

CALCULATING HURST EXPONENT WITH THE USE OF THE SIROKY METHOD IN DEVELOPED AND EMERGING MARKETS

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Abstract

The purpose of the article This paper analysis Hurst exponents calculated with the use of the Siroky method in two time intervals of 625 (H_{625}) and 1250 (H_{1260}) sessions for the following assets: (the number of assets for a given group in brackets): Stock indices (74), currency pairs divided into segments: USD exchange rate in relation to 42 other currencies (USDXXX), EURO exchange rate in relation to 41 other currencies (EURXXX), JPY exchange rate in relation to 40 other currencies (JPYXXX) and other currency pairs (12). In total, 209 financial instruments were analyzed.

Methodology: Hurst coefficient calculation with the use of the following methods; Siroky, Detrended Moving Average (DMA) and Detrended Fluctuation Analysis (DFA).

Results of the research: The Hurst coefficient values calculated with the use of Siroky method are similar to the results obtained using DFA and DMA methods. The second main conclusion that was drawn from the research may be formulated as follows: exchange rates calculated for the developed-developed country currencies are more effective than in the case of the developed-emerging countries group.

Keywords: Hurst exponent, market efficiency, developed countries.

JEL Class: G10, G14.

INTRODUCTION

There is a wide spectrum of mathematical tools constructed to analyze processes characterized by long-term data dependence. One of the important turning points in long-term data series analysis is a method of scaled range analysis, proposed by Hurst [Hurst 1951: 770–799]. The starting point in Hurst's work was Einstein's study of Brownian motions, in which the relationship of the distance r covered by the molecule during time t was expressed in the following form [Einstein 1908: 469–502]:

$$r = c \cdot \sqrt{t}$$

where:

c – represents a constant.

This equation mainly concerns the case when the sequence of increments of the molecule's path in time is random walk, characterized by the independent random variables normally distributed [Weron and Weron 1998; Kapecka 2015: 5–75]. Meanwhile, the results obtained by Hurst led to the conclusion that the overwhelming number of natural phenomena (like: temperature changes, river and sea floods, atmospheric precipitation, sunspot activity) are not subject to random walk, but constitute the so-called processes with long-term memory, called later on as fractional Brownian motion, representing the superposition of trend and noise [Peters 1997: 64; Mastalerz-Kodzis 2003: 37–38].

For the scaled range method, the Hurst exponent is defined by the following formula [Peters 1997: 64]:

$$\left(\frac{R}{S}\right)_n = a \cdot n^H \quad (1)$$

where:

R – range of fluctuations,

S – standard deviation,

$\left(\frac{R}{S}\right)_n$ – so-called scaled range,

$a > 0$ – positive constant,

$n \in N$ – number of time series observations.

Calculating the logarithm of both sides of the equation (1) leads to the following formula for $\ln\left(\frac{R}{S}\right)_n$:

$$\ln\left(\frac{R}{S}\right)_n = H \cdot \ln(n) + \ln(a) \quad (2)$$

To compute the Hurst exponent, first it necessary to estimate the scaled range $\left(\frac{R}{S}\right)_n$ for different n , and then solve equation (2) using linear regression. The Hurst exponent is a regression coefficient, estimated with the use of the least squares method. This method was further improved by Mandelbrot and Wallis [Mandelbrot and Wallis 1968].

Depending on the needs, methods determining the Hurst exponent can be used to study one-dimensional time series:

- a) Long range analysis method (R/S) [Hurst 1951: 770–799],
 - b) Detrended Moving Average (DMA) [Alessio et al. 2002: 197–200; Mantenga and Stanley 1995: 46–49],
 - c) Detrended Fluctuation Analysis (DFA) [Bunde and Havlin 1995: 97–100; Peng et al. 1994],
 - d) Generalized Hurst Exponent (GHE) [Di Matteo et al. 2003: 183–188],
- or multidimensional:

- a) Multifractal Detrended Fluctuation Analysis (MF-DFA) [Abry and Weitch 1998; Kantelhard et al. 2002],
- b) Wavelet Transform Module Maxima (WTMM) [Muzy et al. 1994: 245–302].

Another method for determining the Hurst exponent is the box method, which covers the graph of the analyzed function with a square grid with a certain side and counting the squares having at least one point in common with the examined graph. Then the grid is considered with a side that is k times smaller and the squares having at least one point in common with the analyzed graph are counted again [Voss 1991: 816–817; Mastalerz-Kodzis 2003]. This method was applied, among others by Ehlers to create the Fractal Adaptive Moving Average [Ehlers 2005: 81–82]. In turn, Ehlers and Way modified the box method to include not only closing prices, but also the highest and lowest prices in the analyzed period [Ehlers and Way 2010: 16–20].

There is a close relationship between the Hurst exponent and Minkowski's fractal dimension (d) [Kowgier 2009: 157–167]:

$$d = 2 - H \quad (3)$$

Such relationship with the use of the fractional Brownian motion and self-similarity of the stochastic process was presented in the work of Kowgier [Kowgier 2009: 157–167]. Estimation of the fractal dimension is justified because it enables to gain additional knowledge about the behavior of prices and rates of return, and thus contributes to a better understanding of their nature. A concept of fractal dimension as a measure of risk on the equity market is described in the paper of Orzeszko [Orzeszko 2010: 57–70]. The author presents the application of this method for selected indexes of the Warsaw Stock Exchange. In the further part of this paper, the fractal dimension (d) will be treated as a risk measure.

The method of calculating the Hurst exponent proposed by Siroky [Siroky 2017: 18–21 and 45] directly refers to the method of calculating the fractal dimension using the slope coefficient of sub-segments of the analyzed time window, whose authors are Ehlers and Way [Ehlers and Way 2010: 16–20]. According to the method of segment division, the analyzed time series should be divided into two segments of equal data length, with S_2 as the slope factor in the current data segment and by S_1 in the previous one. The slopes S_2 and S_1 are defined as follows („1” in the denominator means a single calculation period for analyzed segment) [Siroky 2017: 20]:

$$S_2 = \frac{H_2 - L_2}{1} = H_2 - L_2$$

$$S_1 = \frac{H_1 - L_1}{1} = H_1 - L_1$$

where:

H_1 and H_2 – the highest price in the first and second data segments, respectively,
 L_1 and L_2 – the lowest price in the first and second data segments, respectively.

For the entire analyzed range, the slope factor will be equal to („2” in the denominator means both calculation periods forming the entire time horizon) [Siroky 2017: 20]:

$$S_{tot} = \frac{H_{tot} - L_{tot}}{2}$$

where:

H_{tot} and L_{tot} – the highest and lowest prices in the entire data segment, respectively. In turn, the fractal dimension calculated for all data included in the analyzed range will be equal to [Siroky 2017: 20]¹:

$$d = \frac{\ln(S_1+S_2)}{\ln(2)} - \frac{\ln(S_{tot})}{\ln(2)} \quad (4)$$

By dividing the observed range of data into smaller and smaller periods, reaching data for a single session, the slope coefficient becomes equal to the *TR* (*True Range*) value proposed by Wilder and equal to the difference between the highest and lowest price in a given session [Wilder 1978]. With this approach, the value of the slope factor for all observed sessions is equal to the value of R , divided by the number of sessions (n), where R is equal to the difference between the highest and lowest price in the analyzed time window.

Taking into consideration n session window, the equation 4 may be generalized and the expression for the fractal dimension d will take the following form [Siroky 2017: 20]:

$$d = \frac{\ln \sum TR - \ln \frac{R}{n}}{\ln n} = \frac{\ln \left(\frac{\sum TR}{\frac{R}{n}} \right)}{\ln n}$$

Noting that $\sum TR$ is equal to the product of the Mean True Range value (*MTR*) and the number of observations (n), the expression for the fractal dimension takes the form [Siroky 2017: 45]:

$$d = \frac{\ln \left(\frac{MTR \cdot n}{\frac{R}{n}} \right)}{\ln n} = \frac{\ln \left(\frac{MTR \cdot n^2}{R} \right)}{\ln n} = \frac{2 \cdot \ln n - \ln \left(\frac{R}{MTR} \right)}{\ln n} = 2 - \frac{\ln \left(\frac{R}{MTR} \right)}{\ln n}$$

¹ In Siroky's work division by $\ln(2)$ concerns only the second addend of the sum, e.g. $\ln(S_{tot})$, however, in the work of Ehlers and Way, the author quotes, the expression for d takes the form as in formula (4) [Ehlers and Way 2010: 16–20].

Based on the relationship (3), the value of the Hurst exponent is determined by the equation [Siroky 2017: 45]:

$$H = \frac{\ln\left(\frac{R}{MTR}\right)}{\ln n}$$

The MTR value can also be obtained as the Average True Range indicator, averaged over a given time window, with the latter being calculated on the basis of daily data. Siroky proposes following way of applying the ATR indicator [Siroky 2017: 18–21 and 45]:

$$MTR \approx \frac{1}{n} \cdot \sum_{t=1}^{t=n} ATR_t$$

$$ATR_t = \text{Max}(H_t - L_t, |H_t - C_{t-1}|, |L_t - C_{t-1}|)$$

where:

H_t – highest price on the session t ,

L_t – lowest price on the session t ,

C_{t-1} – closing price on the session $t-1$.

The clear advantage of Siroky's method is applying not only closing prices, but also the highest and lowest prices in the calculation process, which should be considered as a method more adequate for use on financial markets than others. Unlike the scaled range method, the Siroky method does not impose the condition that the number of session n has many divisors. Therefore, this method should be considered more appropriate for use on financial markets than others. The Siroky method can therefore be used for shorter time intervals, while other methods often require longer time horizons. One drawback of the Siroky method is the use of the approximate MTR value. This approximation becomes negligible for large n .

Regardless of the type of time series studied, the Hurst exponent assumes values in the range (0; 1), while obtaining boundary values is extremely difficult, therefore they are often considered theoretical. The value of the Hurst coefficient can be interpreted as [Barunik and Kristoufek 2010: 3844–3855]:

a) $H \in (0; 0.5)$ – the time series is defined as anti-persistent, characterized by high variability and high probability of frequent changes in the direction of short-term trends.

b) $H = 0.5$ – the time series does not have a dominant trend, which means that subsequent changes take on a random nature (random walk). The probability of both changing the trend and maintaining the current trend is the same and equal to 0.5.

c) $H \in (0.5; 1)$ – the time series is defined as persistent, characterized by low variability and low probability of frequent changes in the direction of short-term trends. In other words, it is a time series with an ordered course, which maintains the current trend (momentum). The higher the Hurst exponent, the higher the course orderliness.

The purpose of this study is to calculate the Hurst exponent with the use of the Siroky method for specific groups of assets: Stock Indices and Currency Market (pairs: USDXXX2, EURXXX, JPYXXX, Other Currency Pairs). In the second part of the paper two rankings of Hurst exponents were created: the first taking into account the average value of the Hurst exponent and the second based on the distance of the Hurst exponent from the level of 0.5.

The authors of this study are not aware of any scientific papers on the calculation of Hurst exponents based on the method proposed by Siroky. Therefore this paper fills in the existent research gap.

Two main theses presented in the paper were formulated as follows:

T1: The Hurst coefficient can be calculated with the use of the Siroky method for the following market segments: equity and currency.

T2: The extension of the investment horizon (n) leads to a reduction in the differences between the Hurst coefficient values:

$$H_{Siroky} - H_{DMA}$$

$$H_{Siroky} - H_{DFA}$$

where:

H_{Siroky} – Hurst coefficient calculated with the use of the Siroky method,

H_{DMA} – Hurst coefficient calculated with the use of the Detrended Moving Average,

H_{DFA} – Hurst coefficient calculated with the use of the Detrended Fluctuation Analysis.

