4}$$

Based on these assessments, it is possible to rank candidates according to the degree of their professional suitability, as well as to evaluate the criteria by the degree of their feasibility for the entire group of candidates.

The model described above assumes that all evaluation criteria are of equal importance to the employer. However, the model can also be applied to criteria with different importance. Importance is taken into account by introducing weights for each criterion.

Let $w_j$ – be the weight of the $j$-th criterion. To take into account the weights, we propose a model in which, in the residual sum (2), each term will be proportional to its weight. As a result, instead of (2), an optimization problem of the form is solved (Barkalov et al., 2014):

$$\sum_{i=1}^{m}\sum_{j=1}^{n} w_j \cdot (u_{ij} - P_{ij})^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} w_j \cdot \left(u_{ij} - \frac{e^{\theta_i - \beta_j}}{1 + e^{\theta_i - \beta_j}}\right)^2 \to \min . \qquad (5)$$

The solution of the optimization problems (2) and (3) or (5) and (3) can be performed both with the use of specialized software products and in MS Excel using the Solver add-on. How this is done is described in the monograph by Maslak & Moiseev, 2016.

## Model of monitoring the quality of work of a team

Let us now consider the issue of the organization of effective personnel management by the personnel services. For the effective work of a team, a system of comprehensive assessment of the quality of the work of individual performers and of the team as a whole is necessary, taking into account the dynamics of their change over time.

In this section, we will consider models that make it possible to comprehensively evaluate the quality of work of each participant of the team on the one hand, and on the other hand the efficiency of the team as a whole for several periods of time.

We first consider the one-criterion case of an estimate.

Let there be $n$ qualitative participants of the team: $A_1, A_2, \ldots, A_n$, monitoring the quality of work of which is conducted at the current moment and for $m$ previous periods of time.

The evaluation of each employee in each period of time is carried out by a certain criterion. If the criterion is quantitative, the estimate will be in natural units: $x_{ij}$ – an estimate of the work of the $i$-th performer in the $j$-th time period. In this case, all assessments should be measured on a single scale, to use the Rush model, this scale should be a single scale: $x_{ij} \in [0; 1]$. If the scale is different, then it is necessary to conduct the rationing on a single scale.

If the criterion is qualitative and one cannot accurately determine how much the quality of the performer's work has changed at a given time stage, and one can only determine whether the quality has increased or decreased, then the estimate can be made on a dichotomous scale:

$$x_{ij} = \begin{cases} 1, \text{ if the employee has improved quality in the period } j \\ \quad \text{compared to } (j-1); \\ 0, \text{ if the employee has deteriorated the quality in the period } j \\ \quad \text{compared to } (j-1), \end{cases} \tag{6}$$

$$i = 1, 2, \ldots, n; \quad j = 0, 1, \ldots, m-1.$$

It is convenient to use relative indicators of the increase in the quality of work as an indicator of the effectiveness of each performer at each stage of the work. If it is possible to determine the degree of change in the performance of the performer $A_i$ at the $j$-th stage, then we introduce the variable $d_{ij}$ - the relative change in the quality of the work of the performer $A_i$ on the time period $j$ in comparison with the period $j$-1: $d_{ij} = \frac{a_{ij} - a_{ij-1}}{a_{ij-1}}$, where $a_{ij}$ – evaluation of the work of the $i$-th period of time of $j$-th. Let us cite the normalization of this index on the unit scale the necessary condition imposed on the initial data for the Rush model. As the normalization algorithm, let's take the one for which the largest relative increase will give a unit value, the smallest (possibly negative) will give zero, and all the others will be proportional to the relative increments. With such a normalization (Larichev, 2002), we use as $x_{ij}$ the initial data of the form:

$$x_{ij} = \frac{d_{ij} - \min\limits_i(d_{ij})}{\max\limits_i(d_{ij}) - \min\limits_i(d_{ij})} = \frac{\frac{a_{ij} - a_{ij-1}}{a_{ij-1}} - \min\limits_i\left(\frac{a_{ij} - a_{ij-1}}{a_{ij-1}}\right)}{\max\limits_i\left(\frac{a_{ij} - a_{ij-1}}{a_{ij-1}}\right) - \min\limits_i\left(\frac{a_{ij} - a_{ij-1}}{a_{ij-1}}\right)},$$

$$i = 1, 2, \ldots, n; \quad j = 0, 1, \ldots, m-1.$$

In the simplest traditional method for assessing the quality of the work of the i-th performer for the entire observation period, the integral estimation $X_i$ of the quality of work is the result of summation of the partial estimates of the growth of all indicators for all periods, as a result of which the integral estimates of the quality of the work of the performer $A_i$ are given by the formula:

$$X_i = \sum_{j=0}^{m-1} x_{ij} \text{ , } i=1,2,\ldots,n. \tag{7}$$

However, in the calculations of formula (7), all time periods give the same contribution to the overall quality of work performance and do not take into account that usually the importance of effective work in recent periods should be higher than in earlier periods. To account for this fact, we can introduce the weights $w_j$ for the time periods on which observation was required. If we use a linear scale of weights, that is, the contribution of the partial performance measure to the integral scale should be proportional to the time interval between the measurement and the current moment, then to estimate the weight, we can use the formula:

$$w_j = \frac{m - j}{m}, \tag{8}$$

where $j$ – lag or the number of the time period of the measurement.

