Gravitino Condensation and Cosmological Inflation

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Abstract

A new model for inflation based on N=1 supergravity is discussed. By analogy with Nambu-Jona-Lasinio model of superconductivity and its fermion condensate, we use the gravitino condensate and use the induced 1-loop potential as the potential of the inflaton, which gives slow-roll inflation. The model is checked by matching it against the results from the Planck 2018 satellite mission, and it is confirmed that the model is consistent with observations after adjusting some parameters. The formulation of N=1 supergravity with (con)torsion induced by gravitino is explained. The basics of the Nambu-Jona-Lasinio model, especially the derivation of the gap equation and the generation of the quark mass, are explained also. Confirmation of viability of slow-roll inflation is given too.

Contents

1 Introduction

Inflation is known to provide a solution to some problems of BigBang cosmology. Though it provides a proper explanation of the Universe, the mechanism of generation of inflation itself is yet to be established, and many models have been proposed and discussed by researchers. Since the models of inflation must be restricted by observational results, especially from the Planck satellite mission, only the models which match observations are considered as the candidates of an inflation model. I discuss the basic equations for slow-roll inflation and the observational values from the cosmic microwave background (CMB) in section 4.

When constructing a model of inflation, it is worth using supergravity. In supergravity, general relativity is combined with supersymmetry by introducing local supersymmetry. The useful thing about supersymmetry is that it can cancel out the quadratic divergence in perturbation theory, thus it can resolve the hierarchy problems. Supergravity was first proposed in the hope of creating a theory which unifies general relativity and quantum field theory without diverging. However, supergravity is still non-renormalizable. But from the fact that supergravity is the low-energy limit of superstring theory, which is one of the reliable candidates of unification theory, supergravity should play an important role in high energy physics. A formulation of Supergravity can be obtained by introducing the superpartner of graviton which is called gravitino. Supergravity is classified by the number of the gravitinos $N \leq 8$, and we used the simplest one $N = 1$ in our model. The formulation of (N=1) supergravity and (con)torsion induced by gravitino is explained in section 2.

In this thesis, we propose a new inflation model which is based on (N=1) supergravity theory [1]. We use the gravitino condensate as inflaton. The method is similar to the Nambu-Jona-Lasinio model of the theory of superconductivity, which dynamically generates the electron mass from the electron condensate (Cooper pairs) [2, 3]. I explain the basics of the Nambu-Jona-Lasinio model, especially the derivation of the gap equation from its Lagrangian and generation of the quark mass by dynamical chiral symmetry breaking in section 3.

Inflation is induced by the scalar field called inflaton. The inflaton slowly rolls the plateau region of its potential to make quasi-exponential expansion of the Universe and after inflation it decays to generate usual particles, which is call reheating. Currently, the identity of inflaton is yet to be discovered.

In our model, the gravitino condensate, which comes from the quartic interaction term derived from the gravitino contribution to the spacetime (con)torsion, is regarded as the inflaton. Therefore, the inflaton can be naturally generated from the model. Using the gravitino condensate, the 1-loop effective potential is derived. This was first done in [4, 5]; however, the potential which we get differs from those results. We introduce dimensionless quantities for the potential for later use, also the gravitino mass is dynamically generated by dynamical supersymmetry breaking by analogy with Nambu-Jona-Lasinio model. These are discussed in section 5.

Based on the potential we get in section 5, we examine the viability of our model, using the results from Planck 2018 [6]. By fixing the adjustable parameters to give the proper shape of the potential we derive the slow-roll parameters. Using the slow-roll parameters, we fix the other parameters and check the correspondences with the Planck results. This is written down in section 6.

Throughout this paper (except section 4), we adopt the signature of the spacetime metric in the Euclidean form of $(++++)$ for simplicity. This metric can be always transformed to Minkovski metric $(-+++)$ by Wick rotation $t \to it$.

2 SUSY and supergravity

In this section I am going to introduce some basic knowledge of supersymmetry (SUSY) and supergravity[7].

2.1 Formulation of N=1 supergravity

SUSY is a new type of symmetry, which represents a symmetry between the bosonic and fermionic fields in Lagrangian field theory. The local supersymmetry can only be implemented in the field theory if space-time is curved by the presence of gravity. With the presence of supersymmetry, the bosonic gravitational field must have its own fermionic companion, and it is called gravitino, which is a spin 3/2 field.

The Lagrangian in supergravity theory depends on vierbein e_μ^m and spin connection ω_μ^{mn} and gravitino ψ_μ. From a theoretical point of view, supergravity theory can be gained by gauging the space-time symmetry and supersymmetry. The vierbein and spin connection gauge the space-time symmetry and the gravitino gauges supersymmetry.

The vierbein is used to describe a theory which involves spinors and tensor fields such as supergravity theory, and it works as a link between the local Lorentz system and general coordinates. By using the vierbein formalism the metric tensor can be written with the spacetime metric η_{ab} as

$$g_{\mu\nu}(x) = \eta_{ab} e_\mu^a(x) e_\nu^b(x). \tag{1}$$

The greek indices refers to spacetime and the latin indices refer to the local Lorentz frame. The vierbein transformations are as follows: for general coordinate transformations,

$$e'^a_\mu = \frac{\partial x^\nu}{\partial x'_\mu} e^a_\nu, \tag{2}$$

and for local Lorentz transformations,

$$e'^a_\mu(x) = \Lambda^a_b(x) e^b_\mu(x), \tag{3}$$

where Λ^a_b denote the representation of Lorentz group.

Spin connections work as the connections of covariant derivatives for local Lorentz transformations. For a spin 1/2 field, the covariant derivatives can be written with the gamma matrices $\gamma_\mu = e^a_\mu(x)\gamma_a$ as

$$D_\mu \chi = \partial_\mu + \frac{1}{2}\omega_\mu^{mn}\Sigma_{mn}\chi \text{ where } \Sigma_{mn} \equiv \frac{1}{4}[\gamma_m \gamma_n]_-. \tag{4}$$

By using this vierbein formalism, we can write the supergravity Lagrangian.

For the bosonic part of the lagrangian, we will rewrite the Einstein-Hilbert action in this vierbein formalism.

We start with the Ricci scalar inside the action

$$R = \delta^\sigma_\mu g^{\nu\rho} R^\mu_{\nu\rho\sigma}(\Gamma) \text{ where } R^\mu_{\nu\rho\sigma} = \partial_\rho \Gamma^\mu_{\nu\sigma} - \partial_\sigma \Gamma^\mu_{\nu\rho} + \Gamma^\lambda_{\nu\sigma}\Gamma^\mu_{\lambda\rho} - \Gamma^\lambda_{\nu\rho}\Gamma^\mu_{\lambda\sigma}. \tag{5}$$

Here, $\Gamma^\mu_{\nu\lambda}$ is the Christoffel symbol, which can be written as

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\alpha}\left(\frac{\partial g_{\alpha\nu}}{\partial x^\lambda} + \frac{\partial g_{\alpha\lambda}}{\partial x^\nu} - \frac{\partial g_{\nu\lambda}}{\partial x^\alpha}\right) \tag{6}$$

Then we define a new curvature with spin connection as

$$R_{\mu\nu}^{mn}(\omega) = \partial_\mu \omega_\nu^{mn} - \partial_\nu \omega_\mu^{mn} + \omega_\mu^{mc}\omega_{\nu c}^{n} - \omega_\nu^{mc}\omega_{\mu c}^{n}. \tag{7}$$

From the condition that the full covariant derivative of the vierbein vanishes,

$$D_\mu e_\nu^m = \partial_\mu e_\nu^m + \omega_\mu^{mn} e_{n\nu} - \Gamma_{\nu\mu}^{\alpha} e_\alpha^m = 0, \tag{8}$$

the relation between the two curvatures (5), (6) can be derived as

$$R_{\mu\nu mn}(\omega) = R_{\tau\mu\nu}^{\alpha}(\Gamma) e_{m\alpha} e_n^\tau. \tag{9}$$

By using this result, one can write the Einstein-Hilbert action in vierbein formalism as

$$\mathscr{L}^{(2)} = -\frac{M_{\rm Pl}^2}{2}\sqrt{g}R(g,\Gamma) = -\frac{M_{\rm Pl}^2}{2} eR(e,\omega) \tag{10}$$

where $e = \det\, e_\mu^m$ and $R(e,\omega) = e^{m\nu}e^{n\mu}R_{\mu\nu mn}(\omega)$.

Here, it is to be noted that $M_{\rm Pl}$ is the reduced Planck mass, $M_{\rm Pl} = 1/\sqrt{8\pi G_N} \approx 2.4 \times 10^{18}$GeV.

Similarly, for the fermionic part of the Lagrangian, we will start with the Lagrangian of the Rarita-Schwinger equation [8][9]:

$$\mathscr{L}^{(3/2)} = -\frac{1}{2}\epsilon^{mnpq}\bar{\psi}_m \gamma_5 \gamma_n \partial_p \psi_q \tag{11}$$

and extend equation(11) to curved space as

$$\mathscr{L}^{(3/2)} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \text{ where } D_\rho \psi_\sigma = \partial_\rho + \frac{1}{2}\omega_\rho^{mn}\Sigma_{mn}\psi_\sigma. \tag{12}$$

Baced on these Lagrangians, we can formulate the full Lagrangian for supergravity. When we verify the gauge invariance of the supergravity action, we can use the 1.5 order formalism [10][11], which is the combination of the second order formalism by Freedman, van Nieuwenhuizen and Ferrara[12], and the first orde formalism by Deser and Zumino [13].

In the second order formalism, they found that in order to obtain complete invariance, one needs to start with the full lagrangian $\mathscr{L}$ as $\mathscr{L} = \mathscr{L}^{(2)} + \mathscr{L}^{(3/2)}$ and put $\delta\psi_\mu$ as $\delta\psi_\mu = \kappa^{-1}D_\mu(\omega)$ and replace ω with $\omega = \omega(e,\psi)$, which is derived from the equation $\frac{\delta I}{\delta \omega} = 0$, where $\kappa = M_{\rm Pl}^{-1}$.

In the first order formalism, they assume that the spin connection is an independent field and from that assumption they derived the transformation law of ω_μ^{ab}, which makes the full lagrangian to be invariant.

The 1.5 order formalism is a combination of the two previous formalisms. We start by assuming the ω in action to be independent, which the variation of the action can be written, from the chain rule, as

$$\delta I(e,\psi,\omega) = \delta e \left.\frac{\delta I}{\delta e}\right|_{\psi,\omega} + \delta\psi \left.\frac{\delta I}{\delta \psi}\right|_{e,\omega} + \delta\bar{\psi} \left.\frac{\delta I}{\delta \bar{\psi}}\right|_{e,\omega} + \delta\omega \left.\frac{\delta I}{\delta \omega}\right|_{\psi,e}. \tag{13}$$

Requiring that $\frac{\delta I}{\delta \omega} = 0$ we can have $\omega = \omega(e,\psi)$. Also from these facts, one can drop the last term in (13) due to the requirement. This means that only the variations of an vierbein field and a gravitino field are needed.

2.2 Spacetime torsion induced by gravitino

In the supergravity theory, torsion is induced by gravitino and from that, the contorsion tensor, which is the difference between the connections with torsion and without torsion, can be derived. For later use, we derive the torsion.

First, we rewrite the Einstein-Hilbert action (10) as

$$\mathscr{L}^{(2)} = \frac{M_{\rm Pl}^2}{8}\epsilon^{\mu\nu\sigma\rho}\epsilon_{mncd}e^m_\nu e^n_\mu R^{cd}_{\rho\sigma}(\omega). \tag{14}$$

Here we used the identity,

$$\epsilon^{\mu\nu\sigma\rho}\epsilon_{mncd}e^m_\nu e^n_\mu = 2e(e^\rho_c e^\sigma_d - e^\sigma_c e^\rho_d). \tag{15}$$

Also by varying the spin connection of equation (7), one finds

$$\delta R^{cd}_{\rho\sigma}(\omega) = D_\rho \delta\omega^{cd}_\sigma - D_\sigma \delta\omega^{cd}_\rho \text{ where } D_\rho\delta\omega^{cd}_\sigma = \partial_\rho \delta\omega^{cd}_\sigma + \omega^{ce}_\rho \delta\omega^d_{\sigma e} + \omega^{de}_\rho \delta\omega^c_{\sigma e}. \tag{16}$$

Partially integrating equation (14), one finds the variation of the Lagrangian with the spin connection as

$$\delta\mathscr{L}^{(2)} = \frac{M_{\rm Pl}^2}{2}\epsilon^{\mu\nu\sigma\rho}\epsilon_{mncd}(D_\sigma e^m_\mu)e^n_\nu \delta\omega^{cd}_\rho, \text{ where } D_\sigma e^m_\mu = \partial_\sigma e^m_\mu + \omega^{mn}_\sigma e^n_\mu. \tag{17}$$

We also vary the spin connection in the Rarita-Schwinger equation (12) and the result can be written as

$$\delta\mathscr{L}^{(3/2)} = -\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu\Sigma_{cd}\psi_\sigma\delta\omega^{cd}_\rho. \tag{18}$$

Here, $\bar{\psi}_\mu\gamma_5\gamma_\nu\Sigma_{cd}\psi_\sigma$ can be decomposed into the sum of vector terms and axial vector terms as

$$\bar{\psi}_\mu\gamma_5\gamma_\nu\Sigma_{cd}\psi_\sigma = \frac{1}{2}\bar{\psi}_\mu\gamma_5(e_{c\nu}\gamma_d - e_{d\nu}\gamma_c)\psi_\sigma + \frac{1}{2}e^b_\nu\epsilon_{bcdm}\bar{\psi}_\mu\gamma^\mu\psi_\sigma. \tag{19}$$

But since $\bar{\psi}_\mu\gamma_5\gamma_d\psi_\sigma$ is symmetric in μ and σ and $\bar{\psi}_\mu\gamma_m\psi_\sigma$ is antisymmetric, the antisymmetric term remains. Thus, the variation of the Rarita-Schwinger Lagrangian with spin connection can be written as

$$\delta\mathscr{L}^{(3/2)} = -\frac{1}{8}\epsilon^{\mu\nu\sigma\rho}\epsilon_{mncd}(\bar{\psi}_\mu\gamma_m\psi_\sigma)e^n_\nu\delta\omega^{cd}_\rho. \tag{20}$$

Comparing the two equations (17) and (20), one can get

$$D_\mu e^m_\nu - D_\nu e^m_\mu = \frac{1}{2M_{\rm Pl}^2}(\bar{\psi}_\mu\gamma^m\psi_\nu). \tag{21}$$

The torsion $S^\alpha_{\mu\nu}$ is defined by

$$S^\alpha_{\mu\nu} = \frac{1}{2}(\Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\nu\mu}). \tag{22}$$

Hence, in our case (21) the torsion can be written with gravitino as

$$S^\alpha_{\mu\nu} = -\frac{1}{4M_{\rm Pl}^2}(\bar{\psi}_\mu\gamma^\alpha\psi_\nu). \tag{23}$$

We also introduce the contorsion tensor κ^{mn}_μ as

$$\omega^{mn}_\mu = \omega^{mn}_\mu(e) + \kappa^{mn}_\mu. \tag{24}$$

Then by using the equation

$$\partial_\mu e^m_\nu + \omega^m_{\mu\nu} - \partial_\nu e^m_\mu + \omega^m_{\nu\mu} = 0, \tag{25}$$

one can derive the relation of the contortion tensor with gravitino as

$$\kappa_{\mu m\nu} - \kappa_{\nu m\mu} = \frac{1}{2M^2_{\rm Pl}}(\bar{\psi}_\mu \gamma^m \psi_\nu). \tag{26}$$

One can also have a more explicit form for the contorsion by using the identity

$$(\kappa_{\mu m\nu} - \kappa_{\nu m\mu}) + (\kappa_{m\mu\nu} - \kappa_{\nu\mu m}) + (\kappa_{m\nu\mu} - \kappa_{\mu\nu m}) = 2\kappa_{\mu m\nu}. \tag{27}$$

Using this identity, equation (24) can be written as

$$\omega_{\mu mn}(e,\psi) = \omega_{\mu mn}(e) + \frac{1}{4M^2_{\rm Pl}}(\bar{\psi}_\mu\gamma_m\psi_n - \bar{\psi}_\mu\gamma_n\psi_m + \bar{\psi}_m\gamma_\mu\psi_n). \tag{28}$$

The Lagrangian of supergravity $\mathscr{L} = \mathscr{L}^{(2)} + \mathscr{L}^{(3/2)}$ is invariant under the following transformations:

$$\delta e^a_\mu = \frac{\kappa}{2}\bar{\epsilon}\gamma^a\psi_m, \;\; \delta\psi_\mu = \frac{1}{\kappa}D_\mu(\omega(e,\psi))\epsilon. \tag{29}$$

2.3 Auxiliary fields in pure supergravity

There still remains a problem about the degrees of freedom for the bosonic and fermionic fields. On-shell, the degrees of freedom for bosons and fermions are the same, being equal to 2B=2F. But off-shell, they don't match so that SUSY algebra is not closed off-shell (i.e. without using EoM). More concretely, there are 16 bosonic fields in vierbein e^m_μ but the sum of general coordinate transformation parameters and local Lorentz transformation parameters are 10. Therefore there are 6 effective independent bosonic fields. For the fermions, there are 16 fields in gravitino ψ^a_μ but there are only 4 local supersymmetry transformation parameters. Therefore there are 12 effective independent fermionic fields remaining. To match the degrees of freedom of bosonic and fermionic fields with each other even off-shell, we should introduce 6 bosonic auxiliary fields into the Lagrangian. If we introduce an axial vector A_m, a scalar S and a pseudoscalar P, which have the extra degrees of freedom 4+1+1=6B=12F-6B. Then the full supergravity Lagrangian can be written as

$$\mathscr{L} = \mathscr{L}^{(2)}(e,\omega) + \mathscr{L}^{(3/2)}(e,\psi,\omega) - \frac{e}{3}(S^2 + P^2 - A^2_m), \tag{30}$$

which is invariant under the following transformations:

$$\delta e^m_\mu = \frac{1}{2M_{\rm Pl}}\bar{\epsilon}\gamma^m\psi_\mu \tag{31}$$

$$\delta\psi_\mu = M_{\rm Pl}(D_\mu + \frac{i}{2M_{\rm Pl}}A_\mu\gamma_5)\epsilon - \frac{1}{2}\gamma_\mu\eta\epsilon \tag{32}$$

$$\delta S = \frac{1}{4}\bar{\epsilon}\gamma_\mu R^{\mu,cov} \tag{33}$$

$$\delta P = -\frac{i}{4}\bar{\epsilon}\gamma_5\gamma_\mu R^{\mu,cov} \tag{34}$$

$$\delta A_m = \frac{3i}{4}\bar{\epsilon}\gamma_5(R^{cov}_m - \frac{1}{3}\gamma_m\gamma_\mu R^{\mu,cov}) \tag{35}$$

where $\eta = -\frac{1}{3}(S - i\gamma_5 P - i\gamma^m A_m\gamma_5)$ and $R^{\mu,cov} = \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu(D_\rho\psi_\sigma - \frac{i}{2}A_\sigma\gamma_5\psi_\rho + \frac{1}{2}\gamma_\sigma\eta\psi_\rho)$

3 NJL-model and its applications

The Nambu-Jona-Lasinio model (NJL model) is a theory which can give a description of hadrons and generation of their mass. This theory is similar to our model, which uses gravitino instead of quarks. In this section I briefly explain the NJL model.