² Contractual notation denoting the exchange rate of pairs in which USD is one of the currencies. An analogous convention was used for EURXXX and JPYXXX.

1. LITERATURE REVIEW

The long-memory of S&P 500 index have been investigated in numerous empirical studies that have provided confuring results. For example Peters testing the persistence in the US stock market proved the evidence of long memory [Peters 1991; Peters 1994]. Granger and Ding on the basis of 17 000 daily observations of the S&P 500 index concluded that absolute returns exhibited long-memory properties, a later study of Granger and Hyung confirmed these findings [Granger and Ding 1995: 67–91; Granger and Hyungh 2004: 399–421]. Alvarez-Ramirez et al. used daily data for the period 1950–2007 and reported that the long-run memory properties of the S&P 500 index changed over time, especially during crisis periods [Alvarez-Ramirez et al. 2008: 6159–6169]. Similar conclusions were published by Dominique and Rivera, who proved the S&P 500 index to be persistent, but its degree of persistence changes over time [Dominique and Rivera 2011: 1–6]. On the contrary, Chow et al. did not find long-term dependence in stock returns [Chow et al. 1996: 181–194]. The same conclusion was reached by Caporale and Gil-Alana, as well as by Lu and Perron, who did not find evidence of long memory in daily S&P 500 index returns [Caporale et al. 2016: 235–257; Lu and Perron 2010: 138–156].

Financial data time-series and their persistence were analyzed very thorough in case of different types of financial markets:

a) Stock markets [Greene and Fielitz 1977: 339–349; Lo 2004: 15–29; Cheung and Lai 1995: 597–615; Jacobsen 1995: 37–52; Opong et al. 1999: 267–282; McKenzie 2001: 393–406; Costa and Vasconcelos 2003: 231–248; Los and Yalamova 2006: 106–133; Onali and Goddard 2010],

b) Commodities markets [Cheung and Lai 1995: 597–615; Barkoulas et al. 1997: 737–745; Crato and Ray 1999; Alvarez-Ramirez et al. 2002: 651–670; Serletis and Roenberg 2009: 2007–2015],

c) FOREX [Mulligan 2000: 33–49; Kim and Yoon 2004: 272–278; Da Silva et al. 2007: 1–11].

There is popularity of country stock market index research. In many papers the long-term memory of the analyzed market was proved [Greene and Fielitz 1977: 339–349; Peters 1991; Peters 1994; Hja and Lin 2007: 537–540; Lento 2009; Onali and Goddard 2010]. In others the conclusion about random nature of prices fluctuations and absence of the long-term memory of the markets, was presented [Lo 1991: 1279–1313; Jacobsen 1995: 37–52; Berg and Lyhagen 1996; Crato and Ray 1999; Batten et al. 2003; Serletis and Rosenberg 2009: 325–332]. Onali and Goddard analyzing returns for Italian Mibtel and Czech PX indexes found the evidence of long-range dependence in the logarithmic return series [Onali and Goddard 2010]. Batten et al. using the modified rescaled range test for daily Nikkei returns in the period of 1980–2000, accepted the null hypothesis of

lack of long-term dependence for the whole sample and every sub-period [Batten et al. 2003]. The same hypothesis was rejected for classical rescaled adjusted range test. Berg and Lyhagen computing monthly returns in the period of 1919–1995, as well as returns for weekly data of the period 1980–1995, hardly found any evidence of long run dependence [Berg and Lyhagen 1996]. On the basis of three different tests that were robust to short term dependence proved that the modified R/S test and ARFIMA-GARCH tests provided no support for long-run memory in Swedish stock returns. Lo analyzing daily returns for US stock market in the period of 1872–1986 could not find long-term dependence [Lo 1991: 1279–1313]. Jacobsen taking into consideration indexes of five European countries, USA and Japan rejected the hypothesis of long term dependence in the analyzed data series [Jacobsen 1995: 37–52].

Corazza and Malliaris calculating Hurst exponent on the basis of R/S method for forex market in the period of 1972–1994 proved that Hurst exponent was statistically different from 0.5 and changed dynamically over time [Corazza and Malliaris 2002: 387–401]. Glenn assessing H exponent with the use of R/S method for daily returns of NASDAQ index, obtained the value of H equal to 0.59 and proved that the H exponent increased monotonically to a value of 0.87 for 250-session returns [Glenn 2007].

Barkoulas and Baum analyzing data for three stock indices: DJIA, NASDAQ and S&P 500 and seven sectoral monthly stock indices, and daily prices for the 30 companies included in the Dow Jones Industrials Index, did not find evidence of long-range behavior for US equities [Barkoulas and Baum 1996: 253–259].

Hiemstra and Jones applied the modified rescaled range test to the return series of 1 952 common stocks in US and came to the conclusion that there was some evidence consistent with persistent long memory in the returns of a small proportion of stocks [Hiemstra and Jones 1997: 373–401]. On the other hand, Huang and Yang on the basis of companies included in the indexes: NYSE and NASDAQ were using intraday returns proved the existence of long memory [Huang and Yang 1999]. Cheung and Lai analyzed 18 countries founding small support for long memory in the international stock returns [Cheung and Lai 1995: 597–615]. Henry investigated long run dependence for nine international stock indexes and provided evidence of long memory for the German, Japanese, South Korean and Taiwanese stock indexes [Henry 2002: 725–729]. Cajueiro and Tabak studying daily closing prices for equity indexes of 11 emerging markets and two of developed (S&P 500 and Nikkei 225) in the period of January 1992 to December 2002, ranked countries using a „rolling sample” approach [Cajueiro and Tabak 2004a: 349–352; Cajueiro and Tabak 2005: 671–675]. Long-run dependence measures were more significant for Asian than for Latin American countries (with the exception of Chile), while US and Japan ranked as the most efficient ones. Calculating the Hurst exponent over time and using four years

window of data for Latin America and Asia stock exchange indexes, Cajueiro and Tabak found the emerging markets have been becoming more efficient over time with the exception of Brazil, the Philippines and Thailand [Cajueiro and Tabak 2004b: 521–537]. Kyaw et al. calculating Hurst exponents for six Latin markets (Argentina, Brazil, Chile, Colombia, Mexico and Venezuela) with the uses of the MF-DFA method, proved that Latin American equity markets are persistent, except for the Colombian and Mexican stock exchanges, which remain anti-persistent [Kyaw et al. 2004].

Di Matteo et al. performed calculations of the Hurst exponent on the basis of the GHE method for the following market indices (the period covered by the analysis is given in brackets): NASDAQ 100 (1900–2001), NIKKEI 225 (1990–2001), WIG (1990–2001) and JSX (1991–2001), as well as 37 stock indexes (1997–2001) [Di Matteo et al. 2003: 183–188]. They proved that in the case of developed markets, the Hurst exponent values were lower than 0.5 while for less developed markets were higher than 0.5. In addition, by dividing the analyzed period into groups of 3,000 sessions, the authors indicated that the Hurst exponent values tended to fluctuate below 0.5; meanwhile, in the case of JSX and WIG indexes such fluctuations occurred mainly above 0.5.

Lipka and Los analyzing the daily data of eight stock indexes: ATX (Austria), KFX (Denmark), CAC 40 (France), DAX (Germany), OSLO TOTAL INDEX (Norway), IBEX 35 (Spain), MADRID GEN Index (SMSI – Spain) and FTSE 100 (United Kingdom) for various time intervals (with the largest number of observations, i.e. 4 437 collected for the British index, data for the period: April 2, 1984 to October 23, 2000), proved that the values of Hurst exponent, calculated using the modified MF-DFA method, they were lower than 0.5 for all indices, except for the Danish stock exchange index, for which the Hurst exponents was equal to 0.55. For stock indices of Spain, Germany and France, the values of Hurst exponent were slightly lower than 0.5: IBEX (0.46), SMSI (0.48), DAX (0.47) and CAC40 (0.46) [Lipka and Los 2003]. On the other hand, the calculation of Hurst exponent on the basis of the standard MF-DFA procedure³ clearly confirmed that the values of all Hurst exponent were lower than 0.5; wherein for some equity indexes, these values were only slightly lower than 0.5: Austria (0.48), Norway (0.49), Spain IBEX 35 and SMSI (0.46 each).

Jud proved that in the period 1981–July 2017 the value of the Hurst exponent, calculated for the six-month time intervals of the S&P 500 index, fluctuated around the value of 0.5 [Jud 2017: 71–75]. Thus, the author demonstrated the existence of two types of periods on the American market: the first in which short-term upward trends dominate and the second type of periods in which there are

³ IDL Wavelet Toolkit calculation package, developed by Research Systems and available on the website: [www1] <http://ion.researchsystems.com/IONScript/wavelet/website>

no such trends. In addition, the sharp decline in the Hurst exponent in 2015, the highest in twenty years, was recognized as one of the reasons for investors' rapid withdrawal of funds from hedge funds. It resulted that mathematical models for investing in line with the trend began to fail. Of course, customers withdrawing funds had no idea about the historical and current values of the Hurst exponent, and they only pointed to the decrease in returns generated by hedge funds. In addition, the author concluded that the values of the Hurst exponent vary depending on the sector to which the given security or stock index is included. In addition, the occurrence of cycles on the graph smoothed using a half-year double exponential moving average (DEMA) of the Hurst exponent has been proven.

Caporale et al. investigating the degree of persistence of market fear in the VIX index over the sample period 2004–2016, as well as some sub-periods, found that its properties changed over time: in normal periods it exhibited anti-persistence, whilst during crisis period the level of persistence was increasing [Caporale et al. 2018: 140–147]. Mynhardt et al. analyzing the persistence of financial markets in the period of 1990–2007 came to similar as above conclusions [Mynhardt et al. 2014]. The values of Hurst exponents for developed countries are equal to around 0.5, what indicated the adequacy of Efficient Market Hypothesis, while values of Hurst exponents calculated for emerging markets tended to 1, indicating the adequacy of Fractal Market Hypothesis. Similar conclusion were made when studying the foreign markets of analyzed countries. Using the Hurst exponent as a criterion of market efficiency, authors proved that the level of market efficiency is different for pre-crisis and crisis periods.

Jagric et al. based on wavelet analysis, estimating the Hurst exponent with the use of sliding time window, tested the existence of long-range dependence in six emerging markets [Jagric et al. 2005: 79–103]. Authors divided analyzed countries into two groups: with strong long-range dependence (Czech Republic, Hungary, Russia and Slovenia) and markets with a weak form of long-range dependence (Poland, Slovakia), while finding also evidence for the time dependence of the Hurst exponent.

Ferreira investigated the behavior of 18 Eastern European stock indexes with the use of a sliding window DFA and found the most of the considered indexes to be long-range dependent, but the level of the dependence tended to decrease in time run [Ferreira 2018: 454–470]. The analyzed market were ranked with the help of an efficiency index based on the method proposed by Kristoufek and Vosvrda [Kristoufek and Vosvrda 2013: 184–193]. The following markets: Czech Republic, Hungary and Poland resulted to be the most effective. Domino calculating the Hurst exponent for 126 companies listed on the Warsaw Stock Exchange in the period of 1991–2008, concluded that a decreasing value of the Hurst exponent could be regarded as a signal for a potential change in the present trend [Domino 2011: 98–109]. Caraianni testing the presence of multifractality in

the main CEE stock markets (Czech Republic, Poland and Hungary) with the use of MF-DFA method, indicated the long-run dependence on these markets, as well as its stability in time run [Caraiani 2012]. The Hurst exponents calculated for following equity indexes: PX, BUX and WIG were equal to 0.57, 0.55 and 0.54 respectively. Czarnecka and Wilamowska examined the WIG20 index for two time intervals (02.01.2014–17.10.2014 and 02.01.2015–16.10.2015) characterized by Hurst exponent belonging to two different ranges. In each of the analyzed time series, for all analyzed instruments, 200 subsequent changes in the price direction were noted (increase, decrease, increase, decrease, etc.) [Czarnecka and Wilimowska 2018]. In selected time intervals, slight downward trends were observed, with the first of them being the anti-persistent series ($H = 0.41$), and the second one – persistent ($H = 0.6$).

