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Application of Perturbation Theory in Heat Flow Analysis

Neelam Gupta and Neel Kanth

Abstract

Many physical and engineering problems can be modeled using partial differential equations such as heat transfer through conduction process in steady and unsteady state. Perturbation methods are analytical approximation method to understand physical phenomena which depends on perturbation quantity. Homotopy perturbation method (HPM) was proposed by Ji Huan He. HPM is considered as effective method in solving partial differential equations. The solution obtained by HPM converges to exact solution, which are in the form of an infinite function series. Biazar and Eslami proposed new homotopy perturbation method (NHPM) in which construction of an appropriate homotopy equation and selection of appropriate initial approximation guess are two important steps. In present work, heat flow analysis has been done on a rod of length L and diffusivity α using HPM and NHPM. The solution obtained using different perturbation methods are compared with the solution obtained from most common analytical method separation of variables.

Keywords: heat conduction equation, homotopy perturbation method, new homotopy perturbation method, specific heat, diffusivity

1. Introduction

Partial differential equations play a dominant role in applied mathematics. The classical heat conduction equation is second order linear partial differential equation. The solutions of which are obtained by using various analytical and numerical methods [1–3]. This equation describes the heat distribution in each domain over some time. Jean-Joseph Fourier was the first to formulate and describe the heat conduction process [1, 4]. Perturbation methods depending upon small/large parameters have been encountered from past few years. Perturbation methods are analytical approximation method to understand physical phenomena which depends on perturbation quantity. But these methods do not provide an easy way to find out the rapid convergence of approximate series. Therefore, this method is simple, suitable and appropriate method to provide the rapid convergence of series [5–7]. The perturbation method along with the homotopy method has been employed to develop a hybrid method known as homotopy perturbation method (HPM) [1–4]. Ji-Huan was the first to introduce HPM. Homotopy perturbation method provides analytical approximation to linear/nonlinear problems without linearization or discretization. It helps in formulating simpler equations by breaking down the complex problems, which can be solved easily. Since HPM does not depend on small parameters, therefore drawbacks of the existing perturbation

methods can be abolished [8–11]. The solution obtained by HPM converges to exact solution, which are in the form of an infinite function series. Various problems are modeled by linear and non-linear partial differential equations problems in the fields of physics, engineering etc. To solve such kind of partial differential equations (PDE), many methods are used to find the numerical or exact solutions. Homotopy perturbation method (HPM) is one of the methods used in recent years to solve various linear and non-linear PDE [12–15]. Initial and boundary value problems can be solved using HPM extensively. Many researchers and scientists show great interest in homotopy perturbation method. Huan was the first who described homotopy perturbation method. He showed that this method is a one of the powerful tools used to investigate various problems which are arising nowadays. HPM is used for solving linear and non-linear ordinary and partial differential equations [16].

In HPM, complex linear or non-linear problem can be continuously distorted into simpler ones. Perturbation theory and homotopy theory in topology is combined to develop homotopy perturbation method [1]. HPM is applicable to linear and non-linear boundary value problems. The solution obtained by HPM gives the solution approximately near to the universally accepted method of separation of variable [17–19].

Recently, Biazar and Eslami proposed the new homotopy perturbation method (NHPM). Construction of an appropriate homotopy equation and selection of appropriate initial approximation guess are two important steps of NHPM [19, 20]. The study reveals that with less computational work, we can construct proper homotopy by decomposition of source function in a correct way. New homotopy perturbation method is the most powerful tool which can be used to obtain analytical solution of various kinds of linear and nonlinear PDE's. This method is widely used by researchers to obtain solution of various functional Equations [20–22].

To develop this new technique, HPM is combined with the decomposition of source function. The decomposition of a source function is the basis of homotopy used in this method because convergence of a solution is affected by the decomposition of source functions [23]. Different kind of homotopy can be formed using various decomposition of a source functions. This study is aimed at constructing suitable homotopy by decomposition of a source function which requires less computational efforts and made calculations in simpler form unlike other perturbation methods. The obtained results directly imply the fact that NHPM is very influential as compared to HPM or any other perturbation technique. To establish exact solution of linear and non-linear problem with boundary and initial condition, new homotopy method is most appropriate method to apply [23].

