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Maximum Likelihood Estimation for Length biased Burr-XII Distribution with Censored Sample

Abd-Elfattah A. M[a*], Mahdy.M[b], Ismail.G[c]

[a]*Department of Mathematical Statistics, Institute of Statistical Studies & Research, Cairo University, Egypt*
[b]*Department of Statistics, Mathematics and Insurance, Benha University, Egypt*
[c]*Ministry of Insurance and Pensions*
[a]*Email: a_afattah@hotmail.com,* [b]*Email:drmervat.mahdy@fcom.bu.edu.eg,* [c]*Email: gehan_zayan@yahoo.com*

Abstract

In this paper, defined by [1],the maximum likelihood estimation for the parameters of the LBB-XII distribution are studied.Also, different types of censoring, such as, type I, type II. A simulation study is perform to evaluate the maximum likelihood estimates.

Keywords: Length Biased Burr XII Distribution; Maximum Likelihood Estimation; Censored Type- I; Censored Type- II.

1. Introduction

Advances in computational methods and numerical simulations have allowed to incorporate efficient models that are capable of describing real problems. According to [1]theLBB- XII distribution with two parameters, is very flexible model to be fitted by reliability data. Some properties of this model were studied as well as the parameter estimation based on maximum likelihood method. Censored sampling have been attracting great interest due to their wide applications. In life testing, the experimenter may not always obtain complete information on failure times for allexperimental units. Reducing the total test time and the associated cost is one of the major reasons for censoring. In literature, some studies in censored samples.

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* Corresponding author.

According to [13] estimation of the parameters of Burr XII distribution using randomly right censored data with maximum likelihood method.Estimation of parameters of the Burr type XII distribution based on progressive type-II censoring with random removals by using maximum likelihood estimators as [2] . Estimation of Burr-XII. Estimation of the Weibull distribution based on type-II censored samples by using Maximum Likelihood and Bayes method as [8] .According to [5] estimation of the parameters of Laplace (double exponential) distribution based on their maximum likelihood estimators from a Type-II censored sample. Reference [12] present estimation of two parameters of beta-Weibull distribution under type II censored samples with different sample size by using maximum likelihood method. Reference [6] introduced estimation of two-parameter exponential distribution based on complete and Type-I censored samples with Bayesian analysis [10]. Introduced estimation of generalized half Logistic distribution under Type-II Hybrid Censoring by using maximum likelihood estimators and approximate maximum likelihood estimators [15]. Discuss the maximum likelihood equations for Weight Lindley distribution with type I, type II and random censoring mechanism [9]. Discuss estimation of exponential distribution under type -II censoring by maximum likelihood method [7]. Present estimate for the parameters of Burr Type XII distribution under doubly censored sample with Bayesian analysis. Reference [14] present estimation of beta log Weibull distribution under type-II censoring by using Bayesian method.[4] present new log-Burr XII regression model with log-gamma–Weibull distributions for the random effects using maximum likelihood method to estimate the model parameters. In our study, depending on LBB- XII distribution defined by [1] we can present a treatment for different types of censoring samples.The main objective of this paper is to estimate the parameters of the LBB- XII distribution depending on the maximum likelihood estimation considering data based on different types of censoring such as , censored type I, censored type II.The rest of the paper is organized as follows. Review of some properties of length biased burr XII distribution in section 2. The maximum likelihood method and it's properties statistical are provide in section 3. In section 4, we derive the maximum likelihood estimation of unknown parameters for different types of censoring. Simulation study are performed in section 5 and finally conclusion in Section 6.

2. Length Biased BurrXII Distribution

Let *Y*be continuous random variable distributed as length biased Burr XII distribution with parameters[1]and the density function given as:

$$\underset{LBB}{g(y)} = \frac{cy^c\left[1+y^c\right]^{-(k+1)}}{B\left(\frac{1}{c}+1, k-\frac{1}{c}\right)}, \; c,k>0 \text{ and } y>0 \tag{1}$$

The mean and variance of LBB- XII distribution can be given by

$$\underset{LBB}{E(y)} = \frac{\beta\left(\frac{2}{c}+1, k-\frac{2}{c}\right)}{\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)} \tag{2}$$

$$V_{LBB}(y)=\left[\frac{\beta\left(\frac{3}{c}+1,k-\frac{3}{c}\right)}{\beta\left(\frac{1}{c}+1,k-\frac{1}{c}\right)}\right]-\left[\frac{\beta\left(\frac{2}{c}+1,k-\frac{2}{c}\right)}{\beta\left(\frac{1}{c}+1,k-\frac{1}{c}\right)}\right]^2 \quad (3)$$

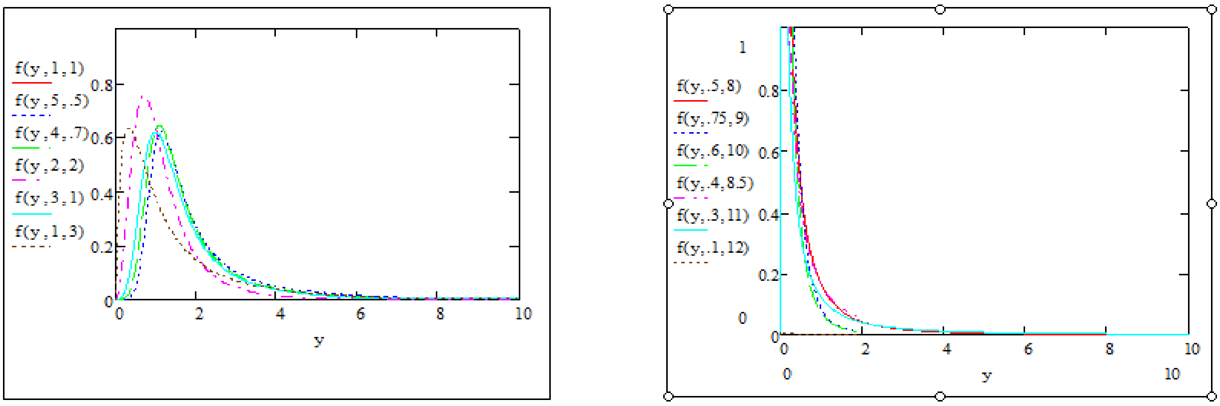
The survival function of LBB XII distribution with the probability of an observation doesn't fail until a spesified time is given

$$S_{LBB}(y)=1-\frac{1}{\beta\left(\frac{1}{c}+1,k-\frac{1}{c}\right)}\int_0^t cy^c\left(1+y^c\right)^{-(k+1)}dy=1-I_y \quad (4)$$

The hazard function is given by

$$H_{LBB}(y)=\frac{cy^c\left[1+y^c\right]^{-(k+1)}}{\beta\left(\frac{1}{c}+1,k-\frac{1}{c}\right)\left(1-I_y\right)} \quad (5)$$

The graphs of LBB-XII for different values are illustrated in Figures 2.1



Figures 2.1: PDF of LBB-XII Distribution for Some Values of the Parameters**.**

3. Maximum Likelihood Estimation

Depending on classical approach, the maximum likelihood estimators was used due to its asymptotic properties. The maximum likelihood method is one of the most important methods which used to estimate the parameters of models. Maximum likelihood estimators are obtained from maximizing the likelihood function [11,16]

The likelihood function of $\theta=(\theta_1,\theta_2,\ldots,\theta_n)$ given y is

$$L(\theta, y) = \prod_{i=1}^{n} f(y_i|\theta) \tag{6}$$

For getting maximum likelihood estimator we must get differentiable of $\ln \ell(\theta_i, t)$ and solving the next equation

$$\frac{\partial \ln \ell(\theta_i, t)}{\partial \theta_i} = 0, \; i = 1,2,\ldots,p \tag{7}$$