Raimundo and Okamoto based on the Hurst exponent and scaled range (R/S) for such currency pairs as: AUDJPY, CHFJPY, EUROJPY, GBPJPY and EUROCHF for one-session, one-hour and fifteen-minute time horizons from the period 01.01.2003–30.12.2014, proved the possibility of use of Hurst exponents as a tool for determining the degree of correlation and persistence on currency markets [Raimundo and Okamoto 2018: 116–124]. Their research confirmed earlier results obtained by Corazza and Malliaris, who revealed that the values of Hurst exponents statistically differ from 0.5 for most of the analyzed currency pairs and change dynamically over time, concluding that changes in exchange rates of currency pairs are subject to Brownian motion [Corazza and Malliaris 2002: 387–401].

Kale and Butar certified that the distribution of 500 Hurst exponents, obtained on the basis of R/S method for the analyzed time series, resulted to be normal [Kale and Butar 2011: 8–19]. In the process of analysis the following tests were implemented: Shapiro-Wilk, Kolmogorov-Smirnov and Anderson-Darling. In all cases, the received values of the p-value parameter were significantly greater than 0.05.

2. METHODOLOGY

The research was conducted in the following groups of assets (the number of assets from a given group after the dash):

- a) Stock indexes – 74,
- b) Currency pairs divided into segments:
 1. USD exchange rate in relation to 42 other currencies (USDXXX),
 2. EURO exchange rate in relation to 41 other currencies (EURXXX),
 3. JPY exchange rate in relation to 40 other currencies (JPYXXX),
 4. Other currency pairs – total 12.

In total, prices of 209 financial instrument were analyzed – see Appendix 1. The selection criterion was the availability of daily data in the following data basis: stooq.pl, investing.pl and bossa.pl.

For each of the assets, the calculations were divided into the following stages:

1. Examination of the normality of the distribution of logarithmic rates of return over the periods:

- a) Date of first listing of the instrument – 31.12.2019⁴ (full time window)
- b) 625 sessions preceding the session on 31.12.2019 (625-sessions window)
- c) 1250 sessions preceding the session on 31.12.2019 (1250-session window)

In the study of Jud, a 125-session interval was used, corresponding to the six-month horizon. Thus, in the case of the two and a half years and in case of five years investment horizon, the following numbers of sessions should be applied: 625 and 1250, respectively [Jud 2017: 74].

Two hypotheses were formulated:

H₀: The distribution of return rates in the analyzed time window is a normal distribution.

H₁: The distribution of return rates in the analyzed time window is not a normal distribution.

Three tests were selected for verification of these statistical hypotheses: Jarque-Barre (JB), Shapiro-Wilk (SW) and d'Agostino-Pearson (DA).

The choice of logarithmic rates of return was supported by their basic advantage, i.e. additivity.

This part of the study is auxiliary and aims to make the reader aware of the normality (or not) of the distribution of analyzed rates of return. More information on the relationship between the returns distribution on financial markets and the Hurst ratios can be found, among others in the paper of Barunik and Kristoufek [Barunik and Kristoufek 2010: 3844–3855].

2. In the following, the H exponents values for 625- and 1250-sessions windows will be designated: H_{625} and H_{1250} , respectively. Calculations of the H exponents were achieved according to the methodology proposed by Siroky. For the first 625 and 1250 prices of a given instrument, the H_{625} and H_{1250} were calculated for the first time (D_0)⁵, after which the calculation window was moved by one session – the next value of the H_{625} and H_{1250} were given for sessions numbers: 2–626 and 2–1251, respectively. Then the calculation window has been moved to the next session. In this way, for the financial instrument listed totally at K sessions, $K - 650 + 1$ exponents of H_{625} and $H - 1250 + 1$ exponents of H_{1250} were computed. In some cases, due to the shorter trading period of a given

⁴ For some assets, the last listing in 2019 took place on 30.12.2019, however in the further part of this article the last session of the year will be consistently designated as 31.12.2019.

⁵ This date will be denoted as D_0 . Of course, for H_{1250} and H_{625} two different D_0 dates are obtained.

instrument, it was not possible to calculate the value of H_{1250} (instruments listed for less than 625 sessions were not included in the analysis). For each of the instruments, the highest and lowest H values were given in the entire analyzed period (for both calculation windows, i.e. 1250 and 625 sessions), the last value H , i.e. as at 31.12.2019 (for both calculation windows), as well as the value of H_{1250} five years earlier (i.e. at the end of the session falling 1250 sessions before 31.12.2019) and the value of H_{625} two and a half years earlier (i.e. at the end of the session falling 625 sessions before 31.12.2019). The average values of H_{1250} and H_{625} were also assessed for each analyzed instrument. Cajueiro and Tabak implemented in their model so called „rolling sample”. In their model: estimates were applied over a given data sample, the statistic were calculated, then the sample was moved up, or „rolled” one observation forward before the procedure was repeated [Cajueiro and Tabak 2004b: 349–352].

3. For all analyzed financial instruments and for 625 and 1250 sessions (before 31.12.2019) Hurst exponents were calculated with the use of the following methods: Siroky (H_{Siroky}), DMA (H_{DMA}) and DFA (H_{DFA}). Then the following differences were examined:

$$H_{Siroky} - H_{DMA}$$

$$H_{Siroky} - H_{DFA}$$

4. The next part of the study compares average H_{1250} and H_{625} for stock indices and FX market, broken down into developed and emerging markets. The average value of the Hurst exponent is understood as the average values of H_{1250} or H_{625} for the time horizon ($D_0, D_{31122019}$)⁶, calculated for each of the analyzed instruments. In the case of stock indexes, it was taken into account whether the stock index belongs to a developed or an emerging country. In turn, the currency market was divided according to the following method. In the case of the exchange rate XXXYYY, two possibilities can occur. The first that the currencies XXX and YYY belong to developed countries, while the second that the currency XXX comes from a developed country and the currency YYY from an emerging market (other cases were not included in the analysis). The further part of this article assumes that the EUR currency originates from developed countries.

Two rankings have been created for both groups of assets, i.e. stock indices and currencies. The first, in which only the Hurst exponent is taken into account – the lower the Hurst exponent, the time series closer to the anti-persistent series and the higher position of a index in the ranking. This methodology is consisted

⁶ D_0 represents the date for which it was possible to provide the first value of the H_{1250} or H_{625} , respectively, according to the methodology described in the second point. $D_{31122019}$ – means the last session in 2019.

with the ranking proposed by Cajueiro and Tabak [Cajueiro and Tabak 2004a: 349–352; Cajueiro and Tabak 2005: 671–675].

The second ranking considers the absolute value of the distance between the Hurst exponent and the level of 0.5. The closer the value of such difference is to zero, the more analyzed time series resemble random walk, represented by the value of 0.5 of the Hurst exponent.

In order to obtain a sufficiently large sample, the widest possible time horizon was selected.

5. In the following part of the study, an analysis of the normality of the distribution of average Hurst exponents (H_{1250} and H_{625}) was developed. The average values were calculated for each of the analyzed financial instruments and then a statistical test was processed in each group (equity indexes and currencies with the division of the latter into: USDXXX, EURXXX, JPYXXX and other currency pairs).

For this purpose, three types of tests were applied: Jarque-Barre (JB), Shapiro-Wilk (SW) and d'Agostino-Pearson (DA).

Two statistic hypotheses were verified:

H₀: In a given segment of the financial market and in the analyzed time window the distribution of average values of Hurst coefficients is a normal distribution.

H₁: In a given segment of the financial market and in the analyzed time window the distribution of average values of Hurst coefficients is not a normal distribution.

3. RESULTS

3.1. Examination of the normality of the distribution of logarithmic rates of return

As a result of statistical tests, the following results were obtained:

a) Regarding the analysis of the normal distribution of returns and the full time window, the null hypothesis was rejected in favor of the alternative hypothesis for all analyzed financial instruments.

b) Regarding the 625- and 1250-session time window, the results are presented in Table 1. For the 625-session time window, there were no reasons for rejecting the null hypothesis in the case of two stock exchange indices (PSEI and WIG) and sixteen currency pairs (including six from the USDXXX segment, eight from the EURXXX segment and two from the other currency pairs segment). Only for the following currency pairs: USDDKK, USDEUR, EUREGP, EURIDR, EURNAD and EURZAR the p-value was higher than 0.05 in the case of one

statistical test. In other cases, the p-value was greater than 0.05 for at least two different statistical tests.

In turn, for the 1250-session time window, there was no reason to reject the null hypothesis in the case of only two exchange rates: USDKRW and USDSGD.

Table 1. Financial instruments for which there were no reason to reject the null hypothesis for 625 and 1250 sessions windows. (p-values > 0.05 in italics)

	Financial Instrument name	Window span	p-value for three types of test			How many times <i>p-value</i> > 0.05
Equity indexes						
1	PSEI	625	<i>0.2265</i>	<i>0.1902</i>	<i>0.2429</i>	3
2	WIG	625	<i>0.0628</i>	<i>0.1591</i>	<i>0.0707</i>	3
FX						
USDXXX						
1	USDDKK	625	<i>0.0534</i>	0.0029	0.0200	1
2	USDEUR	625	<i>0.0509</i>	0.0019	0.0160	1
3	USDKRW	625	<i>0.0717</i>	<i>0.1064</i>	<i>0.1078</i>	3
4	USDKRW	1250	<i>0.2834</i>	<i>0.2250</i>	<i>0.2573</i>	3
5	USDNAD	625	<i>0.2116</i>	<i>0.2082</i>	<i>0.2136</i>	3
6	USDNOK	625	<i>0.1447</i>	<i>0.2713</i>	<i>0.1329</i>	3
7	USDNZD	625	0.0375	<i>0.0895</i>	<i>0.0699</i>	2
8	USDSGD	1250	<i>0.4366</i>	<i>0.4992</i>	<i>0.4222</i>	3
EURXXX						
1	EURBGN	625	<i>0.1229</i>	0.0016	<i>0.0531</i>	2
2	EUREGP	625	0.0027	<i>0.1147</i>	0.0182	1
3	EURHUF	625	<i>0.0810</i>	<i>0.0857</i>	<i>0.1221</i>	3
4	EURIDR	625	0.0486	0.0117	<i>0.0911</i>	1
5	EURNAD	625	0.0153	<i>0.0980</i>	0.0311	1
6	EURNOK	625	0.0281	<i>0.0781</i>	<i>0.0545</i>	2
7	EURUAH	625	<i>0.1579</i>	0.0385	<i>0.1941</i>	2
8	EURZAR	625	0.0173	<i>0.0606</i>	0.0359	1
Other Currency pairs						
1	AUDCHF	625	<i>0.0643</i>	<i>0.1595</i>	<i>0.0691</i>	3
2	CADCHF	625	<i>0.4070</i>	<i>0.1245</i>	<i>0.3713</i>	3

Source: own study.

The distribution of returns in analyzed financial market segments generally is not a normal distribution. It can also be noted that in the case of a 625-session time window, the number of cases when there were no reason to reject the null hypothesis was definitely higher than for a 1250-session time window.