The two most important steps in application of new homotopy perturbation method to construct a suitable homotopy equation and choose a suitable initial guess, we aim in this work to effectively employ the (NHPM) to establish exact solution for two-dimensional Laplace equation with Dirichlet and Neumann boundary condition, the difference between (NHPM) and standard (HPM) is starts from the form of initial approximation of the solution.

In this chapter, the semi analytic solution of one-dimensional heat conduction equation is obtained by means of homotopy perturbation method and new homotopy perturbation method. These methods are effectively applied to obtain the exact solution for the problem in hand which reveals the effectiveness and simplicity of the method. Numerical results have also been analyzed graphically to show the rapid convergence of infinite series expansion. The obtained analytic solution for one dimensional heat conduction equation with boundary and initial conditions using NHPM is same as the universally accepted exact solution. This tells us about the capability and reliability of this method. The solution obtained using NHPM is

considered in the form of an infinite series. The convergence of solution to the exact solution is very rapid.

2. Heat conduction equation

The one-dimensional heat equation

$$\frac{\partial U}{\partial \theta} = \beta \frac{\partial^2 U}{\partial z^2} \quad (1)$$

with boundary conditions

$$U(0, \theta) = 0, U(1, \theta) = 0, \quad (2)$$

and initial condition

$$U(z, 0) = h(z), 0 \leq z \leq 1. \quad (3)$$

3. Basic idea of Homotopy perturbation method

First, we outline the general procedure of the homotopy perturbation method developed and advanced by He. We consider the differential Eq. [2]

$$A(u) - f(r) = 0, r \in \Omega \quad (4)$$

$$B\left(u, \frac{\partial u}{\partial x}\right) = 0, r \in \Gamma \quad (5)$$

where A is a general differential operator, linear or nonlinear, $f(r)$ is a known analytic function, B is a boundary operator and Γ is the boundary of the domain Ω . The operator A can be generally divided into two operators, L and N , where L is linear and N is a nonlinear operator. Eq. (4) can be written as

$$L(u) + N(u) - f(r) = 0 \quad (6)$$

Using the homotopy technique, we can construct a homotopy [1,2]

$v(r, p) : \Omega \times [0, 1] \rightarrow R$ which satisfies the relation

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, r \in \Omega \quad (7)$$

Here $p \in [0, 1]$ is called the homotopy parameter and u_0 is an initial approximation for the solution of Eq. (4), which satisfies the boundary conditions. Clearly, from Eq. (7), we have

$$H(v, 0) = L(v) - L(u_0) \quad (8)$$

$$H(v, 1) = A(v) - f(r) \quad (9)$$

We assume that the solution of Eq. (7) can be expressed as a series in p as follows:

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \dots \quad (10)$$

On setting $p = 1$, we obtain the approximate solution of Eq. (10) as

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + v_3 + \dots \quad (11)$$

4. Basic idea of new Homotopy perturbation method

First, following homotopy is constructed for solving heat conduction equation using NHPM

$$(1-p) \left(\frac{\partial T}{\partial \theta} - U_0 \right) + p \left(\frac{\partial T}{\partial \theta} - \beta \frac{\partial^2 T}{\partial z^2} \right) = 0 \quad (12)$$

Taking $L^{-1} = \int_{\theta_0}^{\theta} (\cdot) d\theta$ i.e. inverse operator on Eq. (12), then

$$T(z, \theta) = \int_{\theta_0}^{\theta} U_0(z, \theta) d\theta - p \int_{\theta_0}^{\theta} \left(U_0 - \beta \frac{\partial^2 T}{\partial z^2} \right) d\theta + T(z, \theta_0) \quad (13)$$

$$\text{Where } T(z, \theta_0) = U(z, \theta_0)$$

Let the solution of Eq. (13) is given by

$$T = T_0 + pT_1 + p^2T_2 + p^3T_3 + \dots \quad (14)$$

where $T_0, T_1, T_2, T_3, \dots$ are to be determined.