The maximum likelihood estimators is the solutions of (7). For large samples the maximum likelihood estimators are unbiased estimators and under some regularity conditions, asymptotically normal joint distribution given by,

$$(\hat{\theta} - \theta) \sim N(0, I^{-1}(\theta)) for(n) \to \infty \tag{8}$$

Where $I(\theta)$ is the Fisher information matrix, $p \times p$ and $I_{i_1 i_2}(\theta)$ is the Fisher information matrix of (θ) in i_1 and i_2 given by,

$$I_{i_1 i_2}(\theta) = E\left[\left(\frac{\partial}{\partial \theta_{i1} \partial \theta_{i2}} \ln(f(x|\theta))^2\right)\right], i_1, i_2 = 1,2,\ldots,p \tag{9}$$

It's so difficult to compute the fisher information matrix in case of censored observation so the alternative is consider the observed information matrix which given by

$$H_{i_1 i_2}(\theta) = \left(\frac{\partial}{\partial \theta_{i1} \partial \theta_{i2}} \ln(f(x|\theta))^2\right), i_1, i_2 = 1,2,\ldots,p \tag{10}$$

For large samples, approximated confidence intervals can be constructed for the individuals parameters θ_i, with confidence coefficient $100(1-\phi)\%$ through marginal distributions given by

$$(\hat{\theta} - \theta) \sim N(0, H_{ii}^{-1}(\theta)) for(n) \to \infty \tag{11}$$

4. Censoring and Parameter Estimation

Censored observations arise whenever the dependent variable of interest represents the time to a terminal event, and the duration of the study is limited in time. There are several mechanisms that can lead to censored data. Under censoring of Type I, a sample of n units is followed for a fixed time T. The number of units experiencing the eventis random, but the total duration of the study is fixed.Under censoring of Type II, a sample of n units is followed as long as necessary until n units have experienced the event. In this design the number of units experiencing the event is fixed and pre-determined and the time of end the experiment is random variable. In this section, we introduce the maximum likelihood estimator for four parameters of LBB-XII distribution censoring type I, type II.

4.1 Type I Censoring

According to the definition of censor type I, the test is finished after a fixed time T_i where this fixed time is predetermined, so the number of failure cases is random variable. Suppose*n*patients at a treatment and suppose that $d < n$ died until the time y_c, then *(n-d)*are still alive and will be censored. The likelihood function for this case is given by

$$\ell = \prod_{i=1}^{n} [g(y)]^{\delta_i} [1 - G(y)]^{1-\delta_i} \tag{12}$$

Where $\sum_{i=1}^{n} \delta_i$ is a random variable and $\delta_i = I(t_i \le t_c)$ is an indicator function. Let $(y_1, y_2, \cdots, y_n)$ be a random sample of *LBB -XII* distribution,the likelihood function with censoring type I is given by,

$$\ell = \prod_{i=1}^{n} \left[\frac{c y_i^c \left(1 + y_i^c\right)^{-(k+1)}}{\beta\left(\frac{1}{c} + 1, k - \frac{1}{c}\right)} \right]^{\delta_i} \left[1 - \frac{1}{\beta\left(\frac{1}{c} + 1, k - \frac{1}{c}\right)} \int_0^t c T^c \left(1 + T^c\right)^{-(k+1)} dT \right]^{1-\delta_i} \tag{13}$$

$$= c^d \prod_{i=1}^{n} \left[y_i^c \right]^{\delta_i} \prod_{i=1}^{n} \left[\left(1 + y_i^c\right)^{-(k+1)} \right]^{\delta_i} \left(\frac{1}{\beta\left(\frac{1}{c} + 1, k - \frac{1}{c}\right)} \right)^d \left[1 - \frac{1}{\beta\left(\frac{1}{c} + 1, k - \frac{1}{c}\right)} \int_0^t c T^c \left(1 + T^c\right)^{-(k+1)} dT \right]^{1-\delta_i} \tag{14}$$

By taking logarithm of the equation (22) we will get

$$\ln \ell = d \ln c + c \sum_{i=1}^{n} \delta_i \ln(y_i) - (k+1) \sum_{i=1}^{n} \delta_i \ln\left(1 + y_i^c\right) - d \ln \beta\left(\frac{1}{c} + 1, k - \frac{1}{c}\right) + (n - d) \ln(I_T) \tag{15}$$

Where

$$I_T = \left[1 - \frac{1}{\beta\left(\frac{1}{c} + 1, k - \frac{1}{c}\right)} \int_0^t c T^c \left(1 + T^c\right)^{-(k+1)} dT \right]$$

From (15) the first partial of c parameter is given by

$$\frac{\partial \ln \ell}{\partial c} = \frac{d}{c} + \sum_{i=1}^{n} \delta_i \ln(y_i) - (k+1)\sum_{i=1}^{n} \delta_i \ln \frac{y_i^c \ln(y_i)}{(1+y_i^c)} - \frac{d\Phi'_{\beta c}}{\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)} - \frac{n-r}{I_{\mathrm{T}}}\left[\Omega_2 \int_0^t Z_C^T dT + \int_0^t \Omega_1^T dT \mathrm{M}_C\right] \tag{16}$$

The first partial of k parameter is given by

$$\frac{\partial \ln \ell}{\partial k} = \sum_{i=1}^{n} \delta_i \ln(1+y_i^c) - \frac{d\Phi'_{\beta k}}{\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)} - \frac{n-r}{I_T}\left[\Omega_2 \int_0^t Z_K^T dT + \int_0^t \Omega_1^T dT \mathrm{M}_K\right] \tag{17}$$

For the observed information matrix of the parameters *(c, k)*,we calculate the second partial derivatives of (15) with respect to c and k respectively as follows:

$$\frac{\partial^2 \ln \ell}{\partial c^2} = -\frac{d}{c^2} - (k+1)\sum_{i=1}^{r} \delta_i \left[\ln(y_i) - \frac{y_i^c \ln(y_i)}{(1+y_i^c)}\right] + d\left[\Omega_2 \Phi''_{\beta C} + \Phi'_{\beta C}\mathrm{M}_C\right]$$
$$+ (n-d)\left\{ (I_T)^{-1}\left[\begin{matrix}\left[\Omega_2 \int_0^t Z_C^{T^2} dT + \int_0^t Z_C^T dT \mathrm{M}_C\right] + \\ \left[\int_0^t \Omega_1^T dT \mathrm{M}_C^2 + \mathrm{M}_C \int_0^t Z_C^T dT\right]\end{matrix}\right] - \left[(I_T)^{-2}\left[\Omega_2 \int_0^t Z_C^T dT + \int_0^t \Omega_1^T dT \mathrm{M}_C\right]^2\right]\right\} \tag{18}$$

$$\frac{\partial^2 \ln \ell}{\partial k^2} = -d\left[\Phi'_{\beta k}\mathrm{M}_K + \Omega_2 \Phi'_{\beta K \beta K}\right] - (n-d)\left[\begin{matrix}\left[(I_T)^{-1}\left[\begin{matrix}\left[\Omega_2 \int_0^t Z_K^{T^2} dT + \mathrm{M}_K \int_0^t Z_K^T dT\right] + \\ \left[\mathrm{M}_{K^2} \int_0^t \Omega_1^T dT + \mathrm{M}_K \int_0^t Z_K^T dT\right]\end{matrix}\right]\right] \\ + (I_T)^{-2}\left[\Omega_2 \int_0^t Z_K^T dT + \mathrm{M}_K \int_0^t \Omega_1^T dT\right]^2\end{matrix}\right] \tag{19}$$

In addition to, the partial derivatives of (17) with respect to c is :

$$\frac{\partial^2 \ln \ell}{\partial k \partial c} = \sum_{i=1}^{n} \delta_i \left[\frac{y_i^c \ln y_i}{1+y_i^c}\right] - d\left[\Omega_2 \Phi'_{\beta k \beta c} + \Phi'_{\beta k}\mathrm{M}_K\right]$$
$$-(n-d)\left\{(I_T)^{-1}\left[\begin{matrix}\left(\mathrm{M}_c \int_0^t Z_K^T dT + \Omega_2 \int_0^t Z_{KC}^T dT\right) + \\ \left(\mathrm{M}_K \int_0^t Z_C^T dT + M_{KC} \int_0^t \Omega_1^T dT\right)\end{matrix}\right]\right\} + \left\{\begin{matrix}(I_T)^{-2}\left[\Omega_2 \int_0^t Z_{KC}^T dT + \int_0^t Z_K^T dT M_C\right] \\ \left[M_{KC} \int_0^t \Omega_1^T dT + M_K \int_0^t Z_C^T dT\right]\end{matrix}\right\} \tag{20}$$