3.2. Comparison of average Hurst exponent H_{1250} and H_{625} for developed and emerging markets

The average values of H_{1250} exponents were higher than 0.5 for the following percentage of cases: 76.32% (developed markets) and 88.89% (emerging markets). In turn, in the case of average values of the H_{625} exponents, the difference between the markets became even more visible, because the percentage of average values of the H_{625} exponents, higher than 0.5 for emerging markets was equal to 94.44%, while for developed markets mounted to 65.79%.

The average H_{1250} values for developed markets oscillated in the range of 0.4727 (FTSE MIBTEL and FTSE MIBTEL) – 0.6596 (NZ50), while for emerging markets in the range: 0.4727 (MEXICIPC) – 0.6409 (SASESLCT). In turn, the average H_{625} values for developed markets belonged to the range: 0.4640 (SMI) – 0.6594 (NZ50), and for emerging markets to: 0.4609 (MEXICIPC) – 0.6615 (SASESLCT).

For the currency market and $H_{1250} < 0.5$, the percentage of cases when the second currency of a currency pair belonged to an emerging country and when both came from developed countries, was equal to 19.45% and 25.93%, respectively. In the group of developed-emerging market pairs, the lowest value was registered for EURRON (0.4258) and the highest for EURBGN (0.7649). In turn, in the group of developed-developed countries' exchanges the lowest and highest value of H_{1250} was registered for AUDCAD (0.4159) and EURCHF (0.6075), respectively. In the case of the condition $H_{625} < 0.5$, the percentage of cases when the second currency in the currency pair belonged to the emerging country and when both currencies belonged to developed countries mounted to 22.22% and 29.79%, respectively. In the group of developed-emerging markets pairs, the lowest value was registered for EURRON (0.4062) and the highest for EURBGN (0.7743). In turn, in the group of developed-developed countries the lowest and highest value of H_{1250} was registered for EURDKK (0.3665) and EURCHF (0.6049), respectively. Thus, for both H_{1250} and H_{625} exponents, the maximum and minimum values were obtained for the same currency pairs of developed and emerging countries, except EURDKK.

3.3 Comparison of average Hurst exponent H_{1250} and H_{625} calculated with the following methods: Siroky, DMA and DFA

For each of the analyzed instrument and $N = 625$ and $N = 1250$, the Hurst exponents were calculated with the use of the following methods: Siroky, DMA, and DFA. The obtained results are presented in Tables 2, 3 and 4.

Table 2. The percentage of cases when the H_{Siroky} was greater than that H_{DFA} and H_{DMA} , broken down into different market segments ($H = 625$ or $H = 1250$)

	Percentage of cases when $H_{Siroky} - H_{DMA} > 0$ N = 625	Percentage of cases when $H_{Siroky} - H_{DFA} > 0$ N = 625	Percentage of cases when $H_{Siroky} - H_{DMA} > 0$ N = 1250	Percentage of cases when $H_{Siroky} - H_{DFA} > 0$ N = 1250
Equity indexes	81.25	82.16	89.36	90.25
FX	86.36	87.42	93.56	92.69

Source: own study.

Table 3. Average values of differences: $H_{Siroky} - H_{DMA}$ and $H_{Siroky} - H_{DFA}$, broken down into different market segments ($H = 625$ or $H = 1250$)

	Average value of the difference $H_{Siroky} - H_{DMA}$ N = 625	Average value of the difference $H_{Siroky} - H_{DFA}$ N = 625	Average value of the difference $H_{Siroky} - H_{DMA}$ N = 1250	Average value of the difference $H_{Siroky} - H_{DFA}$ N = 1250
Equity indexes	0.0056	0.0170	0.0178	0.0141
FX	0.0276	0.0285	0.0251	0.0188

Source: own study.

In the vast majority of cases, H_{Siroky} values were slightly higher than H_{DMA} and H_{DFA} . The change in the number of sessions from $N = 625$ to $N = 1250$ led to an increase of the percentage of cases when where following inequalities occurred: $H_{Siroky} - H_{DMA} > 0$ and $H_{Siroky} - H_{DFA} > 0$.

Table 4. Percentage of differences $H_{Siroky} - H_{DMA} > 0$ for different market segments

	N = 625					N = 1250				
	<0	(0.00-0.1)	(0.1-0.2)	(0.2-0.3)	>0.3	<0	(0.0-0.1)	(0.1-0.2)	(0.2-0.3)	>0.3
Equity indexes	12.83	50.97	21.41	4.09	10.71	8.60	55.25	16.64	5.00	14.51
Currencies	14.92	50.90	18.62	2.62	12.94	10.06	52.60	18.34	6.60	12.40

Source: own study.

Table 5. Percentage of differences $H_{Siroky} - H_{DFA} > 0$ for different market segments

	N = 625					N = 1250				
	<0	(0.0-0.1)	(0.1-0.2)	(0.2-0.3)	>0.3	<0	(0.0-0.1)	(0.1-0.2)	(0.2-0.3)	>0.3
Equity indexes	7.46	57.91	13.74	6.17	14.72	6.93	58.96	20.83	6.96	6.32
Currencies	11.31	50.46	23.67	4.07	10.48	17.12	52.20	18.47	4.65	7.56

Source: own study.

The distribution of differences $H_{Siroky} - H_{DMA} > 0$ and $H_{Siroky} - H_{DFA} > 0$ clearly indicates that their highest percentage was registered in the range 0.0–0.1 (for all financial instruments). In addition, the percentage of analyzed differences registered in the range 0.0–0.1 was higher for N = 1250 sessions than for N = 625 sessions.

3.4 The first ranking for average Hurst exponents

3.4.1 RANKING FOR EQUITY INDEXES

Rankings of average values H_{1250} and H_{625} are presented in Appendix 2. For both rankings, the lowest value of the average values H_{1250} and H_{625} was observed for the MEXICIPC index, mounting to 0.4473 and 0.4609, respectively, while the highest values were noted for index NZ50, and were equal to 0.6596 and 0.6594, respectively. In the ranking of twenty stock indexes with the lowest average values of H_{1250} and H_{625} exponents, 12 and 13 (respectively), belonged to developed equity markets. What's more interesting, the indexes of countries such as the USA and Germany were not included in the top twenty group. For example DJIA was classified at 56th (H_{1250}) and 58th (H_{625}) and DAX at 34th (H_{1250}) and 23rd (H_{625}) positions, In turn, the WIG index was ranked at positions 28th (H_{1250}) and 37th (H_{625}).

3.4.2 RANKINGS FOR CURRENCY PAIRS

In the top twenty ranking of the lowest H_{1250} and H_{625} values, there were 8 currency pairs belonging to developed markets. In addition, the AUDCAD and EURDKK pairs were classified in the highest positions in the ranking for H_{1250} , while for H_{625} ranking the order was the reverse. In turn, in the ranking of the twenty highest H_{1250} and H_{625} values there was only one currency pair (EURCHF), when both currencies (EUR and CHF) belong to developed countries.

The highest (the lowest) classified currency pair, when the second currency belongs to the developing country was EURRON (EURBGN). This relationship is true both for H_{1250} and H_{625} .

The average value of H_{1250} , calculated for pairs in which both currencies belonged to developed countries was equal to 0.5102. The average value of H_{1250} for the group in which the second currency pertains a developing country, amounted to 0.5467. For the H_{625} these values were equal to 0.5009 and 0.5380 respectively.

3.5 The second ranking for average Hurst exponents

Rankings of average values H_{1250} and H_{625} are presented in Appendix 3.

3.5.1 RANKING FOR EQUITY INDEXED

In case of the ranking based on the distance between the Hurst exponent from the level 0.5, for H_{1250} and H_{625} , there were 12 and 13 indices, respectively (out of 20) belonging to stock exchanges localized in developed countries. The equity index for New Zealand was classified on the last place in the same ranking. Relatively low positions in the ranking of such indices as NASDAQ, NASDAQ100 or SDAX (all from developed countries) are noteworthy.

3.5.2 RANKING FOR CURRENCY PAIRS

In the top twenty currency pairs with the lowest values $|H_{1250}-0.5|$ and $|H_{625}-0.5|$, there were 10 and 8 currency pairs, respectively belonging to developed markets. In addition, the JPYAUD and GPBCAD pairs were classified in the highest positions in the ranking for $|H_{1250}-0.5|$ and $|H_{625}-0.5|$, respectively. In turn, in the last twenty (i.e. with the highest values of $|H_{1250}-0.5|$ and $|H_{625}-0.5|$), there was one currency pair (EURCHF) for $|H_{1250}-0.5|$ and 3 currency pairs (AIDCAD, EURCHF and EURDKK) for $|H_{1250}-0.5|$, when both currencies belong to developed countries.

In the case of $|H_{1250}-0.5|$ and $|H_{625}-0.5|$, the highest classified currency pair, when the second currency belongs to the developing country was EURXPD and USDPLN, respectively. The EURBGN currency pair was classified in the last position in rankings for $|H_{1250}-0.5|$ as well as for $|H_{625}-0.5|$.

The average value of $|H_{1250}-0.5|$ calculated for pairs, in which both currencies belonged to developed countries was equal to 0.0258. In the group when the second currency pertains a developing country, the average value mounted to 0.0568. For the $|H_{625}-0.5|$, these values were equal to 0.0247 and 0.0554 respectively.

In the case of a cumulative ranking (created for both stock indexes and currency pairs), the extreme values, i.e. the highest and lowest in the first and second ranking, belonged to currency pairs. This fact allows to draw the conclusion that the values of the H_{1250} and H_{625} , calculated for the stock indexes, belonged to a narrower range than the same exponents obtained for currency pairs.

3.6 The relationship between following pairs of Hurst exponents: H_{625} vs H_{1250} and $|H_{1250}-0.5|$ vs $|H_{625}-0.5|$

Relationships between average Hurst exponents H_{625} vs H_{1250} and $|H_{1250}-0.5|$ vs $|H_{625}-0.5|$, calculated for stock indices and currency pairs, are shown in Figures no. 1–4.

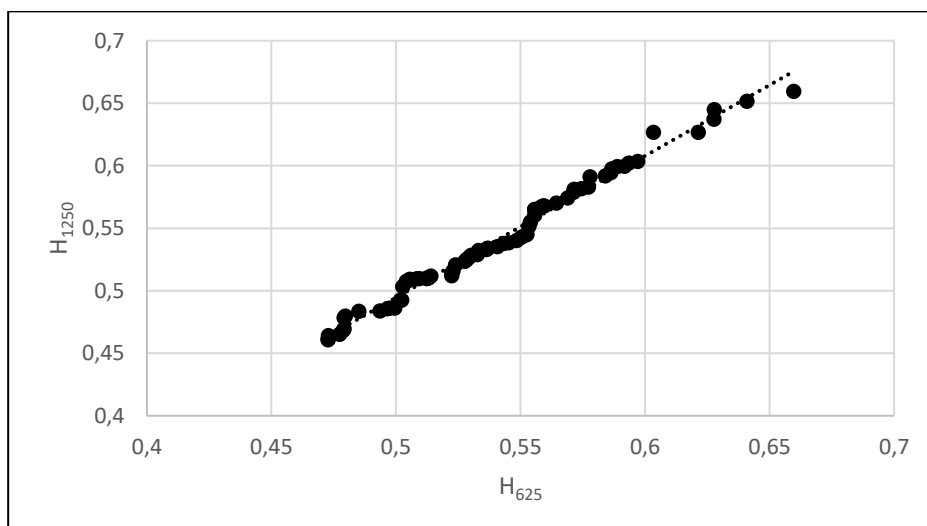


Chart 1. Relationship between the average values H_{625} and H_{1250} for stock indices

Source: own calculations.