The NJL model was first invented by Yoichiro Nambu and Giovanni Jona-Lasino in 1961, which was inspired by the BCS theory of superconductivity [2, 3]. It is a chiral effective theory and can be interpreted as a low energy approximation of Quantum Chromodynamics (QCD). It is also a non-renormalizable theory and therefore a regularization parameter cannot be removed.

The NJL model has the same flavor symmetric as the QCD: the NJL model has symmetries $SU(3)_c \otimes SU(N_f)_V \otimes SU(N_f)_A \otimes U(1)_V \otimes \mathcal{C} \otimes \mathcal{P} \otimes \mathcal{T}$, where N_f denotes the number of flavors in the system. Therefore the NJL lagrangian should be symmetric under the transformations

$$\begin{aligned} SU(N_f)_V &: \psi \to e^{-i\mathbf{t}\cdot\boldsymbol{\theta}_V}\psi \quad \bar{\psi} \to \bar{\psi} e^{i\mathbf{t}\cdot\boldsymbol{\theta}_V} \\ SU(N_f)_A &: \psi \to e^{-i\gamma_5\mathbf{t}\cdot\boldsymbol{\theta}_A}\psi \quad \bar{\psi} \to \bar{\psi} e^{-i\gamma_5\mathbf{t}\cdot\boldsymbol{\theta}_A} \\ U(1)_V &: \psi \to e^{-i\theta}\psi \quad \bar{\psi} \to \bar{\psi} e^{i\theta} \end{aligned} \tag{36}$$

(For the later calculation, I will take the number of flavors as 2 and that of colors as 3 for simplicity.)

For their Lagrangian, Nambu and Jona-Lasinio choose the one, which satisfies the symmetries, as

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]. \tag{37}$$

By using the Euler-Lagrange equation in the Lagrangian for $\bar{\psi}$ and considering the mean-field approximation, the equation of motion (EoM) for the Lagrangian can be written as

$$[-i\partial\!\!\!/ + m - 2G\langle\bar{\psi}\psi\rangle]\psi = 0. \tag{38}$$

Therefore the constituent quark mass M turns out to be

$$M = m - 2G\langle\bar{\psi}\psi\rangle. \tag{39}$$

This is called the gap equation. The quark condensate $\langle\bar{\psi}\psi\rangle$ is

$$\langle\bar{\psi}\psi\rangle = -\int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{Tr}[iS(k)] \tag{40}$$

in which $S(k)$ is the quark propagator that can be written as

$$S(k) = \frac{1}{k\!\!\!/ - M + i\epsilon} = \frac{k\!\!\!/ + M}{k^2 - M^2 + i\epsilon}. \tag{41}$$

The gap equation (39) in more explicit form reads

$$M = m + 48iGM\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2 - M^2 + i\epsilon}. \tag{42}$$

The NJL model is non-renormalizable as mentioned before; therefore a regularization, such as cut-off regularization and Pauli-Villars regularization, is needed to avoid divergences. The integral in equation (42) will diverge. For this reason, I use the proper-time regularization [14]

$$\frac{1}{X^n} = \frac{1}{(n-1)!}\int_0^\infty \mathrm{d}\tau\tau^{n-1}e^{-\tau X} \to \frac{1}{(n-1)!}\int_{\tau_{UV}}^\infty \mathrm{d}\tau\tau^{n-1}e^{-\tau X} \tag{43}$$

to avoid this problem. Introducing the parameter τ_{UV}, the integral can be written as

$$48iGM\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2-M^2}\to 48iGM\int_{-\infty}^{\infty}\frac{\mathrm{d}^4k}{(2\pi)^4}\int_{\tau_{UV}}^{\infty}\mathrm{d}\tau e^{-\tau(k^2-M^2)}. \tag{44}$$

Solving this equation and substituting the result in equation (42) gives us the gap equation with the parameter τ_{UV},

$$M=m+M\frac{3G}{\pi^2}\int_{\tau_{UV}}^{\infty}\mathrm{d}\tau\frac{e^{-\tau M^2}}{\tau^2}. \tag{45}$$

For the case $m=0$, the equation has two solutions: the trivial solution, $M=0$, and the non-trivial solution, $M\neq 0$.

To see which solution is valid, we derive the vacuum energy density ϵ for each solution, and subtracting one from the other, we obtain

$$\epsilon(M)-\epsilon(M=0)=-\frac{3}{4\pi^2}\int\mathrm{d}\tau\frac{1}{\tau^3}(e^{-\tau M^2}-1)+\frac{M^2}{4G}. \tag{46}$$

Therefore for $G>G_{cri}$, the lowest energy has $M\neq 0$, which means that chiral symmetry is dynamically broken and the quark mass is generated as a result.

We apply this mechanism of fermion condensation and generation of mass to supergravity in section 5.

4 Cosmological inflation and CMB

In this section, I introduce basic knowledge of cosmological inflation and the cosmic microwave background (CMB), which can provide a discrimination to models of inflation.

Cosmological inflation was first introduced as a solution to the initial conditions problems in BigBang cosmology, specifically the horizon problem and flatness problem, etc. Inflation is an quasi-exponential expansion of the Universe at its very beginning. Inflation was driven by the scalar field called inflaton, and its potential energy filled up the early Universe to cause the exponential expansion.

The Friedmann equation is written as

$$H^2=\left(\frac{\dot{a}}{a}\right)^2=\frac{1}{3M_{Pl}^2}\rho, \tag{47}$$

which can be derived from the Einstein equation with the Friedmann-Lemaître-Robertson-Walker metric(FLRW metric), which describes the homogeneous and isotropic space-time,

$$ds^2=-c^2dt^2+a(t)^2\left(\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\phi^2)\right)\ \text{where } k=+1,-1,0 \tag{48}$$

For a flat universe, $k=0$, which we use also.

Using the energy density of the homogeneous scalar field ϕ as

$$\rho_\phi=\frac{1}{2}\dot{\phi}^2+V(\phi), \tag{49}$$

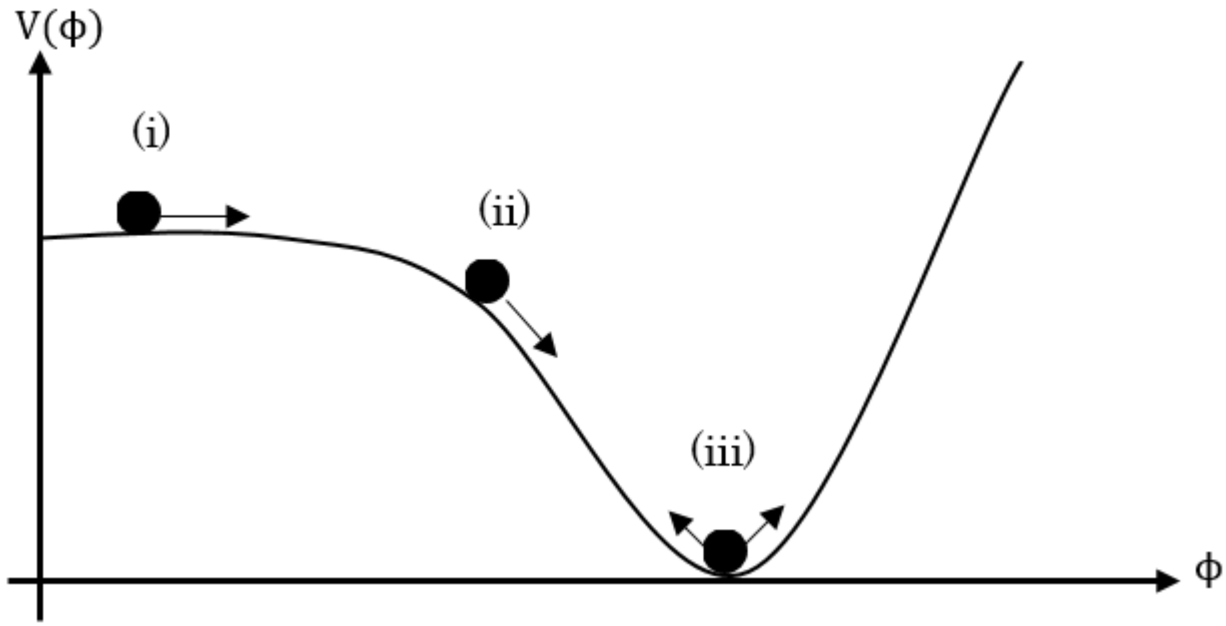

Figure 1: The graph of the potential of typical inflaton field.

the equation of motion of the inflaton can be derived as

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0. \tag{50}$$

Therefore the shape of the potential $V(\phi)$ determines the characteristics of inflation.

There are some conditions for the inflaton that a legitimate account of inflation has to confirm, that is: (i) the potential has to have a sufficiently long plateau region to make a slowly changing potential, which is called slow-roll, (ii) the plateau region ends and the inclination of the potential increases meaning the end of inflation, (iii) at the bottom of the potential, the inflaton oscillates which starts the particle production (see Figure 1).

If we assume the slow-roll inflation, the motion term in equation (50) can be ignored for the plateau region. For this reason, the slow-roll approximation can be applied for the region as

$$\dot{\phi} \simeq -\frac{V'}{3H}. \tag{51}$$

In order to verify the plateau for the slow-roll inflation, some parameters called the slow-roll parameters ϵ, η are used, and they can be defined as

$$\epsilon = \frac{1}{2} M_{Pl}^2 \left(\frac{V'}{V} \right)^2, \ \eta = M_{Pl}^2 \left| \frac{V''}{V} \right|. \tag{52}$$

To have the proper slow-roll inflation, the condition $\epsilon, \eta << 1$ is needed, and also for the length of the inflation, e-folding number N_e for the inflation needs to be $50 \sim 60$ to solve the flatness problem. The e-folding number is defined as

$$N_e = \frac{1}{M_{Pl}^2} \int_{\phi_f}^{\phi} \frac{V}{V'} \mathrm{d}\phi. \tag{53}$$

Inflation not only solves the problems of cosmology, but it can generate the primordial density fluctuations which enable the large-scale structure of the Universe we see today, and the slight anisotropy of the CMB.

The scalar field in de Sitter space has the quantum fluctuation as follows [15]:

$$\delta\phi \equiv \sqrt{< \delta\phi^2 >} = \frac{H_{\rm inf}}{2\pi}, \tag{54}$$

where $H_{\rm inf}$ corresponds to the Hubble constant for the inflation era, and this quantum fluctuation was enlarged by the rapid accelerating expansion, caused by the inflation, and became classical fluctuation accounting for the density fluctuations of matter and radiation.

The energy densities for the radiation and matter differ in $\rho \propto 1/a^n$, as $n = -4$ and $n = -3$, respectively. Their fluctuations can be written as

$$\delta \sim \frac{\delta\rho}{\rho} \sim \frac{\delta a}{a} \sim \frac{\dot{a}}{a}\delta t. \tag{55}$$

and if this δt was the result from the fluctuation of scalar field ϕ mentioned before, by using the slow-roll approximation (51), the fluctuation (55) can be written with the potential V as

$$\delta \sim H\frac{\delta\phi}{\dot{\phi}} \sim \frac{H^3}{V'} \sim \frac{1}{M_{Pl}^3}\frac{V^{3/2}}{V'}. \tag{56}$$

Practically we use the curvature fluctuation ζ for the calculation of the primordial fluctuations, where the curvature fluctuation ζ can be written as

$$\zeta = \frac{1}{\sqrt{12\pi^2 M_{Pl}^3}}\frac{V^{3/2}}{|V'|}. \tag{57}$$

In order to describe the characteristics of the curvature fluctuation, we define the power spectrum P_ζ as the 3-dimensional Fourier transformation of the correlation function,

$$\langle\zeta(\mathbf{k}_1)\ \zeta(\mathbf{k}_2)\rangle \equiv (2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2)P_\zeta(k_1). \tag{58}$$

By using the non-dimensional power spectrum $\mathcal{P}$ where

$$\mathcal{P} = \frac{k^3}{2\pi^2}P_\zeta(k), \tag{59}$$

to describe the dependance of power spectrum on the scale k, one defines the spectrum index n_s as

$$n_s - 1 \equiv \frac{\mathrm{d}\ \ln\mathcal{P}_\zeta(k)}{\mathrm{d}\ \ln k}. \tag{60}$$

Also, using the slow-roll parameters in (52), for the standard single-field inflation model, this spectrum index is related to the slow-roll parameters as

$$n_s = 1 - 6\epsilon + 2\eta. \tag{61}$$

During inflation, density fluctuations were not the only ones, but also the tensor fluctuations existed, which are the primordial gravitational waves. The tensor fluctuations can be derived

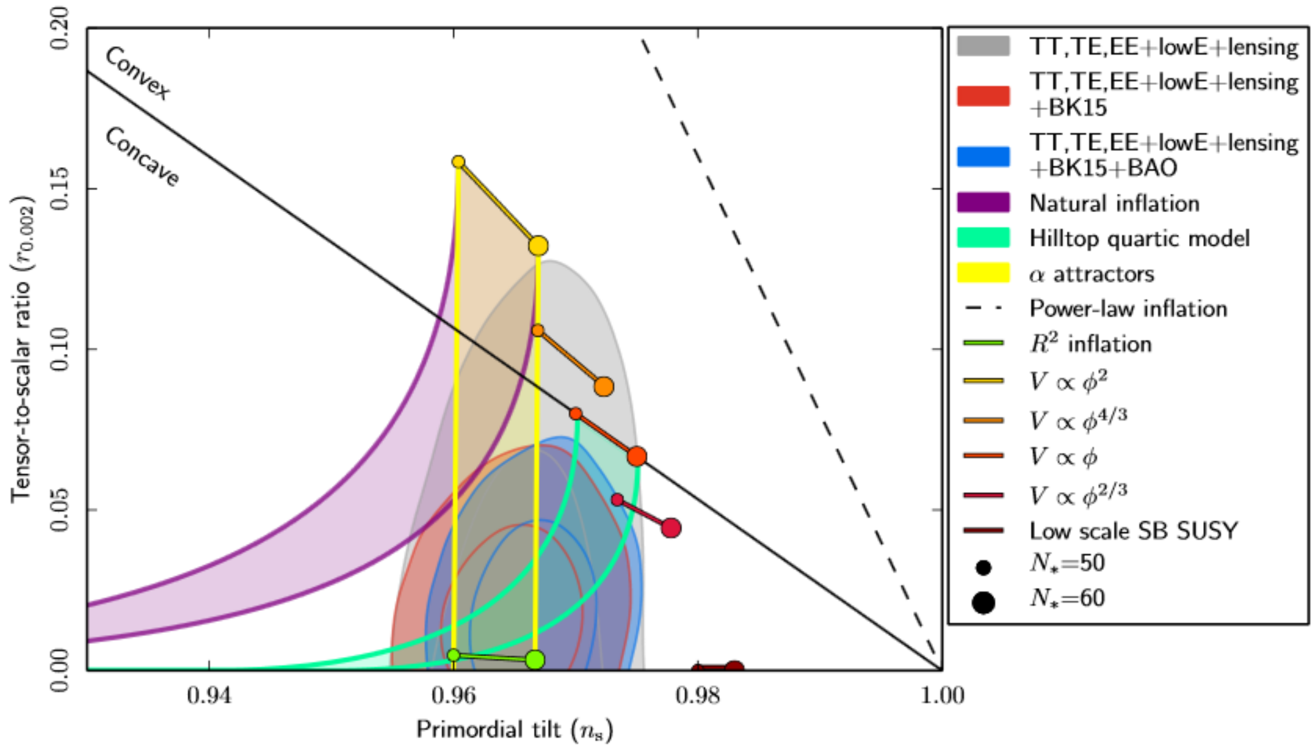

Figure 2: The graph of viable regions of the spectrum index n_s and the tensor-scalar ratio r in 68% and 95% CL, respectively, in Planck observation alone, combination with BK15 and combination with BK15+BAO for some inflation models [6].

similarly as the density fluctuations as written above, and the power spectrum for the tensor fluctuation can be written as

$$\mathcal{P}_T(k) = \frac{8}{M_{Pl}^2}\left(\frac{H}{2\pi}\right)^2. \tag{62}$$

Normally we write the amplitude of the tensor fluctuation in the form of tensor-scalar ratio r, where

$$r \equiv \frac{\mathcal{P}_T(k)}{\mathcal{P}_\zeta(k)}. \tag{63}$$

For the single-field inflation model, this tensor-scalar ratio is also related to the slow roll parameters in (52) as

$$r = 16\epsilon. \tag{64}$$

The important thing is that these two values n_s and r are the observables, and by the Planck satellite mission in 2018 [6], their values were measured and restricted. The values are as follows:

$$\begin{aligned} n_s &= 0.9649 \pm 0.0042 \; (68\% CL, \mathit{Planck}\text{TT+lowE+lensing}) \\ r &< 0.064 \; (95\% CL, \mathit{Planck}\text{TT+lowE+lensing+BK15+BAO}) \end{aligned} \tag{65}$$

The useful thing for these observables is that these values do provide restrictions to any inflation model. Calculating these values for given models and comparing them with the observational results can verify the accuracy of the model (Figure 2).

See Ref.[16] for more details.

5 Gravitino condensate

Our model of inflation originates from the gravitino condensate in $N = 1$ supergravity theory, which is similar to the Nambu-Jona-Lasinio model, which was introduced in section 3. The contribution of the gravitino to the spacetime (con)torsion leads to the quartic gravitino interaction. The interaction gives rise to a one-loop effective action, and we used the potential part of the real scalar field in the action as the potential of the inflaton, which makes the slow-roll inflation.

In this section, we derive the one-loop effective action for the gravitino condensate, and derive the one-loop contribution of the scalar field potential. Also in the next section, we examine the validity of the potential concerning whether it can survive the observational results.

