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# Sampling Plans Designing with Simulation When Life Time Distributed the Logistic Distribution


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**Abstract**

Sampling design is a very important topic; it is the most efficient when it comes to costs and convenience. Time live distribution should be identified to give the best estimator of sampling plans. This research discuses designing sampling plans when life time follow logistic distribution, so we can use distribution parameters to calculate the required sample size and number of groups. This will enable us to decide to accepting or rejecting the whole lot. The findings of this research show the specific number of group and the specific size of these samples that give the lowest costs for accepting or rejecting the lot. Future research papers could be done on other distributions to investigate how sampling plans can be affected by distributing life time. Designing sequent and multiplied sampling plans can guarantee the decision of accepting or rejecting the lot through hiring the less numbers of groups and smallest size of the sample.



***Keywords:*** Sampling design plan; Cumulative Function; Logistical Distribution; Probability density function (;).


## 1. Aim and Objective of research

The sampling plans are increasingly important for having a significant relationship with the cost of the sampling.

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The decision to accept or reject the lot should be based on sufficient information and consequently the lowest risk. The research aims:

1- to design sampling plans based on the time of inspection until the failure occurs, as well as
2- to find sampling plans that have the lowest number of groups and sample size to reach the decision to accept or reject the whole lot.

## 2. Introduction

The subject of sampling plan designing and determining the optimal number of items (subject to inspection to reach a decision to reject or accept the lot) is highly correlated with the cost required for the inspection. Whereas sampling plans with fewer items certainly accompany lower sampling cost compared to their counterparts. Sometimes the sampling plan damages the inspected items and consequently a reduction in the final lot output. In this field, many researches were carried out, Reference [1] sampling plan according to an algorithm based on binary and hypergeometric distribution. The research solutions developed a single sampling plan for specific cases according to the assumed distributions. Reference [2] presented a research that includes a sampling plan for monitoring data according to Gama distribution. The research shows the difference in the sampling plans in two cases: the first when the life time has a Gama distribution and the second when it is assumed to fall under the hypothesis of the amputated life line during the inspection. While the search results proved the applicability. Reference [3] and others presented research for a double sampling plan and for tests of the amputated life line based on the (Bure Type XII) distribution. In this research, the sampling plan was presented according to the assumed characteristics based on the assumptions of the assumed distribution, the research showed that double sampling plans had more numbers than the rest. As a result, this research is an attempt to build a sampling plan based on a number of hypotheses, including having the inspection time of the logistic distribution according to specific parameters within the simulations to generate different volumes of lots. The results of the simulation proved that the sampling plans submitted had fewer items to accept or reject the whole lot. It is known that the increase in the cost of the test leads to an increase in the total cost of rejecting or accepting the whole lot. Further research can also be conducted if we assume that the time of the inspection has other distributions in addition to the distribution of logistics with the design of sampling plans according to other assumptions for comparison purposes The aim of the research is to arrive at the designs of sampling plans based on specific sampling conditions that the distribution of the inspection time be logistical distribution until the failure occurs with specific parameters (a, b) and thus reduce the risk by accepting a bad lot or rejecting a valid lot. The results showed that the sampling plans have the lowest number of groups and the lowest sample size to reach the decision of rejecting or accepting the whole lot according to the probability of pre-acceptable risk. Further research can be carried out by assuming that the time of the inspection (until the first failure is reached) possesses other distributions with different parameters. Sampling plans can be applied to real data to observe the accuracy of the decision to accept or reject the lot and the costs associated with the sampling.

## 3. Logistic Distribution

This distribution is a set of continuous statistical distributions where the random variable is a continuous type.

This distribution has a probability density function according to the following formulas:

$$g(y) = \frac{e^{-\frac{(y-a)}{b}}}{b[1+e^{-\frac{(y-a)}{b}}]^2} \ldots\ldots (1)$$