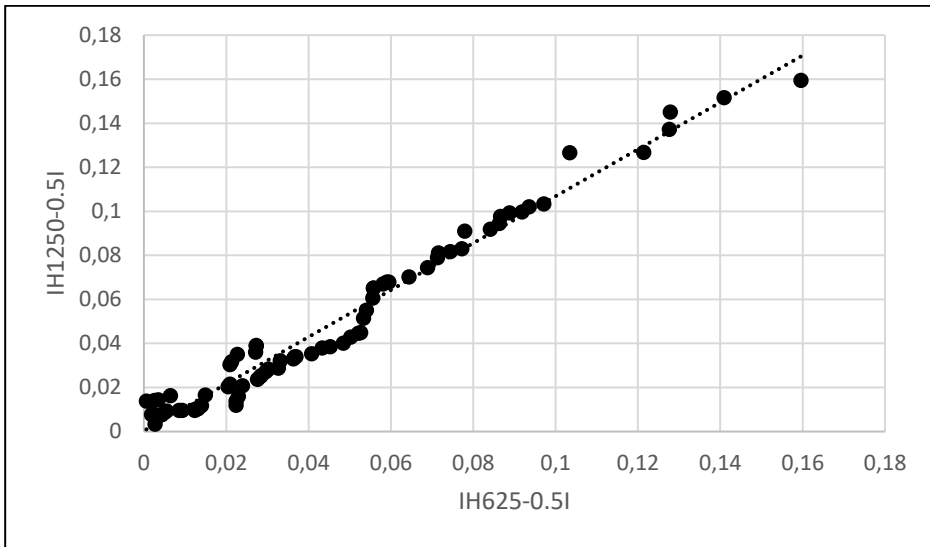


Chart 2. Relationship between the average values $|H_{625}-0.5|$ and $|H_{1250}-0.5|$ for stock indices

Source: own calculations.

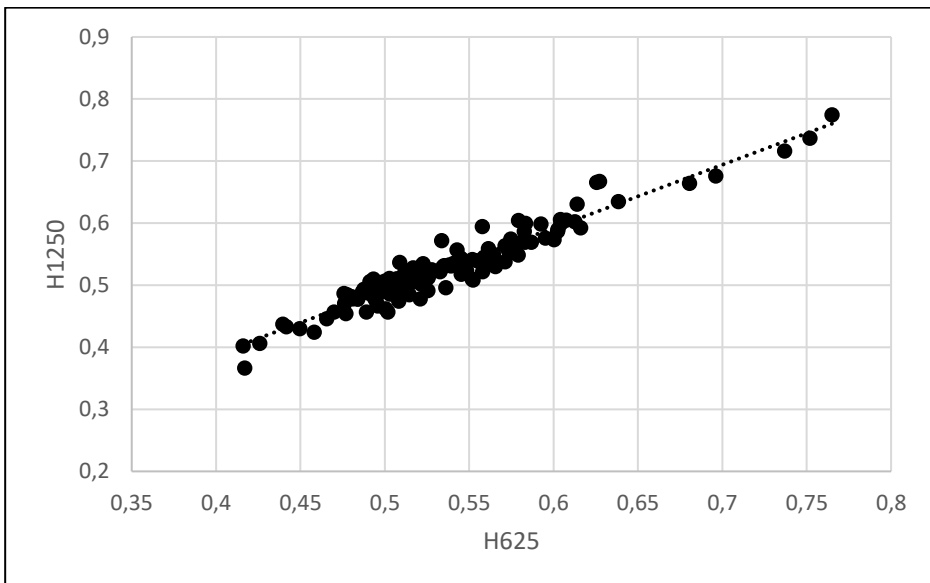


Chart 3. Relationship between the average values H_{625} and H_{1250} for currency pairs

Source: own calculations.

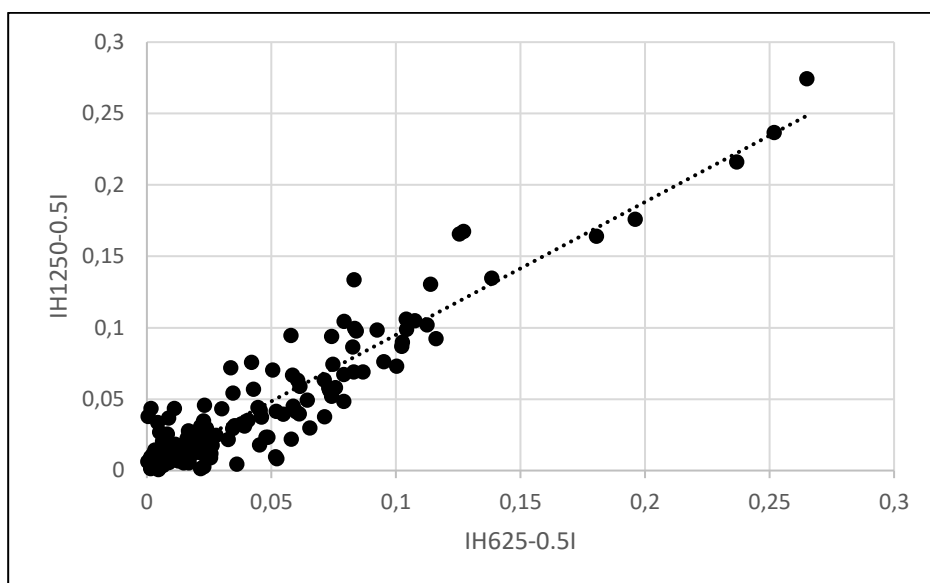


Chart 4. Relationship between the average values $|H_{625}-0.5|$ and $|H_{1250}-0.5|$ for currency pairs

Source: own calculations.

Determination coefficients R^2 calculated for average Hurst exponents H_{625} and H_{1250} , as well as for $|H_{1250}-0.5|$ and $|H_{625}-0.5|$ mounted to the following values:

- Equity indexes, H_{1250} vs. H_{625} : $R^2 = 0.98731$
- Equity indexes, $|H_{1250}-0.5|$ vs $|H_{625}-0.5|$: $R^2 = 0.97150$
- Currency pairs, H_{1250} vs. H_{625} : $R^2 = 0.92461$
- Currency pairs, $|H_{1250}-0.5|$ vs $|H_{625}-0.5|$: $R^2 = 0.87406$

In the segment of stock indices, the values of R^2 coefficients were higher than in the currency segment both for the Hurst exponents, as well as when analyzing Hurst exponents' distance from the level of 0.5, e.g. for $|H_{1250}-0.5|$ and $|H_{625}-0.5|$. These results confirm the conclusion obtained in point 3.4.2 on greater compression of the average Hurst exponent values for the capital market index segment than for currencies.

Analysis of average values of H_{125} vs. H_{625} and $|H_{1250}-0.5|$ vs. $|H_{625}-0.5|$, broken down into developed and emerging markets, proved that determination coefficients R^2 are higher in the group of emerging markets – see Table 6.

Table 6. R^2 coefficients for average values of H_{125} vs. H_{625} and $|H_{125}-0.5|$ vs. $|H_{625}-0.5|$ broken down into developed and emerging markets

Stock indexes	Developed markets	Emerging markets
H_{1250} vs. H_{625}	$R^2 = 0.98241$	$R^2 = 0.99033$
$ H_{1250}-0.5 $ vs $ H_{625}-0.5 $	$R^2 = 0.96438$	$R^2 = 0.97151$
Currencies	Developed-developed countries	Developed-emerging countries
H_{1250} vs. H_{625}	$R^2 = 0.82306$	$R^2 = 0.93267$
$ H_{1250}-0.5 $ vs $ H_{625}-0.5 $	$R^2 = 0.63984$	$R^2 = 0.89273$

Source: own calculations.

3.7. Normal distribution of the average values of the Hurst exponents: H_{1250} and H_{625}

The results obtained are presented in Table 7.

The distribution of average values of Hurst exponents is a normal distribution except for the following groups of assets:

- USDXXX segment for H_{1250}
- USDXXX segment for H_{625}
- JPYXXX segment for H_{625}

In all four cases, the p-value was lower than 0.05 for all three tests (JB, SW and DA). In turn, in the case of the stock indexes and H_{625} , the null hypothesis was rejected with the use of SW test, when the p-value was equal to 0.0499, while in the case of the remaining JB and SW tests, p-value, was higher than 0.05 and amounted to: 0.2079 and 0.1970 respectively.

Table 7. The p-value parameters calculated with the use of normal distribution tests (JB, SW and DA) for average values of H_{1250} and H_{625} (p-value < 0.05 in italics).

Instrument	H_{1250}			H_{625}		
	JB	SW	DA	JB	SW	DA
Equity indexes	0.2062	0.0730	0.1904	0.2079	<i>0.0499</i>	0.1970
USDXXX	<i>0.0000</i>	<i>0.0000</i>	<i>0.0000</i>	<i>0.0000</i>	<i>0.0001</i>	<i>0.0000</i>
JPYXXX	0.1543	0.4084	0.0638	<i>0.0305</i>	<i>0.0227</i>	<i>0.0066</i>
Other currency pairs	0.6583	0.7737	0.2743	0.8339	0.8772	0.7660

Source: own study.

CONCLUSIONS

The main conclusions obtained in the study may be formulated as follows:

a) The percentage of cases, when the condition $H_{1250} > 0.5$ is met, was higher for emerging than for developed markets. This relationship was even more apparent for H_{625} . Given the relationship between the fractal dimension and the Hurst exponent, this means that investment risk in emerging markets was lower than in developed markets and this tendency has increased in the past two and a half years (assuming that the fractal dimension is a risk measure).

b) The Hurst coefficient values calculated with the use of Siroky method are similar to the results obtained on the basis of DFA and DMA methods. The differences in the obtained values of the Hurst exponents, calculated with the use of three different methods, result from the approximation of MTR applied in the Siroky method. As the number of sessions increases, the differences in Hurst coefficient values calculated on the basis of Siroky's method are similar to those obtained with the application of the DMA and DFA methods.

c) The average values of Hurst exponents for the capital market indexes were more compressed than those calculated for currency pairs.

d) In the case of the forex market and the first ranking, among the currency pairs with the lowest H values, there were pairs in which both currencies belong to developed countries, as well as pairs, in which the second currency derived from the emerging market. In turn, among currency pairs with the highest H values dominated currency pairs in which the second currency belonged to the emerging market country.

e) Analyzing the distribution of developed-developed and developed-emerging currency pairs in the first ranking, it can be concluded that these exchange rates in the developed-developed group are more effective than in the case of the second group. However, it should be remembered that this is a general conclusion, and the effectiveness of the exchange rate of a given currency pair is its individual feature.

f) With few exceptions, the distribution of average values of Hurst exponents is a normal distribution.

The obtained results regarding the increase in investment risk proved conclusions presented by Przekota, who verified the hypothesis the lower values of the fractal dimension are consistent with lower risk, and higher values of the fractal dimension with higher risk [Przekota 2012: 186–187].

The presented results are in line with those published in relation to the S&P 500 index by Alvarez-Ramirez et al. and Dominique and Rivera, and in the case of the NASDAQ index by Glenn [Alvarez-Ramirez et al. 2008: 6159–6169; Dominique and Rivera 2011: 1–6; Glenn 2007]. The results also confirm the conclusions obtained by Huang and Yang in the process of analyzing the Hurst

exponents for the indices: DJIA, NASDAQ and S&P 500, as well as results revealed by Henry for DAX, TOPIX and Nikkei 225 [Huang and Yang 1999; Henry 2002: 725–729].