We start with the $N = 1$ supergravity Lagrangian (30) which we derived in section 1. Using the equation for the contortion (28) in section 1, and imposing the SUSY gauge condition:

$$\gamma^\mu \psi_\mu = 0, \tag{66}$$

the Lagrangian of supergravity becomes

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(3/2)},\ \mathcal{L}^{(2)} = -\frac{\tilde{M}_{\text{Pl}}^2}{2} eR(e,\psi) \text{ and } \mathcal{L}^{(3/2)} = -\frac{1}{2}\varepsilon^{\mu\nu\lambda\rho}\bar{\psi}_\mu \gamma_5 \gamma_\nu D_\lambda \psi_\rho \tag{67}$$

$$\text{Where } D_\lambda = \partial_\lambda + \frac{1}{2}\omega_\lambda^{ab}(e,\psi)\Sigma_{ab}.$$

Here we changed the Planck scale from M_{Pl} to the *effective* scale of quantum gravity, $\tilde{M}_{\text{Pl}}$, which may be lower than the previous one. But from the negative results of the Large Hadron Collider (LHC), the effective Planck scale should also be much higher than the TeV scale:

$$1\text{TeV} \ll \tilde{M}_{\text{Pl}} \ll M_{\text{Pl}}, \tag{68}$$

which constitutes our assumption. The (reduced) Planck mass is given as

$$M_{\text{Pl}} = \frac{1}{\sqrt{8\pi G_N}} \approx 2.4 \times 10^{18}\text{GeV}. \tag{69}$$

By rewriting the full Lagrangian by application of contorsion, the quartic gravitino coupling terms explicitly appear as

$$\mathcal{L}_{\text{quartic}} = \frac{11}{16}\tilde{M}_{\text{Pl}}^{-2}\left[(\bar{\psi}_\mu\psi^\mu)^2 - (\bar{\psi}_\mu\gamma_5\psi^\mu)^2\right] - \frac{33}{64}\tilde{M}_{\text{Pl}}^{-2}(\bar{\psi}^\mu\gamma_5\gamma_\nu\psi^\mu)^2. \tag{70}$$

By introducing the Lagrangian multiplier auxiliary filed, the first quartic gravitino interaction term can be eliminated [17]. Using the equation

$$(\bar{\psi}_\mu\Sigma^{\mu\nu}\psi_\nu) = -\frac{1}{2}\bar{\psi}_\mu\psi^\mu, \tag{71}$$

which derives from the gauge condition (66), application of the auxiliary field to the *non-chiral* (first) quartic gravitino term can be written as

$$\mathcal{L}_{\text{quartic}} = \sqrt{11}\tilde{M}_{\text{Pl}}^{-1}\rho(\bar{\psi}_\mu\Sigma^{\mu\nu}\psi_\nu) - \rho^2 \tag{72}$$

Here ρ is the real *scalar* field. The gravitino condensate means the non-vanishing Vacuum Expectation Value, $< \rho > \equiv \rho_0 \neq 0$ and the ρ_0 contributes to the gravitino mass.

The 1-loop effective potential for ρ field was first calculated by R.S.Jasinschi and A.W.Smith in Ref.[5, 4]. However, our calculations resulted in a different answer, which is qualitatively similar, but is quantitatively different.

First we quantize the theory using the path integral. The full Lagrangian using the auxiliary fields ρ, π, λ_μ can be written as

$$\begin{aligned}
\mathcal{L} \;&=\; -\frac{e\tilde{M}_{\rm Pl}^2}{2}R(e) + \frac{1}{4}e\{\left(\frac{1}{e}\right)\partial_\mu[e_a^\mu e_b^\nu e][\bar{\psi}_\nu\gamma^a\psi^b - \bar{\psi}_\nu\gamma^b\psi^a + \bar{\psi}^b\gamma_\nu\psi^a]\} \\
&\quad -\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu[\partial_\lambda + \frac{1}{2}\omega_\lambda^{ab}(e)\Sigma_{ab}]\psi_\sigma \\
&\quad +\sqrt{11}\tilde{M}_{\rm Pl}^{-1}\rho(\bar{\psi}_\mu\Sigma^{\mu\nu}\psi_\nu) - \rho^2 - \frac{\sqrt{11}i}{2}\tilde{M}_{\rm Pl}^{-1}(\bar{\psi}_\mu\gamma_5\psi^\mu)\pi - \pi^2 \\
&\quad -\frac{\sqrt{33}i}{2}\tilde{M}_{\rm Pl}^{-1}(\bar{\psi}_\mu\gamma_5\gamma_\nu\psi^\mu)\lambda^\nu - \frac{e}{3}(S^2 + P^2 - A_\mu^2)
\end{aligned} \tag{73}$$

Quantizing the theory by the path integral, we take the Gaussian integral over ψ and using the gauge condition (66), the 1-loop contribution to the quantum effective action yields

$$\Gamma_{1-\text{loop}} = -\frac{i}{2}\text{Tr}\ln\Delta(\rho), \tag{74}$$

where $\Delta(\rho)$ is the kinetic operator in the gravitino action. The interaction with gravity is ignored by the replacement of e_μ^a with δ_μ^a, because it is irrelevant here.

By using the massless gravitino propagator in momentum space

$$P_{ab} = -\frac{i}{2}\frac{\gamma_b\gamma^\mu p_\mu\gamma_a}{p^2}, \tag{75}$$

the 1-loop contribution to the ρ-scalar potential can then be derived as

$$\begin{aligned}
V_{1-\text{loop}} \;&=\; \lim_{\mathcal{V}\to\infty}\left[\frac{1}{2\mathcal{V}}\sum_{n=1}^{\infty}\frac{(\sqrt{11}\tilde{M}_{\rm Pl}^{-1})^n}{n}(-1)^{n+1}\text{Tr}(P_{ab}\rho)^{2n}\right] &(76)\\
&=\; \lim_{\mathcal{V}\to\infty}\left[\frac{-1}{2\mathcal{V}}\sum_{n=1}^{\infty}\frac{(\sqrt{11}\tilde{M}_{\rm Pl})^{2n}}{2n}\text{Tr}(P_{ab}\rho)^{2n}\right]. &(77)
\end{aligned}$$

Here, $\mathcal{V}$ is the spacetime 4-volume regulator, and the trace Tr acts on all variables. The last equation came from the fact that the trace of the odd product of P_{ab} vanishes.

The potential (77) in integral form with the Ultra-Violet cutoff Λ reads

$$V_{1-\text{loop}} = -\frac{4}{(2\pi)^4}\int^\Lambda d^4p\,\ln\left(1 + 11\tilde{M}_{\rm Pl}^{-2}\frac{\rho^2}{p^2}\right), \tag{78}$$

and the full potential becomes

$$V(\rho) \equiv V_{\text{classical}}(\rho) + V_{1-\text{loop}}(\rho) = \rho^2 - \frac{4}{(2\pi)^4}\int^\Lambda d^4p\,\ln\left(1 + 11\tilde{M}_{\rm Pl}^{-2}\frac{\rho^2}{p^2}\right). \tag{79}$$

We calculated this four-dimensional integral and the result we get is

$$V(\rho) = \rho^2 + \frac{1}{8\pi^2}\left\{\frac{121\rho^4}{\tilde{M}_{\rm Pl}^4}\ln\left(1 + \frac{\tilde{M}_{\rm Pl}^2\Lambda^2}{11\rho^2}\right) - \frac{11\rho^2\Lambda^2}{\tilde{M}_{\rm Pl}^2} - \Lambda^4\ln\left(1 + \frac{11\rho^2}{\tilde{M}_{\rm Pl}^2\Lambda^2}\right)\right\} \tag{80}$$

The wave function renormalization of ρ in the 1-loop approximation $Z(\rho)$, can be described by logarithmic scaling using the renormalization scale μ as

$$Z[\rho] \sim \text{const.} \times \ln\left(\frac{\Lambda^2}{\mu^2}\right). \tag{81}$$

From this fact, the canonical scalar ϕ is

$$\phi = \text{const.}\sqrt{\ln\left(\frac{\Lambda^2}{\mu^2}\right)}\tilde{M}_{\text{Pl}}^{-1}\rho \equiv \omega\tilde{M}_{\text{Pl}}\sigma \equiv \tilde{\omega}M_{\text{Pl}}\sigma. \tag{82}$$

By introducing the dimensionless quantities as

$$\sigma = \tilde{M}_{\text{Pl}}^{-2}\rho,\ \ \tilde{M}_{\text{Pl}}^{-1}\Lambda = \tilde{\Lambda} \text{ and } \tilde{M}_{\text{Pl}}^{-1}\tilde{M}_{\text{SUSY}} = \alpha, \tag{83}$$

the full scalar potential is rewritten as

$$V(\sigma)\tilde{M}_{\text{Pl}}^{-4} = \sigma^2 - \frac{1}{8\pi^2}\left\{\tilde{\Lambda}^4\ln\left(1+\frac{11\sigma^2}{\tilde{\Lambda}^2}\right) - 121\sigma^4\ln\left(1+\frac{\tilde{\Lambda}^2}{11\sigma^2}\right) + 11\sigma^2\tilde{\Lambda}^2\right\} + \alpha^4. \tag{84}$$

The renormalization quantities $\tilde{\Lambda}$ and $\tilde{\omega}$ are the phenomenological parameters that are not derived but chosen to get the desired results. The α term in the full scalar potential is needed to compensate the cosmological constant.

The hierarchy between inflationary scale $H_{\text{inf.}}$, the SUSY breaking scale M_{susy}, the GUT scale M_{GUT}, the effective gravitational scale $\tilde{M}_{\text{Pl}}$ and the Planck scale M_{Pl} is

$$H_{\text{inf.}} \ll M_{\text{susy}} \approx M_{\text{GUT}} \approx \tilde{M}_{\text{Pl}} \ll M_{\text{Pl}}. \tag{85}$$

Incidentally, for the GUT scale, we adopt $M_{GUT} \approx \mathcal{O}(10^{15})$GeV.

The gravitino mass can be gained from the point where the scalar potential vanishes $V(\sigma_c) = 0$. According to equation (72), the $\rho_c \neq 0$ determines the gravitino mass as

$$m_{3/2} = \sqrt{11}\rho_c\tilde{M}_{\text{Pl}}^{-1} = \sqrt{11}\tilde{M}_{\text{Pl}}\sigma_c. \tag{86}$$

Here, the non-vanishing values of ρ_c and σ_c are determined by solving the equation with the first derivative of the potential $V(\sigma)$ equal to zero, which leads to a transcendental equation.

6 Gravitino condensate as inflaton

In this section, we proceed with the potential (84) which we derived in the previous section. The idea of a slow-roll inflation from gravitino condensation in supergravity was proposed and studied in the papers by Ellis and Mavromatos [4, 18]. Since the scalar potential we derived in the previous section differs from that of [4, 18], we ask whether our potential can survive the observational results of Planck and can be a candidate of an inflation model.

From equation (84) it follows that the shape of the potential depends of the value of Λ. To have the double-well shaped potential, which is required for inflation, a local maximum at $\rho = \sigma = 0$ with the positive height α is needed. We differentiate $V_{1-loop}(\sigma)\tilde{M}_{\text{Pl}}^{-4}$ with respect to σ^2

$$\frac{\text{d}}{\text{d}\sigma^2}V_{1-loop}(\sigma)\tilde{M}_{\text{Pl}}^{-4} = \frac{4\pi^2 - 11\tilde{\Lambda}^2 + 121\sigma^2\ln(1+\frac{\tilde{\Lambda}^2}{11\sigma^2})}{4\pi^2}, \tag{87}$$

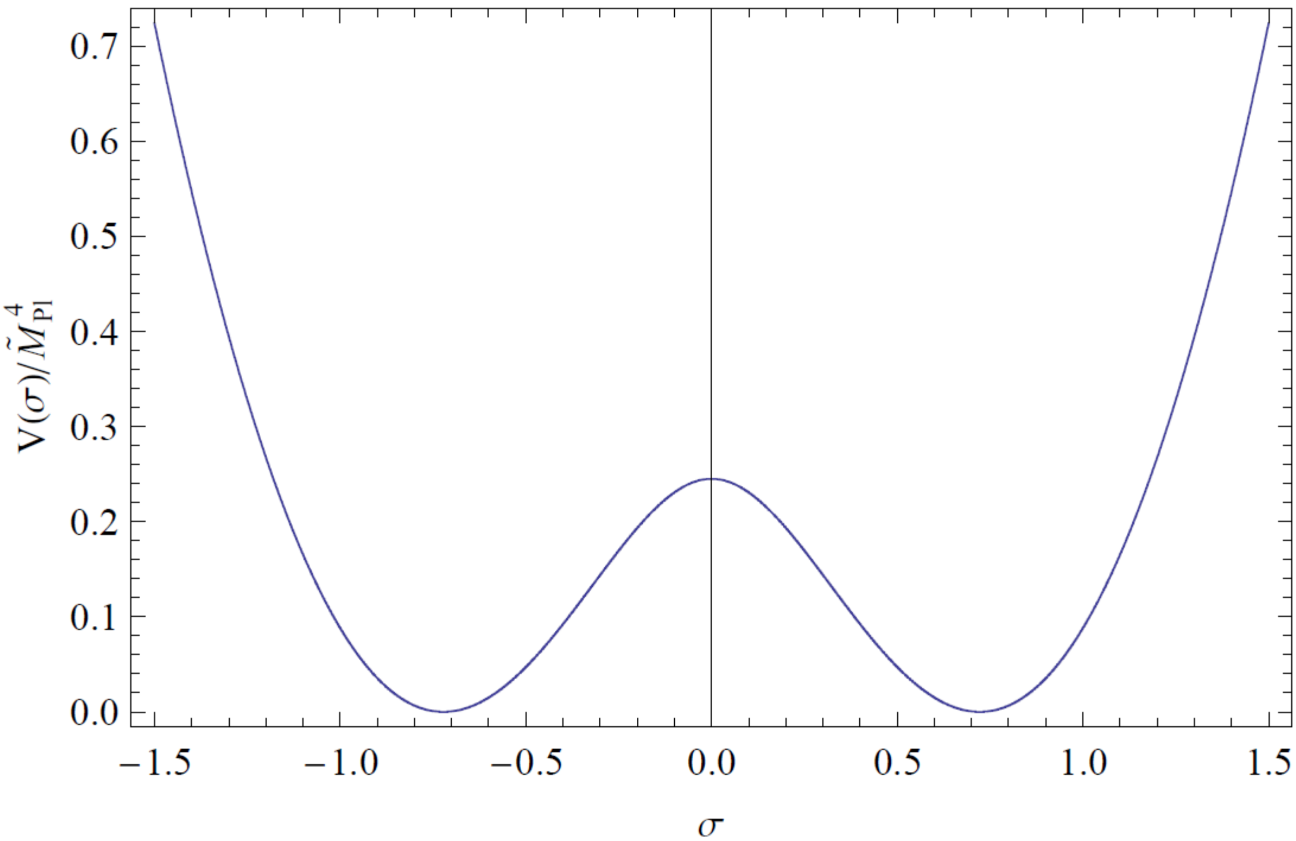

Figure 3: The shape of our potential in equation (84) with $\tilde{\Lambda}$=3.

and take the limit $\sigma \to +0$ and make the result smaller than zero, which leads to the condition

$$\frac{4\pi^2 - 11\tilde{\Lambda}^2}{4\pi^2} < 0 \;\Leftrightarrow\; \tilde{\Lambda}^2 > \frac{4\pi^2}{11} \approx 3.59. \tag{88}$$

By forcing the condition, the shape of the potential in equation (84) looks as in Figure 3.

Next, we derive the order of the ratio of the (reduced) Planck mass scale and the effective gravitational scale ($\tilde{M}_{\rm Pl}/M_{\rm Pl}$). Though inflation requires a potential-dominated expansion $\dot{\phi}^2 < V$, from the Friedmann equation (47) and energy density for a scalar field (49), the height of the potential at maximum $V_{\rm max}(\sigma = 0)$ is given by

$$V_{\rm max} = 3M_{\rm Pl}^2 H_{\rm inf.}^2 \tag{89}$$

From the magnitude of the primordial density perturbations, the constraint

$$\left(\frac{V}{\epsilon}\right)^{\frac{1}{4}} = 0.0275 \times M_{\rm Pl} \tag{90}$$

is imposed on the value of the inflationary potential [18]. Using these equations (89), (90) and the equation (64), inflationary Hubble scale $H_{\rm inf.}$ is related to the tensor-scalar ratio r as

$$\frac{H_{\rm inf.}}{M_{\rm Pl}} = 1.06 \times 10^{-4}\sqrt{r}. \tag{91}$$

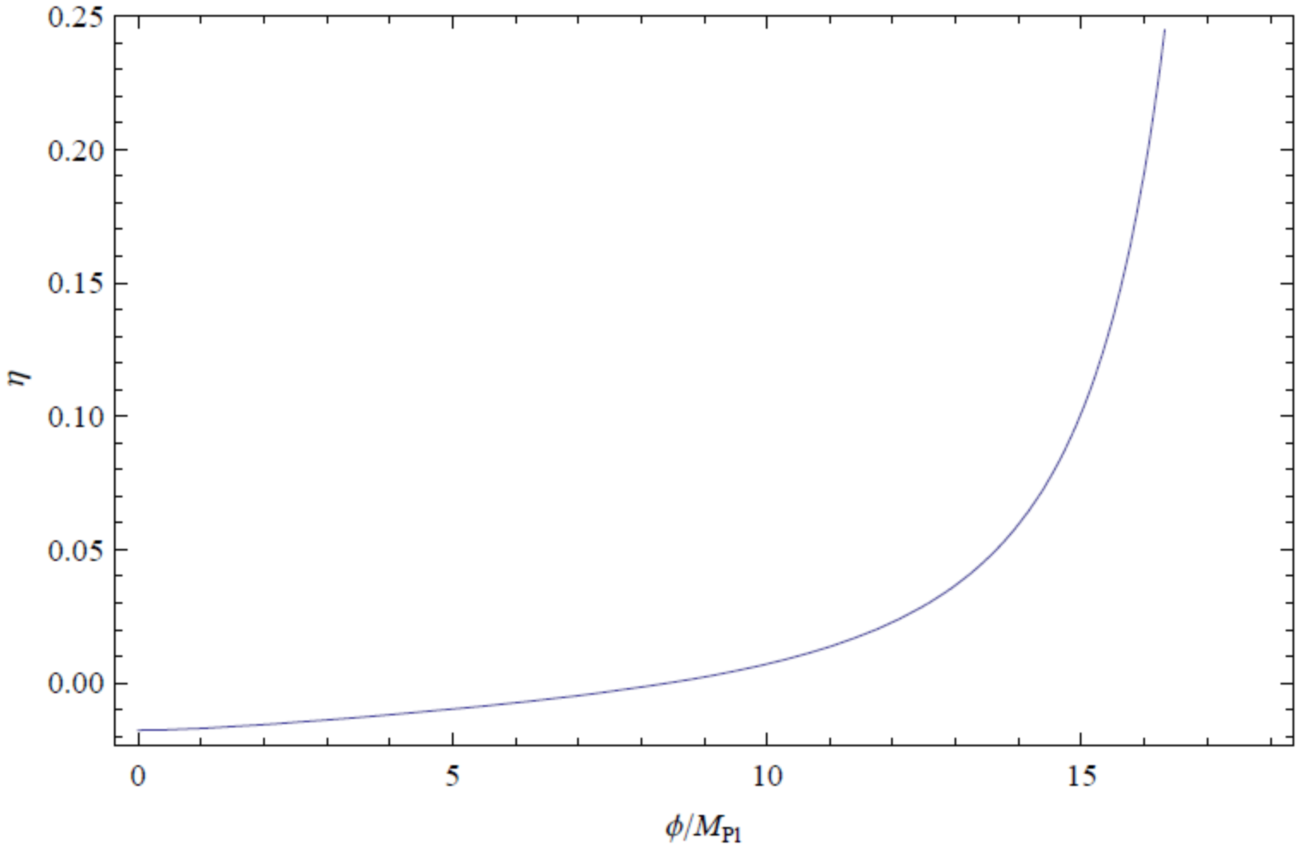

Figure 4: The running of the slow-roll parameter η with $\gamma = 0.5$ and $\tilde{\omega} = 13$

The tensor-scalar ratio r is restricted by the observational results of the Planck 2018 satellite mission (65), and from the restriction, the inflationary Hubble scale is, in turn, restricted. The equation is derived using the previous equation (91) and the value of the (reduced) Planck mass as

$$H_{\text{inf.}} < 6 \times 10^{13}\text{GeV}. \tag{92}$$

Therefore, from the magnitude correlation in equation (85), the ratio $(\tilde{M}_{\text{Pl}}/M_{\text{Pl}})$ should be of the order $10^{-2} \sim 10^{-3} \ll 1$ for viable inflation. We then define a dimensionless parameter γ as

$$\frac{\tilde{M}_{\text{Pl}}}{M_{\text{Pl}}} \equiv \frac{10^{-3}}{\gamma} \tag{93}$$

so that γ becomes of the order one.