Then formula (7) takes the form:

$$X_i = \sum_{j=0}^{m-1} w_j x_{ij} = \sum_{j=0}^{m-1} \frac{m - j}{m} x_{ij} \text{ , } i=1,2,\ldots,n. \tag{9}$$

However, this approach has a number of drawbacks, for example, estimates will be non-linear, depends on the set of performers and the number of time periods. To eliminate these shortcomings, we can use an estimation model based on the method of estimating latent variables.

To assess the dynamics of changes in performance efficiency, latent variables will be: $\theta_i$ – integral evaluation of the $i$-th performer for the whole observation period, and $\beta_j$ – some indicator characterizing the cumulative change of labour efficiency for the whole group of workers in the $j$-th time period, and, the smaller the value of $\beta$, the higher the efficiency of work. In such a model, the probability $p_{ij}$ that, in the $j$-th time period, the $i$-th employee improved the indicator of labour efficiency, will be determined by the logistic function of the form (1).

To find the latent variables $\theta_i$ and $\beta_j$ without taking into account the weights, it is necessary to solve an optimization problem of the form (2) and (3). If it is necessary to take into account the weights $w_j$, obtained, for example, by the formula (8), then instead of the optimization problem (2) it is necessary to solve the problem (5).

Let's consider now the model allowing to carry out multicriteria estimation of quality of work of executors in dynamics of their changes. In this case criteria: $K_1, K_2, \ldots, K_l$, are used for evaluating the effectiveness of work at each stage, and at each stage a matrix of the form $x_{ik}^{(j)}$, equal to the quality estimate of the performance of the performer $A_i$ for the $k$-th criterion in the $j$-th time period is formed, if the criteria are quantitative. If the criteria are qualitative, then, by analogy with (6), we can use as the initial data a dichotomous matrix of the form:

$$x_{ik}^{(j)} = \begin{cases} 1, \text{ if the employee } A_i \text{ has improved quality} \\ \quad \text{in the period } j \text{ compared to } (j-1) \text{ by criterion } k; \\ 0, \text{ if the employee } A_i \text{ has deteriorated the quality} \\ \quad \text{in the period } j \text{ compared to } (j-1) \text{ by criterion } k, \\ \quad i = 1, 2, \ldots, n; \;\; j = 0, 1, \ldots, m-1; \;\; k = 1, 2, \ldots, l. \end{cases} \tag{10}$$

On the basis of the data $x_{ik}^{(j)}$ a matrix of private assessments of the quality of the performers' work is formed according to the evaluation criteria at all stages of the observations: $\tilde{x}_{ik} = \sum_{j=0}^{m-1} x_{ik}^{(k)}$ - without taking into account the weights, and $\tilde{x}_{ik} = \sum_{j=0}^{m-1} w_j x_{ik}^{(k)}$ - taking into account the weights. To calculate the integral estimates of the quality of work of each performer, we use formulas (2), (3), but we use the matrix as the initial data $\tilde{x}_{ik}$.

## Model of selection of the optimal vacancy for an employee

Let us now turn to the task of recruitment and placement of personnel - one of the most important functions of the management cycle carried out by the management of the organization. Consider one of its elements - choosing the best vacancy for some jobseeker. The selection of the best vacancy is based on the calculation of the degree of suitability of the employee for each vacancy, and the selection of the vacancy with the best estimate.

Suppose that in some organization there are a number of vacancies, one of which is claimed by the employee. The task is to select this vacancy for the employee, which will best match the employee's personal indicators to the requirements for the position corresponding to these vacancies. The selection of a vacancy is that for an arbitrary employee, it is necessary to obtain an assessment of the degree of compliance with the requirements of vacancies, if possible, in a probabilistic interpretation. At the same time, it is desirable to obtain estimates of the influence of the criteria on the vacancy estimates. To solve this problem, we will also use the Rasch model of estimation of latent variables.

Let there be *n* kinds of vacancies, the degree of correspondence to which a particular employee needs to determine: $B_1$, $B_2$, …, $B_n$. To find the degree of employee compliance, vacancies use *m* criteria: $K_1$, $K_2$, …, $K_m$.