On the other hand, the obtained results are in contradiction to the conclusions published by: Onali and Goddard for the FTSEMIBTEL and PX indices, by Henry for the Kospi index, Berg and Lyhagen for OMX Stockholm Index, by Barkoulas and Baum for three indices: DJIA, NASDAQ and S&P 500 and by Cajueiro and Tabak for the MEXIPC and IPC indexes [Onali and Goddard 2010; Henry 2002: 725–729; Barkoulas and Baum 1996: 253–259; Cajueiro and Tabak 2004b: 349–352; Cajueiro and Tabak 2005: 671–675]. In the case of Latin American stock exchange indices, the two Mexican stock exchange indices MEXIPC and IPC were ahead of the highest qualified Asian index, i.e. Kospi. The latter was classified in the first ranking at the higher position than the Bovespa index. It should be noted, however, that only a few Latin American stock exchange indices were applied in the study.

In the case of the S&P 500 index, the obtained results contradict the conclusions drawn from the work of Granger and Ding, Granger and Hyung, as well as Lo and Jacobsen [Granger and Ding 1995: 67–91; Granger and Hyung 2004: 399–421; Lo 2004: 15–29; Jacobsen 1995: 37–52].

The received results regarding the percentage of cases when the average H values were higher than 0.5 for emerging markets confirm those by Di Matteo et al. and by Kapecka [Di Matteo et al. 2003: 183–188; Kapecka 2015: 59–75]. In the part regarding the percentage of cases when the average H values were higher than 0.5 for developed markets, the obtained results are consistent with the research of Kapecka and Barunik and Kristoufek, while contradicting the conclusions presented by Mateo et al. as well as by Lipka and Los [Kapecka 2015: 59–75; Barunik and Kristoufek 2010: 3844–3855; Di Matteo et al. 2003: 183–188; Lipka and Los 2003]. However, it should be remembered that the first research concerns only four stock indexes. In addition, in the current study the average values of the Hurst exponent were taken into account, while in both cited papers only one Hurst exponent was calculated for each of the analyzed stock indices. The results of the study referring to the persistence of Hurst exponents for Latin American markets are consistent with the conclusions published by Kyawa et al. [Kyawa et al. 2004].

On the basis of the presented research it was found that the periods with large Hurst exponents can be predicted more accurately than those with H values close to random series. This suggests that analyzed markets (equity and forex) are not totally random in all periods. Some periods are characterized by strong trend structure and this structure can be used to open and close investment positions.

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APPENDIXES

Appendix 1

The list of analyzed financial assets and the dates of their first listing disclosed in the database (portals: stooq.pl and bossa.pl)

No.	Equity indexes	First quotation	FX market		FX market	
1	AEX	04.01.1983	USDXXX	First quotation	EURSEK	04.01.1971
2	ALL ORDINARIES	31.10.1989	USDAUD	04.01.1971	EURTHB	04.01.1993
3	AMEX	03.01.1995	USDBGN	23.10.1995	EURTRY	02.01.1991
4	ATHEX COMPOSITE	02.01.1987	USDBRL	02.01.1995	EURTWD	02.01.1991
5	ATX	11.11.1992	USDCAD	04.01.1971	EURUAH	02.09.1996
6	BEL20	29.10.1991	USDCHF	04.01.1971	EURXAG	02.01.1991
7	BET	31.10.2000	USDCLP	25.10.1993	EURXAU	02.01.1991
8	BIST 100	03.01.1988	USDCNY	09.01.1984	EURXDR	02.01.1991
9	BOVESPA	02.01.1995	USDCZK	23.10.1995	EURXPD	02.01.1991
10	BSE SENSEX	03.04.1979	USDDKK	02.01.1984	EURXPT	02.01.1991
11	BSHARES	11.05.1998	USDEGP	16.11.1995	EURZAR	02.01.1991
12	BUENOS	08.10.1996	USDEURO	04.01.1971	JPYXXX	First quotation
13	BUX	02.01.1991	USDGBP	04.01.1971	JPYCNV	02.01.1991
14	CAC40	02.01.1969	USDHKD	09.01.1984	JPYHKD	02.01.1991
15	China A50	19.05.2010	USDHRK	23.10.1995	JPYIDR	25.10.1993
16	CSE ALL SHARES	13.07.2000	USDHUF	23.10.1995	JPYILS	25.10.1993
17	DAX	28.09.1959	USDIDR	25.10.1993	JPYINR	02.01.1991
18	DJIA	27.05.1896	USDILS	25.10.1993	JPYKRW	02.01.1991
19	DJTA	26.10.1896	USDINR	02.01.1973	JPYMYR	02.01.1991
20	DJUA	02.01.1929	USDISK	23.10.1995	JPYNZD	02.01.1991
21	EOE	02.01.1995	USDJPY	21.01.1974	JPYPHP	25.10.1993
22	EURO STOXX 50	15.08.2011	USDKRW	13.04.1981	JPYSGD	02.01.1991
23	FTSE 100	22.10.1992	USDMXN	09.11.1989	JPYTHB	04.01.1993
24	FTSE MIBTEL	02.01.1998	USDMYR	08.01.1990	JPYTWD	02.01.1991
25	FTSE250	31.12.1985	USDNAD	09.01.1984	JPYBGN	23.10.1995
26	HANG SENG	24.11.1969	USDNOK	04.01.1971	JPYCHF	04.01.1971
27	HEX	02.01.1995	USDNZD	04.01.1971	JPYCZK	23.10.1995
28	IBEX 35	05.01.1987	USDPHP	25.10.1993	JPYDKK	02.01.1991
29	IDX COMPOSITE	09.04.1990	USDPLN	16.05.1991	JPYGBP	04.01.1971
30	IPC	08.11.1991	USDRON	23.10.1995	JPYHRK	23.10.1995
31	IPSA	02.01.1987	USDRUB	23.10.1995	JPYHUF	23.10.1995
32	JCI	04.04.1983	USDSGD	02.01.1981	JPYISK	23.10.1995
33	KLCI	03.01.1977	USDSEK	04.01.1971	JPYNOK	02.01.1991
34	KOSPI	01.05.1981	USDTHB	04.01.1993	JPYPLN	16.05.1991

35	MDAX	29.02.1996	USDTRY	02.01.1984	JPYRON	23.10.1995
36	MERVAL	16.05.1988	USDTRW	02.01.1984	JPYRUB	23.10.1995
37	MEXICIPC	20.12.1993	USDUAH	02.09.1996	JPYSEK	04.01.1971
38	MOEX	23.09.1997	USDXAG	09.01.1984	JPYTRY	02.01.1991
39	NASDAQ	03.05.1971	USDXAU	09.01.1984	JPYUAH	02.09.1996
40	NASDAQ 100	01.10.1985	USDXDR	02.01.1991	JPYBRL	03.01.1995
41	NIFTY 50	06.11.1995	USDXPB	09.01.1984	JPYCAD	04.01.1971
42	NIKKEI	01.06.1949	USDXPB	09.01.1984	JPYCLP	25.10.1993
43	NZ 50	03.01.2001	USDZAR	04.01.1971	JPYMXN	02.01.1991
44	OMX RIGA	03.01.2000	EURXXX	First quotation	JPYEGP	16.11.1995
45	OMX S30	01.10.1986	EURAUD	02.01.1980	JPYNAD	02.01.1991
46	OMX STOCKHOLM	30.09.1986	EURBGN	23.10.1995	JPYZAR	02.01.1991
47	OMX TALLINN	03.01.2000	EURBRL	03.01.1995	JPYXAG	02.01.1991
48	OMX VILNIUS	03.01.2000	EURCAD	04.01.1971	JPYXAU	02.01.1991
49	OSE ALL	03.01.1983	EURCHF	04.01.1971	JPYXPB	02.01.1991
50	PSEI	02.01.1986	EURCLP	25.10.1993	JPYXPT	02.01.1991
51	PSI 20	03.01.1983	EURCNY	02.01.1991	JPYXDR	02.01.1991
52	PX	07.09.1993	EURCZK	23.10.1995	JPYAUD	02.01.1984
53	RTSI	04.09.1995	EURDKK	02.01.1991	Other Currency Pairs	First quotation
54	RUSSEL	22.10.2001	EUREGP	16.11.1995	AUDCHF	02.01.1980
55	S&P ASX 200	01.06.1992	EURGBP	04.01.1971	AUDNZD	02.01.1991
56	S&P TSX COMPOSITE	03.01.1977	EURHKD	02.01.1991	AUDCAD	02.01.1980
57	SASESLCT	02.01.2003	EURHRK	23.10.1995	CADCHF	04.01.1971
58	SAX	03.07.1995	EURHUF	23.10.1995	GBP AUD	02.01.1980
59	SDAX	15.03.1999	EURIDR	25.10.1993	GBP CAD	04.01.1971
60	SET	05.01.1982	EURILS	25.10.1993	GBP CHF	04.01.1971
61	Shanghai Composite	20.12.1990	EURINR	02.01.1991	GBP PLN	16.05.1991
62	SMI	01.07.1988	EURISK	23.10.1995	NZD JPY	02.01.1991
63	SOFIX	26.11.2001	EURJPY	04.01.1971	GBP NZD	09.01.1984
64	SP500	17.02.1885	EURKRW	13.04.1981	AUD PLN	16.05.1991
65	STRAITS TIMES	28.12.1987	EURMXN	02.01.1991	CHF PLN	16.05.1991
66	TA 35	20.10.1992	EURMYR	02.01.1991		
67	TAIEX	05.01.1995	EURNAD	02.01.1991		
68	TECDAX	16.09.1999	EURNOK	02.01.1991		
69	TOPIX	22.10.2001	EURNZD	02.01.1991		
70	TSE300	15.08.1989	EURPHP	25.10.1993		
71	UK 100	13.11.1935	EURPLN	16.05.1991		
72	UX	03.11.1997	EURRON	23.10.1995		
73	WIG	16.04.1991	EURRUB	23.10.1995		
74	XU 100	02.01.1990	EURSDG	02.01.1981		

Source: own calculations.