We calculate the quantities for the potential and mass of the gravitino numerically from the previous equations by fixing the adjustable parameter $\tilde{\Lambda}$. We choose the cutoff scale as $\tilde{\Lambda} = 3$ to satisfy the condition in equation (88). Therefore, from the equation (84), the height of the potential at maximum $V_{\max} \equiv V(\sigma = 0)$, where the slow-roll occur, is

$$V_{\max}\tilde{M}_{\text{Pl}}^{-4} = 0.245. \tag{94}$$

Also, the value of σ_c, which refers to the Minkovski vacua, is derived from the condition $dV/d(\sigma^2) = 0$, and the result is

$$\sigma_c = \tilde{\omega}^{-1}(\phi_c/M_{\text{Pl}}) = 0.722. \tag{95}$$

In turn, from these values the gravitino mass $m_{3/2}$ and the gravitino condensate mass $M_{\rm cond.}$ are

$$m_{3/2} = 2.39\tilde{M}_{\rm Pl} \text{ and } m_{\rm cond.} = \sqrt{\frac{8}{11}} m_{3/2} = 2.038\tilde{M}_{\rm Pl}. \tag{96}$$

We finally investigate the slow-roll inflation in our model. In order to derive the running of the slow-roll parameters, we rewrite our potential using canonical variable ϕ using equation (82) and (83). The result is

$$V(\phi)\tilde{M}_{\rm Pl}^{-4} = \frac{121\phi^4 \ln(1+\frac{x\tilde{\Lambda}^2}{11\phi^2}) - x\left((-8\pi^2+11\tilde{\Lambda}^2)\phi^2 + x\tilde{\Lambda}^4\ln(1+\frac{11\phi^2}{x\tilde{\Lambda}^2})\right)}{8\pi^2 x^2} + \alpha(\tilde{\Lambda}), \tag{97}$$
$$\text{where } x \equiv \omega^2\tilde{M}_{\rm Pl}^2.$$

By differentiating this potential with respect to the inflaton field ϕ, we get

$$V'(\phi)\tilde{M}_{\rm Pl}^{-4} = \frac{\phi\left(4\pi^2 x - 11x\tilde{\Lambda}^2 + 121\phi^2\ln(1+\frac{x\tilde{\Lambda}^2}{11\phi^2})\right)}{2\pi^2x^2}, \tag{98}$$
$$V''(\phi)\tilde{M}_{\rm Pl}^{-4} = \frac{x\left(4\pi^2(x\tilde{\Lambda}^2+11\phi^2) - 11(x\tilde{\Lambda}^4 + 33\tilde{\Lambda}^2\phi^2)\right) + 363(x\tilde{\Lambda}^2\phi^2 + 11\phi^4)\ln(1+\frac{x\tilde{\Lambda}^2}{11\phi^2})}{2\pi^2x^2(x\tilde{\Lambda}^2+11\phi^2)}.$$

Then we substitute these results for the slow-roll parameters (52) and use the value of $\tilde{\Lambda} = 3$ and the result from equation (94), to get

$$\epsilon = 8\times10^6\gamma^2\left(\Phi\left(-99\omega^2 + 4\pi^2\omega^2 + 121\Phi^2\ln(1+\frac{9\omega^2}{11\Phi^2})\right)\right. \tag{99}$$
$$\left.\Big/\left(121\Phi^4\ln(1+\frac{9\omega^2}{11\Phi^2}) + \omega^2\left(-99\Phi^2 + 8\pi^2(0.245\omega^2+\Phi^2) - 81\omega^2\ln(1+\frac{11\Phi^2}{9\omega^2})\right)\right)\right)^2.$$
$$\eta = -4\times10^6\gamma^2\left(4\pi^2\omega^2(9\omega^2+11\Phi^2) - 11\omega^2(81\omega^2+297\Phi^2) + 363(9\omega^2\Phi^2+11\Phi^4)\ln(1+\frac{9\omega^2}{11\Phi^2})\right)$$
$$\Big/\left((9\omega^2+11\Phi^2)\times\left(-121\Phi^4\ln(1+\frac{9\omega^2}{11\Phi^2}) + \omega^2\left(99\Phi - 8\pi^2(0.245\omega^2+\Phi^2)\right.\right.\right. \tag{100}$$
$$\left.\left.\left. +81\omega^2\ln(1+\frac{11\Phi^2}{9\omega^2})\right)\right)\right),$$
$$\text{where } \Phi \equiv \frac{\phi}{\tilde{M}_{\rm Pl}}.$$

By fixing the parameter γ as 0.1, 0.5 and 1, we have found that ϵ is always under $\mathcal{O}(10^{-4})$, and from (64) it is within the bound of the Planck 2018 data [6]. Also for η, from the value of the scalar index n_s based on the Planck 2018 data, and equation (61), while ignoring ϵ from the result as explained above, the restriction for η becomes $\eta = -0.0177$ at the horizon crossing. This can be satisfied by fixing the parameter $\tilde{\omega}$ as of order one, specifically $\tilde{\omega} = 13$. For the e-folding number (53), without demanding any other constraints on parameters γ and $\tilde{\omega}$, we can have the value between $50 \sim 60$ as is desired for viable candidates of inflation, when assigning the inflaton field $\Phi \equiv \frac{\phi}{\tilde{M}_{\rm Pl}}$ to run somewhere between 0 to 5. The graph for the running of the slow-roll parameter η is shown in Figure 4. The results can be plotted inside the blue region in Figure2, in the middle for n_s and at the bottom for r.

7 Conclusion

We studied a new model of inflation baced on the gravitino condensate in the framework of supergravity. The results obtained in section 6 were compared to the results in [18], and we concluded that they are qualitatively matched but are quantitatively different. The slow-roll parameter, η has the same order in the inflation region $\eta = \mathcal{O}(10^{-2})$, but ϵ has a considerably higher value than $\epsilon = \mathcal{O}(10^{-9})$ in [18], because we get $\epsilon = \mathcal{O}(10^{-4})$. From equation (64), the same result can apply to the tensor-scalar ratio r. Also for the inflationary scale $H_{\text{inf.}}$, we found it gets as high as 10^{12} GeV where $H_{\text{inf.}} = \mathcal{O}(10^{10})$GeV in [18]. Therefore the gravitino condensate can be considered as viable inflaton in supergravity when assuming the effective quantum gravity scale, the (super) GUT scale and the SUSY breaking scale all close to 10^{15} GeV. Comparing to other well known models of inflation, such as R^2 inflationary model, we get lower values of r by one order of the magnitude.

This Master thesis is based on my research conducted together with my supervisor Dr. S.V. Ketov. It is part of a larger project devoted to an investigation of the gravitino condensate in supergravity coupled to the supersymmetric Born-Infeld theory that also includes the goldstino field described by Akulov-Volkov theory [1]. Some important physical issues, such as supersymmetry breaking and cosmological constant, were only briefly mentioned in the text because their detailed study is beyond the scope of this Master thesis. According to [1], supersymmetry is dynamically broken by the gravitino condensate, and the cosmological constant in the classical Born-Infeld action is exactly compensated by the gravitino condensate contribution.

References

[1] **R. Ishikawa** and S. V. Ketov. Gravitino condensate in $N = 1$ supergravity coupled to the $N = 1$ supersymmetric Born-Infeld theory. arXiv:1904.08586 [hep-th]. Prog. Theor. Exp. Phys. 2020, 013B05 (13 pages); DOI:10.1093/ptep/plz152.

[2] Y. Nambu and G. Jona-Lasinio. Dynamical model of elementary particles based on an analogy with superconductivity. I. *Phys. Rev.*, 122:345–358, 1961.

[3] Y. Nambu and G. Jona-Lasinio. Dynamical model of elementary particles based on an analogy with superconductivity. II. *Phys. Rev.*, 124:246–254, 1961.

[4] R.S. Jasinschi and A. W. Smith. Effective potential in N = 1, d = 4 supergravity coupled to the Volkov-Akulov field. *Physics Letters B*, 174(2):183 – 185, 1986.

[5] R.S. Jasinschi and A. W. Smith. Dynamical mass generation for the gravitino in N = 1 supergravity. *Physics Letters B*, 173(3):297 – 302, 1986.

[6] Y. Akrami et al. Planck 2018 results. X. Constraints on inflation. 2018. arXiv:1807.06211 [astro-ph.CO].

[7] P. van Nieuwenhuizen. Supergravity. *Physics Reports*, 68(4):189 – 398, 1981.

[8] W. Rarita and J. Schwinger. On a theory of particles with half-integral spin. *Phys. Rev.*, 60:61–61, Jul 1941.

[9] S. Weinberg. *The quantum theory of fields. Vol. 3: Supersymmetry*. Cambridge University Press, 2013.

[10] P. K. Townsend and P. van Nieuwenhuizen. Geometrical Interpretation of Extended Supergravity. *Phys. Lett.*, 67B:439–442, 1977.

[11] A. H. Chamseddine and P. C. West. Supergravity as a Gauge Theory of Supersymmetry. *Nucl. Phys.*, B129:39–44, 1977.

[12] D. Z. Freedman, P. van Nieuwenhuizen, and S. Ferrara. Progress Toward a Theory of Supergravity. *Phys. Rev.*, D13:3214–3218, 1976.

[13] S. Deser and B. Zumino. Consistent Supergravity. *Phys. Lett.*, 62B:335, 1976. [,335(1976)].

[14] Zhu-Fang Cui, Jin-Li Zhang, and Hong-Shi Zong. Proper time regularization and the QCD chiral phase transition. *Sci. Rep.*, 7:45937, 2017.

[15] D. Langlois. Inflation, quantum fluctuations and cosmological perturbations. In *Particle physics and cosmology: The interface. Proceedings of NATO Advanced Study Institute, School, Cargese, France, August 4-16, 2003*, pages 235–278, 2004. arXiv:hep-th/0405053.

[16] Y. Aldabergenov, **R. Ishikawa**, S. V. Ketov, and S. I. Kruglov. Beyond Starobinsky inflation. *Phys. Rev.*, D98(8):083511, 2018. DOI: 10.1103/PhysRevD.98.083511.

[17] A. W. Smith, J. A. Helayël-Neto, and F. A. B. Rabelo de Carvalho. Gravitino auxiliary fields of $N=1$, $d=10$ supergravity theory. *Phys. Rev. D*, 33:3779–3780, Jun 1986.

[18] J. Ellis and N. E. Mavromatos. Inflation induced by gravitino condensation in supergravity. *Phys. Rev.*, D88(8):085029, 2013. arXiv:1308.1906 [hep-th].

Beyond Starobinsky inflation

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A supergravity extension of the $(R + R^2)$ gravity with the additional (Born-Infeld) structure of a massive vector multiplet gives rise to the specific $F(R)$ gravity, whose structure is investigated in detail. The massive vector multiplet has an inflaton (scalaron), goldstino, and massive vector field as its field components. The model describes Starobinsky inflation and allows us to extrapolate the $F(R)$ function beyond the inflationary scale (up to Planck scale). We observe some differences versus the $(R + R^2)$ gravity and several breaking patterns of the well-known correspondence between the $F(R)$ gravity and the scalar-tensor gravity.

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I. INTRODUCTION

An ultimate theory of cosmological inflation should be based on quantum gravity and is yet to be constructed. This is related to another open problem of finding an ultraviolet (UV) completion of any phenomenologically viable inflationary model. Among the most successful and popular inflationary models, the Starobinsky inflationary model of $(R + R^2)$ gravity [1] is special because it is entirely based on gravitational interactions. This model is, however, nonrenormalizable and has the UV cutoff given by Planck scale. In addition, when extrapolating the $(R + R^2)$ gravity beyond the inflationary scale of about 10^{13} GeV, i.e., when going to the very large curvature regime, we are left with the scale-invariant R^2 gravity. The original motivation in [1] was to get rid of the initial singularity of Einstein-Friedmann gravity, in addition to describing inflation in the early Universe. However, demanding the asymptotical scale invariance at very high energies is clearly not the only option. Hence, the open question remains: what should we expect beyond Starobinsky inflation?

To address this question at least partially, one needs a motivated extension of the $(R + R^2)$ gravity in a specific framework. In this paper, we address the issue in four-dimensional $N = 1$ supergravity. The importance of the inflationary model building in supergravity stems from the natural objective to unify gravity with particle physics beyond the standard model of elementary particles and beyond the standard (ΛCDM) model of cosmology; see, e.g., [2,3] for a review.

Though supergravity can be considered as the low-energy effective theory of (compactified) superstrings, and the latter can be viewed as a consistent theory of quantum gravity, we obviously need more specific assumptions.

Our additional specific assumptions in this paper are the following:

(i) Starobinsky inflationary model should be embedded into a four-dimensional $N = 1$ supergravity, with linearly realized (manifest) local supersymmetry,
(ii) inflaton (scalaron) should belong to a massive $N = 1$ vector supermultiplet,
(iii) the kinetic terms of the vector supermultiplet should have the Born-Infeld (or Dirac-Born-Infeld) structure, inspired by superstrings and D-branes.

This leads to the specific (modified) $F(R)$ gravity model, whose peculiar structure is in the focus of our investigation in this paper.

Our paper is organized as follows. In Sec. II, we outline Born-Infeld (BI) nonlinear electrodynamics and

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the supergravity theory with the BI structure. In Sec. III, we review the Starobinsky inflationary model. In Sec. IV, we study in detail the $F(R)$ gravity extension of the $(R+R^2)$ gravity, originating from the supergravity theory. In Sec. V, we present the dual description of the same $F(R)$ gravity in terms of the scalar-tensor gravity. Our conclusion is in Sec. VI. In the Appendix, we formulate the full supergravity theory in terms of superfields in curved superspace.

II. BORN-INFELD STRUCTURE IN GRAVITY AND SUPERGRAVITY

The Born-Infeld (BI) Lagrangian was originally introduced [4] as a nonlinear generalization of the Lagrangian of Maxwell electrodynamics in terms of the Abelian field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$,

$$L_{\rm BI} = -b^{-2}\left[\sqrt{-\det\left(\eta_{\mu\nu} + \frac{b}{e}F_{\mu\nu}\right)} - 1\right] = -\frac{1}{4e^2}F^{\mu\nu}F_{\mu\nu} + \mathcal{O}(F^4), \qquad (1)$$

where we have introduced the dimensional (BI) coupling constant $b = M_{\rm BI}^{-2}$ and the gauge (dimensionless) coupling constant e. Being minimally coupled to gravity, the BI action reads

$$S_{\rm BI} = b^{-2}\int d^4x\left[\sqrt{-g} - \sqrt{-\det\left(g_{\mu\nu} + \frac{b}{e}F_{\mu\nu}\right)}\right]. \qquad (2)$$

This BI structure also arises (i) in the bosonic part of the open superstring effective action [5], (ii) as part of the Dirac-Born-Infeld effective action of a D3-brane [6], and (iii) as part of the Maxwell-Goldstone action describing partial supersymmetry breaking of $N=2$ supersymmetry to $N=1$ supersymmetry [7,8]. In string theory, $b = 2\pi\alpha'$, while the BI scale $M_{\rm BI}$ does not have to coincide with $M_{\rm Pl}$.[1]

In $N=1$ supersymmetry and supergravity, a vector field belongs to an $N=1$ vector multiplet, whose supergravity couplings are naturally (off-shell) described in superconformal tensor calculus [11] and in curved superspace [12]. A massive $N=1$ vector multiplet has a single (real) scalar field amongst its bosonic field components, in addition to a massive vector field. In this paper, we identify this real scalar with inflaton, and unify it with the massive vector field whose kinetic terms are assumed to have the BI structure in $N=1$ supergravity (we do not assume any relation between our massive vector field and electromagnetic field).

The full action of the self-interacting massive vector multiplet with the BI structure in supergravity is very complicated: it was found by using the superconformal tensor calculus in [13], and we present this action in the Appendix, by using superfields in curved superspace.[2] In particular, local supersymmetry (SUSY) is spontaneously broken in this theory (after inflation also), while goldstino is identified with a massive "photino" in the same vector multiplet with inflaton.

For our purposes in this paper, it is enough to notice that in the dual (modified supergravity) picture the BI structure just leads to the presence of the contribution $12R^2/(e^2M_{\rm BI}^4)$ under the square root of the BI term, in addition to the $F_{\mu\nu}$-dependent terms there. When ignoring all other interactions besides the modified gravity itself (i.e., keeping only the R-dependent terms), it gives rise to the following $F(R)$ gravity model (see Ref. [13] and the Appendix):

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \frac{M_{\rm BI}^4}{3}\left(\sqrt{1+\frac{12R^2}{e^2M_{\rm BI}^4}} - 1\right)\right]. \qquad (3)$$

It is this modified gravity theory that is the main subject of our investigation in this paper. It is worth noticing that it does not imply the upper bound on the values of R, unlike the original BI theory (1) that limits the maximal values of the gauge field strength components.

It is worth noticing here that the idea of finding a "BI-extension" of Einstein gravity is old but still popular, although it lacks a good definition and guiding principles; see, e.g., [17] for classification of many such extensions in gravitational theory and [18] for other proposals to an $F(R)$ gravity function of the BI-type.

A "BI-extension" of $N=1$ supergravity is more restrictive, but it suffers similar problems; see, e.g., [19] for some specific proposals of BI supergravity in curved superspace. Equation (3) is just the specific extension of Starobinsky $(R+R^2)$ gravity in the framework of $F(R)$ gravity derived from supergravity and inspired by string theory. It is directly related to the BI action (1) that arises together with the $F(R)$ gravity (3) in the same supergravity theory having the BI structure.

It is also worth mentioning that Starobinsky inflation is equivalent to the so-called Higgs inflation in gravity and supergravity, because both lead to the same inflationary observables [20].

III. STAROBINSKY INFLATION AND $F(R)$ GRAVITY

The Starobinsky model of inflation is defined by the action [1]

$$S_{\rm Star} = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}\left(R + \frac{1}{6m^2}R^2\right), \qquad (4)$$

[1]See also [9,10] for more about special properties of the BI action and its supersymmetric extensions.