Suppose solution given by Eq. (14) is the solution of Eq. (13). On comparing the coefficients of powers of p and equating to zero and using Eq. (14) in Eq. (13), following are obtained:

$$p^0 : T_0(z, \theta) = \int_{\theta_0}^{\theta} U_0(z, \theta) d\theta + T(z, \theta_0)$$

$$p^1 : T_1(z, \theta) = - \int_{\theta_0}^{\theta} \left(U_0(z, \theta) - \beta \frac{\partial^2 T_0}{\partial z^2} \right) d\theta$$

$$p^2 : T_2(z, \theta) = \int_{\theta_0}^{\theta} \left(\beta \frac{\partial^2 T_1}{\partial z^2} \right) d\theta$$

$$p^3 : T_3(z, \theta) = \int_{\theta_0}^{\theta} \left(\beta \frac{\partial^2 T_2}{\partial z^2} \right) d\theta$$

$$\text{and so on ...} \quad (15)$$

Consider the initial approximation of Eq. (1) as

$$U_0(z, \theta) = \sum_{n=0}^{\infty} c_n(z) P_n(\theta), T(z, 0) = U(z, 0), P_k(\theta) = \theta^k, \quad (16)$$

where, $P_1(\theta), P_2(\theta), P_3(\theta), \dots$ and $c_0(z), c_1(z), c_2(z), \dots$ are specified functions and unknown coefficients respectively, depending on the problem.

Using Eq. (16) in (15), following are obtained:

$$T_0(z, \theta) = \left(c_0(z)\theta + c_1(z)\frac{\theta^2}{2} + c_2(z)\frac{\theta^3}{3} + c_3(z)\frac{\theta^4}{4} + \dots \right) + U(z, 0)$$

$$T_1(z, \theta) = (-c_0(z) - \beta\pi^2 \sin \pi z)\theta + \left(-\frac{1}{2}c_1(z) + \frac{1}{2}\beta c_0''(z)\right)\theta^2 + \left(-\frac{1}{3}c_2(z) + \frac{1}{3}c_1''(z)\right)\theta^3 + \dots$$

and so on ... (17)

Now solving the above equations in such a manner that, $T_1(z, \theta) = 0$. Therefore Eq. (17) reduces to

$$T_1(z, \theta) = T_2(z, \theta) = \dots = 0.$$

So $U(z, \theta) = T_0(z, \theta) = \sum_{n=0}^{\infty} c_n(z)P_n(\theta)$ is obtained solution which is found to be exactly same as the exact solution obtained through method of separation of variable.

If $U_0(z, \theta)$ is analytic at $\theta = \theta_0$,

$U_0(z, \theta) = \sum_{n=0}^{\infty} c_n(z)(\theta - \theta_0)^n$ is the Taylor series expansion which can be used in Eq. (9).

5. Applications of Homotopy perturbation method and new Homotopy perturbation method

For understanding the application of HPM and NHPM, we will solve the one-dimensional heat equation given by

$$\frac{\partial U}{\partial \theta} = \beta \frac{\partial^2 U}{\partial z^2} \quad (18)$$

with boundary conditions

$$U(0, \theta) = 0, U(1, \theta) = 0, \quad (19)$$

and initial condition

$$U(z, 0) = \sin \frac{2\pi z}{L}, 0 \leq z \leq L. \quad (20)$$

The homotopy for the diffusion equation given by (18) is obtained as follows [2].

$$\left(\frac{\partial v}{\partial \theta} - \frac{\partial u_0}{\partial \theta}\right) + \beta \left(\frac{\partial u_0}{\partial \theta} - \beta \frac{\partial^2 v}{\partial z^2}\right) = 0 \quad (21)$$

Let $u_0 = \sin \frac{2\pi z}{L} \cos \pi^2 \theta$ be the initial approximation, which satisfies boundary conditions given by (19).