Appendix 2

First and the second ranking of equity indexes
Indexes from developed countries marked in bold

Ranking	First index ranking				Second index ranking			
	Index	Average value of H_{1250}	Index	Average value of H_{625}	Index	$ H_{1250}-0.5 $	Index	$ H_{625}-0.5 $
1	MEXICIPC	0.4727	MEXICIPC	0.4609	S&P ASX 200	0.0006	PX	0.0138
2	FTSE MIBTEL	0.4727	IPC	0.4612	BIST 100	0.0018	ALL ORDINARIES	0.0076
3	IPC	0.4729	SMI	0.464	XU 100	0.0023	KLCI	0.0075
4	EURO STOXX 50	0.4773	FTSE MIBTEL	0.465	MOEX	0.0025	OMX S30	0.014
5	IBEX 35	0.4787	EURO STOXX 50	0.4683	STRAITS TIMES	0.0027	KOSPI	0.0033
6	FTSE 100	0.4791	IBEX 35	0.4696	BOVESPA	0.0035	OMX STOCKHOLM	0.0144
7	KOSPI	0.4791	S&P ASX 200	0.4785	TSE300	0.0041	TSE300	0.0075
8	UK 100	0.4796	CAC40	0.4797	S&P TSX COMPOSITE	0.0044	S&P TSX COMPOSITE	0.0076
9	PX	0.4851	FTSE 100	0.4835	ALL ORDINARIES	0.0055	BIST 100	0.0092
10	CAC40	0.4936	UK 100	0.4837	CAC40	0.0064	UK 100	0.0163
11	BOVESPA	0.4965	OMX STOCKHOLM	0.4856	SMI	0.0085	SET	0.0096
12	MOEX	0.4975	OMX S30	0.486	ATX	0.0093	XU 100	0.0096
13	S&P ASX 200	0.4994	PX	0.4862	OMX STOCKHOLM	0.0123	STRAITS TIMES	0.0096
14	BIST 100	0.5018	ALL ORDINARIES	0.4924	OMX S30	0.0125	DAX	0.0101
15	XU 100	0.5023	KLCI	0.4925	HANG SENG	0.0131	BEL20	0.0102
16	STRAITS TIMES	0.5027	KOSPI	0.5033	TAIEX	0.014	AEX	0.0117
17	TSE300	0.5041	TSE300	0.5075	PX	0.0149	FTSE 100	0.0165
18	S&P TSX COMPOSITE	0.5044	S&P TSX COMPOSITE	0.5076	UK 100	0.0204	CAC40	0.0203
19	ALL ORDINARIES	0.5055	BIST 100	0.5092	FTSE 100	0.0209	IBEX 35	0.0304
20	SMI	0.5085	SET	0.5096	KOSPI	0.0209	S&P ASX 200	0.0215
21	ATX	0.5093	XU 100	0.5096	IBEX 35	0.0213	EURO STOXX 50	0.0317
22	OMX STOCKHOLM	0.5123	STRAITS TIMES	0.5096	AEX	0.0224	EOE	0.0119
23	OMX S30	0.5125	DAX	0.5101	EOE	0.0224	PSEI	0.0138
24	HANG SENG	0.5131	BEL20	0.5102	EURO STOXX 50	0.0227	FTSE MIBTEL	0.035
25	TAIEX	0.514	AEX	0.5117	WIG	0.0229	DJUA	0.0159
26	AEX	0.5224	EOE	0.5119	KLCI	0.0239	RTSI	0.0207
27	EOE	0.5224	PSEI	0.5138	IPC	0.0271	SMI	0.036
28	WIG	0.5229	DJUA	0.5159	FTSE MIBTEL	0.0273	IPC	0.0388
29	KLCI	0.5239	RTSI	0.5207	MEXICIPC	0.0273	MEXICIPC	0.0391
30	HEX	0.5276	HANG SENG	0.5236	HEX	0.0276	HANG SENG	0.0236
31	DJUA	0.5283	BOVESPA	0.5249	DJUA	0.0283	BOVESPA	0.0249
32	BEL20	0.5285	HEX	0.5254	BEL20	0.0285	HEX	0.0254
33	RTSI	0.5296	TAIEX	0.5271	RTSI	0.0296	TAIEX	0.0271

34	DAX	0.5302	ATX	0.5283	DAX	0.0302	ATX	0.0283
35	SET	0.5326	TA 35	0.5288	SET	0.0326	TA 35	0.0288
36	IPSA	0.5331	MOEX	0.5321	IPSA	0.0331	MOEX	0.0321
37	IDX COMPOSITE	0.5363	WIG	0.5329	IDX COMPOSITE	0.0363	WIG	0.0329
38	JCI	0.5366	JCI	0.5338	JCI	0.0366	JCI	0.0338
39	TA 35	0.5369	IDX COMPOSITE	0.534	TA 35	0.0369	IDX COMPOSITE	0.034
40	China A50	0.5407	DJTA	0.5352	China A50	0.0407	DJTA	0.0352
41	ATHEX COMPOSITE	0.5433	NIKKEI	0.5379	ATHEX COMPOSITE	0.0433	NIKKEI	0.0379
42	RUSSEL	0.5452	China A50	0.5384	RUSSEL	0.0452	China A50	0.0384
43	DJTA	0.5484	TOPIX	0.5401	DJTA	0.0484	TOPIX	0.0401
44	BUX	0.5502	ATHEX COMPOSITE	0.5428	BUX	0.0502	ATHEX COMPOSITE	0.0428
45	PSEI	0.5521	FTSE250	0.5446	PSEI	0.0521	FTSE250	0.0446
46	BET	0.5526	RUSSEL	0.5449	BET	0.0526	RUSSEL	0.0449
47	NIKKEI	0.5533	BET	0.5514	NIKKEI	0.0533	BET	0.0514
48	TOPIX	0.554	UX	0.5551	TOPIX	0.054	UX	0.0551
49	AMEX	0.5556	IPSA	0.5605	AMEX	0.0556	IPSA	0.0605
50	OSE ALL	0.5557	OSE ALL	0.565	OSE ALL	0.0557	OSE ALL	0.065
51	PSI 20	0.5557	PSI 20	0.565	PSI 20	0.0557	PSI 20	0.065
52	FTSE250	0.5581	MDAX	0.5669	FTSE250	0.0581	MDAX	0.0669
53	BSE SENSEX	0.559	BSE SENSEX	0.5678	BSE SENSEX	0.059	BSE SENSEX	0.0678
54	NIFTY 50	0.5595	AMEX	0.5679	NIFTY 50	0.0595	AMEX	0.0679
55	UX	0.5644	NIFTY 50	0.5702	UX	0.0644	NIFTY 50	0.0702
56	DJIA	0.5689	SP500	0.5743	DJIA	0.0689	SP500	0.0743
57	BSHARES	0.5713	Shanghai Composite	0.5789	BSHARES	0.0713	Shanghai Composite	0.0789
58	Shanghai Composite	0.5715	DJIA	0.5811	Shanghai Composite	0.0715	DJIA	0.0811
59	SP500	0.5744	CSE ALL SHARES	0.5816	SP500	0.0744	CSE ALL SHARES	0.0816
60	OMX TALLINN	0.5772	BSHARES	0.583	OMX TALLINN	0.0772	BSHARES	0.083
61	MDAX	0.5779	SOFIX	0.591	MDAX	0.0779	SOFIX	0.091
62	SAX	0.5841	NASDAQ	0.5918	SAX	0.0841	NASDAQ	0.0918
63	CSE ALL SHARES	0.5863	SAX	0.5944	CSE ALL SHARES	0.0863	SAX	0.0944
64	NASDAQ	0.5866	NASDAQ 100	0.5976	NASDAQ	0.0866	NASDAQ 100	0.0976
65	OMX RIGA	0.5888	OMX TALLINN	0.5992	OMX RIGA	0.0888	OMX TALLINN	0.0992
66	NASDAQ 100	0.5918	BUX	0.5996	NASDAQ 100	0.0918	BUX	0.0996
67	SOFIX	0.5935	TECDAX	0.602	SOFIX	0.0935	TECDAX	0.102
68	TECDAX	0.5971	SDAX	0.6033	TECDAX	0.0971	SDAX	0.1033
69	SDAX	0.6034	BUENOS	0.6266	SDAX	0.1034	BUENOS	0.1266
70	OMX VILNIUS	0.6214	MERVAL	0.6267	OMX VILNIUS	0.1214	MERVAL	0.1267
71	BUENOS	0.6276	OMX VILNIUS	0.6371	BUENOS	0.1276	OMX VILNIUS	0.1371
72	MERVAL	0.6278	OMX RIGA	0.645	MERVAL	0.1278	OMX RIGA	0.145
73	SASESLCT	0.6409	SASESLCT	0.6516	SASESLCT	0.1409	SASESLCT	0.1516
74	NZ 50	0.6596	NZ 50	0.6594	NZ 50	0.1596	NZ 50	0.1594

Source: own calculations.

Appendix 3

First and the second ranking of currency pairs
Cases when both currencies are form developed countries are marked in bold

Ranking	First ranking				Second ranking			
	Currency pair	H_{1250}	Currency pair	H_{625}	Currency pair	$ H_{1250}-0.5 $	Currency pair	$ H_{625}-0.5 $
1	AUDCAD	0.4159	EURDKK	0.3665	EURXPD	0.0003	USDPLN	0.0005
2	EURDKK	0.4168	AUDCAD	0.4022	USDXAG	0.0005	GBPCAD	0.0010
3	EURRON	0.4258	EURRON	0.4062	JPYZAR	0.0009	JPYSEK	0.0013
4	EURPLN	0.4395	JPYXAG	0.4243	JPYAUD	0.0014	JPYTHB	0.0013
5	EURHUF	0.4415	JPYXAU	0.4298	JPYSEK	0.0015	EURILS	0.0020
6	JPYXAU	0.4495	EURHUF	0.4332	EURILS	0.0016	JPYCLP	0.0025
7	JPYXAG	0.4580	EURPLN	0.4370	AUDNZD	0.0016	EURTHB	0.0035
8	AUDPLN	0.4654	AUDPLN	0.4458	JPYNAD	0.0024	USDEURO	0.0035
9	EURCAD	0.4699	EURAUD	0.4544	EURGBP	0.0027	JPYSGD	0.0043
10	EURSEK	0.4758	AUDNZD	0.4566	USDILS	0.0030	USDSGD	0.0044
11	EURSDG	0.4762	EURXAG	0.4567	USDPHP	0.0030	JPYZAR	0.0053
12	EURAUD	0.4769	EURCAD	0.4568	USDSGD	0.0030	USDCZK	0.0053
13	GBPPLN	0.4775	USDXAG	0.4621	EURNOK	0.0031	EURJPY	0.0054
14	USDHKD	0.4775	USDXAU	0.4665	EURTHB	0.0031	JPYNOK	0.0055
15	JPYXPT	0.4802	EURSDG	0.4704	USDXAU	0.0043	EURXPD	0.0063
16	EURCZK	0.4809	NZDJPY	0.4731	USDPLN	0.0046	USDKRW	0.0063
17	EURXPT	0.4839	JPYNZD	0.4745	GBPCAD	0.0047	USDNZD	0.0067
18	EURNZD	0.4866	EURXPT	0.4774	JPYCZK	0.0050	USDAUD	0.0075
19	USDNZD	0.4875	EURNAD	0.4777	USDKRW	0.0051	JPYHKD	0.0081
20	EURXAG	0.4890	EURCZK	0.4779	NZDJPY	0.0051	EURCLP	0.0086
21	USDHUF	0.4904	EURXAU	0.4796	JPYHUF	0.0055	JPYHUF	0.0088
22	JPYNOK	0.4911	GBPPLN	0.4808	EURXAU	0.0063	USDJPY	0.0089
23	EURMXN	0.4932	JPYXPT	0.4816	USDEURO	0.0065	JPYPLN	0.0090
24	EURXAU	0.4937	EURZAR	0.4845	EURMXN	0.0068	JPYCZK	0.0093
25	NZDJPY	0.4949	USDHKD	0.4846	USDXDR	0.0071	JPYAUD	0.0094
26	USDKRW	0.4949	EURNOK	0.4858	JPYNZD	0.0082	JPYCAD	0.0096
27	USDPLN	0.4954	EURSEK	0.4867	USDGBP	0.0087	JPYILS	0.0096
28	USDXAU	0.4957	USDHUF	0.4870	JPYNOK	0.0089	EURMXN	0.0098
29	EURTHB	0.4969	EURNZD	0.4888	USDHUF	0.0096	USDILS	0.0103
30	USDSGD	0.4970	JPYNAD	0.4888	EURXAG	0.0110	USDPHP	0.0103
31	USDILS	0.4970	USDILS	0.4897	EURCLP	0.0113	USDXDR	0.0106
32	USDPHP	0.4970	USDPHP	0.4897	USDSEK	0.0114	EURGBP	0.0108
33	JPYNAD	0.4976	JPYAUD	0.4906	JPYPLN	0.0121	JPYNAD	0.0112
34	EURILS	0.4984	JPYCZK	0.4907	USDNZD	0.0125	EURNZD	0.0112
35	JPYAUD	0.4986	USDJPY	0.4911	EURNZD	0.0134	USDXPT	0.0119
36	JPYZAR	0.4991	JPYHUF	0.4912	EURZAR	0.0145	EURHKD	0.0120
37	EURXPD	0.4997	EURCLP	0.4914	USDCZK	0.0147	JPYXPD	0.0127
38	USDXAG	0.5005	USDNZD	0.4933	JPYCAD	0.0148	USDHUF	0.0130
39	JPYSEK	0.5015	USDKRW	0.4937	EURXPT	0.0161	EURSEK	0.0133
40	AUDNZD	0.5016	JPYZAR	0.4947	USDXPD	0.0166	EURNOK	0.0142
41	EURGBP	0.5027	USDSGD	0.4956	EURJPY	0.0168	GBPCHF	0.0148
42	EURNOK	0.5031	JPYSGD	0.4957	USDAUD	0.0169	USDHKD	0.0154
43	GBPCAD	0.5047	EURTHB	0.4965	GBPCHF	0.0177	EURZAR	0.0155