[2]See also [14–16] for related papers.

where we have introduced the reduced Planck mass $M_{\rm Pl} = 1/\sqrt{8\pi G_{\rm N}} \approx 2.4 \times 10^{18}$ GeV, and the scalaron (inflaton) mass m as the only parameter. We use the spacetime signature $(-,+,+,+,)$. The $(R+R^2)$ gravity model (4) can be considered as the simplest extension of the standard Einstein-Hilbert action in the context of (modified) $F(R)$ gravity theories with an action

$$S_F = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}F(R), \qquad (5)$$

in terms of the function $F(R)$ of the scalar curvature R.

The $F(R)$ gravity action (5) is classically equivalent to

$$S[g_{\mu\nu},\chi] = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}[F'(\chi)(R-\chi)+F(\chi)] \qquad (6)$$

with the real scalar field χ, provided that $F'' \neq 0$ that we always assume. Here the primes denote the derivatives with respect to the argument. The equivalence is easy to verify because the χ-field equation implies $\chi = R$. In turn, the factor F' in front of the R in (6) can be (generically) eliminated by a Weyl transformation of metric $g_{\mu\nu}$, that transforms the action (6) into the action of the scalar field χ minimally coupled to Einstein gravity and having the scalar potential

$$V = \left(\frac{M_{\rm Pl}^2}{2}\right)\frac{\chi F'(\chi) - F(\chi)}{F'(\chi)^2}. \qquad (7)$$

Differentiating this scalar potential yields

$$\frac{dV}{d\chi} = \left(\frac{M_{\rm Pl}^2}{2}\right)\frac{F''(\chi)[2F(\chi)-\chi F'(\chi)]}{(F'(\chi))^3}. \qquad (8)$$

The kinetic term of χ becomes canonically normalized after the field redefinition $\chi(\varphi)$ as

$$F'(\chi) = \exp\left(\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right), \qquad \varphi = \frac{\sqrt{3}M_{\rm Pl}}{\sqrt{2}}\ln F'(\chi), \qquad (9)$$

in terms of the canonical inflaton field φ, with the total acton

$$S_{\rm quintessence}[g_{\mu\nu},\varphi] = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}R - \int {\rm d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right]. \qquad (10)$$

The classical and quantum stability conditions of $F(R)$ gravity theory are given by [3]

$$F'(R) > 0 \quad \text{and} \quad F''(R) > 0, \qquad (11)$$

and they are obviously satisfied for the Starobinsky model (4) for $R > 0$.

Differentiating the scalar potential V in Eq. (7) with respect to φ yields

$$\frac{dV}{d\varphi} = \frac{dV}{d\chi}\frac{d\chi}{d\varphi} = \frac{M_{\rm Pl}^2}{2}\left[\frac{\chi F''+F'-F'}{F'^2} - 2\frac{\chi F'-F}{F'^3}F''\right]\frac{d\chi}{d\varphi}, \qquad (12)$$

where we have

$$\frac{d\chi}{d\varphi} = \frac{d\chi}{dF'}\frac{dF'}{d\varphi} = \frac{dF'}{d\varphi}\Big/\frac{dF'}{d\chi} = \frac{\sqrt{2}}{\sqrt{3}M_{\rm Pl}}\frac{F'}{F''}. \qquad (13)$$

This implies

$$\frac{dV}{d\varphi} = M_{\rm Pl}\frac{2F-\chi F'}{\sqrt{6}F'^2}. \qquad (14)$$

Combining Eqs. (7) and (14) yields R and F in terms of the scalar potential V,

$$R = \left[\frac{\sqrt{6}}{M_{\rm Pl}}\frac{dV}{d\varphi} + \frac{4V}{M_{\rm Pl}^2}\right]\exp\left(\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right), \qquad (15)$$

$$F = \left[\frac{\sqrt{6}}{M_{\rm Pl}}\frac{dV}{d\varphi} + \frac{2V}{M_{\rm Pl}^2}\right]\exp\left(2\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right). \qquad (16)$$

These equations define the function $F(R)$ in the parametric form, in terms of a scalar potential $V(\varphi)$, i.e., the inverse transformation to (7). This is known as the classical equivalence (duality) between the $F(R)$ gravity theories (5) and the scalar-tensor (quintessence) theories of gravity (10).

In the case of Starobinsky model (4), one gets the famous potential

$$V(\varphi) = \frac{3}{4}M_{\rm Pl}^2 m^2\left[1-\exp\left(-\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right)\right]^2. \qquad (17)$$

This scalar potential is bounded from below (non-negative and stable), and it has the absolute minimum at $\varphi = 0$ corresponding to a Minkowski vacuum. The scalar potential (17) also has a plateau of positive height (related to inflationary energy density) that gives rise to the slow roll of the inflaton in the inflationary era. The Starobinsky model (4) is the particular case of the so-called α-attractor inflationary models [21] and is also a member of the close family of viable inflationary models of $F(R)$ gravity, originating from higher dimensions [22].

A duration of inflation is measured in the slow roll approximation by the e-foldings number

$$N_e \approx \frac{1}{M_{\rm Pl}^2} \int_{\varphi_{\rm end}}^{\varphi_*} \frac{V}{V'} d\varphi, \tag{18}$$

where φ_* is the inflaton value at the reference scale (horizon crossing), and $\varphi_{\rm end}$ is the inflaton value at the end of inflation when one of the slow roll parameters

$$\varepsilon_V(\varphi) = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta_V(\varphi) = M_{\rm Pl}^2\left(\frac{V''}{V}\right), \tag{19}$$

is no longer small (close to 1).

The amplitude of scalar perturbations at horizon crossing is given by [23]

$$A = \frac{V_*^3}{12\pi^2 M_{\rm Pl}^6 (V_*')^2} = \frac{3m^2}{8\pi^2 M_{\rm Pl}^2} \sinh^4\left(\frac{\varphi_*}{\sqrt{6}M_{\rm Pl}}\right). \tag{20}$$

The Starobinsky model (4) is the excellent model of cosmological inflation, in very good agreement with the Planck data [24–26]. The Planck satellite mission measurements of the cosmic microwave background (CMB) radiation [24–26] give the scalar perturbations tilt as $n_s \approx 1 + 2\eta_V - 6\varepsilon_V \approx 0.968 \pm 0.006$ and restrict the tensor-to-scalar ratio as $r \approx 16\varepsilon_V < 0.08$. The Starobinsky inflation yields $r \approx 12/N_e^2 \approx 0.004$ and $n_s \approx 1\text{–}2/N_e$, where N_e is the e-foldings number between 50 and 60, with the best fit at $N_e \approx 55$ [27,28].

The Starobinsky model (4) is geometrical (based on gravity only), while its (mass) parameter m is fixed by the observed CMB amplitude (COBE, WMAP) as

$$m \approx 3 \times 10^{13}\ \text{GeV} \quad \text{or} \quad \frac{m}{M_{\rm Pl}} \approx 1.3 \times 10^{-5}. \tag{21}$$

A numerical analysis of (18) with the potential (17) yields [23]

$$\sqrt{\frac{2}{3}}\varphi_*/M_{\rm Pl} \approx \ln\left(\frac{4}{3}N_e\right) \approx 5.5$$

$$\text{and} \quad \sqrt{\frac{2}{3}}\varphi_{\rm end}/M_{\rm Pl} \approx \ln\left[\frac{2}{11}(4+3\sqrt{3})\right] \approx 0.5, \tag{22}$$

where we have used $N_e \approx 55$.

IV. BI-MODIFIED STAROBINSKY MODEL

In accordance to (5), the modified gravity theory (3) has

$$F(R) = R + \frac{2g^2}{3\beta}\left(\sqrt{1+12\beta R^2} - 1\right), \tag{23}$$

where we have introduced the parameters $g = 1/(eM_{\rm Pl})$ and $\beta = 1/(e^2 M_{\rm BI}^4)$. In this parametrization, our F-function (23) exactly agrees with Eq. (37) of Ref. [13].

When assuming $12\beta R^2 \ll 1$, the function (23) gives rise to the $(R + R^2)$ gravity model of Starobinsky in (4), as it should. It allows us to identify

$$g^2 = \frac{1}{24m^2} \quad \text{and} \quad e^2 = 24\left(\frac{m}{M_{\rm Pl}}\right)^2 \approx 4 \times 10^{-9}, \tag{24}$$

where we have used (21). In terms of the dimensionless quantities $\tilde{F} = F/M_{\rm Pl}^2$ and $\tilde{R} = R/M_{\rm Pl}^2$, and the dimensionless parameters

$$\alpha = \frac{M_{\rm BI}}{M_{\rm Pl}} \quad \text{and} \quad \tilde{\gamma} = e\alpha^2, \tag{25}$$

we have the dimensionless function,

$$\tilde{F}(\tilde{R}) = \tilde{R} + \frac{2}{3}\alpha^4\left(\sqrt{1+12\tilde{R}^2/\tilde{\gamma}^2} - 1\right) \tag{26}$$

A global shape of this function is given in Fig. 1.

The physical conditions imply the range $\tilde{R} \in [-1, 1]$ (i.e., up to the UV cutoff) and $\alpha \in [0.01, 1]$ (i.e., between the grand unification scale and Planck scale), so that $\tilde{\gamma} \in 6.3 \cdot [10^{-7}, 10^{-5}]$. The Starobinsky inflation takes place for $0 < \tilde{R} \ll \tilde{\gamma}$.

The function (23) is well defined for any values of R and implies three physical regimes:

(i) the small curvature regime, where gravity is described by the standard Einstein-Hilbert action,

(ii) the inflationary regime, where gravity is described by Starobinsky $(R + R^2)$ action (4),

(iii) the high curvature regime, where gravity is again described by the Einstein-Hilbert action, though with the different (larger) effective Planck scale $M_{\rm Pl,effective} = M_{\rm Pl}(1 + 4g^2/\sqrt{3\beta})^{1/2} \le 189 M_{\rm Pl}$ for large positive values of R.

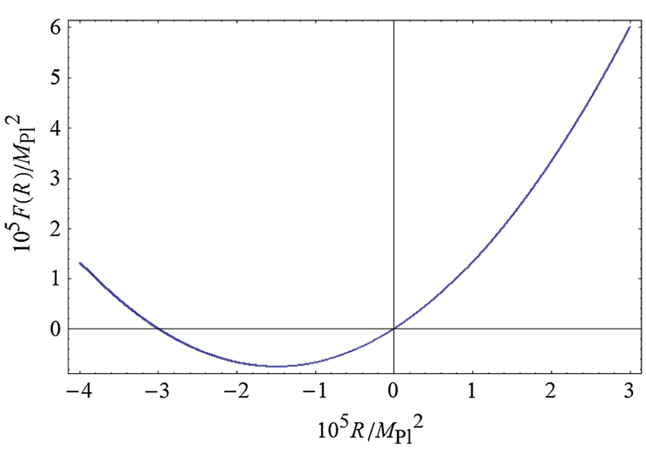

FIG. 1. The profile of the $F(R)$ gravity function (23) for $\alpha = 1$ and $\tilde{\gamma}^{-2} = 10^5$. This value of the parameter $\tilde{\gamma}$ is only chosen to demonstrate the global shape of the function.

Static solutions to the $F(R)$ gravity field equations with $R = \mathrm{const} \equiv R_0$ follow from our Eqs. (8) and (14) and are given by solutions to the algebraic equation [29]

$$RF'(R) = 2F(R), \tag{27}$$

In our case (23), with

$$F'(R) = 1 + \frac{8g^2 R}{\sqrt{1+12\beta R^2}} > 0 \quad \text{for } R \geq 0, \tag{28}$$

we find

$$\frac{8g^2 R_0^2}{\sqrt{1+12\beta R_0^2}} = R_0 + \frac{4g^2}{3\beta}\left(\sqrt{1+12\beta R_0^2} - 1\right) \tag{29}$$

that gives rise to the condition

$$R_0\left[4(16g^4 - 3\beta)R_0^3 + 32g^2 R_0^2 - R_0 + \frac{8g^2}{3\beta}\right] = 0. \tag{30}$$

Besides the trivial solution $R_0 = 0$ corresponding to a stable Minkowski vacuum, any other real positive solution $(R_0 > 0)$ must obey the cubic equation,

$$aR_0^3 + bR_0^2 + cR_0 + d = 0, \tag{31}$$

whose coefficients are $a = 4(16g^4 - 3\beta)$, $b = 32g^2$, $c = -1$ and $d = 8g^2/(3\beta)$. By using the standard replacement,

$$y = R_0 + \frac{b}{3a}, \tag{32}$$

we can bring (31) to the canonical form,

$$y^3 + 3py + 2q = 0, \tag{33}$$

where we have

$$2q = \frac{2b^3}{27a^3} - \frac{bc}{3a^2} + \frac{d}{a} = \frac{4g^2(1152g^8 - 104g^4\beta + 27\beta^2)}{27\beta(16g^4 - 3\beta)^3}, \tag{34}$$

and

$$3p = \frac{3ac - b^2}{3a^2} = \frac{9\beta - 304g^4}{12(16g^4 - 3\beta)^2}. \tag{35}$$

The number of real solutions depends upon the sign of the cubic discriminant $D = q^2 + p^3$ that in our case reads

$$D = \frac{(144g^4 + \beta)(32g^4 + 3\beta)^2}{5184\beta^2(16g^4 - 3\beta)^4}. \tag{36}$$

Since $D > 0$, there is only one real solution. Our numerical studies show that this root R_0 is negative (e.g., with $\alpha = 1$ we find $R_0 \approx -8.7 \times 10^{-7} M_{\mathrm{Pl}}^2$).

The second derivative of the $F(R)$ gravity function (23),

$$F''(R) = \frac{8g^2}{(1+12\beta R^2)^{3/2}} > 0, \tag{37}$$

can be compared to the laboratory bound of the Eöt-Wash experiment [30], $F''(0) \leq 2 \times 10^{-6}\ \mathrm{cm}^2$, or

$$g < 0.5 \times 10^{-3}\ \mathrm{cm}^2, \tag{38}$$

which is well satisfied because of (21) and (24).

V. SCALAR-TENSOR GRAVITY AND THE INFLATON SCALAR POTENTIAL

It is instructive to study the same gravitational model (3) in the dual (scalar-tensor gravity) picture defined by (7), (9), and (10). The classical equivalence (duality) between the $F(R)$ gravity theories and their scalar-tensor gravity (or quintessence) counterparts is well known; see, e.g., [31].

Our Eq. (9) implies

$$\frac{\tilde{R}}{\tilde{\gamma}} = \frac{\frac{1}{2}\tilde{\gamma}(1 - e^{-\sqrt{2/3}\tilde{\varphi}})}{\sqrt{16\alpha^2 - 3\tilde{\gamma}^2(1 - e^{-\sqrt{2/3}\tilde{\varphi}})^2}}, \tag{39}$$

where we have introduced the dimensionless inflaton field $\tilde{\varphi} = \varphi/M_{\mathrm{Pl}}$. Actually, (9) determines R^2 as the function of φ, and our sign choice in (39) comes from demanding a plateau of the scalar potential at positive values of R.

In turn, our Eq. (7) yields

$$\tilde{V} = \frac{\alpha^4}{3}\sqrt{1 + 12\tilde{R}^2/\tilde{\gamma}^2}\,\frac{\sqrt{1+12\tilde{R}^2/\tilde{\gamma}^2} - 1}{(8\alpha^4\tilde{\gamma}^{-1}(\tilde{R}/\tilde{\gamma}) + \sqrt{1+12\tilde{R}^2/\tilde{\gamma}^2})^2}, \tag{40}$$

where we have introduced the dimensionless scalar potential $\tilde{V} = V/M_{\mathrm{Pl}}^4$. The scalar potential $\tilde{V}(\tilde{\varphi})$ is obtained via a substitution of (39) into (40), while the value of the parameter $\tilde{\gamma}$, according to Secs. III and IV, is given by $\tilde{\gamma} \approx 6.3 \times 10^{-5}\alpha^2$.

A profile of the scalar potential is given in Fig. 2.

As expected, the scalar potential $V(\varphi)$ has a plateau for positive values of φ and R, which corresponds to Starobinsky inflation (Sec. III). As is clear from (39), the higher the values of φ and R are, the closer the potential $V(\varphi)$ to the Starobinsky potential (17) with $V_{\max} = \frac{3}{4}m^2 M_{\mathrm{Pl}}^2$ is. Hence, the BI structure does not play a significant role for positive values of φ and R.

When formally sending $\varphi \to +\infty$ in (39), we get $\tilde{R}_{\max} = \frac{\tilde{\gamma}^2}{2\sqrt{16\alpha^2 - 3\tilde{\gamma}^2}} > 0$. The scalar-tensor gravity description does not exist for $\tilde{R} > \tilde{R}_{\max}$, whereas the $\tilde{F}(\tilde{R})$ gravity

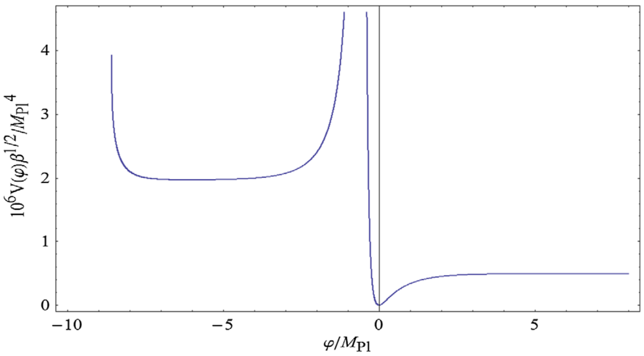

FIG. 2. The profile of the $V(\varphi)$ function (40) for $\alpha = 1$ and $\tilde{\gamma} = 6.3 \times 10^{-5}$. This function is not well defined for all values of $\tilde{\varphi}$. It reproduces the inflationary potential (17) for the relevant values of $\tilde{\varphi}$ (Sec. III). The infinite maximum occurs at $\tilde{\varphi} \approx -0.6$ that corresponds to $\tilde{R} \approx -5 \times 10^{-10}$. The only minimum occurs at $\tilde{\varphi} \approx -6.5$ that corresponds to the root $\tilde{R}_0 \approx -8.7 \times 10^{-7}$ found in Sec. IV. The wall on the left-hand side, where V sharply goes up to infinity, appears at $\tilde{\varphi} \approx -9$.

description (26) is well defined there. This is an explicit example of breaking the naive equivalence between the two dual descriptions.

Though the scalar potential $V(\varphi)$ cannot be trusted for large negative values of φ and R, because of intense particle production (reheating) starting near the absolute minimum of the scalar potential, it is instructive to illustrate two more breaking patterns of the naive equivalence between $F(R)$ gravity theories and scalar-tensor gravity theories in our specific example.[3]

First, we observe the infinite maximum of the scalar potential in Fig. 2. It happens when the expression under the root in the denominator of (40) vanishes, that corresponds to zero of $F'(R)$ in (7) at a negative value of R. Since this occurs at a finite value of R, it represents an example of the broken correspondence, when the $F(R)$ gravity description is regular, but the scalar-tensor description is singular.