Let solution of (18) has the following form

$$v = v_0 + \beta v_1 + \beta^2 v_2 + \beta^3 v_3 + \beta^4 v_4 + \dots \quad (22)$$

On substituting the value of v in Eq. (21) and comparing the coefficients of like powers of β we obtain

$$\beta^0 : \frac{\partial v_0}{\partial \theta} = \frac{\partial u_0}{\partial \theta}$$

$$\begin{aligned}
 \beta^1 : \frac{\partial v_1}{\partial \theta} &= \beta \frac{\partial^2 v_0}{\partial z^2}, v_1(0, \theta) = 0 = v_1(L, \theta) \\
 \beta^2 : \frac{\partial v_2}{\partial \theta} &= \beta \frac{\partial^2 v_1}{\partial z^2}, v_2(0, \theta) = 0 = v_2(L, \theta) \\
 \beta^3 : \frac{\partial v_3}{\partial \theta} &= \beta \frac{\partial^2 v_2}{\partial z^2}, v_3(0, \theta) = 0 = v_3(L, \theta) \\
 \beta^n : \frac{\partial v_n}{\partial \theta} &= \beta \frac{\partial^2 v_{n-1}}{\partial z^2}, v_n(0, \theta) = 0 = v_n(L, \theta)
 \end{aligned} \tag{23}$$

On solving the system of Eq. (23) using Mathematica 5.2

$$v_0 = u_0 = \sin \frac{2\pi z}{L} \cos \pi^2 \theta$$

$$\begin{aligned}
 \frac{\partial v_1}{\partial \theta} &= -\frac{4\beta\pi^2}{L^2} v_0 \Rightarrow v_1 = -\frac{\beta \sin \left[\frac{2\pi z}{L} \right] \sin [\pi^2 \theta]}{L^2} + \sin \left[\frac{\pi z}{L} \right] \\
 \frac{\partial v_2}{\partial \theta} &= -\frac{4\beta\pi^2}{L^2} v_1 \Rightarrow v_2 = -\frac{\beta(L^2 \pi^2 \theta + \alpha \cos [\pi^2 \theta]) \sin \left[\frac{2\pi z}{L} \right]}{L^4} + \frac{L^4 \sin \left[\frac{2\pi z}{L} \right] + \beta^2 \sin \left[\frac{2\pi z}{L} \right]}{L^4} \\
 \frac{\partial v_3}{\partial \theta} &= -\frac{4\beta\pi^2}{L^2} v_2 \Rightarrow v_3 = \frac{\beta(\pi^2 \theta(-2L^4 + L^2 \pi^2 \beta \theta - 2\beta^2) + 2\beta^2 \sin [\pi^2 \theta]) \sin \left[\frac{2\pi z}{L} \right]}{2L^6} + \sin \left[\frac{2\pi z}{L} \right] \\
 \frac{\partial v_4}{\partial \theta} &= -\frac{4\beta\pi^2}{L^2} v_3 \Rightarrow v_4 \\
 &= \frac{1}{6L^8} \left(\beta(\pi^2 \theta(-6L^6 + 3L^4 \pi^2 \beta \theta - L^2 \pi^4 \theta^2 \beta^2 + 3\beta^3 \pi^2 \theta)) 6\beta^3 \cos [\pi^2 \theta] \right) \sin \left[\frac{2\pi z}{L} \right] \\
 &\quad + \frac{L^8 \sin \left[\frac{2\pi z}{L} \right] - \beta^4 \sin \left[\frac{2\pi z}{L} \right]}{L^8} \\
 \frac{\partial v_5}{\partial \theta} &= -\frac{4\beta\pi^2}{L^2} v_4 \Rightarrow v_5 \\
 &= \frac{-1}{24L^{10}} \left(\beta(\pi^2 \theta(24L^8 - 12L^6 \pi^2 \beta \theta + 4L^4 \pi^4 \theta^2 \beta^2 - L^2 \pi^6 \theta^3 \beta^3 + 4(-6 + \pi^4 \theta^2) \beta^4) \right. \\
 &\quad \left. + 24\beta^4 \sin [\pi^2 \theta]) \sin \left[\frac{2\pi z}{L} \right] \right) + \sin \left[\frac{2\pi z}{L} \right] \\
 \frac{\partial v_6}{\partial \theta} &= -\frac{\beta\pi^2}{L^2} v_5 \Rightarrow v_6 = \frac{-1}{120L^{12}} \left(\beta(\pi^2 \theta(120L^{10} - 60L^8 \pi^2 \beta \theta + 20L^6 \pi^4 \theta^2 \beta^2 \right. \\
 &\quad \left. - 5L^4 \pi^6 \theta^3 \beta^3 + L^2 \pi^8 \theta^4 \beta^4 - 5\pi^2 \theta(-12 + \pi^4 \theta^2) \beta^5) \right. \\
 &\quad \left. + 120\beta^5 \cos [\pi^2 \theta]) \sin \left[\frac{2\pi z}{L} \right] \right) \\
 &\quad + \frac{L^{12} \sin \left[\frac{2\pi z}{L} \right] + \beta^6 \sin \left[\frac{2\pi z}{L} \right]}{L^{12}}
 \end{aligned} \tag{24}$$