The cumulative distribution function has the following formulas:

$$G(Y \leq y) = \frac{1}{1+e^{-\frac{(y-a)}{b}}} \ldots\ldots (2)$$

This distribution is highly used in one of the regression types, called logistic regression, where it is used to process specific types of models. The Quintiles function for logistical distribution is:

$$G^{-1} = a + bLn\left(\frac{Q}{1-Q}\right) \ldots\ldots (3)$$

$$such\ that : 0 \leq Q \leq 1$$

***3.1 Parameter estimation***

The researcher (Fisher) between the years (1912-1922) found the estimation of logistic distribution parameters based on the likelihood function method. This method is based on the probability density function (1). In the case of the availability of (k) elements, each of which distributes the logistic distribution according to the parameters (α, β), the Likelihood function would be as follows:

$$\prod_{j=1}^{k} \frac{e^{-\frac{(y_j-a)}{b}}}{b[1+e^{-\frac{(y_j-a)}{b}}]^2} \ldots\ldots (4)$$

- Taking the logarithm of the function according to the formulas (4) will be

$$Ln(\prod_{j=1}^{k} g(y_j)) = \frac{e^{-\frac{\sum_{j=1}^{k}(y_j-a)}{b}}}{b^k \prod_{j=1}^{k}\left[1+e^{-\frac{(y_j-a)}{b}}\right]^2} \ldots\ldots (5)$$

$$Ln(\prod_{j=1}^{k} g(y_j)) = -\,kLn(b) - \frac{\sum_{j=1}^{k}(y_j - a)}{b} - 2\sum_{j=1}^{k} Ln\left[1 + e^{-\frac{(y_j - a)}{b}}\right] \ldots\ldots (6)$$

- Taking the derivative of the previous formula for parameter (a) once and parameter (b) again. We get:

$$\frac{\partial Ln(\prod_{j=1}^{k} g(y_j))}{\partial \alpha} = \frac{k}{\hat{b}} - \frac{2}{\hat{b}}\sum_{j=1}^{k} \frac{e^{(y_j - \hat{a})/\hat{b}}}{1 + e^{(y_j - \hat{a})/\hat{b}}} \ldots\ldots (7)$$

$$\frac{\partial Ln(\prod_{j=1}^{k} g(y_j))}{\partial b} = -\frac{k}{\hat{b}} + \frac{1}{\hat{b}^2}\sum_{j=1}^{k}(y_j - \hat{a}) - \frac{2}{\hat{b}^2}\sum_{j=1}^{k} \frac{(y_j - \hat{a})e^{-\frac{(y_j - \hat{\alpha})}{\hat{b}}}}{1 + e^{-\frac{(y_j - \hat{a})}{\hat{b}}}} \left(\frac{y_j - \hat{a}}{\hat{b}^2}\right) \ldots (8)$$

It is possible to find the Likelihood estimators of the logistic distribution parameters by solving equations (7,8) according to the numerical method of Newton Ravson because they cannot be solved according to the method of substitution and elimination. Likelihood estimators can be obtained according to Newton Ravson's method according to the following steps:

$$\beta_{i+1} = \beta_i - I.R \ \ldots (9)$$

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*Such that:*

$$\beta_{i+1} = \begin{bmatrix} a_{i+1} \\ b_{i+1} \end{bmatrix}, \qquad \beta_i = \begin{bmatrix} a_i \\ b_i \end{bmatrix}, I = \begin{bmatrix} \frac{\partial g_1(a)}{\partial a} & \frac{\partial g_1(a)}{\partial b} \\ \frac{\partial g_2(b)}{\partial a} & \frac{\partial g_2(b)}{\partial b} \end{bmatrix}, R = \begin{bmatrix} g_1(a) \\ g_2(b) \end{bmatrix}$$

$$g_1(a) = \frac{k}{\hat{b}} - \frac{2}{\hat{b}}\sum_{j=1}^{k} \frac{e^{(y_j - \hat{a})/\hat{b}}}{1 + e^{(y_j - \hat{a})/\hat{b}}} \ldots (10)$$

$$g_2(b) = -\frac{k}{\hat{b}} + \frac{1}{\hat{b}^2}\sum_{j=1}^{k}(y_j - \hat{a}) - \frac{2}{\hat{b}^2}\sum_{j=1}^{k} \frac{(y_j - \hat{a})e^{-\frac{(y_j - \hat{a})}{\hat{b}}}}{1 + e^{-\frac{(y_j - \hat{a})}{\hat{b}}}} \left(\frac{y_j - \hat{a}}{\hat{b}^2}\right) \ldots (11)$$

**Logistic distribution simulation**

Formulas (2) can be based to simulate the logistic distribution according to the predefined (a, b) parameters in addition to the sample size (k) defined as follows: -