Second, yet another example of the broken correspondence is given by the wall on the left-hand side of Fig. 2. This wall appears when the expression under the root in the denominator of (39) vanishes at a finite value of φ that gives rise to the infinite values of R and the scalar potential $V(\varphi)$, although the value of $V(R)$ remains finite. Beyond the wall, the scalar-tensor gravity description does not exist in our case.

VI. CONCLUSION

Our main results are given in Secs. IV and V. They provide a viable extension of Starobinsky $(R + R^2)$ inflationary model, motivated by the Born-Infeld structure in supergravity, in turn, motivated by string theory.

Our physical motivation is to explore the range of energies beyond the Starobinsky inflationary scale of approximately 10^{13} GeV up to the (reduced) Planck scale of approximately 10^{18} GeV, by using the specific modified gravity function (3) derived from the supergravity model under our assumptions formulated in Sec. I.

The significant deviation between our modified $F(R)$ gravity model and Starobinsky $(R + R^2)$ gravity model takes place only for very large positive curvature, with the asymptotic R^2 gravity being replaced by the asymptotic Einstein-Hilbert gravity having a larger effective Planck scale. The corresponding values of the inflaton field are trans-Planckian, so that the asymptotic gravity is supposed to be considered with a grain of salt, because it may be affected by quantum gravity effects.

On the other side, we found explicit examples of breaking the naive correspondence between the $F(R)$ gravity theories and the scalar-tensor gravity theories in our model. They are, however, of academic interest in the inflationary physics context, because they occur at large negative values of the curvature.

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APPENDIX: SUPERGRAVITY WITH BI STRUCTURE IN SUPERSPACE

The supersymmetric extension of the $(R + R^2)$ gravity (with Maxwell structure) in the new-minimal formulation of $N = 1$ supergravity is given by Eq. (38) of Ref. [13] in the superconformal tensor calculus. In curved superspace, with $M_{\mathrm{Pl}} = 1$, the Lagrangian reads [32,33]

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\left(-\frac{3}{16}\bar{\mathcal{D}}^2 V_{\mathrm{R}} + \frac{\gamma}{4} W^\alpha(V_{\mathrm{R}})W_\alpha(V_{\mathrm{R}})\right) + \mathrm{H.c.}, \tag{A1}$$

where V_{R} is the gauge multiplet of SUSY algebra, representing the new-minimal set of supergravity field components, W_α is its superfield strength, and $\gamma \sim e^{-2}$ is the R^2 parameter. The superfield V_{R} has the following bosonic components (in a Wess-Zumino gauge):

$$\bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{D}_\alpha V_{\mathrm{R}}| = 2\sigma^m_{\alpha\dot{\alpha}} A_m,$$
$$\bar{\mathcal{D}}^2\mathcal{D}^2 V_{\mathrm{R}}| = \frac{32}{3} b_m A^m + 16 D_{\mathrm{R}}, \tag{A2}$$

[3] Our considerations are formally based on Eqs. (39) and (40) only, ignoring Eq. (9).

where A_m is the (dynamical) gauge field,

$$D_{\rm R} = \frac{1}{3}\left(R + \frac{3}{2}B_m B^m\right)$$

is the gravitational D-term, and B_m is the auxiliary vector field of supergravity multiplet. The old-minimal set of supergravity is also present via $\mathcal{E}$ and $\mathcal{R}$ that is hidden in the definition of the superfield strength $W_\alpha \equiv -\frac{1}{4}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{D}_\alpha V_{\rm R}$.

After identifying the "old" auxiliary field b_m with the "new" auxiliary field B_m as $b_m = -\frac{3}{2}B_m$, we can expand the Lagrangian (A1) as follows:

$$e^{-1}\mathcal{L} = \frac{1}{2}R + \frac{3}{4}B_m B^m - \frac{3}{2}B_m A^m - \frac{1}{4e^2}F_{mn}F^{mn} + \frac{2}{e^2}\left(R + \frac{3}{2}B_m B^m\right)^2 + \cdots, \tag{A3}$$

where we have kept only the relevant terms. When allowing the superfield $V_{\rm R}$ to be massive (or not using a WZ gauge), the complex scalar M of the old-minimal set [12] also appears.

The BI extension of the supergravity theory (A1) can be written down as follows:

$$\mathcal{L} = \left(-\frac{3}{16}\int d^2\Theta 2\mathcal{E}\bar{\mathcal{D}}^2 V_{\rm R} + {\rm H.c.}\right) + \frac{\gamma}{4}\int d^4\theta E W^2 \bar{W}^2 \Lambda, \tag{A4}$$

where the BI structure function Λ is given by (see, e.g., Ref. [9])

$$\Lambda \equiv \frac{\kappa}{1 + \kappa(\omega + \bar{\omega}) + \sqrt{1 + \kappa(\omega + \bar{\omega}) + \frac{\kappa^2}{4}(\omega - \bar{\omega})^2}}, \tag{A5}$$

with $\omega \equiv \mathcal{D}^2 W^2/8$ and the BI coupling $\kappa = b^{-2} = M_{\rm BI}^{-4}$. The Lagrangian (A4) can be expanded as

$$e^{-1}\mathcal{L} = \frac{1}{2}R + \frac{3}{4}B_m B^m - \frac{3}{2}B_m A^m + \frac{M_{\rm BI}^4}{3}\left(\sqrt{1 - \frac{3}{2M_{\rm BI}^4 e^2}\left(F^2 - 8\left(R + \frac{3}{2}B_m B^m\right)^2\right) + \left(\frac{3}{4M_{\rm BI}^4 e^2}\right)^2 (F\tilde{F})^2} - 1\right) + \cdots, \tag{A6}$$

where we have kept only the relevant terms. Using $B_m = F_{mn} = 0$ as a solution, we get (3).

[1] A. A. Starobinsky, A new type of isotropic cosmological models without singularity, Phys. Lett. **91B**, 99 (1980).
[2] M. Yamaguchi, Supergravity based inflation models: A review, Classical Quantum Gravity **28**, 103001 (2011).
[3] S. V. Ketov, Supergravity and early universe: The meeting point of cosmology and high-energy physics, Int. J. Mod. Phys. **A28**, 1330021 (2013).
[4] M. Born and L. Infeld, Foundations of the new field theory, Proc. R. Soc. **A144**, 425 (1934).
[5] E. S. Fradkin and A. A. Tseytlin, Nonlinear electrodynamics from quantized strings, Phys. Lett. B **163**, 123 (1985).
[6] R. G. Leigh, Dirac-Born-Infeld action from Dirichlet sigma model, Mod. Phys. Lett. **A4**, 2767 (1989).
[7] J. Bagger and A. Galperin, A new Goldstone multiplet for partially broken supersymmetry, Phys. Rev. D **55**, 1091 (1997).
[8] M. Roček and A. A. Tseytlin, Partial breaking of global $D = 4$ supersymmetry, constrained superfields, and three-brane actions, Phys. Rev. D **59**, 106001 (1999).
[9] S. V. Ketov, Many faces of Born-Infeld theory, in the Proceedings of 7th International Wigner Symposium, College Park, Maryland, 2001, arXiv:hep-th/0108189.
[10] S. V. Ketov, Solitons, monopoles and duality: From sine-Gordon to Seiberg-Witten, Fortschr. Phys. **45**, 237 (1997).
[11] D. Z. Freedman and A. Van Proeyen, *Supergravity* (Cambridge University Press, Cambridge, England, 2012).
[12] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, 1992).
[13] H. Abe, Y. Sakamura, and Y. Yamada, Massive vector multiplet inflation with Dirac-Born-Infeld type action, Phys. Rev. D **91**, 125042 (2015).
[14] Y. Aldabergenov and S. V. Ketov, SUSY breaking after inflation in supergravity with inflaton in a massive vector supermultiplet, Phys. Lett. B **761**, 115 (2016).
[15] Y. Aldabergenov and S. V. Ketov, Higgs mechanism and cosmological constant in $N = 1$ supergravity with inflaton in a vector multiplet, Eur. Phys. J. **C77**, 233 (2017).
[16] A. Addazi, S. V. Ketov, and M. Yu. Khlopov, Gravitino and Polonyi production in supergravity, Eur. Phys. J. C **78**, 642 (2018).
[17] J. B. Jimenez, L. Heisenberg, G. J. Olmo, and D. Rubiera-Garcia, Born-Infeld inspired modifications of gravity, Phys. Rep. **727**, 1 (2018).
[18] S. I. Kruglov, Notes on Born-Infeld-like modified gravity, Astrophys. Space Sci. **361**, 73 (2016).

[19] S. J. Gates, Jr. and S. V. Ketov, 4D, $N = 1$ Born-Infeld supergravity, Classical Quantum Gravity **18**, 3561 (2001).
[20] S. V. Ketov and A. A. Starobinsky, Inflation and non-minimal scalar-curvature coupling in gravity and supergravity, J. Cosmol. Astropart. Phys. 08 (2012) 022.
[21] M. Galante, R. Kallosh, A. Linde, and D. Roest, Unity of Cosmological Inflation Attractors, Phys. Rev. Lett. **114**, 141302 (2015).
[22] H. Nakada and S. V. Ketov, Inflation from higher dimensions, Phys. Rev. D **96**, 123530 (2017).
[23] J. Ellis, M. A. G. Garcia, D. V. Nanopoulos, and K. A. Olive, Calculations of inflaton decays and reheating: With applications to no-scale inflation models, J. Cosmol. Astropart. Phys. 07 (2015) 050.
[24] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XIII. Cosmological parameters, Astron. Astrophys. **594**, A13 (2016).
[25] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XX. Constraints on inflation, Astron. Astrophys. **594**, A20 (2016).
[26] P. A. R. Ade *et al.* (BICEP2 and Keck Array Collaborations), Improved Constraints on Cosmology and Foregrounds from BICEP2 and Keck Array Cosmic Microwave Background Data with Inclusion of 95 GHz Band, Phys. Rev. Lett. **116**, 031302 (2016).
[27] V. F. Mukhanov and G. V. Chibisov, Quantum fluctuations and a nonsingular universe, Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981) [JETP Lett. **33**, 532 (1981)].
[28] S. Kaneda, S. V. Ketov, and N. Watanabe, Fourth-order gravity as the inflationary model revisited, Mod. Phys. Lett. **A25**, 2753 (2010).
[29] J. D. Barrow and A. C. Ottewill, The stability of general relativistic cosmological theory, J. Phys. **A16**, 2757 (1983).
[30] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle, and H. E. Swanson, Tests of the Gravitational Inverse-Square Law below the Dark-Energy Length Scale, Phys. Rev. Lett. **98**, 021101 (2007).
[31] Y. Fujii and K.-I. Maeda, *The Scalar-Tensor Theory of Gravitation* (Cambridge University Press, Cambridge, 2007).
[32] S. Cecotti, S. Ferrara, M. Porrati, and S. Sabharwal, New minimal higher derivative supergravity coupled to matter, Nucl. Phys. **B306**, 160 (1988).
[33] F. Farakos, A. Kehagias, and A. Riotto, On the Starobinsky model of inflation from supergravity, Nucl. Phys. **B876**, 187 (2013).

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Gravitino condensate in $N = 1$ supergravity coupled to the $N = 1$ supersymmetric Born–Infeld theory

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The $N = 1$ supersymmetric Born–Infeld theory coupled to $N = 1$ supergravity in four spacetime dimensions is studied in the presence of a cosmological term with spontaneous supersymmetry breaking. The consistency is achieved by compensating a negative contribution to the cosmological term from the Born–Infeld theory by a positive contribution originating from the gravitino condensate. This leads to an identification of the Born–Infeld scale with the supersymmetry-breaking scale. The dynamical formation of the gravitino condensate in supergravity is reconsidered and the induced one-loop effective potential is derived. Slow-roll cosmological inflation with the gravitino condensate as the inflaton (near the maximum of the effective potential) is viable against the Planck 2018 data and can lead to the inflationary (Hubble) scale as high as 10^{12} GeV. Uplifting the Minkowski vacuum (after inflation) to a de Sitter vacuum (dark energy) is possible by the use of the alternative Fayet–Iliopoulos term. Some major physical consequences of our scenario for reheating are also briefly discussed.

Subject Index B11, B12, B16, B32, E81

1. Introduction

The gravitino condensate and the gravitino mass gap in $N = 1$ supergravity [1] coupled to the Volkov–Akulov field [2] in four spacetime dimensions arise as the one-loop effect due to the quartic gravitino interaction coming from the gravitino contribution to the spacetime (con)torsion [3,4]. This is similar to the Nambu–Jona-Lasinio model [5] of the dynamical generation of electron mass and the formation of Cooper pairs near the Fermi surface in superconductivity. The dynamical gravitino mass also leads to a positive contribution to the vacuum energy and, hence, the dynamical supersymmetry breaking too [6]. Given the standard (reduced) Planck mass as the only (dimensional) coupling constant, the gravitino mass gap should be of the order of the Planck scale also, which prevents phenomenological applications of the gravitino condensate to physics under the Planck scale.

However, the effective scale of quantum gravity may be considerably lower than its standard value associated with the (reduced) Planck mass $M_{\rm Pl} = 1/\sqrt{8\pi G_{\rm N}} \approx 2.4 \times 10^{18}$ GeV. This may happen

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because the effective strength of gravity can depend upon either large or warped extra dimensions in the braneworld, or the dilaton expectation value in string theory, or both these factors together [7–9].[1] The negative results of the Large Hadron Collider (LHC) searches for copious production of black holes imply that the low-scale gravity models may have to be replaced by high-scale gravity (or supergravity) models, whose effective Planck scale $\tilde{M}_{\rm Pl}$ is much higher than the TeV scale but is still under the standard scale $M_{\rm Pl}$, i.e.

$$1\ \text{TeV} \ll \tilde{M}_{\rm Pl} \ll M_{\rm Pl}. \tag{1}$$

This can be of particular importance to the early Universe cosmology, where the Newtonian limit does not apply, as well as for high-energy particle physics well above the electroweak scale.

In this scenario, supergravity may play the crucial role in the description of cosmological inflation, reheating, dark energy, and dark matter; see, e.g., Ref. [11] and the references therein. For instance, it is unknown which physical degrees of freedom were present during inflation, while supergravity may be the answer. Describing inflation and a positive cosmological constant (dark energy) in supergravity is non-trivial, especially when one insists on the minimalistic hidden sector. Inflation is driven by positive energy so that it breaks supersymmetry (SUSY) spontaneously. As a (model-independent) consequence, the goldstino should be present during inflation in supergravity cosmology. The goldstino effective action is *universal* and is given by the Akulov–Volkov (AV) action up to field redefinition [12,13]. As was demonstrated in Refs. [14,15], the viable description of inflation and dark energy in supergravity can be achieved by employing an $N = 1$ *vector* multiplet with its $N = 1$ supersymmetric Born–Infeld (BI) action [16] in the presence of the alternative Fayet–Iliopoulos (FI) term [17–21] without gauging the R-symmetry.[2]

In this paper we also employ an $N = 1$ vector multiplet with its $N = 1$ supersymmetric BI action that automatically contains the goldstino (AV) action, but we choose the gravitino condensate as the inflaton. A dynamical SUSY breaking is achieved at the very high scale with the vanishing cosmological constant. The extra (FI) mechanism of spontaneous SUSY breaking is then used to uplift a Minkowski vacuum to a de Sitter (dS) vacuum.

The BI theory has solid motivation. It is expected that Maxwell electrodynamics does not remain unchanged up to the Planck scale, because of its internal problems related to the Coulomb singularity and the unlimited values of electromagnetic field. This motivated Born and Infeld [26] to propose the non-linear vacuum electrodynamics with the Lagrangian (in flat spacetime)

$$\mathcal{L}_{\rm BI} = -M_{\rm BI}^4\sqrt{-\det\left(\eta_{\mu\nu} + M_{\rm BI}^{-2}F_{\mu\nu}\right)} = -M_{\rm BI}^4 - \tfrac{1}{4}F^2 + \mathcal{O}(F^4), \tag{2}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $F^2 = F^{\mu\nu}F_{\mu\nu}$. The constant term on the right-hand side of Eq. (2) can be ignored in flat spacetime. The BI theory has the new scale $M_{\rm BI}$ whose value cannot exceed the GUT scale where electromagnetic interactions merge with strong and weak interactions. On the other hand, we need $M_{\rm BI} < \tilde{M}_{\rm Pl}$ in order to ignore quantum gravity corrections. The BI theory naturally emerges (i) in the bosonic part of the open superstring effective action [27],

[1] The effective Planck scale may also be dynamically generated [10].

[2] In Refs. [22–24], the $N = 1$ massive vector multiplet, unifying the Starobinsky inflaton (scalaron) [25] and the goldstino (photino), was used together with the BI action, the FI term, the chiral (Polonyi) multiplet representing the hidden SUSY-breaking sector, and the massive gravitino as the lightest SUSY particle (LSP) for dark matter.

(ii) as part of the Dirac–Born–Infeld (DBI) effective action of a D3-brane [28], and (iii) as part of the Maxwell–Goldstone action describing partial supersymmetry breaking of $N = 2$ supersymmetry to $N = 1$ supersymmetry [29,30].[3] The peculiar non-linear structure of the BI theory is responsible for its electricmagnetic (Dirac) self-duality, taming the Coulomb self-energy of a point-like electric charge, and causal wave propagation (no shock waves and no superluminal propagation)—see, e.g., Refs. [39,40] and the references therein for a review and non-Abelian extensions of BI theory. All this adds more reasons for using the BI structure.

In a curved spacetime with metric $g_{\mu\nu}$ the BI action is usually defined as the difference between two spacetime densities,

$$S_{\rm BI,standard} = M_{\rm BI}^4 \int d^4x \left[\sqrt{-\det(g_{\mu\nu})} - \sqrt{-\det\left(g_{\mu\nu} + M_{\rm BI}^{-2} F_{\mu\nu}\right)} \right], \tag{3}$$

where the first term has been added "by hand" in order to eliminate the cosmological constant arising from the second term and in Eq. (2). In this paper we propose the gravitino condensation as the origin and the mechanism of such cancellation in the supergravity extension of the BI theory with spontaneously broken SUSY.

The $N = 1$ (rigid) supersymmetric extension of BI theory is also self-dual [41]. The supersymmetric BI theory coupled to $N = 1$ supergravity [i.e. the locally supersymmetric extension of Eq. (3)] was constructed in Ref. [42].

Our paper is organized as follows. In Sect. 2 we provide more details on how to deal with a cosmological constant and spontaneous supersymmetry breaking in the context of a supersymmetric BI theory coupled to supergravity, and relate the BI scale to the spontaneous SUSY-breaking scale. Most of the comments in Sect. 2 are known in the literature and are recalled to justify the consistency of our approach. In Sect. 3 we study the dynamical gravitino condensate arising from the one-loop effective action of pure supergravity, and investigate the induced scalar potential. Slow-roll inflation with the gravitino condensate playing the role of inflaton is studied numerically in Sect. 4. Uplifting the Minkowski vacuum to a de Sitter vacuum using the alternative FI term is proposed in Sect. 5. Our conclusion is presented in Sect. 6. We use the supergravity notation of Ref. [1].

