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Raise the (Proportion) Bar!

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Abstract

This article, drawing mainly on references to teacher preparation textbooks, proposes proportion bars as a somewhat novel graphical approach to solving simple (direct) proportion problems and to illustrate the advantages of such an approach, which include accessibility with materials at early grade levels, allowance of students to better develop number sense and estimation, facilitation of setting up proportions, allowance for conceptual understanding and motivation of the procedure for solving direct proportions, assistance with part-to-part and part-to whole comparisons, and drawing of connections among mathematical topics. The emphasis is on teaching with understanding, rather than procedural knowledge.

Keywords: proportions, proportional reasoning

Proportionality and proportional reasoning are significant ideas introduced in the elementary grades and consolidated in the middle grades. The 2000 *Principles and Standards for School Mathematics* states, "Proportionality is an important integrative thread that connects many of the mathematics topics studied in grades 6-8" [8, page 217]. This point is made more explicit by the 2006 *Curriculum Focal Points for Pre-Kindergarten through Grade 8 Mathematics: A Quest for Coherence* [7], which indicates how proportionality connects topics in Geometry and Measurement (similarity and conversions), Number and Operations (percentages), Data Analysis (estimation), Probability (approximations), and Algebra (rates of change). Finally, the 2010 *Common Core State Standards for Mathematics* [3] highlights ratios and proportional relationships in grade 6, understanding and applying proportional relationships in grade 7, and understanding the connections between proportional relationships, lines, and linear equations in grade 8.

While it is difficult to overestimate the importance of proportional reasoning in the mathematics curriculum, a full understanding of the development of proportional reasoning in adolescents remains elusive. In 1998, Post, Behr, and Lesh suggested that “[p]roportional reasoning encompasses a wider and more complex spectrum of cognitive abilities” [10, page 79], and involves both quantitative and qualitative thinking. Further, they proposed, “[a]lgebraic thought and understanding often involve different modes of representation” and that “proportional situations and the reasoning that accompanies them provide an excellent vehicle within which to illustrate these multimodal situations” [10, page 81]. So, multiple representations of proportions might enhance students’ understanding of proportional reasoning. In particular, a graphical representation has potential to link the skill of solving proportion problems with the development of proportional reasoning. The purpose of this article is to propose a novel graphical approach to solving simple (direct) proportion problems and to illustrate the advantages of such an approach.

1. Visual Representation of a Proportion

A proportion bar is a visual representation of a proportion using a rectangular bar with different sides of the bar representing different variables, with the long sides of the bar having different scales. Like a percent bar, with which the reader may be familiar, a proportion bar may be drawn with its long side horizontal or vertical. Unlike a percent bar, however, a proportion bar can usually be set up in multiple ways.

While percent bars are common, proportion bars are nearly absent in American teacher preparation textbooks. Three notable exceptions are a 1994 text by VandeWalle [13], who proposes a double line segment diagram, a 2011 text by Beckmann [1], with a double number line diagram and a forthcoming 2020 e-text by Gaze & Brown [4], who proposes a number line diagram.

Consider a typical proportion problem:

> A 175 pound man on earth would weigh 28 pounds on the moon.
> How much would his 30 pound dog weigh? [6, page 280]

A common approach for solving this problem might involving setting up the equivalence of two ratios (*i.e.*, a proportion):

Let d represent the weight of the dog on the moon. Then, we have:

$$\frac{175}{28} = \frac{30}{d}.$$

By cross-multiplication, $175d = 28 \cdot 30$, or $175d = 840$. So, $d = 840 \div 175 = 4.8$. So, the dog would weigh 4.8 pounds on the moon.

Such an approach is concise and efficient. Perhaps more importantly, it is correct! But, consider the alternative of using an approach with a proportion bar: Let d represent the weight of the dog on the moon. A proportion bar can be created (see Figure 1) with the two sides representing "Earth" weight and "Moon" weight:

Figure 1: Proportion bar for the Earth/Moon problem.

The scale may be explicit or (in this case) implied. A weight of 0 pounds on the Earth corresponds to a weight of 0 pounds on the moon and a weight of 175 pounds on Earth corresponds to a weight of 28 pounds on the moon. The 30 pound weight of the dog on the Earth is approximately placed and the variable d is placed across from it.