Where

- $\Omega_1^T = cT_i^c\left(1+T_i^c\right)^{-(k+1)}$
- $\frac{\partial\Omega_1^T}{\partial c} = -(k+1)\ln(T_i)cT_i^{2c}\left(1+T_i^c\right)^{-(k+2)} + \left(1+T_i^c\right)^{-(k+1)}\left[T_i^c\left[c\ln(T_i)+1\right]\right] = Z_C^T$
- $\frac{\partial\Omega_1^T}{\partial c_2} = -(k+1)\ln(T_i)\left\{\left[1+T_i^c\right]^{-(k+2)}\left[cT_i^{2c}\ln(T_i)+T_i^{2c}\right] - cT_i^{2c}(k+2)\left[1+T_i^c\right]^{-(k+3)}T_i^c\ln(T_i)\right\}$
  $+\left[1+T_i^c\right]^{-(k+1)}\left[T_i^c\ln(T_i)+\left[c\ln(T_i)+1\right]T_i^c\ln(T_i)\right] = Z_C^{T^2}$
- $\frac{\partial\Omega_1^T}{\partial k} = -cT_i^c\left[1+T_i^c\right]^{-(k+1)}\ln\left[1+T_i^c\right] = Z_K^T$
- $\frac{\partial^2\Omega_1^T}{\partial k^2} = cT_i^c\left[\ln\left(1+T_i^c\right)\right]^2\left[1+T_i^c\right]^{-(k+1)} = Z_K^{T^2}$
- $\frac{\partial^2\Omega_1}{\partial k\partial c} = \left[\ln\left[1+y_i^c\right]\right]^2\frac{y_i^c\ln(y_i)}{\left[1+y_i^c\right]}cy_i^c\left[1+y_i^c\right]^{-(k+1)} + \left[cy_i^c\ln(y_i)\right]\left[\left[\ln\left[1+y_i^c\right]\right]^2\right]\left[1+y_i^c\right]^{-(k+1)}$
  $-(k+1)\left[1+y_i^c\right]^{-(k-2)}y_i^c\ln(y_i)cy_i^c\left[\left[\ln\left[1+y_i^c\right]\right]^2\right]$
- $\frac{\partial^2\Omega_1}{\partial k\partial c} = \left[\left[\ln\left[1+y_i^c\right]\right]^2\right]\left\{\frac{cy_i^{2c}\ln(y_i)}{\left[1+y_i^c\right]^{(k+2)}} + \frac{\left[cy_i^c\ln(y_i)\right]}{\left[1+y_i^c\right]^{(k+1)}} - \frac{(k+1)cy_i^{2c}\ln(y_i)}{\left[1+y_i^c\right]^{(k+2)}}\right\} = Z_{CK}$
- $\frac{\partial\Omega_2}{\partial c\partial k} = \frac{1}{\left[\beta\left(\frac{1}{c}+1,k-\frac{1}{c}\right)\right]^2}\Phi'_{\beta C\beta K} - \frac{2\Phi'_{\beta c}\Phi'_{\beta k}}{\left[\beta\left(\frac{1}{c}+1,k-\frac{1}{c}\right)\right]^3} = M_{CK}$
- $\Phi''_{\beta c\beta k} = \beta\left(\frac{1}{c}+1,k-\frac{1}{c}\right)\left[\left[\Psi_0\left(\frac{1}{c}+1\right)-\Psi_0(k+1)\right]\times\left[\Psi_0\left(k-\frac{1}{c}\right)-\Psi_0(k+1)-\Psi_1(k+1)\right]\right]$

4.2 Type II Censoring

Usually in industrial process, the study of some items electronic components are finished after a predetermine fixed number of failures (r), so (n − r) items will be censored. This type of censoring is call censor type II, [15], and it's likelihood function is given by

$$\ell(y_i|c,k) = \frac{n!}{(n-r)!}\prod_{i=1}^{r} g(y_i|c,k)\left[S(y_{(r)}|c,k)\right]^{n-r} \quad (21)$$

Where $y_{(r)}$ is order statistics.

Let $(y_1, y_2, \ldots, y_n)$ be a random sample of LBB -XII distribution , the likelihood function is given by

$$\ell(y_i|c,k) = \frac{n!}{(n-r)!}\frac{c^r \prod_{i=1}^{r} y_i^c \prod_{i=1}^{r}\left(1+y_i^c\right)^{-(k+1)}}{\left[\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)\right]^r}\left[1-\frac{1}{\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)}\int_0^t c y_i^c\left(1+y_i^c\right)^{-(k+1)}dy\right]^{n-r} \quad (22)$$

The logarithm of the likelihood function (22) is given by,

$$\ln \ell(y_i|c,k) = \ln(n!) - \ln[(n-r)!] + r\ln(c) + c\sum_{i=1}^{r}\ln(y_i) - (k+1)\sum_{i=1}^{r}\ln\left(1+y_i^c\right) - r\ln\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right) + (n-r)\ln I_Y$$

(23)

Where,

$$I_Y = \left[1-\frac{1}{\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)}\int_0^t c y_i^c\left(1+y_i^c\right)^{-(k+1)}dy\right]$$

Differentiating (23) with respect to $c$and k respectively as follow

$$\frac{\partial \ln \ell}{\partial c} = \frac{r}{c} + \sum_{i=1}^{r}\ln(y_i) - (k+1)\sum_{i=1}^{r}\ln\frac{y_i^c \ln(y_i)}{\left(1+y_i^c\right)} - \frac{r\Phi'_{\beta c}}{\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)} - \frac{n-r}{I_Y}\left[\Omega_2\int_0^t Z_C\,dy + \int_0^t \Omega_1 dy \mathrm{M}_C\right]$$

; (24)

Where

- $\Omega_1 = c y_i^c\left(1+y_i^c\right)^{-(k+1)}$
- $\frac{\partial \Omega_1}{\partial c} = -(k+1)\ln(y_i) c y_i^{2c}\left(1+y_i^c\right)^{-(k+2)} + \left(1+y_i^c\right)^{-(k+1)}\left[y_i^c\left[c\ln(y_i)+1\right]\right] = Z_C$
- $\Omega_2 = \frac{1}{\beta\left(\frac{1}{c}+1.k-\frac{1}{c}\right)}$

- $$\frac{\partial\Omega_2}{\partial c}=-\frac{\Phi'_{\beta c}}{\left[\beta\left(\frac{1}{c}+1.k-\frac{1}{c}\right)\right]^2}=-\frac{\left[\Psi_0\left(\frac{1}{c}+1\right)-\Psi_0(k+1)\right]}{\beta\left(\frac{1}{c}+1.k-\frac{1}{c}\right)}=\mathrm{M}_C$$
- $$\Phi'_{\beta c}=\frac{\partial\beta\left(\frac{1}{c}+1.k-\frac{1}{c}\right)}{\partial c}=\beta\left(\frac{1}{c}+1.k-\frac{1}{c}\right)\left[\Psi_0\left(\frac{1}{c}+1\right)-\Psi_0(k+1)\right]$$

Since;

$\Phi'_{\beta c}=\frac{\partial\beta\left(\frac{1}{c}+1.k-\frac{1}{c}\right)}{\partial c}$ is the first partial of the first parameter for beta function.

$$\frac{\partial\ln\ell}{\partial k}=\sum_{i=1}^{r}\ln\left(1+y_i^c\right)-\frac{r\Phi'_{\beta k}}{\beta\left(\frac{1}{c}+1,k-\frac{1}{c}\right)}-\frac{n-r}{I_Y}\left[\Omega_2\int_0^t Z_K\,dy+\int_0^t\Omega_1 dy\mathrm{M}_K\right] \tag{25}$$

Where

- $$\frac{\partial\Omega_1}{\partial k}=-cy_i^c\left[1+y_i^c\right]^{-(k+1)}\ln\left[1+y_i^c\right]=Z_k$$
- $$\frac{\partial\Omega_2}{\partial k}=\frac{-1}{\left[\beta\left(\frac{1}{c}+1,k-\frac{1}{c}\right)\right]^2}\Phi'_{\beta k}=M_k$$

The second partial is given by:

$\Phi'_{\beta k}=\frac{\partial\beta\left(\frac{1}{c}+1.k-\frac{1}{c}\right)}{\partial k}=\beta\left(\frac{1}{c}+1.k-\frac{1}{c}\right)\left[\Psi_0\left(k-\frac{1}{c}\right)-\Psi_0(k+1)\right]$ is the first partial of the second parameter for beta function.