2. Spontaneous SUSY breaking, AV and BI actions, and their coupling to supergravity

We recall that the AV Lagrangian in flat spacetime is given by [2]

$$\mathcal{L}_{\rm AV} = -M_{\rm susy}^4 \det\left(\delta_b^a + \frac{i}{2M_{\rm susy}^4}\bar{\lambda}\gamma^a\partial_b\lambda\right) = -M_{\rm susy}^4 - \tfrac{i}{2}\bar{\lambda}\gamma\cdot\partial\lambda + \mathcal{O}(\lambda^4), \tag{4}$$

where $\lambda(x)$ is a Majorana fermion field of spin 1/2. This fermionic field is called the *goldstino* because the AV action has spontaneously broken non-linearly realized rigid SUSY under the transformations

$$\delta\lambda = M_{\rm susy}^2\varepsilon + \frac{i}{M_{\rm susy}^2}(\bar{\varepsilon}\gamma^a\lambda)\partial_a\lambda \tag{5}$$

[3] See Refs. [31–34] for the extensions of BI theory to extended supersymmetry and higher dimensions.

with the infinitesimal Majorana spinor parameter ε, so that the goldstino is indeed a Nambu–Goldstone fermion. The AV theory of Eq. (4) has the spontaneous SUSY-breaking scale $M_{\rm susy}$.

A coupling of the AV action to supergravity is supposed to generate a gravitino mass via the so-called super-Higgs effect [1] when the gravitino "eats up" the goldstino and thus gets the right number of physical degrees of freedom. However, it is impossible to couple the AV action to supergravity in a manifestly supersymmetric way (i.e. with the linearly realized SUSY) when using standard supermultiplets or unconstrained superfields because of the mismatch in the numbers of bosonic and fermionic physical degrees of freedom.[4] We embed the goldstino into a standard *vector* supermultiplet, i.e. identify the goldstino with the photino, and use an $N = 1$ supersymmetric BI action for the vector multiplet, because it is well motivated at very high energies and includes the goldstino AV action up to a field redefinition [12,13].

The supersymmetric extension of the BI action in Eq. (3) minimally coupled to supergravity in curved superspace of the (old-minimal) supergravity (in a superconformal gauge) with a vanishing cosmological constant, and the vanishing gravitino mass is given by

$$S_{\rm SBI}[V] = \tfrac{1}{4}\left(\int d^4x d^2\theta \mathcal{E} W^2 + \text{h.c.}\right) + \tfrac{1}{4}M_{\rm BI}^{-4}\int d^4x d^2\theta d^2\bar{\theta} E \frac{W^2\bar{W}^2}{1+\frac{1}{2}A+\sqrt{1+A+\frac{1}{4}B^2}},$$

$$A = \tfrac{1}{8}M_{\rm BI}^{-4}\left(\mathcal{D}^2W^2 + \text{h.c.}\right), \quad B = \tfrac{1}{8}M_{\rm BI}^{-4}\left(\mathcal{D}^2W^2 - \text{h.c.}\right), \tag{6}$$

where $\mathcal{E}$ is the chiral (curved) superspace density, E is the full (curved) superspace density, $\mathcal{D}^\alpha$ are the covariant spinor derivatives in superspace, W^α is the chiral gauge-invariant field strength,

$$W_\alpha = -\tfrac{1}{4}\left(\bar{\mathcal{D}}^2 - 4\mathcal{R}\right)\mathcal{D}_\alpha V, \tag{7}$$

of the gauge real scalar superfield pre-potential V describing an $N = 1$ vector multiplet, $\mathcal{R}$ is the chiral (scalar curvature) supergravity superfield, $W^2 = W^\alpha W_\alpha$, and $\mathcal{D}^2 = \mathcal{D}^\alpha\mathcal{D}_\alpha$ [1].

The action in Eq. (6) is obtained from the standard (Bagger–Galperin) action [29]

$$S_{\rm BG}[W,\overline{W}] = \frac{1}{4}\int d^4x d^2\theta\, X + \text{h.c.}, \quad X + \frac{1}{4M_{\rm BI}^4}X\overline{D}^2\overline{X} = W^2 \tag{8}$$

in terms of the constrained chiral superfield X after solving the constraint in Eq. (8) and then minimally coupling the resulting action with the supergravity in curved superspace [19,39], where the spacetime metric $g_{\mu\nu}$ is replaced by the vierbein e^a_μ and is extended to an off-shell supermultiplet $(e^a_\mu, \psi_\mu, M, b_\mu)$, with ψ_μ as the Majorana gravitino field, whereas the complex scalar M and the real vector field b_μ are the auxiliary fields.[5]

The gauge vector (photon) field A_μ is extended in SUSY to an off-shell (real) gauge vector multiplet (or a general real superfield) V with the field components

$$V = (C, \chi, H, A_\mu, \lambda, D), \tag{9}$$

where λ is the Majorana fermion called the photino, D is the auxiliary field, while the rest of the fields (C, χ, H) are the super-gauge degrees of freedom that are ignored in what follows.

[4] The manifestly supersymmetric description is, nevertheless, possible at low energies when embedding the goldstino into the constrained chiral superfield $\tilde{X}$ obeying the nilpotency condition $\tilde{X}^2 = 0$ [35–38]. We avoid that goldstino superfield because it is problematic at higher energies and in quantum theory.

[5] The auxiliary fields of the supergravity multiplet do not play a significant role in our investigation and are ignored below.

Disturbing the action in Eq. (6) by adding a negative cosmological constant $-M_{\rm BI}^4$ to restore the original BI action in Eq. (2) explicitly breaks SUSY, which, however, can be restored by modifying the action and the SUSY transformation laws [6,21]. As a result, it was found that the deformed (new) BI action cannot have a non-vanishing cosmological constant but can have a spontaneously broken local SUSY with a non-vanishing gravitino mass related to the SUSY-breaking scale $M_{\rm BI}$. This does not explain, however, the physical origin of the necessary compensating positive term $+M_{\rm BI}^4$. We explain its origin by gravitino condensation (Sect. 3). To illustrate those features, we add a few simple arguments below.

In order to cancel the SUSY variation of the cosmological constant multiplied by $\sqrt{-\det(g_{\mu\nu})} = e$ due to $\delta_{\rm susy} e_\mu^a = -i\tilde{M}_{\rm Pl}^{-1}(\bar{\varepsilon}\gamma^a\psi_\mu)$ with the infinitesimal SUSY parameter $\varepsilon(x)$, we have to add the photino–gravitino mixing term

$$-ie\frac{M_{\rm BI}^2}{\tilde{M}_{\rm Pl}}(\bar{\lambda}\gamma^\mu\psi_\mu) \tag{10}$$

to the Lagrangian, and simultaneously demand the supersymmetric variation of the photino λ as

$$\delta_{\rm susy}\lambda = M_{\rm BI}^2\varepsilon + \cdots , \tag{11}$$

where the dots stand for the other field-dependent terms. The identification of the photino λ with the goldstino of the spontaneously broken local SUSY already requires

$$M_{\rm BI} = M_{\rm susy} \tag{12}$$

by comparison of Eqs. (5) and (11). This may be not surprising after taking into account that the initial (rigid) Bagger–Galperin action of Eq. (8) has a second (spontaneously broken and non-linearly realized) SUSY whose transformation law is similar to that of Eq. (5). However, our deformed super-BI action in supergravity does not respect another SUSY by construction.

The SUSY-restoring deformation comes together with the gravitino mass term having the mass parameter $m^2 = \frac{1}{3}M_{\rm BI}^4/\tilde{M}_{\rm Pl}^2$, and the modification of the gravitino SUSY transformation law as $\delta_{\rm susy}\psi_\mu = -2\tilde{M}_{\rm Pl}(D_\mu\varepsilon + \frac{1}{2}m\gamma_\mu) + \cdots$. This also implies (by local SUSY) the presence of the goldstino mass term in the Lagrangian with the same mass parameter m [6]. Hence, the super-Higgs effect is in place.

The recovery of the AV action from the super-BI action is possible by identifying the goldstino λ_α with the leading field component of the superfield W_α and projecting the other fields out, $F_{\mu\nu}(A) = D = \psi_\mu = 0$ in the absence of gravity, $e_\mu^a = \delta_\mu^a$. Then, the action in Eq. (8) reduces to the AV action in Eq. (4) up to a field redefinition in the higher-order terms—see Ref. [43] for details. The same conclusions are supported by the superconformal tensor calculus in supergravity [44]. In our approach, the AV action is thus the fermionic *fragment* of the supersymmetric BI theory coupled to supergravity with the spontaneously broken SUSY at the scale M_{BI}. In Sect. 3 we concentrate on the pure supergravity sector of our theory, ignoring the gravitino–photino mixing (i.e. taking into consideration only spin-3/2 gravitino components), just for simplicity. Accounting of a spin-1/2 photino contribution is beyond the scope of our investigation in this paper.

3. One-loop effective action and gravitino condensate

The classical supergravity Lagrangian $\mathcal{L}_{\rm SUGRA}$ besides the Einstein–Hilbert and Rarita–Schwinger terms,

$$\mathcal{L}_{\rm EH} = -\frac{\tilde{M}_{\rm Pl}^2}{2} eR \tag{13}$$

and

$$\mathcal{L}_{\rm RS} = -\tfrac{1}{2}\varepsilon^{\mu\nu\lambda\rho}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\lambda\psi_\rho, \tag{14}$$

respectively, also has the quartic gravitino coupling,

$$\mathcal{L}_{\rm quartic} = \tfrac{11}{16}\tilde{M}_{\rm Pl}^{-2}\left[(\bar{\psi}_\mu\psi^\mu)^2 - (\bar{\psi}_\mu\gamma_5\psi^\mu)^2\right] - \tfrac{33}{64}\tilde{M}_{\rm Pl}^{-2}(\bar{\psi}^\mu\gamma_5\gamma_\nu\psi^\mu)^2, \tag{15}$$

originating from the spacetime (con)torsion in the covariant derivative of the gravitino field in its kinetic term in the second-order formalism for supergravity [1].[6]

Since the supergravity action is invariant under the local SUSY, whose gauge field is ψ_μ, one can choose the (physical) gauge condition $\gamma^\mu\psi_\mu = 0$, which implies $(\bar{\psi}_\mu\Sigma^{\mu\nu}\psi_\nu) = -\frac{1}{2}\bar{\psi}_\mu\psi^\mu$, in the notation $\Sigma_{\mu\nu} = \frac{1}{4}[\gamma^\mu,\gamma^\nu]_-$, and rewrite the (non-chiral) quartic gravitino term in Eq. (6) as

$$\mathcal{L}_{\rm quartic} = \sqrt{11}\tilde{M}_{\rm Pl}^{-1}\rho(\bar{\psi}_\mu\Sigma^{\mu\nu}\psi_\nu) - \rho^2, \tag{16}$$

where the real scalar field ρ has been introduced. As is clear from Eq. (16), a gravitino condensate leads to a non-vanishing vacuum expectation value (VEV), $\langle\rho\rangle \equiv \rho_0 \neq 0$, whereas ρ_0 contributes to the gravitino mass.

The one-loop contribution to the effective potential $V_{1\text{-loop}}(\rho)$ of the scalar field ρ together with its kinetic term arise after quantizing the gravitino sector and taking the Gaussian integral over ψ_μ in the gauge $\gamma^\mu\psi_\mu = 0$. This yields the one-loop contribution to the quantum effective action in the standard form,

$$\Gamma_{1\text{-loop}} = -\tfrac{i}{2}{\rm Tr}\ln\Delta(\rho), \tag{17}$$

where $\Delta(\rho)$ stands for the kinetic operator in the gravitino action, and the interaction with gravity is ignored ($e_\mu^a = \delta_\mu^a$). The one-loop contribution to the ρ-scalar potential [i.e. the terms without the spacetime derivatives in Eq. (17)] was first computed in Refs. [3,4], with the result

$$V_{1\text{-loop}} = \lim_{\mathcal{V}\to\infty}\left[\frac{-1}{2\mathcal{V}}\sum_{n=1}^{\infty}\frac{(\sqrt{11}\tilde{M}_{\rm Pl})^{2n}}{2n}{\rm Tr}(P_{ab}\rho)^{2n}\right] = -\frac{4}{(2\pi)^4}\int^\Lambda d^4p\,\ln\left(1 + 11\tilde{M}_{\rm Pl}^{-2}\frac{\rho^2}{p^2}\right) \tag{18}$$

in terms of the standard massless gravitino propagator (in momentum space)

$$P_{ab} = -\frac{i}{2}\frac{\gamma_b\gamma^\mu p_\mu\gamma_a}{p^2}, \tag{19}$$

the spacetime four-volume regulator $\mathcal{V}$, and the ultraviolet (UV) cutoff Λ, with the trace Tr acting on all variables.

[6] We separate the quartic terms from the minimal term in Eq. (14).

The one-loop contribution in Eq. (17) expanded up to the second order in the spacetime derivatives also yields the ρ-kinetic term subject to the wave function renormalization (i.e. with the Z factor), so that the initially auxiliary scalar field ρ becomes dynamical with a mass M_c. The specific calculations can be found in the literature [3,4,45–47], and the effective potential reads[7]

$$V(\rho) \equiv V_{\text{classical}}(\rho) + V_{\text{1-loop}}(\rho) = \rho^2 - \frac{4}{(2\pi)^4}\int^{\Lambda} d^4p \, \ln\left(1 + 11\tilde{M}_{\text{Pl}}^{-2}\frac{\rho^2}{p^2}\right). \quad (20)$$

Our result of taking the four-dimensional integral in Eq. (20) is given by (cf. Refs. [3,4])

$$V(\rho) = \rho^2 + \frac{1}{8\pi^2}\left\{\frac{121\rho^4}{\tilde{M}_{\text{Pl}}^4}\ln\left(1 + \frac{\tilde{M}_{\text{Pl}}^2\Lambda^2}{11\rho^2}\right) - \frac{11\rho^2\Lambda^2}{\tilde{M}_{\text{Pl}}^2} - \Lambda^4\ln\left(1 + \frac{11\rho^2}{\tilde{M}_{\text{Pl}}^2\Lambda^2}\right)\right\}. \quad (21)$$

The logarithmic scaling of the wave function renormalization of ρ in the one-loop approximation yields the factor proportional to $\ln\left(\frac{\Lambda^2}{\mu^2}\right)$, where μ is the renormalization scale. Hence, the canonical (physical) scalar ϕ is given by [46]

$$\phi = \text{const.}\sqrt{\ln\left(\frac{\Lambda^2}{\mu^2}\right)}\tilde{M}_{\text{Pl}}^{-1}\rho \equiv \tilde{w}M_{\text{Pl}}\sigma, \quad (22)$$

where we have introduced the dimensionless (renormalization) constant $\tilde{w}$ as the parameter. We also use the other dimensionless quantities

$$\sigma = \tilde{M}_{\text{Pl}}^{-2}\rho, \quad \tilde{M}_{\text{Pl}}^{-1}\Lambda = \tilde{\Lambda}, \quad \text{and} \quad \tilde{M}_{\text{Pl}}^{-1}M_{\text{BI}} = \alpha, \quad (23)$$

which allow us to rewrite the *full* scalar potential as

$$V(\sigma)\tilde{M}_{\text{Pl}}^{-4} = \sigma^2 - \frac{1}{8\pi^2}\left\{\tilde{\Lambda}^4\ln\left(1 + \frac{11\sigma^2}{\tilde{\Lambda}^2}\right) - 121\sigma^4\ln\left(1 + \frac{\tilde{\Lambda}^2}{11\sigma^2}\right) + 11\sigma^2\tilde{\Lambda}^2\right\} + \alpha^4, \quad (24)$$

where we have added the contribution of the first term on the right-hand side of Eq. (2).

The scalar potential of Eq. (24) has the double-well shape and is bounded from below, see Fig. 1, provided that

$$\tilde{\Lambda}^2 > \frac{4\pi^2}{11} \approx 3.59, \quad \text{or} \quad \tilde{\Lambda} > \frac{2\pi}{\sqrt{11}} \approx 1.89. \quad (25)$$

There is a local maximum at $\rho = \sigma = 0$ with the positive height M_{BI}^4. A similar potential near its maximum was used for describing slow-roll inflation with the inflaton field ϕ [46]; see Sect. 4 for more. There are also two stable Minkowski vacua at $\rho_c \neq 0$.

According to the previous section, supersymmetry requires the scalar potential of Eq. (24) to vanish at the minimum, i.e. $V(\sigma_c) = 0$. In addition, according to Eq. (16), $\rho_c \neq 0$ determines the gravitino mass

$$m_{3/2} = \sqrt{11}\rho_c/\tilde{M}_{\text{Pl}} = \sqrt{11}\tilde{M}_{\text{Pl}}\sigma_c. \quad (26)$$

[7] The quantum effective action may have the *imaginary* part (sometimes lost in perturbation theory) that contributes to the decay of the gravitino condensate after inflation. Our considerations are limited to the inflationary era by assuming the scale of the imaginary part to be much less than the scale of inflation.

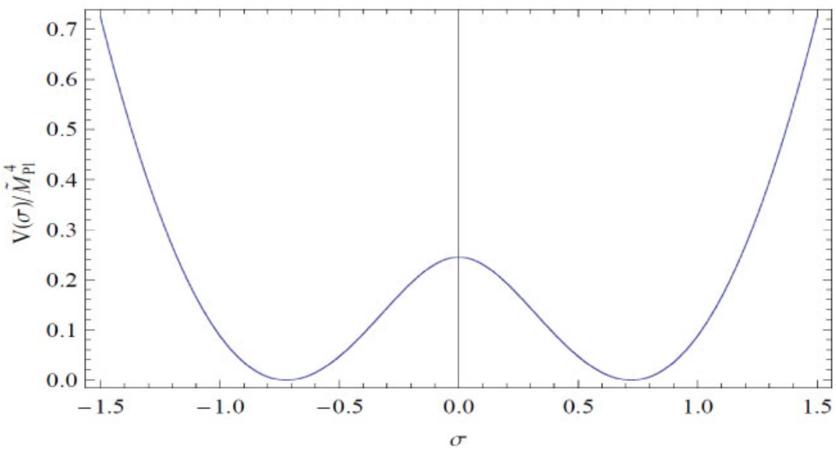

Fig. 1. The profile of the $V(\sigma)$ function in Eq. (24).

The non-vanishing values of ρ_c and σ_c are determined by the condition $dV/d(\sigma^2) = 0$, which yields the transcendental equation

$$121\sigma_c^2 \ln\left(1 + \frac{\tilde{\Lambda}^2}{11\sigma_c^2}\right) = 11\tilde{\Lambda}^2 - 4\pi^2 > 0. \tag{27}$$

The hierarchy between the inflationary scale $H_{\rm inf.}$, the BI scale $M_{\rm BI}$, the SUSY-breaking scale $M_{\rm susy}$, the (super-)GUT scale $M_{\rm GUT}$, the effective gravitational scale $\tilde{M}_{\rm Pl}$, and the Planck scale $M_{\rm Pl}$ in our approach reads

$$H_{\rm inf.} \ll M_{\rm BI} = M_{\rm susy} \approx M_{\rm GUT} \approx \tilde{M}_{\rm Pl} \ll M_{\rm Pl}, \tag{28}$$

where "much less" means two to three orders of magnitude "less" (in GeV), and "approximately" means the same order of magnitude; see the next section for our numerical estimates. As regards the GUT scale, we take $M_{\rm GUT} \approx \mathcal{O}(10^{15})$ GeV.