A quick estimate might be found by reasoning that $30 \cdot 5 = 150 \approx 175$ and $28 \div 5 \approx 25 \div 5 = 5$. So, the dog should weigh about 5 pounds on the moon.

Now, the proportion may be set up in many different ways, using the bar as reference:

$$\frac{30}{d} = \frac{175}{28} \quad \text{or} \quad \frac{30}{175} = \frac{d}{28} \quad \text{or} \quad \frac{d}{30} = \frac{28}{175} \quad \text{or} \quad \frac{28}{d} = \frac{175}{30},$$

all of which give equivalent answers of 4.8 by solving a proportion.

2. Advantages of the Proportion Bar

While not as efficient as simply setting up a proportion, the approach of using a proportion bar to solve proportion problems has several pedagogical advantages.

1. *This approach is accessible and can be actualized with materials in early grade levels.*

A common set of centimeter rods may be used to illustrate proportions in a way very similar to the way a proportion bar may be set up. Consider the problem:

> If three workers can mow two lawns in an hour, how many lawns of the same size can be mowed in a hour by nine workers?

One approach to solving this problem might be to use centimeter rods to represent the workers and different longer centimeter rods to represent the lawns, then extending the workers' line to nine total workers. Placed side-by-side, we can represent the problem as in Figure 2, with n representing the number of lawns that nine workers can mow in an hour:

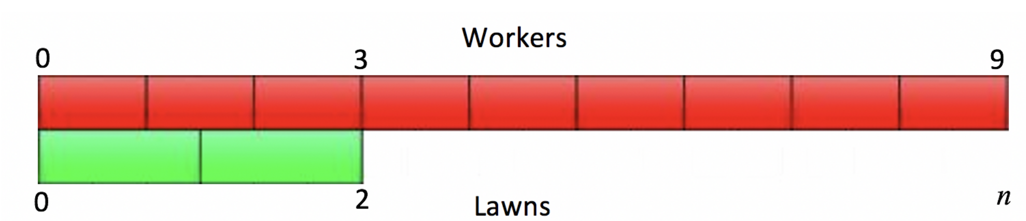

Figure 2: Centimeter rod representation of the lawn problem.

Now, a student can place the longer rods to represent the total number of lawns that can be mowed by nine workers. In this case, the answer is six, as seen in Figure 3.

Of course, the answer can be found by setting up a proportion as before, but our solution involving centimeter rods leading up to proportion bars illustrates how we can facilitate proportional reasoning in a tactile environment, accessible to students in earlier grade levels.

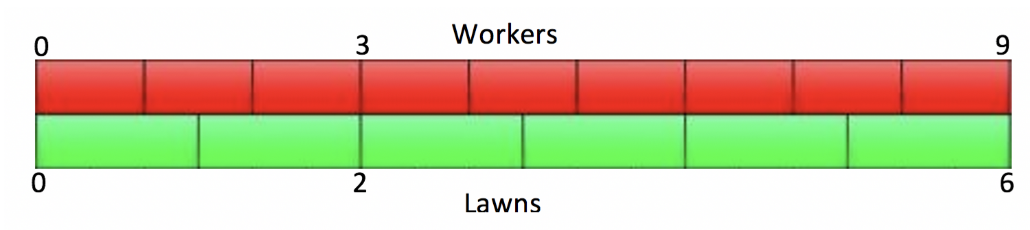

Figure 3: Centimeter rod solution of the lawn problem.

2. *This approach allows students to make a guess, supporting number sense and estimation.*

Too often students solve problems and make arithmetic mistakes when a reasonable estimate could reduce the number of those mistakes. With proportion problems especially, an incorrect setup can lead to drastically incorrect results (a common error is to set up an inverse proportion when a direct proportion is required).

Consider the following proportion problem:

> Last year at the Laundromat Users convention, 3,216 people ate 1,011 chickens at the Saturday afternoon picnic. This year, 3,800 people are expected to attend. How many chickens should be ordered? [11, page 351]

One common incorrect way to set up this problem would be:

$$\frac{3,216}{3,800} = \frac{c}{1,011}.$$

By cross multiplication, we have $3,800c = 3,216 \cdot 1,011$ or

$$c = \frac{3,216 \cdot 1,011}{3,800} \approx 856 \text{ chickens.}$$

In this case, the number of chickens would vary inversely with the number of people, rather than directly as the problem requires. A proportion bar can help students avoid this mistake and further develop quantitative reasoning since, as the number of people expected increases, the number of chickens required should increase as well.