Assuming (Z = G (≤ Y)) to be

$$Z = \frac{1}{1 + e^{-\frac{(y-a)}{b}}} \ldots \ldots (12)$$

Where (Z) is a randomization function, whose values are between zero and one

$$Z\left[1 + e^{-\frac{(y-a)}{b}}\right] = 1$$

$$e^{-\frac{(y-a)}{b}} = \frac{1-Z}{Z}$$

$$-\frac{(y-a)}{b} = Ln(\frac{1-Z}{Z})$$

$$-y + a = bLn\left(\frac{1-Z}{Z}\right)$$

$$y = a - bLn\left(\frac{1-Z}{Z}\right) \ldots \ldots (13)$$

Formulas (9) are based to generate a sample that distributes the logistic distribution according to the distribution parameters (a, b) and the number (k)

### *3.2 Design Sampling Plan*

The sampling plan can be designed based on the availability of (K) elements that represent the total size of the lot to be inspected. The life line distribution for each element within the batch is the logistic distribution according to the parameters (a, b). The sampling plan (to accept the whole batch of K) is based on an inspection plan (k) of the elements at a time, as assumed by the researcher [4]. The assumed sampling plan is based on finding (k, h) such that:

$$P(Y \leq h/k, p_1) = (1 - E_1) \ldots (14)$$

And surely: -

$$P(Y \leq h/k, p_2) = E_2$$

Where ((E1, E2) represents the sampling error of type I and II respectively. The sampling plan can be obtained based on the following formula:

$$P\left(\leq \frac{r}{k}, p \quad\right) = \sum_{l=0}^{h} \frac{k!}{z!\,(k-l)!} p^{l} q^{k-l} \ldots (15)$$

1. Such that p + q=1
2. According to the assumed sampling plan, the lot is accepted if there are (h) or fewer failures for each (u) of groups of each size (s). And the probability of accepting the lot is equal to: -

$$L(A) = [\sum_{j=1}^{h} \frac{s!}{j!\,(s-j)!} p^{j}\, q^{s-j}]^{u} \ldots (16)$$

So that (q) represents the probability of failure of unit (i) within group (u) before the time of the test (t) reaches the failure time average ( $t_0$ ), the total volume of the lot is equal: $K = us$ So that (s) represents the number of elements within the group (u) Represents the number of groups

(p) represents the probability of the time of failure, which is distributed logistical distribution according to the following formula: -

$$p = \frac{e^{-\frac{(y_j - a)}{b}}}{b[1 + e^{-\frac{(y_j - a)}{b}}]^2} \ldots (17)$$

## 4. Experimental Results

After applying the formulas in the theoretical side to the simulation dataset whose experiments were assumed according to the preconditions, the agency inspection plans were obtained:

**Table 1:** Sampling plans for (s = (4), a = (0.5,1) = (0.01,0.03,0.05),y = (2,3))

| **5.** $\gamma$ | **6.** $b$ | **7.** $h_2$ | **8.** $a = 0.5$ | | | **9.** $a = 1$ | | |
|---|---|---|---|---|---|---|---|---|
| | | | **10.** $u$ | **11.** $h$ | **12.** $L(A)$ | **13.** $u$ | **14.** $h$ | **15.** $R(Q$ |
| **16.** 2 | **17.** 0. 05 | **18.** 2 | **19.** 2 9 | **20.** 3 | **21.** 0.9 72 | **22.** 1 8 | **23.** 4 | **24.** 0.9 87 |
| | | **25.** 4 | **26.** 6 | **27.** 2 | **28.** 0.9 84 | **29.** 3 | **30.** 2 | **31.** 0.9 89 |
| | | **32.** 6 | **33.** 3 | **34.** 1 | **35.** 0.9 92 | **36.** 2 | **37.** 1 | **38.** 0.9 82 |
| | | **39.** 8 | **40.** 1 | **41.** 1 | **42.** 0.9 51 | **43.** 1 | **44.** 1 | **45.** 0.9 98 |
| | **46.** 0. 03 | **47.** 2 | **48.** 4 5 | **49.** 3 | **50.** 0.9 61 | **51.** 2 5 | **52.** 4 | **53.** 0.9 54 |
| | | **54.** 4 | **55.** 6 | **56.** 2 | **57.** 0.9 72 | **58.** 4 | **59.** 2 | **60.** 0.9 82 |
| | | **61.** 6 | **62.** 4 | **63.** 1 | **64.** 0.9 86 | **65.** 2 | **66.** 1 | **67.** 0.9 86 |
| | | **68.** 8 | **69.** 2 | **70.** 1 | **71.** 0.9 91 | **72.** 1 | **73.** 1 | **74.** 0.9 91 |
| | **75.** 0. 01 | **76.** 2 | **77.** 8 9 | **78.** 3 | **79.** 0.9 65 | **80.** 4 | **81.** 3 | **82.** 0.9 82 |
| | | **83.** 4 | **84.** 8 | **85.** 2 | **86.** 0.9 62 | **87.** 3 | **88.** 2 | **89.** 0.9 78 |
| | | **90.** 6 | **91.** 6 | **92.** 1 | **93.** 0.9 82 | **94.** 2 | **95.** 1 | **96.** 0.9 88 |
| | | **97.** 8 | **98.** 2 | **99.** 1 | **100.** 0.9 91 | **101.** 2 | **102.** 1 | **103.** 0.9 92 |
| **104.** 3 | **105.** 0. 05 | **106.** 2 | **107.** 1 8 | **108.** 2 | **109.** 0.9 57 | **110.** 4 | **111.** 3 | **112.** 0.9 78 |
| | | **113.** 4 | **114.** 5 | **115.** 1 | **116.** 0.9 77 | **117.** 2 | **118.** 2 | **119.** 0.9 95 |
| | | **120.** 6 | **121.** 3 | **122.** 1 | **123.** 0.9 87 | **124.** 1 | **125.** 1 | **126.** 0.9 72 |
| | | **127.** 8 | **128.** 1 | **129.** 1 | **130.** 0.9 98 | **131.** 1 | **132.** 1 | **133.** 0.9 82 |
| | **134.** 0. 03 | **135.** 2 | **136.** 2 9 | **137.** 2 | **138.** 0.9 77 | **139.** 7 | **140.** 3 | **141.** 0.9 76 |
| | | **142.** 4 | **143.** 8 | **144.** 1 | **145.** 0.9 98 | **146.** 4 | **147.** 2 | **148.** 0.9 92 |
| | | **149.** 6 | **150.** 4 | **151.** 1 | **152.** 0.9 81 | **153.** 3 | **154.** 1 | **155.** 0.9 98 |
| | | **156.** 8 | **157.** 2 | **158.** 1 | **159.** 0.9 98 | **160.** 1 | **161.** 1 | **162.** 0.9 83 |
| | **163.** 0. 01 | **164.** 2 | **165.** 4 2 | **166.** 2 | **167.** 0.9 76 | **168.** 1 1 | **169.** 3 | **170.** 0.9 54 |
| | | **171.** 4 | **172.** 9 | **173.** 1 | **174.** 0.9 98 | **175.** 7 | **176.** 2 | **177.** 0.9 72 |
| | | **178.** 6 | **179.** 4 | **180.** 1 | **181.** 0.9 82 | **182.** 3 | **183.** 1 | **184.** 0.9 81 |
| | | **185.** 8 | **186.** 2 | **187.** 1 | **188.** 0.9 96 | **189.** 1 | **190.** 1 | **191.** 0.9 76 |

K= $u * h$

K=87

The results can be seen in Table (1), and in case of:

, Then the sampling plan assume :)$s_2 = 2, s = (4), a = (0.5), b = (0.01), \gamma = (2)$ (

$(u = 29, h = 3, L(A) = 0.972)$ Thus, the number of elements to be inspected (K) is equal:

K= $u * h$

K=87

The probability of risk is equivalent to (0.972) and if the cost of the unit inspection is equal to (k), then the total cost of the inspection plan for this case will be (87k). And so for other sampling plans. We note from the results that the probability of risk varies according to the remaining plans so that they are increasing with increasing value (s)