4. Gravitino condensate as inflaton

A slow-roll inflation induced by gravitino condensation in supergravity was proposed and studied by Ellis and Mavromatos in Ref. [46]. Since our induced scalar potential differs from that of Ref. [46], we reconsider this inflation here by using $\tilde{\Lambda}$ and $\tilde{w}$ as the phenomenologically adjustable parameters.

A slow roll is possible near the maximum of the scalar potential of Eq. (24). Since the height of the potential at the maximum is related to the inflationary Hubble scale $H_{\rm inf.}$ by Friedmann equation,

$$V_{\rm max.} = 3M_{\rm Pl}^2 H_{\rm inf.}^2, \tag{29}$$

the value of $H_{\rm inf.}/M_{\rm Pl}$ is suppressed by the factor $(\tilde{M}_{\rm Pl}/M_{\rm Pl})^2$. On the other hand, the inflationary Hubble scale is related to the cosmic microwave background (CMB) tensor-to-scalar ratio r as

$$\frac{H_{\rm inf.}}{M_{\rm Pl}} = 1.06 \cdot 10^{-4}\sqrt{r}. \tag{30}$$

In turn, r is restricted by Planck (2018) measurements [49] as $r < 0.064$ (with 95% CL), which implies $H_{\rm inf.} < 6 \cdot 10^{13}$ GeV. Therefore, the ratio $(\tilde{M}_{\rm Pl}/M_{\rm Pl})$ should be of order $10^{-2} \div 10^{-3} \ll 1$ for viable inflation. This justifies our setup in Sect. 1. We define the dimensionless parameter γ as $(\tilde{M}_{\rm Pl}/M_{\rm Pl}) \equiv 10^{-3}/\gamma$, where γ is of order one.

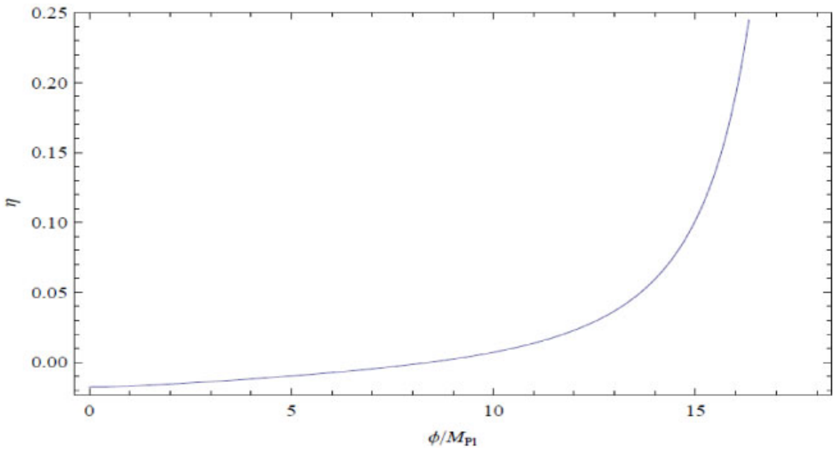

Fig. 2. The running of the slow-roll parameter η for $\gamma = 0.5$ and $\tilde{w} = 13$.

In our numerical calculations we have chosen the cutoff scale $\tilde{\Lambda} = 3$, so that the restriction in Eq. (25) is satisfied. Then, Eqs. (24) and (27) imply that

$$V_{\rm max}\tilde{M}_{\rm Pl}^{-4} = 0.245 \quad \text{and} \quad \sigma_{\rm cr.} = \tilde{w}^{-1}(\phi_{\rm cr.}/M_{\rm Pl}) = 0.722. \tag{31}$$

In turn, this yields the gravitino mass $m_{3/2}$ and the gravitino condensate mass $m_{\rm cond.}$ as follows:

$$m_{3/2} = 2.39\tilde{M}_{\rm Pl} \quad \text{and} \quad m_{\rm cond.} = m_\phi = \sqrt{8/11}\, m_{3/2} = 2.038\tilde{M}_{\rm Pl}. \tag{32}$$

We numerically studied the running of the slow inflationary parameters $\varepsilon = \frac{1}{2}M_{\rm Pl}^2(V'/V)^2$ and $\eta = M_{\rm Pl}^2(V''/V)$ with respect to the inflaton field ϕ for values of the parameter γ of 0.1, 0.5, and 1, and found that ε is always under $\mathcal{O}(10^{-4})$ so that it can be ignored within the errors of the Planck 2018 data. Then the value of the scalar index $n_{\rm s} = 1 - 6\varepsilon + 2\eta = 0.9649 \pm 0.0042$ (with 68% CL) [49] can be reached with $\eta = -0.0177$ at the horizon crossing by using the parameter $\tilde{w}$ of order one. There are no additional constraints on the parameters γ and $\tilde{w}$ from demanding that the e-folding number,

$$N_e = -\frac{1}{M_{\rm Pl}^2}\int_{\phi_{\rm ini.}}^{\phi_{\rm end}} \frac{V}{V'}d\phi, \tag{33}$$

be between 50 and 60, as is desired for viable inflation, when assigning the inflaton field $\phi/M_{\rm Pl}$ to run somewhere between 0 and 5 during inflation. The running of the slow-roll parameter η is displayed in Fig. 2.

In summary, our results qualitatively agree with those of Ref. [46], but quantitatively allow considerably higher values of ε and r up to order $\mathcal{O}(10^{-4})$, contrary to the $\mathcal{O}(10^{-8})$ of Ref. [46], with Planckian values of the inflaton ϕ during inflation, contrary to its sub-Planckian values of $\mathcal{O}(10^{-3})M_{\rm Pl}$ in Ref. [46]. Hence, the inflationary scale $H_{\rm inf.}$ can be as high as 10^{12} GeV versus the 10^{10} GeV of Ref. [46].

5. Adding the FI term

In order to uplift the Minkowski vacuum to a de Sitter vacuum (dark energy) in our approach, we need an extra tool of spontaneous SUSY breaking. In the BI theory (without chiral matter) coupled to supergravity such a tool can be provided by the (alternative) FI terms [18–21] that do not require the gauged R-symmetry, unlike the standard FI term [17] whose extension to supergravity is severely restricted [48].

The (Abelian) gauge vector multiplet superfield V can be decomposed into a sum of the reduced gauge superfield $\mathcal{V}$ including the gauge field A_μ, and the nilpotent gauge-invariant goldstino superfield $\mathcal{G}$ that contains only th egoldstino λ and the auxiliary field D [19],

$$V = \mathcal{V} + \mathcal{G}, \quad \mathcal{G}^2 = 0. \tag{34}$$

The simplest examples of the goldstino superfield are given by [18,19]

$$\mathcal{G}_1 = -4\frac{W^2\bar{W}^2}{\mathcal{D}^2 W^2 \bar{\mathcal{D}}^2 \bar{W}^2}(\mathcal{D}W) \tag{35}$$

and

$$\mathcal{G}_2 = -4\frac{W^2\bar{W}^2}{(\mathcal{D}W)^3}, \tag{36}$$

respectively, in terms of the standard $N = 1$ gauge superfield strength

$$W_\alpha = -\frac{1}{4}\left(\bar{\mathcal{D}}^2 - 4\mathcal{R}\right)\mathcal{D}_\alpha V, \tag{37}$$

where $\mathcal{R}$ is the chiral scalar curvature superfield. The W_α obeys the Bianchi identities

$$\bar{\mathcal{D}}_{\dot{\beta}} W_\alpha = 0 \quad \text{and} \quad \bar{\mathcal{D}}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \equiv \bar{\mathcal{D}}\bar{W} = \mathcal{D}^\alpha W_\alpha \equiv \mathcal{D}W. \tag{38}$$

The field components are given by $W_\alpha| = \lambda_\alpha$, $\mathcal{D}W| = -2D$, and $\mathcal{D}_{(\alpha} W_{\beta)}\big| = i(\sigma^{ab})_{\alpha\beta}F_{ab} + \cdots$. The difference between the superfields $\mathcal{G}_1$ and $\mathcal{G}_2$ is only in the gauge sector, and is not essential for our purposes here.

The extra FI term with the coupling constant $\xi \neq 0$ is given by

$$S_{\rm FI} = \xi \int d^4x d^4\theta E\mathcal{G}, \tag{39}$$

where E is the supervielbein (super)determinant [1]. This FI term is manifestly SUSY- and gauge-invariant, does *not* include the higher spacetime derivatives of the field components, but leads to the inverse powers of the auxiliary field D (up to the fourth order) in the non-scalar sector of the theory.[8] Integrating out the auxiliary field D leads to a *positive* contribution to the cosmological constant,

$$V_\xi = \frac{1}{2}\xi^2 > 0. \tag{40}$$

Matching V_ξ with the observed cosmological constant allows us to include a viable description of the dark energy into our approach. The phenomenological values of the cosmological constant and the related contribution (ξ) to the VEV of the auxiliary field D are tiny, so that they do not affect our considerations of the high-scale SUSY breaking in the previous sections.

[8] The limit $\xi \to 0$ does not lead to a well-defined theory, so that $\langle D\rangle = \xi$ must be non-vanishing.

The nilpotent goldstino superfield $\mathcal{G}$ introduced above is composed of the usual (standard) superfields and, hence, is very different from the intrinsically nilpotent goldstino superfield introduced in Refs. [35–38].

As the FI term affects the quartic and higher-order terms with respect to the gauge field and its fermionic (spin-1/2) superpartner, back reaction of the FI term on the effective action should be examined (work in progress). This should be done together with quantum renormalization of those terms and, perhaps, requires a field-dependent FI parameter ξ. The D-type scalar potential and the associated dark energy are expected to be unaffected because of cancellation of (perturbative) quartic and quadratic (ultraviolet) divergences due to supersymmetry of the action.

6. Conclusion

The gravitino condensate can be considered as a viable candidate for the inflaton in supergravity, when assuming the effective (quantum) gravity scale to be close to the (super-)GUT scale that is also close to the SUSY-breaking scale in our approach, with all scales close to 10^{15} GeV. Actually, in this scenario we have the hyper-GUT where *all* fundamental interactions merge, including gravity. At the same time, it is the weak point of our calculations because we ignored (other) quantum gravity corrections.

The inflationary (Hubble) scale is well below the GUT scale, and can be as large as 10^{12} GeV. The gravitino mass is above the inflationary scale, so that there is no gravitino overproduction problem in the early Universe. The constraints from proton decay and big bang nucleosynthesis are very weak because of high-scale SUSY. Then, SUSY is not a solution to the hierarchy problem with respect to the electroweak scale. This is similar to the setup studied in Refs. [50,51]. Our scenario is consistent with the known Higgs mass of about 125 GeV after taking into account the extreme possible values of the gaugino mixing parameter $\tan\beta$ in the context of SUSY extensions of the Standard Model [52].

As regards reheating after inflation, the inflaton (gravitino condensate) field decays into other matter and radiation, which is highly model dependent, as usual. Unlike Ref. [24], the inflaton as the gravitino condensate cannot decay into gravitinos because Eq. (32) leads to the kinematical constraint $2m_{3/2} > m_{\text{cond.}}$. It also implies that the gravitino cannot be a dark matter particle in this scenario. A detailed study of reheating requires knowledge of the couplings of the gravitino and gravitino condensate to the Standard Model particles, which is beyond the scope of this paper.

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References

[1] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, 1992).
[2] D. V. Volkov and V. P. Akulov, Phys. Lett. B **46**, 109 (1973).

[3] R. S. Jasinschi and A. W. Smith, Phys. Lett. B **173**, 297 (1986).
[4] R. S. Jasinschi and A. W. Smith, Phys. Lett. B **174**, 183 (1986).
[5] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[6] S. Deser and B. Zumino, Phys. Rev. Lett. **38**, 1433 (1977).
[7] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315] [Search INSPIRE].
[8] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398] [Search INSPIRE].
[9] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221] [Search INSPIRE].
[10] J. Kubo, M. Lindner, K. Schmitz, and M. Yamada, Phys. Rev. D **100**, 015037 (2019) [arXiv:1811.05950 [hep-ph]] [Search INSPIRE].
[11] S. V. Ketov and M. Yu. Khlopov, Symmetry **11**, 511 (2019).
[12] T. Hatanaka and S. V. Ketov, Phys. Lett. B **580**, 265 (2004) [arXiv:hep-th/0310152] [Search INSPIRE].
[13] S. M. Kuzenko, Phys. Rev. D **81**, 085036 (2010) [arXiv:0911.5190 [hep-th]] [Search INSPIRE].
[14] H. Abe, Y. Aldabergenov, S. Aoki, and S. V. Ketov, J. High Energy Phys. **1809**, 094 (2018) [arXiv:1808.00669 [hep-th]] [Search INSPIRE].
[15] H. Abe, Y. Aldabergenov, S. Aoki, and S. V. Ketov, Class. Quantum Grav. **36**, 075012 (2019) [arXiv:1812.01297 [hep-th]] [Search INSPIRE].
[16] S. Cecotti and S. Ferrara, Phys. Lett. B **187**, 335 (1987).
[17] P. Fayet and J. Iliopoulos, Phys. Lett. B **51**, 461 (1974).
[18] N. Cribiori, F. Farakos, M. Tournoy, and A. Van Proeyen, J. High Energy Phys. **1804**, 032 (2018) [arXiv:1712.08601 [hep-th]] [Search INSPIRE].
[19] S. M. Kuzenko, Phys. Lett. B **781**, 723 (2018) [arXiv:1801.04794 [hep-th]] [Search INSPIRE].
[20] Y. Aldabergenov, S. V. Ketov, and R. Knoops, Phys. Lett. B **785**, 284 (2018) [arXiv:1806.04290 [hep-th]] [Search INSPIRE].
[21] N. Cribiori, F. Farakos, and M. Tournoy, J. High Energy Phys. **1903**, 050 (2019) [arXiv:1811.08424 [hep-th]] [Search INSPIRE].
[22] Y. Aldabergenov and S. V. Ketov, Phys. Lett. B **761**, 115 (2016) [arXiv:1607.05366 [hep-th]] [Search INSPIRE].
[23] Y. Aldabergenov and S. V. Ketov, Eur. Phys. J. C **77**, 233 (2017) [arXiv:1701.08240 [hep-th]] [Search INSPIRE].
[24] A. Addazi, S. V. Ketov, and M. Yu. Khlopov, Eur. Phys. J. C **78**, 642 (2018) [arXiv:1708.05393 [hep-th]] [Search INSPIRE].
[25] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).
[26] M. Born and L. Infeld, Proc. Roy. Soc. Lond. A **144**, 425 (1934).
[27] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. B **163**, 123 (1985).
[28] R. G. Leigh, Mod. Phys. Lett. A **4**, 2767 (1989).
[29] J. Bagger and A. Galperin, Phys. Rev. D **55**, 1091 (1997) [arXiv:hep-th/9608177] [Search INSPIRE].
[30] M. Roček and A. A. Tseytlin, Phys. Rev. D **59**, 106001 (1999) [arXiv:hep-th/9811232] [Search INSPIRE].
[31] S. V. Ketov, Mod. Phys. Lett. A **14**, 501 (1999) [arXiv:hep-th/9809121] [Search INSPIRE].
[32] S. V. Ketov, Nucl. Phys. B **553**, 250 (1999) [arXiv:hep-th/9812051] [Search INSPIRE].
[33] S. Bellucci, E. Ivanov, and S. Krivonos, Phys. Lett. B **502**, 279 (2001) [arXiv:hep-th/0012236] [Search INSPIRE].
[34] S. Bellucci, S. Krivonos, A. Shcherbakov, and A. Sutulin, Phys. Lett. B **721**, 353 (2013) [arXiv:1212.1902 [hep-th]] [Search INSPIRE].
[35] M. Roček, Phys. Rev. Lett. **41**, 451 (1978).
[36] U. Lindström and M. Roček, Phys. Rev. D **19**, 2300 (1979).
[37] Z. Komargodski and N. Seiberg, J. High Energy Phys. **0909**, 066 (2009) [arXiv:0907.2441 [hep-th]] [Search INSPIRE].
[38] I. Antoniadis, E. Dudas, S. Ferrara, and A. Sagnotti, Phys. Lett. B **733**, 32 (2014) [arXiv:1403.3269 [hep-th]] [Search INSPIRE].
[39] S. V. Ketov, arXiv:hep-th/0108189 [Search INSPIRE].
[40] S. V. Ketov, Fortsch. Phys. **45**, 237 (1997) [arXiv:hep-th/9611209] [Search INSPIRE].

[41] S. M. Kuzenko and S. Theisen, J. High Energy Phys. **0003**, 034 (2000) [arXiv:hep-th/0001068] [Search INSPIRE].
[42] S. M. Kuzenko and S. A. McCarthy, J. High Energy Phys. **0505**, 012 (2005) [arXiv:hep-th/0501172] [Search INSPIRE].
[43] S. M. Kuzenko and S. J. Tyler, J. High Energy Phys. **1105**, 055 (2011) [arXiv:1102.3043 [hep-th]] [Search INSPIRE].
[44] D. Z. Freedman and A. Van Proeyen, *Supergravity* (Cambridge University Press, Cambridge, 2012).
[45] I. L. Buchbinder and S. D. Odintsov, Class. Quantum Grav. **6**, 1955 (1989).
[46] J. Ellis and N. E. Mavromatos, Phys. Rev. D **88**, 085029 (2013) [arXiv:1308.1906 [hep-th]] [Search INSPIRE].
[47] J. Alexandre, N. Houston, and N. E. Mavromatos, Phys. Rev. D **88**, 125017 (2013) [arXiv:1310.4122 [hep-th]] [Search INSPIRE].
[48] P. Binétruy, G. Dvali, R. Kallosh, and A. Van Proeyen, Class. Quantum Grav. **21**, 3137 (2004) [arXiv:hep-th/0402046] [Search INSPIRE].
[49] Y. Akrami et al. [Planck Collaboration], arXiv:1807.06211 [astro-ph.CO] [Search INSPIRE].
[50] E. Dudas, Y. Mambrini, and K. A. Olive, Phys. Rev. Lett. **119**, 051801 (2017) [arXiv:1704.03008 [hep-ph]] [Search INSPIRE].
[51] E. Dudas, T. Gherghetta, Y. Mambrini, and K. A. Olive, Phys. Rev. D **96**, 115032 (2017) [arXiv:1710.07341 [hep-ph]] [Search INSPIRE].
[52] G. F. Giudice and A. Strumia, Nucl. Phys. B **858**, 63 (2012) [arXiv:1108.6077 [hep-ph]] [Search INSPIRE].