For the observed information matrix of the parameters *(c, k)*, we calculate the second partial derivatives of (24) with respect to *c* and *k* respectively as follows:

$$\frac{\partial^2 \ln \ell}{\partial c^2} = -\frac{r}{c^2} - (k+1)\sum_{i=1}^{r}\left[\ln(y_i) - \frac{y_i^c \ln(y_i)}{\left(1+y_i^c\right)}\right] + r\left[\Omega_2 \Phi''_{\beta C} + \Phi'_{\beta C} \mathrm{M}_C\right]$$
$$+(n-r)\left\{(I_Y)^{-1}\left[\begin{array}{l}\left[\Omega_2 \int_0^t Z_C^2 dy + \int_0^t Z_C dy \mathrm{M}_C\right] + \\ \left[\int_0^t \Omega_1 dy \mathrm{M}_C^2 + \mathrm{M}_C \int_0^t Z_C dy\right]\end{array}\right] - \left[(I_Y)^{-2}\left[\Omega_2 \int_0^t Z_C dy + \int_0^t \Omega_1 dy \mathrm{M}_C\right]^2\right]\right\} \quad ; \quad (26)$$

Where

$\Phi''_{\beta C} = \dfrac{\partial^2 \beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)}{\partial c^2}$ is the second partial of the first parameter for beta function.

$$\frac{\partial \Omega_1}{\partial c_2} = -(k+1)\ln(y_i)\left\{\left[1+y_i^c\right]^{-(k+2)}\left[c y_i^{2c}\ln(y_i) + y_i^{2c}\right] - c y_i^{2c}(k+2)\left[1+y_i^c\right]^{-(k+3)} y_i^c \ln(y_i)\right\}$$
$$+\left[1+y_i^c\right]^{-(k+1)}\left[y_i^c \ln(y_i) + \left[c\ln(y_i)+1\right] y_i^c \ln(y_i)\right] = Z_C^2$$

and

$$\frac{\partial \Omega_2}{\partial c^2} = -\left[\frac{1}{\left[\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)\right]^2}\Phi''_{\beta c} - \frac{2\left(\Phi'_{\beta c}\right)^2}{\left[\beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)\right]^3}\right] = M_C^2$$

$$\frac{\partial^2 \beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)}{\partial k^2} = \beta\left(\frac{1}{c}+1, k-\frac{1}{c}\right)\left[\left[\Psi_0\left(k-\frac{1}{c}\right) - \Psi_0(k+1)\right]^2 + \Psi_1\left(k-\frac{1}{c}\right) - \Psi_1(k+1)\right]$$

$$\frac{\partial^2 \ln \ell}{\partial k^2} = -r\left[\Phi'_{\beta k}\mathrm{M}_K + \Omega_2 \Phi'_{\beta K \beta K}\right] - (n-r)\left[\left[(I_Y)^{-1}\left[\begin{array}{l}\left[\Omega_2 \int_0^t Z_{K^2} dy + \mathrm{M}_K \int_0^t Z_K dy\right] + \\ \left[\mathrm{M}_{K^2}\int_0^t \Omega_1 dy + \mathrm{M}_K \int_0^t Z_K dy\right]\end{array}\right]\right] + (I_Y)^{-2}\left[\Omega_2 \int_0^t Z_K dy + \mathrm{M}_K \int_0^t \Omega_1 dy\right]^2\right]$$

$$\frac{\partial^2 \ln \ell}{\partial k^2} = -r\left[\Phi'_{\beta k}\mathrm{M}_K + \Omega_2 \Phi'_{\beta K\beta K}\right] - (n-r)\left[\begin{matrix} \left[(I_Y)^{-1} \left[\begin{matrix} \left[\Omega_2 \int_0^t Z_{K^2} dy + \mathrm{M}_K \int_0^t Z_K dy\right] + \\ \left[\mathrm{M}_{K^2} \int_0^t \Omega_1 dy + \mathrm{M}_K \int_0^t Z_K dy\right] \end{matrix}\right]\right] \\ + (I_Y)^{-2}\left[\Omega_2 \int_0^t Z_K dy + \mathrm{M}_K \int_0^t \Omega_1 dy\right]^2 \end{matrix}\right] ,(27)$$

$$\frac{\partial^2 \Omega_1}{\partial k^2} = c y_i^c \left[\ln\left(1 + y_i^c\right)\right]^2 \left[1 + y_i^c\right]^{-(k+1)} = Z_K^2$$

$$\frac{\partial^2 \Omega_2}{\partial k^2} = -\frac{1}{\left[\beta\left(\frac{1}{c} + 1; k - \frac{1}{c}\right)\right]^2} \Phi''_{\beta k \beta k} - \left[\frac{2\left(\Phi'_{\beta k}\right)^2}{\left[\beta\left(\frac{1}{c} + 1; k - \frac{1}{c}\right)\right]^3}\right] = \mathrm{M}_K^2$$

In addition to, the partial derivatives of(25) with respect to c is :

$$\frac{\partial^2 \ln \ell}{\partial k \partial c} = \sum_{i=1}^{r}\left[\frac{y_i^c \ln y_i}{1 + y_i^c}\right] - r\left[\Omega_2 \Phi'_{\beta k\beta c} + \Phi'_{\beta k}\mathrm{M}_K\right]$$
$$-(n-r)\left\{ (I_Y)^{-1} \left[\begin{matrix} \left(\mathrm{M}_c \int_0^t Z_K dy + \Omega_2 \int_0^t Z_{KC} dy\right) + \\ \left(\mathrm{M}_K \int_0^t Z_C dy + M_{KC} \int_0^t \Omega_1 dy\right) \end{matrix}\right]\right\} + \left\{\begin{matrix} (I_Y)^{-2}\left[\Omega_2 \int_0^t Z_{KC} dy + \int_0^t Z_K dy M_C\right] \\ \left[M_{KC} \int_0^t \Omega_1 dy + M_K \int_0^t Z_C dy\right] \end{matrix}\right\} \quad (28)$$

5. Simulation Study

In this section, a numerical study is performed, to investigate the properties of estimates based on type II and type I censored samples.

5.1 Simulation study in case of censor type I

For simulation, Generating a random sample $y_1, y_2, \ldots, y_n$ size $n = 10,20,30,50,100,150$ from LBB-XII distribution. and predetermine time , $T = 0.5$ and 0.7 are selected, with three different set values of the parameters $s_1 = (c = 0.9, k = 5)$, $s_2 = (c = 1.5, k = 5)$ and $s_3 = (c = 0.9, k = 5.6)$. To examine the estimation accuracies, the total biases and the total RMSE (RMSE) are computed. Figures (5.1) to (5.3) show a graphical representation of the total biases and total RMSE of the parameter estimates as a function of sample size n. Figures (5.4) to (5.5) show total biases at T=0.5,0.7 for different values of parameters c and k and figures (5.6) to (5.7) show Total RMSE For different values of parameters c and k.