and so on ...

The approximate solution of (1) by setting $\beta = 1$ in (23) is given by

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + v_3 + v_4 + \dots \tag{25}$$

On substituting values of v_i 's in Eq. (25), solution is obtained in terms of a summation of infinite series which gives results near to the exact solution.

Now we will solve the Eq. (18) using NHPM. First of all, following homotopy is constructed for solving heat conduction equation using NHPM

$$(1-p)\left(\frac{\partial T}{\partial \theta} - U_0\right) + p\left(\frac{\partial T}{\partial \theta} - \beta \frac{\partial^2 T}{\partial z^2}\right) = 0 \quad (26)$$

Taking $L^{-1} = \int_{\theta_0}^{\theta} (\cdot) d\theta$ i.e. inverse operator on Eq. (26), then

$$T(z, \theta) = \int_0^{\theta} U_0(z, \theta) d\theta - p \int_0^{\theta} \left(U_0 - \beta \frac{\partial^2 T}{\partial z^2} \right) d\theta + T(z, 0). \quad (27)$$

Let the solution of the (27) is

$$T = T_0 + pT_1 + p^2T_2 + p^3T_3 + \dots, \quad (28)$$

where, T_0, T_1, T_2, \dots are to be determined.

Suppose Eq. (25) is the solution of Eq. (24). Comparing the coefficients of powers of p and equating to zero and using Eq. (25) in Eq. (24), following are obtained:

$$\begin{aligned} p^0 : T_0(z, \theta) &= \int_0^{\theta} U_0(z, \theta) d\theta + T(z, 0) \\ p^1 : T_1(z, \theta) &= - \int_0^{\theta} \left(U_0(z, \theta) - \beta \frac{\partial^2 T_0}{\partial z^2} \right) d\theta \\ p^2 : T_2(z, \theta) &= \int_0^{\theta} \left(\beta \frac{\partial^2 T_1}{\partial z^2} \right) d\theta \\ p^3 : T_3(z, \theta) &= \int_0^{\theta} \left(\beta \frac{\partial^2 T_2}{\partial z^2} \right) d\theta \\ &\text{and so on.} \end{aligned} \quad (29)$$

Consider initial approximation of Eq. (18) as

$$U_0(z, \theta) = \sum_{n=0}^{\infty} c_n(z) P_n(\theta), T(z, 0) = U(z, 0), P_k(\theta) = \theta^k, \quad (30)$$

where, $P_1(\theta), P_2(\theta), P_3(\theta), \dots$ and $c_0(z), c_1(z), c_2(z), \dots$ are specified functions and unknown coefficients respectively, depending on the problem.