**192.** Thus, the number of elements to be inspected (K) is equal. Whereas (u, h) decreases with increasing value (h).

**Table 2:** sampling plans for (s = (8), a= (0.5,1),b = (0.01,0.03,0.05),y = (2,3))

| $\gamma$ | $b$ | $s_2$ | $a = 0.5$ | | | $a = 1$ | | |
|---|---|---|---|---|---|---|---|---|
| | | | $u$ | $h$ | $L(A)$ | $u$ | $h$ | $R(Q)$ |
| 2 | 0.05 | 2 | 21 | 4 | 0.972 | 4 | 3 | 0.968 |
| | | 4 | 5 | 2 | 0.987 | 2 | 2 | 0.987 |
| | | 6 | 3 | 1 | 0.962 | 2 | 1 | 0.966 |
| | | 8 | 1 | 0 | 0.982 | 2 | 1 | 0.987 |
| | 0.03 | 2 | 20 | 3 | 0.973 | 5 | 3 | 0.952 |
| | | 4 | 7 | 1 | 0.992 | 4 | 2 | 0.987 |
| | | 6 | 6 | 1 | 0.975 | 3 | 0 | 0.962 |
| | | 8 | 2 | 0 | 0.989 | 2 | 0 | 0.975 |
| | 0.01 | 2 | 38 | 3 | 0.973 | 6 | 3 | 0.982 |
| | | 4 | 6 | 1 | 0.989 | 2 | 2 | 0.995 |
| | | 6 | 4 | 1 | 0.962 | 2 | 1 | 0.958 |
| | | 8 | 2 | 0 | 0.998 | 1 | 1 | 0.977 |
| 3 | 0.05 | 2 | 22 | 3 | 0.972 | 7 | 3 | 0.979 |
| | | 4 | 6 | 1 | 0.979 | 4 | 2 | 0.993 |
| | | 6 | 2 | 1 | 0.991 | 3 | 1 | 0.991 |
| | | 8 | 2 | 0 | 0.959 | 2 | 1 | 0.988 |
| | 0.03 | 2 | 19 | 3 | 0.978 | 6 | 3 | 0.973 |
| | | 4 | 2 | 1 | 0.973 | 2 | 2 | 0.987 |
| | | 6 | 2 | 1 | 0.991 | 2 | 1 | 0.992 |
| | | 8 | 2 | 0 | 0.959 | 1 | 1 | 0.996 |
| | 0.01 | 2 | 12 | 3 | 0.953 | 4 | 3 | 0.973 |
| | | 4 | 2 | 1 | 0.961 | 3 | 2 | 0.986 |
| | | 6 | 2 | 1 | 0.989 | 2 | 2 | 0.989 |
| | | 8 | 1 | 0 | 0.967 | 1 | 0 | 0.996 |

Note that the results presented in Table (2), the assumed sampling plans were (s = 8), We note from the results that the probability of risk increases with increasing value (c), whereas (u, h) decreases with increasing value (s).

## 5. Conclusions and Suggestions

After the assumed sampling plans were designed according to the research hypotheses, a number of conclusions and recommendations, the most important of them are: -

1. The assumed sampling plans contribute effectively to reduce the cost and time of the inspection to reach the decision of accepting or rejecting the lot.
2. The life-time distribution of the samples under inspection can contribute to better sampling plans by studying the behavior of that distribution and thus we can estimate the distribution parameters.
3. To reach the best sampling plan requires specific assumptions, including the probability of risk and the distribution of time off in addition to the size of the batch.
4. It is possible to find sampling plans for the used samples previously, and thus the subject of reliability may be taken into consideration during the study of the distribution of the failure time to reach a decision to accept or reject the lot.
5. It is possible to design sequential sampling plans after making assumptions for the failure time distribution and compare the design costs associated with each case.
6. Assumptions of time distributions can be made in addition to the logistical distribution, and the sample plans provided for each distribution can also be compared.

## References

[1]. R. W. Samohyl, "Acceptance sampling for attributes via hypothesis testing and the hypergeometric distribution," J. Ind. Eng. Int., vol. 14, no. 2, pp. 395–414, 2018.

[2]. B. S. Ramirez and J. G. Ramírez, Douglas Montgomery's Introduction to Statistical Quality Control : a JMP Companion. 2018.

[3]. W. Gui and M. Xu, "Double acceptance sampling plan based on truncated life tests for half exponential power distribution," Stat. Methodol., vol. 27, no. 67, pp. 123–131, 2015.

[4]. C. H. Jun, S. Balamurali, and S. H. Lee, "Variables sampling plans for Weibull distributed lifetimes under sudden death testing," IEEE Trans. Reliab., vol. 55, no. 1, pp. 53–58, 2006.