Suppose that for some employee, the degree of suitability for which vacancies should be evaluated, data are obtained on the criteria. In accordance with the requirements of the Rasch model, for the identification, criterial assessments of the employee's suitability for each type of vacancy for each criterion, normalized on a single scale, will be used. The estimates for the following scales can be used for the calculation.

If the scale is dichotomous, then the empirical data will be:

$$x_{ij} = \begin{cases} 1, \text{ if an employee satisfies vacancies } i \text{ by criterion } j; \\ 0, \text{ if the employee does not meet the vacancy } i \text{ by criterion } j; \end{cases} \quad (11)$$
$$i = 1,2,...,n; \quad j = 1,2,...,m.$$

Similarly, but with a greater number of gradations, a polytomic rating scale can be used.

A situation is possible when the criterion makes an estimation on a certain continuous scale. There are two possible options. If the degree of correspondence of the vacancy is known at once, then the matrix $x_{ij}$ is equal to the fraction of employee's suitability for the *i*-th vacancy by the *j*-th criterion, and its elements should be measured on a single scale.

A more complicated situation arises if the degree of correspondence is not explicitly known, but it can be estimated by some identification feature that determines the employee's vacancy by hitting the values of this characteristic in an interval typical for each type of job. Suppose that the estimate $U_j$ of an arbitrary employee according to the criterion $K_j$ should fall into the interval for the vacancy $B_i$ of the form ($a_{ij}$, $b_{ij}$), $i = 1,2,...,n;$ $j = 1,2,...,m$, where the middle of this interval corresponds to the highest degree of correspondence vacancy $B_i$ by the criterion $K_j$: $x_{ij}$=1, and with the distance from the middle, the degree of correspondence decreases. It is also possible to have a small value of the degree of correspondence xij and the output of the estimate $U_j$ for the interval ($a_{ij}$, $b_{ij}$). Suppose that the estimate of $U_j$ is distributed according to the normal law, its mathematical expectation is equal to $m_{ij}=(a_{ij}+b_{ij})/2$. We choose the root-mean-square deviation $\sigma_{ij}$ so that the interval ($a_{ij}$, $b_{ij}$) is equal to 2*l* standard deviations, that is

$\sigma_{ij} = \frac{b_{ij} - a_{ij}}{2l}$. The parameter $l$ will regulate the probability of the output of $U_j$ in the interval ($a_{ij}$, $b_{ij}$). Then, in the case of unit valuation, we obtain:

$$x_{ij} = \frac{\frac{1}{\sqrt{2\pi}\sigma_{ij}} \exp\left( -\frac{(U_j - m_{ij})^2}{2\sigma_{ij}^2} \right)}{\frac{1}{\sqrt{2\pi}\sigma_{ij}} \exp(0)} = \exp\left( -\frac{(U_j - m_{ij})^2}{2\sigma_{ij}^2} - 1 \right). \qquad (12)$$

According to the Rasch model, we introduce latent variables:

$\theta_i$ – assessment of the degree of the employee's suitability for the *i*-th kind of vacancy;

$\beta_j$ – degree of importance, dominance of the *j*-th criterion for the evaluation of this employee.

Then the probability that the worker will be determined as the corresponding vacancy $B_i$ by the criterion $K_j$ will be determined by the formula (1).

To find the values of the latent variables $\theta_i$ and $\beta_j$ we need to solve an optimization problem of the form (2) and (3).

After finding the estimates, we can carry out their normalization by formula (4). In view of the fact that the Rasch model gives estimates on a linear scale, the valuations normalized in this way $\tilde{\theta}_i$ can be interpreted as the proportion of the employee's correspondence to the *i*-th vacancy, and the estimates $\tilde{\beta}_j$ as a fraction of the influence of the *j*-th criterion on the evaluation of this employee.

## Conclusions

Some problems of estimating employees based on the theory of latent variables were considered. Employee evaluation approaches based on the Rasch method have several advantages over traditional, what follows from the papers of (Maslak et al., 2015) and (Maslak et al., 2017), which describes the estimates properties of latent variables by the Rasch model, based on the method of least squares:

1. Assessments of employees are their unique properties and do not depend on the set of criteria for evaluation.
2. Estimates are measured on linear dimensionless scales that can easily be translated into other scales, for example, in probabilistic.
3. In addition to evaluations of employees, it is possible to evaluate the properties of evaluation criteria on a linear scale.

All this will make assessments of the main indicators of personnel policy more objective. This, in turn, should provide a supportive environment in which the labor potential is realized, personal abilities of employees develop, which receive satisfaction from the work performed and public recognition of their achievements.

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