Using Eq. (30) in (29), following are obtained:

$$\begin{aligned} T_0(z, \theta) &= \left(c_0(z)\theta + c_1(z)\frac{\theta^2}{2} + c_2(z)\frac{\theta^3}{3} + c_3(z)\frac{\theta^4}{4} + \dots \right) + \sin \frac{2\pi z}{L} \\ T_1(z, \theta) &= \left(-c_0(z) - \frac{4\beta\pi^2}{L} \sin \frac{2\pi z}{L} \right) \theta + \left(-\frac{1}{2}c_1(z) + \frac{1}{2}\beta c_0''(z) \right) \theta^2 \\ &\quad + \left(-\frac{1}{3}c_2(z) + \frac{1}{3}c_1''(z) \right) \theta^3 + \dots \\ &\text{and so on ...} \end{aligned} \quad (31)$$

Now solving the above equations in such a manner that, $T_1(z, \theta) = 0$.
Therefore Eq. (31) reduces to

$$\begin{aligned}
 c_0(z) &= -\frac{2^2 \beta \pi^2}{L} \sin \frac{2\pi z}{L} \\
 c_1(z) &= \frac{2^4 \beta^2 \pi^4}{L^2} \sin \frac{2\pi z}{L} \\
 c_2(z) &= -\frac{2^6 \beta^3 \pi^6}{L^3} \sin \frac{2\pi z}{L} \\
 U(z, \theta) &= T_0(z, \theta) = \sin \frac{2\pi z}{L} + c_0(z)\theta + c_1(z)\frac{\theta^2}{2} + c_2(z)\frac{\theta^3}{3} + c_3(z)\frac{\theta^4}{4} + \dots \\
 &= \sin \frac{2\pi z}{L} \left[1 - \frac{2^2 \beta \pi^2}{L} \theta + \frac{2^4 \beta^2 \pi^4}{L^2} \frac{\theta^2}{2} - \frac{2^6 \beta^3 \pi^6}{L^3} \frac{\theta^3}{3} + \dots \right] = \sin \frac{2\pi z}{L} e^{-\frac{4\beta \pi^2}{L} \theta}
 \end{aligned}
 \tag{32}$$

which is same as the universally accepted exact solution for the problem which is shown in **Figure 1**.

The solution of one-dimensional heat conduction equation is solved using HPM and NHPM and then compared with the universally accepted exact solution obtained from method of separation of variable. **Figure 2** represents the comparison of solution of heat equation using HPM, NHPM and method of separation of variable. It is found that the solution obtained using HPM gives result near to the exact solution whereas solution using NHPM gives same results as the exact solution.