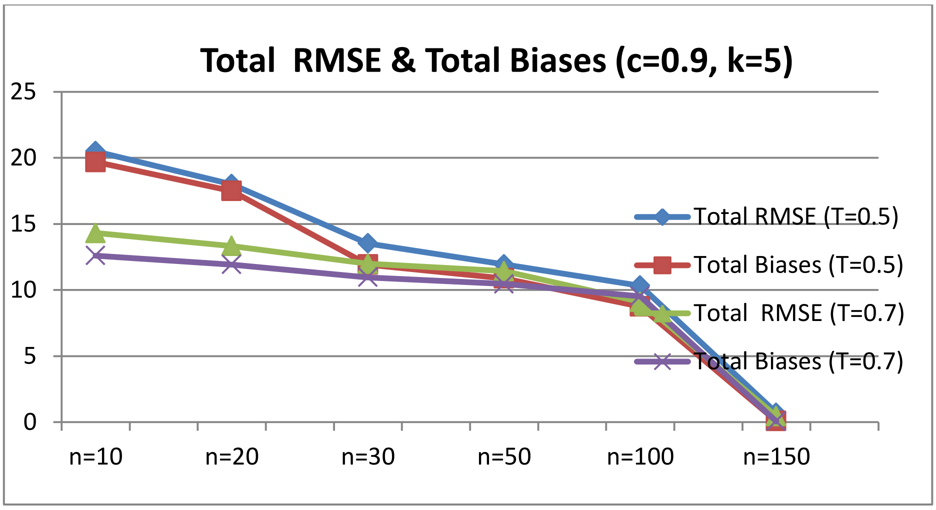


Figure 5.1: The Total RMSE and Total Biases for *c=0.9, k=5*

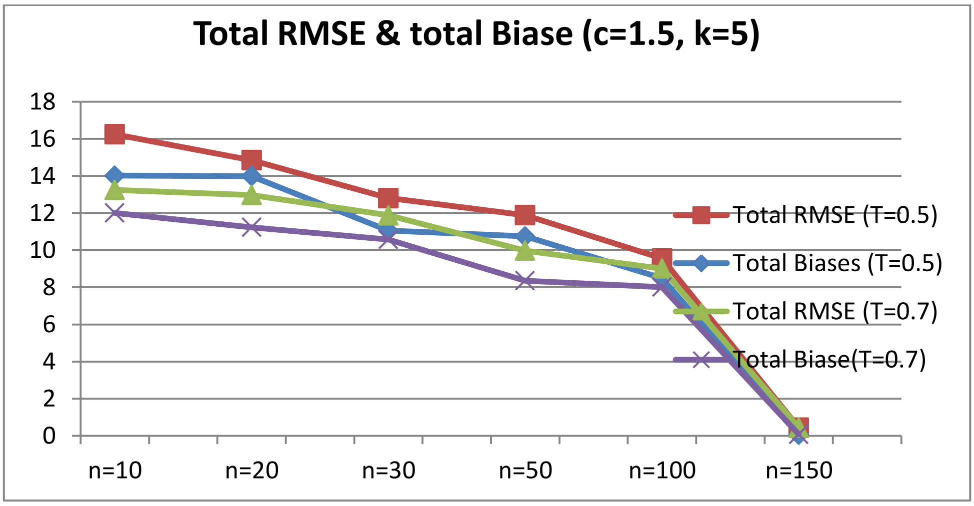


Figure 5.2: The Total RMSE and Total Biases for *c=1.5, k=5*

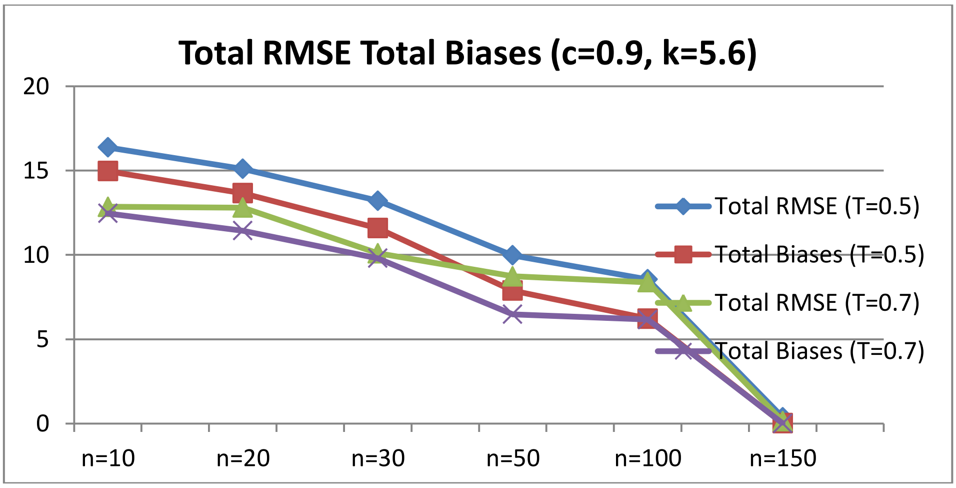


Figure 5.3: The Total RMSE and Total Biases for *c=0.9, k=5.6*

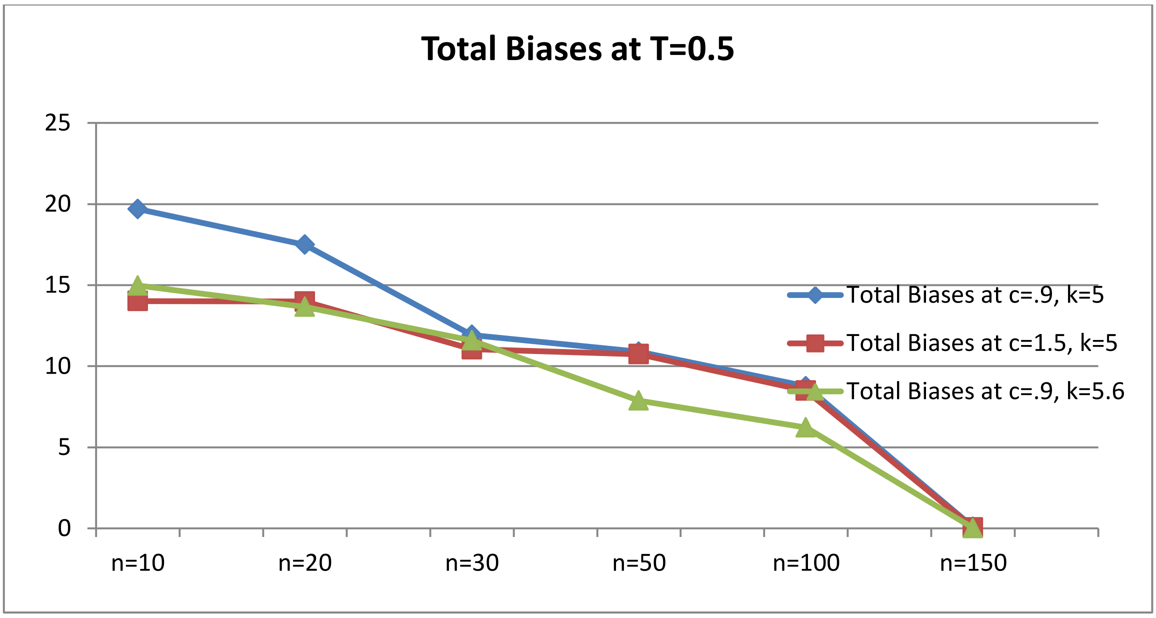


Figure 5.4: The Total Biases for at different values of c and k for *T=0.5*

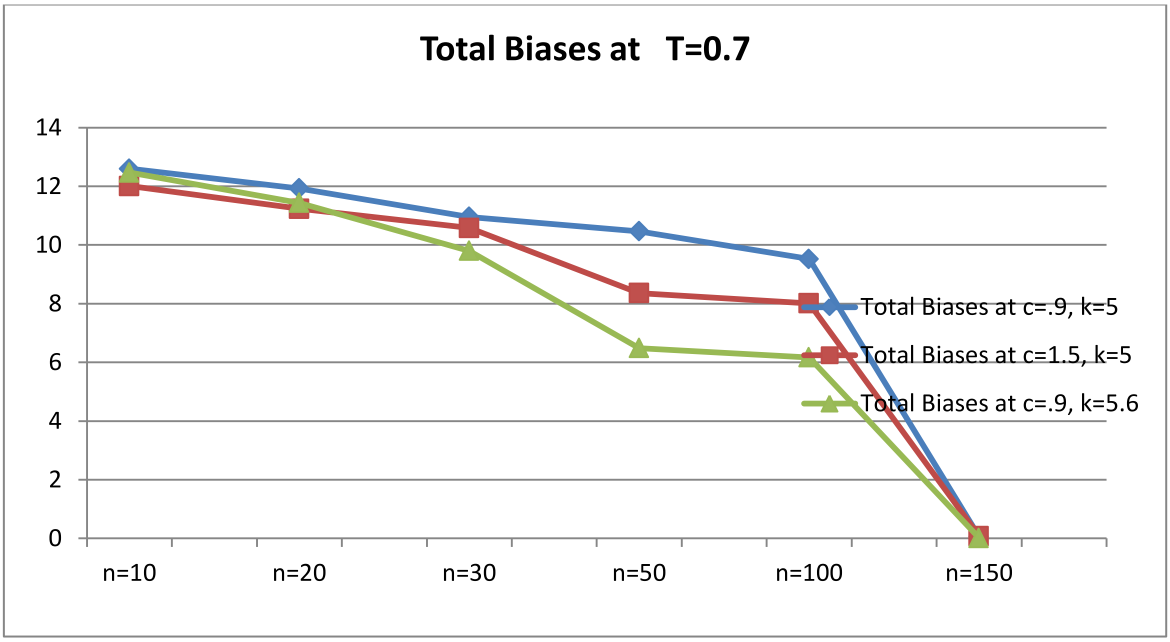


Figure 5.5: The Total Biases for at different values of c and k for T=0.7

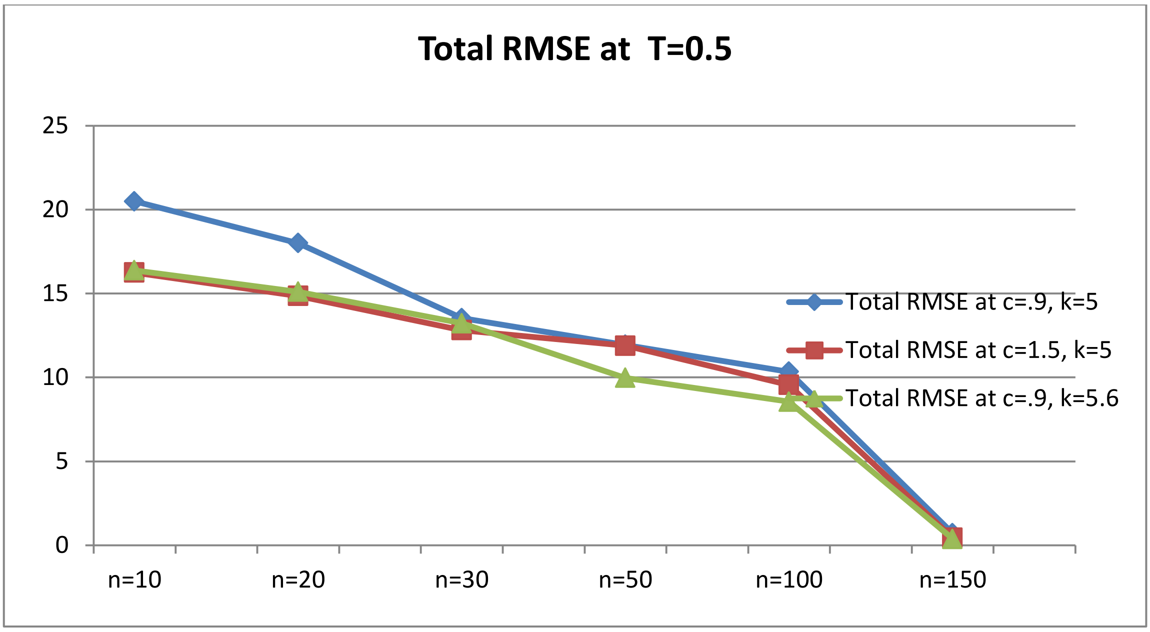


Figure 5.6: The Total RMSE for at different values of c and k for T=0.5

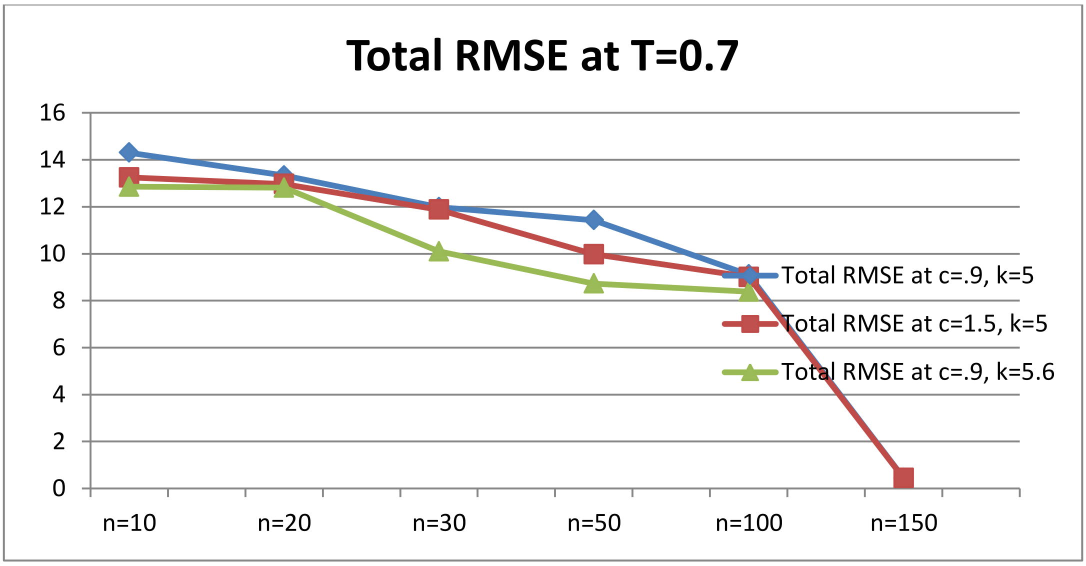


Figure 5.7: The Total RMSE for at different values of c and k for T=0.7

We can summarized the figures from(5.1) to (5.7) as follows :

- Figures (5.1) to (5.3) show that the relationship between sample size and both total RMSE and total biases, as sample size increase, total biases and total RMSE are decreases.
- Figures (5.4) to (5.5) show that the lowest curve of total biases curves occurs at $(c = 0.9, k = 5.6)$ in case of $T = (0.5, 0.7)$.
- Figures (5.6) to (5.7) show that the lowest curve of total RMSE curves occurs at $(c = 0.9, k = 5.6)$ in case of $T = (0.5, 0.7)$ and its mean the best values of the parameters in case of censor type I occur

at $(c = 0.9, k = 5.6)$.

5.2 Simulation study in case of censor type II

For simulation, Generating a random sample $y_1, y_2, \ldots, y_n$ size $n = 10,20,30,50,100,150$ from LBB-XII distribution and predetermine time , $(r/n) = 70\%$ and 90% are selected, with three different set values of the parameters $s_1 = (c = 0.9, k = 5)$, $S_2 = (c = 1.5, k = 5)$ and $S_3 = (c = 0.9, k = 5.6)$. To examine the estimation accuracies, the total biases and the total RMSE (RMSE) are computed. Figures (5.8) to (5.10) show a graphical representation of the total biases and total RMSE of the parameter estimates as a function of sample size n. Figures (5.11) to (5.12) show total biases at $(r/n) = 70\%, 90\%$ for different values of parameters c and k and figures (5.13) to (5.14) show Total RMSE For different values of parameters c and k.