Also, in this problem, an exact answer is not necessarily required and a good guess might be sufficient. The proportion bar helps with the guess; see Figure 4:

Figure 4: Proportion bar for the people/chickens problem.

Let c represent the number of chickens to be ordered. We first note that more chickens are required than last year since more people are expected to attend. Now, $3,800 - 3,216 \approx 3,800 - 3,200 = 600$. So, approximately 600 more people are expected to attend. One also notices $3,216 \div 1,011 \approx 3,000 \div 1,000 = 3$. So, for every 3 people increase, approximately 1 additional chicken is needed (a unit rate increase). So, for 600 additional people, approximately 200 additional chickens would be required. So, approximately $1,011 + 200 = 1,211$ chickens should be ordered.

For an exact answer, we illustrate the proportion

$$\frac{3,216}{1,011} = \frac{3,800}{c},$$

and solve by cross-multiplication: $3,216 \cdot c = 3,800 \cdot 1,011$, or $c = (3,800 \cdot 1,011) \div 3,216 \approx 1,195$ chickens. Note this answer is close to our estimate from earlier.

3. *This approach facilitates setting up the proportion or the equation describing the relevant proportional relationship.*

As we mentioned earlier, a common mistake for students solving problems involving proportional reasoning is to set up the proportion incorrectly, often creating an inverse, rather than direct, proportion. A clearly labeled proportion bar, however, can help eliminate this error. Consider the following proportion problem:

> The Spruce Goose, a wooden flying boat built for Howard Hughes, had the world's largest wingspan, 319 ft 11 in. according to the Guinness Book of World Records. It flew only once in 1947, for a distance of about 1000 yards. Shelly wants to build a scale model of the 218 ft 8 in.-long Spruce Goose. If her model will be 20 inches long, what will its wingspan be (to the nearest inch)? [6, page 280]

The proportion bar approach to solving this problem can assist students in setting up the correct proportion:

First, to handle the mixed units, we change all units to inches:

$$\begin{aligned} 319 \text{ ft } 11 \text{ in} &= 319 \cdot 12 + 11 = 3,839 \text{ in.} \\ 218 \text{ ft } 8 \text{ in} &= 218 \cdot 12 + 8 = 2,624 \text{ in.} \end{aligned}$$

Now, let w represent the wingspan of the model. Then, a proportion bar can be created with two sides representing "Length" and "Wingspan"; see Figure 5:

Figure 5: Proportion bar for the length/wingspan problem.

A reasonable estimate for the wingspan of the model might be 30 in. since $20 \cdot 100 = 2,000 \approx 2,624$ and $3,839 \div 1000 \approx 3,000 \div 100 = 30$. Again, using the bar as a reference, we may set up the proportion in several (equivalent) ways:

$$\frac{20}{w} = \frac{2,624}{3,839} \quad \text{or} \quad \frac{20}{2,624} = \frac{w}{3,839} \quad \text{or} \quad \frac{w}{20} = \frac{3,839}{2,624} \quad \text{or} \quad \frac{2,624}{20} = \frac{3,839}{w},$$

all of which give equivalent answers of approximately 29 in.

An alternate method for setting up the problem, again facilitating setting up the proportion, is to let the two sides represent the "Real" Spruce Goose and the "Model".

To see this solution path, let w represent the wingspan of the model and set up the bar; see Figure 6.

Figure 6: Alternate proportion bar for the length/wingspan problem.

Note that this representation will facilitate setting up the same proportions given before, leading to the same guess and exact answer.

4. *This approach is conceptual and motivates the procedure.*

As teachers, we sometimes focus on presenting a single procedure for solving particular types of problems without motivating the procedure. With proportion problems especially, we are tempted to illustrate the solution method of setting up a proportion to be solved by cross-multiplication, and we expect our students to develop proportional reasoning from this illustration. This can be dangerous since, "If students over-practice procedures before they understand them, it is more difficult to make sense of them later" [5, page 17]. The proportion bar approach gives meaning to the proportion with a visual representation. A proportion bar is the missing component between a proportion problem and its solution, and it makes the connection explicit by facilitating the setup of equivalent ratios and leading more readily to the solution by cross-multiplication.