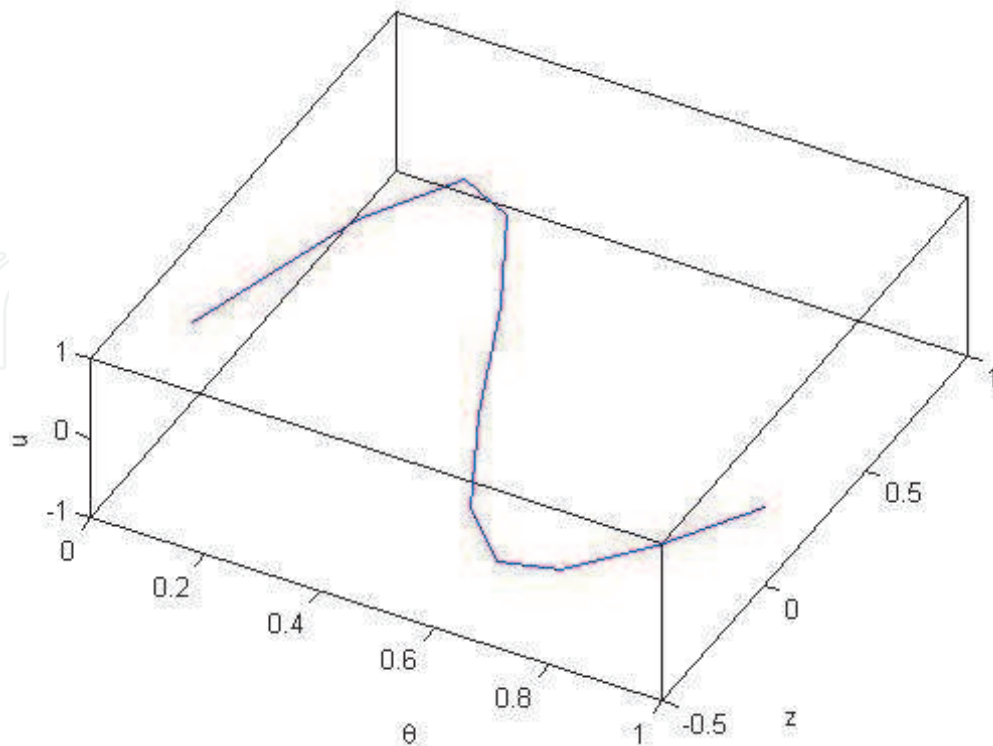


Figure 1.
Solution using NHPM.

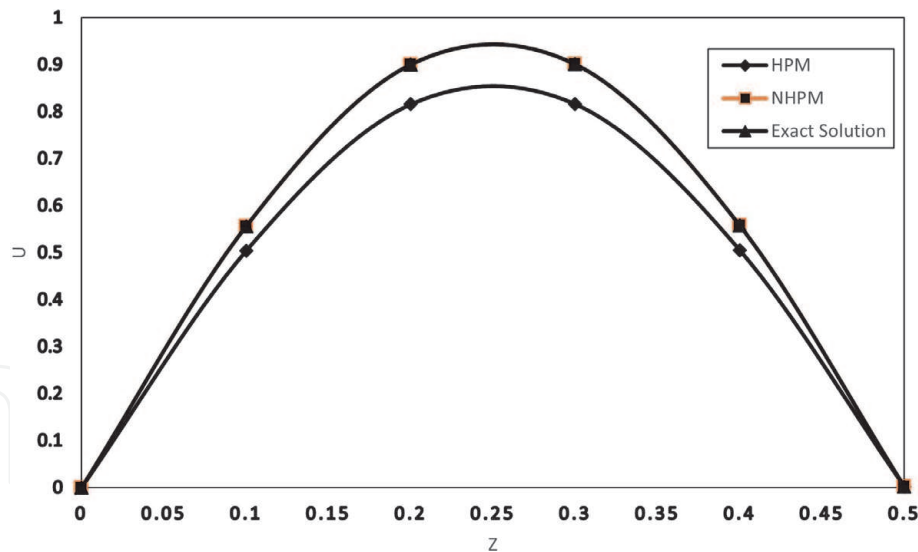


Figure 2.
Comparison of HPM, NHPM and the exact solution.

6. Conclusion

The analytical approximate solutions of one-dimensional heat conduction equation are obtained by applying new homotopy perturbation method and new homotopy perturbation method. It is found that new homotopy perturbation method (NHPM) converges very rapidly as compared to homotopy perturbation method (HPM) and other traditional methods. The exact solutions are obtained up to more accuracy using NHPM. An infinite convergent series solution for particular initial conditions are obtained using these methods which shows the effectiveness and efficiency of NHPM and HPM. The convergence rate of NHPM is much faster than traditional methods which directly indicates that this method is better than other methods. The solution of heat equation obtained by homotopy perturbation method and new homotopy perturbation method are exactly same and very close to the solution obtained by universally accepted and tested analytical method of separation of variables. If the initial guess in homotopy perturbation method is effective and properly chosen which satisfy boundary and initial condition, homotopy perturbation method provides solution with rapid convergence. It is illustrated that NHPM is very prominent, when accuracy has a vital role to play. The numerical results also reflect the remarkable applicability of NHPM to linear and non-linear initial and boundary value problems. NHPM provides the rapid convergence of the series solution for linear as well as non-linear problems with less computational work.

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