44	JPYCZK	0.5050	JPYSEK	0.4987	EURCZK	0.0191	EURKRW	0.0166
45	JPYHUF	0.5055	GBPCAD	0.4990	JPYXPD	0.0197	GBPAUD	0.0173
46	USDEURO	0.5065	USDPLN	0.5005	JPYXPT	0.0198	USDNOK	0.0176
47	USDXDR	0.5071	JPYTHB	0.5013	USDCHF	0.0198	USDDKK	0.0178
48	JPYNZD	0.5082	EURILS	0.5020	EURNAD	0.0209	USDSEK	0.0182
49	USDGBP	0.5087	JPYCLP	0.5025	JPYTHB	0.0215	JPYXPT	0.0184
50	EURCLP	0.5113	USDEURO	0.5035	USDTHB	0.0216	GBPPLN	0.0192
51	USDSEK	0.5114	USDCZK	0.5053	EURHKD	0.0220	EURMYR	0.0195
52	JPYPLN	0.5121	EURJPY	0.5054	GBPNZD	0.0221	EURXAU	0.0204
53	EURZAR	0.5145	JPYNOK	0.5055	GBPAUD	0.0222	JPYINR	0.0217
54	USDCZK	0.5147	EURXPD	0.5063	USDHKD	0.0225	USDNAD	0.0219
55	JPYCAD	0.5148	USDAUD	0.5075	GBPPLN	0.0225	EURCZK	0.0221
56	USDXPD	0.5166	JPYHKD	0.5081	USDISK	0.0227	EURNAD	0.0223
57	EURJPY	0.5168	JPYPLN	0.5090	JPYCLP	0.0229	USDCHF	0.0224
58	USDAUD	0.5169	JPYCAD	0.5096	EURAUD	0.0231	EURXPT	0.0226
59	GBPCHF	0.5177	JPYILS	0.5096	EURKRW	0.0232	JPYRON	0.0231
60	JPYXPD	0.5197	EURMXN	0.5098	EURSDG	0.0238	USDZAR	0.0232
61	USDCHF	0.5198	USDXDR	0.5106	EURSEK	0.0242	USDZAR	0.0247
62	EURNAD	0.5209	EURGBP	0.5108	EURMYR	0.0249	JPYNZD	0.0255
63	JPYTHB	0.5215	USDXPT	0.5119	USDJPY	0.0255	NZDJPY	0.0269
64	USDTHB	0.5216	EURHKD	0.5120	USDXPT	0.0256	USDXPD	0.0277
65	EURHKD	0.5220	JPYXPD	0.5127	USDNOK	0.0261	AUDCHF	0.0293
66	GBPNZD	0.5221	GBPCHF	0.5148	USDZAR	0.0277	EURSDG	0.0296
67	GBPAUD	0.5222	EURKRW	0.5166	EURCAD	0.0301	GBPNZD	0.0297
68	USDISK	0.5227	GBPAUD	0.5173	JPYINR	0.0326	JPYDKK	0.0298
69	JPYCLP	0.5229	USDNOK	0.5176	USDCNY	0.0337	USDTHB	0.0309
70	EURKRW	0.5232	USDDKK	0.5178	AUDCHF	0.0344	CADCHF	0.0310
71	EURMYR	0.5249	USDSEK	0.5182	AUDPLN	0.0346	JPYBRL	0.0313
72	USDJPY	0.5255	EURMYR	0.5195	JPYBRL	0.0352	JPYMXN	0.0327
73	USDXPT	0.5256	JPYINR	0.5217	JPYSGD	0.0361	USDXAU	0.0335
74	USDNOK	0.5261	USDNAD	0.5219	JPYMXN	0.0380	USDISK	0.0346
75	USDZAR	0.5277	USDCHF	0.5224	CADCHF	0.0391	JPYGBP	0.0352
76	JPYINR	0.5326	JPYRON	0.5231	JPYGBP	0.0406	USDGBP	0.0366
77	USDCNY	0.5337	USDZAR	0.5232	JPYXAG	0.0420	JPYMYR	0.0374
78	AUDCHF	0.5344	USDZAR	0.5247	JPYIDR	0.0428	JPYKRW	0.0376
79	JPYBRL	0.5352	USDXPD	0.5277	EURUSD	0.0446	USDZAR	0.0379
80	JPYSGD	0.5361	AUDCHF	0.5293	USDDKK	0.0453	USDZAR	0.0393
81	JPYMXN	0.5380	GBPNZD	0.5297	USDCAD	0.0455	JPYISK	0.0396
82	CADCHF	0.5391	JPYDKK	0.5298	JPYMYR	0.0460	EURINR	0.0411
83	JPYGBP	0.5406	USDTHB	0.5309	USDZAR	0.0478	EURCNY	0.0414
84	JPYIDR	0.5428	CADCHF	0.5310	JPYRON	0.0485	USDCAD	0.0421
85	EURUSD	0.5446	JPYBRL	0.5313	JPYXAU	0.0505	EURCAD	0.0432
86	USDDKK	0.5453	JPYMXN	0.5327	JPYILS	0.0516	EURXAG	0.0433
87	USDCAD	0.5455	USDISK	0.5346	EURCNY	0.0519	AUDNZD	0.0434
88	JPYMYR	0.5460	JPYGBP	0.5352	JPYHKD	0.0522	EURUSD	0.0440
89	USDZAR	0.5478	USDGBP	0.5366	USDZAR	0.0547	EURBRL	0.0449
90	JPYRON	0.5485	JPYMYR	0.5374	JPYTRY	0.0578	EURAUD	0.0456
91	JPYILS	0.5516	JPYKRW	0.5376	USDNAD	0.0579	JPYTWD	0.0483
92	EURCNY	0.5519	USDZAR	0.5393	EURHUF	0.0585	USDZAR	0.0493
93	JPYHKD	0.5522	JPYISK	0.5396	EURBRL	0.0587	JPYPHP	0.0520

94	USDMXN	0.5547	EURINR	0.5411	EURINR	0.0599	AUDPLN	0.0542
95	JPYTRY	0.5578	EURCNY	0.5414	EURPLN	0.0605	JPYRUB	0.0559
96	USDNAD	0.5579	USDCAD	0.5421	JPYISK	0.0612	JPYIDR	0.0569
97	EURBRL	0.5587	EURTWD	0.5440	EURPHP	0.0614	JPYCNY	0.0577
98	EURINR	0.5599	EURBRL	0.5449	USD TWD	0.0644	USDCLP	0.0579
99	JPYISK	0.5612	JPYTWD	0.5483	JPYDKK	0.0654	EURPHP	0.0589
100	EURPHP	0.5614	USD TWD	0.5493	JPYCHF	0.0712	EURPLN	0.0630
101	USD TWD	0.5644	JPYPHP	0.5520	JPYKRW	0.0713	JPYCHF	0.0634
102	JPYDKK	0.5654	JPYRUB	0.5559	JPYCNY	0.0730	EURHUF	0.0668
103	JPYCHF	0.5712	JPYIDR	0.5569	JPYRUB	0.0733	EURHRK	0.0672
104	JPYKRW	0.5713	JPYCNY	0.5577	JPYPHP	0.0742	JPYXDR	0.0689
105	JPYCNY	0.5730	USDCLP	0.5579	EURRON	0.0742	USDINR	0.0690
106	JPYRUB	0.5733	EURPHP	0.5589	CHFPLN	0.0746	JPYXAU	0.0702
107	JPYPHP	0.5742	JPYCHF	0.5634	USDCLP	0.0756	USDCNY	0.0720
108	CHFPLN	0.5746	EURHRK	0.5672	JPYTWD	0.0790	EURRUB	0.0731
109	USDCLP	0.5756	JPYXDR	0.5689	EURHRK	0.0790	CHFPLN	0.0744
110	JPYTWD	0.5790	USDINR	0.5690	EURTRY	0.0791	JPYXAG	0.0757
111	EURHRK	0.5790	USDCNY	0.5720	EURXDR	0.0826	USDBRL	0.0761
112	EURTRY	0.5791	EURRUB	0.5731	JPYXDR	0.0829	EURXDR	0.0865
113	EURXDR	0.5826	CHFPLN	0.5744	EURDKK	0.0832	EURIDR	0.0868
114	JPYXDR	0.5829	USDBRL	0.5761	EURISK	0.0834	JPYHRK	0.0899
115	EURISK	0.5834	EURXDR	0.5865	AUDCAD	0.0841	USD RUB	0.0924
116	USDINR	0.5867	EURIDR	0.5868	USDINR	0.0867	EURRON	0.0938
117	USDMYR	0.5924	JPYHRK	0.5899	USDMYR	0.0924	JPYTRY	0.0945
118	USDBRL	0.5951	USD RUB	0.5924	USDBRL	0.0951	AUDCAD	0.0978
119	EURRUB	0.6002	JPYTRY	0.5945	EURRUB	0.1002	USDMYR	0.0983
120	EURIDR	0.6023	USDMYR	0.5983	EURIDR	0.1023	USDHRK	0.0988
121	JPYHRK	0.6025	USDHRK	0.5988	JPYHRK	0.1025	EURISK	0.0995
122	USDBGN	0.6041	EURISK	0.5995	USDBGN	0.1041	JPYBGN	0.1020
123	USDHRK	0.6042	JPYBGN	0.6020	USDHRK	0.1042	EURTRY	0.1045
124	EURCHF	0.6075	EURTRY	0.6045	EURCHF	0.1075	EURCHF	0.1049
125	JPYBGN	0.6125	EURCHF	0.6049	JPYBGN	0.1125	USDBGN	0.1059
126	USDTRY	0.6138	USDBGN	0.6059	USDTRY	0.1138	USDTRY	0.1305
127	USD RUB	0.6160	USDTRY	0.6305	USD RUB	0.1160	EURDKK	0.1335
128	JPYEGP	0.6254	USDIDR	0.6345	JPYEGP	0.1254	USDIDR	0.1345
129	EUREGP	0.6271	JPYUAH	0.6640	EUREGP	0.1271	JPYUAH	0.1640
130	USDIDR	0.6385	JPYEGP	0.6656	USDIDR	0.1385	JPYEGP	0.1656
131	JPYUAH	0.6805	EUREGP	0.6673	JPYUAH	0.1805	EUREGP	0.1673
132	EURUAH	0.6961	EURUAH	0.6758	EURUAH	0.1961	EURUAH	0.1758
133	USDUAH	0.7368	USDUAH	0.7161	USDUAH	0.2368	USDUAH	0.2161
134	USDEGP	0.7518	USDEGP	0.7366	USDEGP	0.2518	USDEGP	0.2366
135	EURBGN	0.7649	EURBGN	0.7743	EURBGN	0.2649	EURBGN	0.2743

Source: own calculations.