Consider the following problem:

> The ratio of apples to oranges in a gift box is 3 to 2, and there are 18 apples. How many oranges are there? [2, page 406]

We begin by setting up an appropriate proportion bar (see Figure 7), with O representing the number of oranges.

Figure 7: Proportion bar for the apples/oranges problem.

Next, instead of immediately setting up the proportion, we add additional labels to the bar in equal increments (see Figure 8).

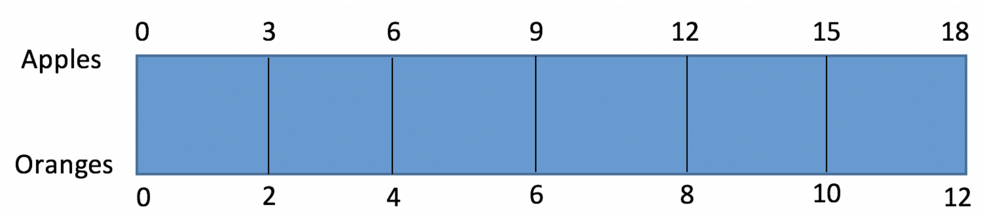

Figure 8: Proportion bar with equal increments for the apples/oranges problem.

To find the answer, one can count by 3s on the top portion of the bar to get 18, while counting by 2s on the bottom to get O. Alternately, we can solve the proportion

$$\frac{3}{2} = \frac{18}{O}$$

by creating equivalent ratios:

$$\frac{3}{2} = \frac{6}{4} = \frac{9}{6} = \frac{12}{8} = \frac{15}{10} = \frac{18}{12},$$

as suggested by the bar. So, there are 12 oranges for 18 apples. Note that the procedure of solving a proportion by cross-multiplication is absent here, replaced by reasoning proportionally.

5. *This approach assists with part-to-part and part-to-whole comparisons.*

The following proportion problem is often incorrectly solved by students who equate part-to-part and part-to-whole relationships.

> Frozen orange juice concentrate is usually mixed with water in a ratio of 3 parts water to 1 part concentrate. How much orange juice can be made from a 12-oz can of concentrate? [11, page 352]

One way to solve this problem using proportion bars is shown by Figure 9, with w representing the amount of water that we will need:

Figure 9: Proportion bar for the water/concentrate problem.

Notice the bar clearly represents the "Water" and "Concentrate" parts of the whole "Juice." Juice concentrate and water are parts of the whole orange juice, which is absent from the bar. A guess might yield the "answer" of 36 parts water without setting up the proportion. However, the answer to the problem is not 36. The bar indicates that 36 ounces of water must be added to 12 ounces of concentrate to make 48 ounces of juice.

A third segment on the bar could productively be added to illustrate this more complicated relationship, indicating how all three variables vary directly (see Figure 10), with j representing the amount of juice:

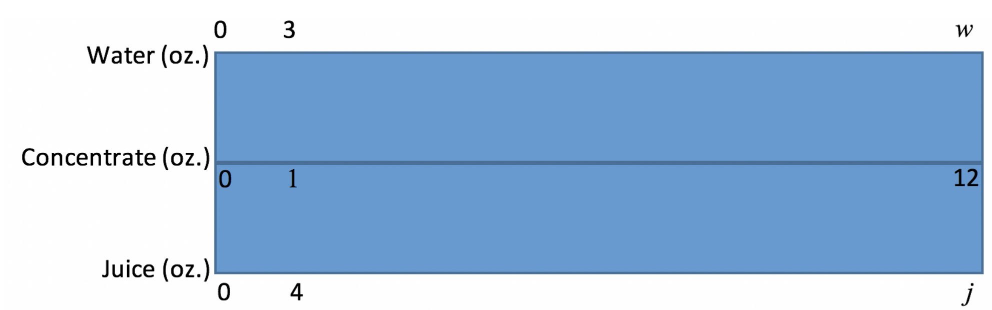

Figure 10: Proportion bar for the water/concentrate problem with additional segment.

Now, the proportion between the amount of concentrate used and the resulting amount of juice can be set up directly:

$$\frac{1}{4} = \frac{12}{j}.$$

At this point, cross multiplication gives the correct answer of 48 ounces.