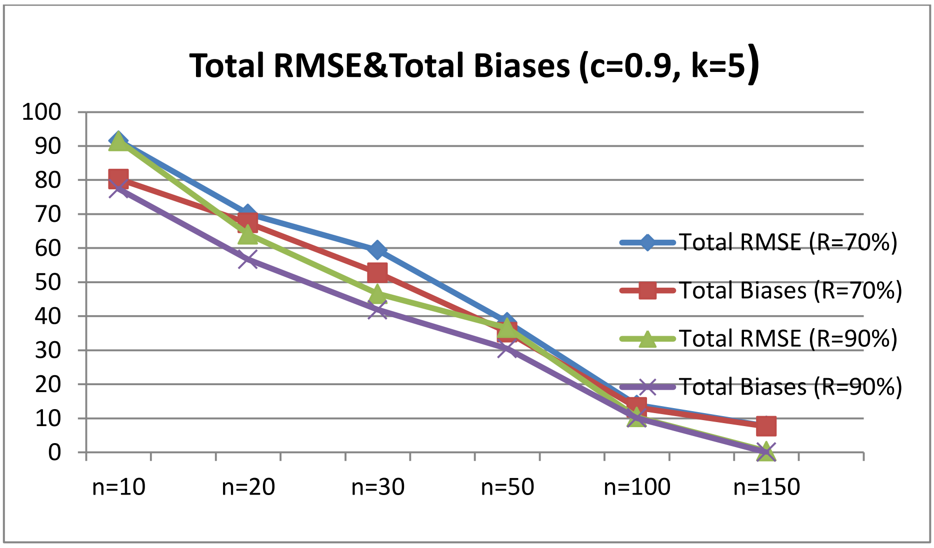


Figure 5.8: The Total RMSE and Total Biases for c=.9, k=5

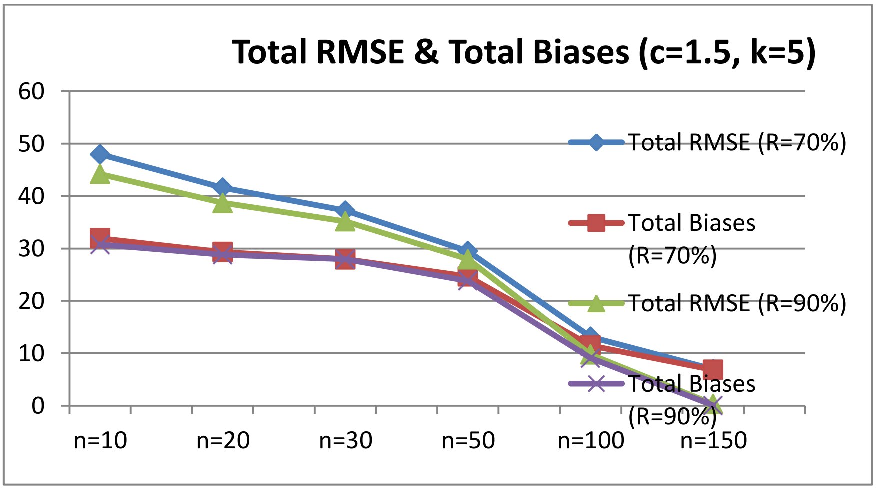


Figure 5.9: The Total RMSE and Total Biases for c=1.5, k=5

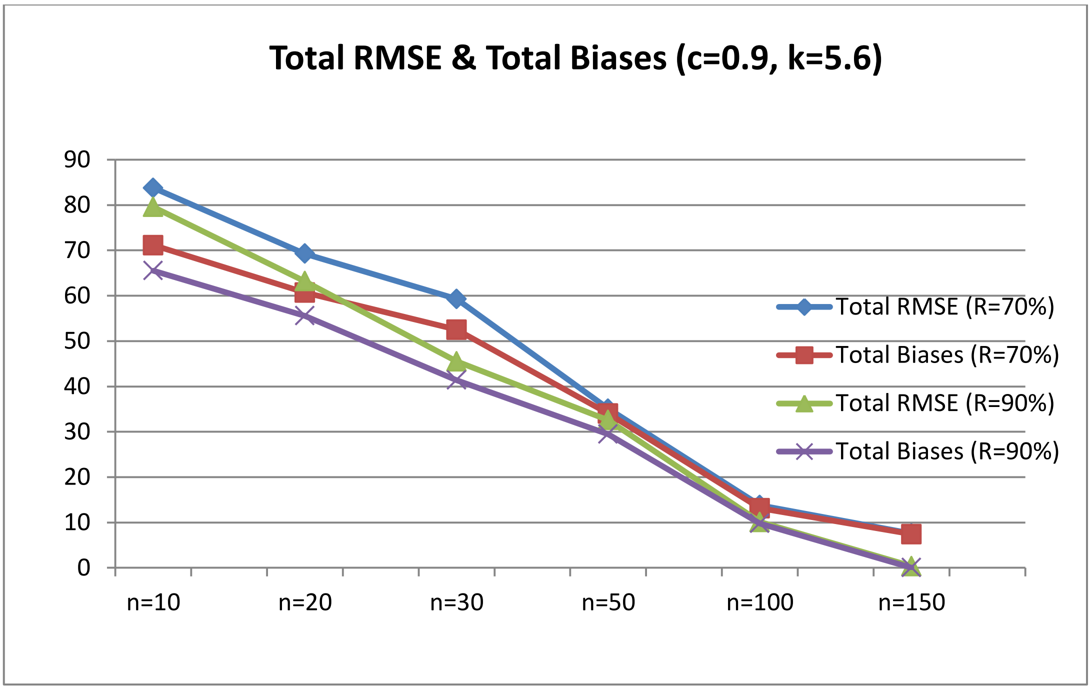


Figure 5.10: The Total RMSE and Total Biases for c=.9, k=5.6

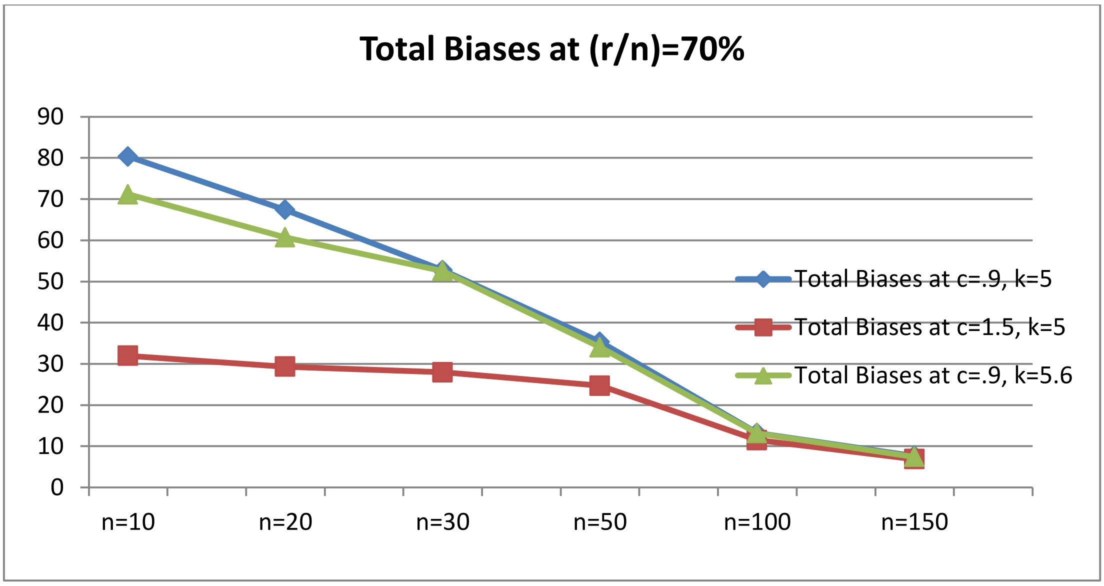


Figure 5.11: The Total Biases at different values of c and k for(r/n)=70%

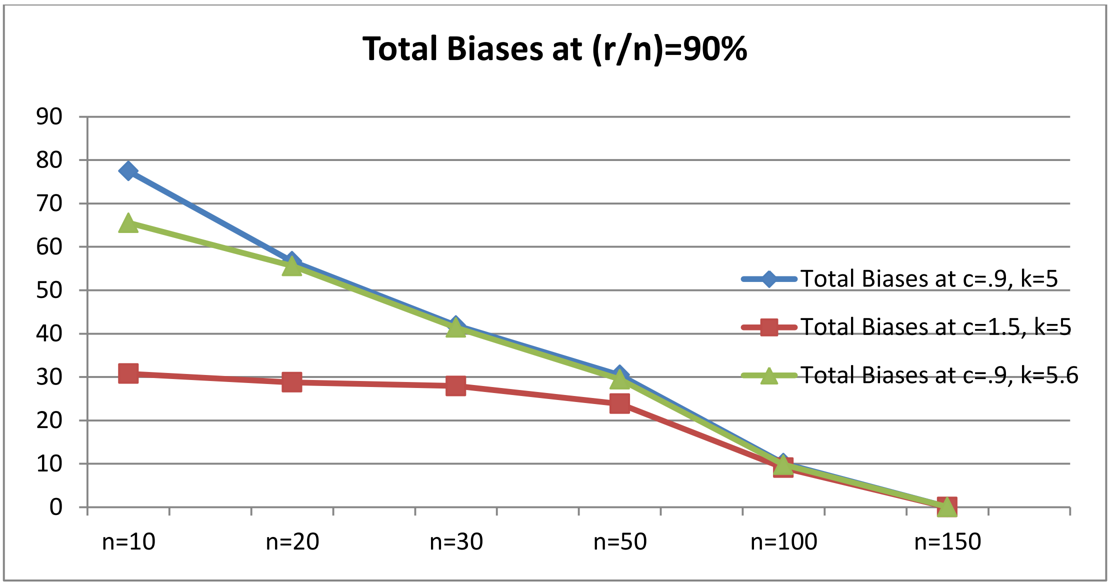


Figure 5.12: The Total Biases at different values of c and k for(r/n)=90%

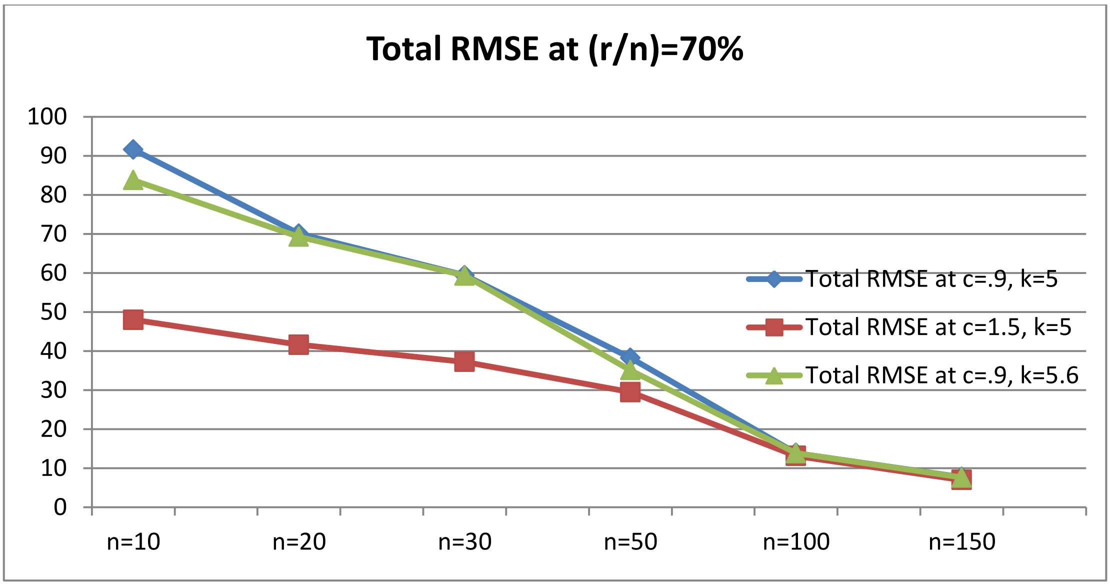


Figure 5.13: The Total RMSE at different values of c and k for(r/n)=70%

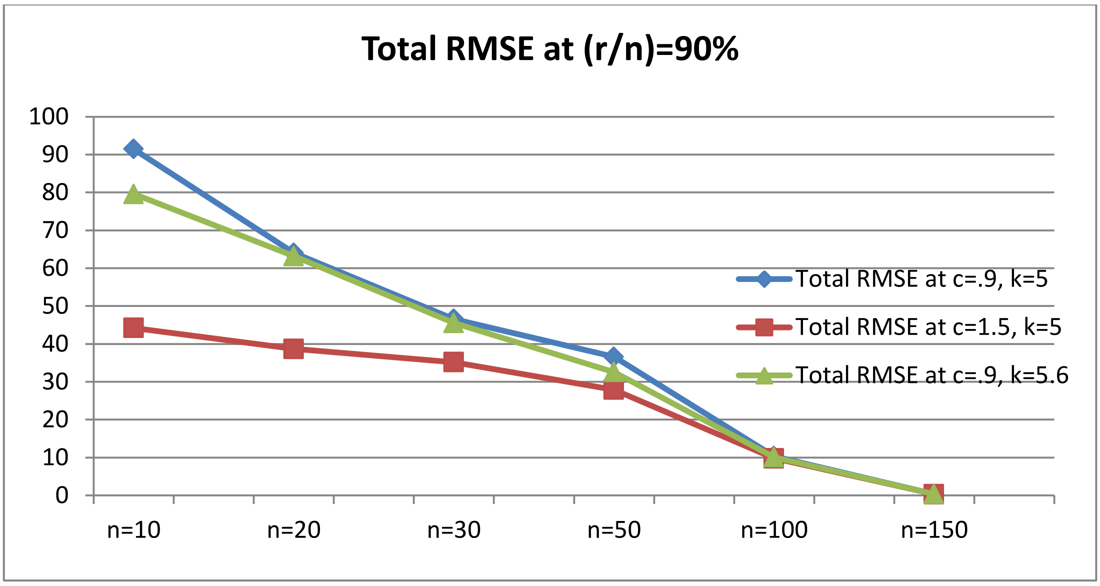


Figure 5.14: The Total RMSE at different values of c and k for(r/n)=90%

We can summarized the figures from(5.8) to (5.14) as follows :

- Figures (5.8) to (5.10) show that the relationship between sample size and both total RMSE and total biases, as sample size increase, total biases and total RMSE are decreases.
- Figures (5.11) to (5.12) show that the lowest curve of total biases curves occurs at $(c = 0.9, k = 5.6)$ in case of $(r / n) = 70\%,90\%$
- Figures (5.13) to (5.14)) show that the lowest curve of total RMSE curves occurs at $(c = 0.9, k = 5.6)$ in case of $(r / n) = 70\%$ and 90% and its mean the best values of the parameters in case of censor type II occur at $(c = 0.9, k = 5.6)$.

6. Conclusion

This paper discusses the estimation problem for the unknown parameters of the LBBXII distribution using censored type I and censored type II samples. Simulation study are performed to investigate the properties of estimates for censored samples. In general, as show from the simulation study results of LBB-XII distribution in case of censored type I and censored type II, the total RMSE and total biases decrease as sample size increases based on censored samples. According to censored samples, the best values of parameters which decreased the total biased and total RMSE where $(c = 0.9, k = 5.6)$.

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