6. *This approach draws connections among mathematical topics.*

The proportion bar approach connects topics in mathematics to one another. First, let us see how it can connect ratios with rates of change. Consider the following proportion problem:

> A donut machine produces 60 donuts every 5 minutes. How many donuts does it produce in an hour? [12, page 166]

To solve this problem, we let d stand for the number of donuts and handle the mixed units by changing one hour to 60 minutes and setting up a proportion bar; see Figure 11:

Figure 11: Proportion bar with equal increments for the apples/oranges problem.

A quick estimate for the number of donuts produced might be 720 since $60 \div 5 = 12$ and $60 \cdot 2 = 720$. In this case, the estimate is also the exact answer. We note that the machine produces 720 donuts per hour, which is a unit rate of change, building a connection with the slope concept and algebra. Again, a third segment could be added to the bar to illustrate the time in hours; see Figure 12.

The proportion may be now set up as follows:

$$\frac{60}{1/2} = \frac{d}{1}.$$

Again, cross multiplication gives the correct answer of 720 donuts (and provides a visual conversion from minutes to hours).

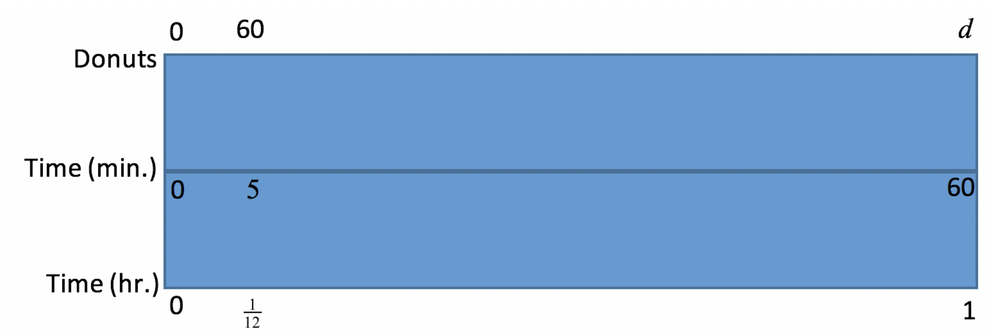

Figure 12: Proportion bar for the donuts/time problem with added segment.

Now, consider the following percent problem:

> Jere learned that 72 of her 150-member senior class went to college. What percent of her senior class went to college? [9, page 411]

We may solve this using the more familiar approach of a percent bar, letting p represent the percent that went to college; see Figure 13.

A quick estimate would be a bit less than 50% since 72 is a bit less than one-half of 150. Setting up the proportion (with assistance from the bar) gives

$$\frac{72}{p} = \frac{150}{100},$$

which we solve by cross multiplication: $150 \cdot p = 72 \cdot 100$, or

$$p = \frac{72 \cdot 100}{150} = 48 \text{ percent}.$$

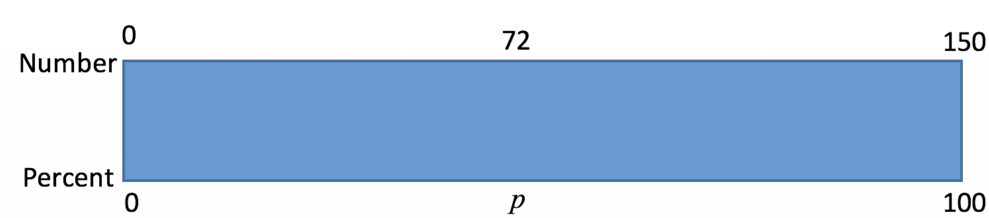

Figure 13: Percent bar for the senior class problem.

We notice that a percent bar is a special case of a proportion bar (since a percent is a ratio out of 100), with one side of the bar always representing the percent. So, the representation becomes more meaningful as it can be used for both proportion and percent problems, building a connection between the two.

3. Conclusion

The development of proportional reasoning by students — especially for middle grades students — is one of the most fundamental in mathematics. This development is important not only for solving proportion problems, but for drawing connections among mathematics topics and supporting number sense and estimation. Proportion bars assist with this by providing a concrete visual representation by which students can reason proportionally before tackling the procedure of solving proportions, giving meaning and understanding to their work. So, raise the proportion bar!

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