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Free-surface flow behind elastic plate impacting on a thin liquid layer

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Abstract. The problem of an inclined impact by an elastic plate on a thin liquid layer is considered. Evolution of the flow behind the plate is studied. The main input parameters are the position of the separation point of the liquid from the plate and the speed of the liquid under this point. The flow in the wake behind the plate is described by shallow water equations without gravity. Analytical formulae for the shape of the free surface behind the plate are derived. The study is focused on the possibility of the formation of jets arising from the wake perpendicular to the liquid layer. The problem is solved in two stages: before and after the formation of the first such a jet. The effects of the flow speed at the beginning of the wake, its time derivative, and the law of motion of the separation point on the formation of jets are investigated. The positions of the jets, their speeds and shapes are determined. Using the obtained results, mechanisms of the thin layer aeration behind the plate are discussed.

1. Introduction

This study is motivated by the experiments on droplet deposition in annular gas-liquid flow and mass exchange between the gas core and the liquid film [1]. Bubbles entrapped in the liquid film were observed for some conditions of oblique droplet impacts onto the film. The bubbles were created during the impacts. Then, some of the bubbles collapsed behind the impacting droplet, but others survived inside the liquid film.

This work is a continuation of the paper [2], where the nonlinear non-stationary two-dimensional problem of oblique impact by an elastic plate on a thin liquid layer was considered. The theory of impact on a thin liquid layer was used [3]. The conditions of matching the flows under the impacting plate and outside of the plate accounted for spray jet at the leading edge of the wetted part of the plate and the flow separation from the plate at the trailing edge of wetted area. The position of the separation point was determined by the Brillouin-Villat condition, see [4, 5]. The study in [2] was focused on the plate dynamics. It was shown that the air can be trapped in front of the plate as the result of elastic deformations of the plate and the formation of a jet at the leading edge of the wetted part of the body and under the plate due to its elastic vibrations. The liquid in front of the moving plate was at rest. The influence of the flow in the wake behind the plate on the plate motion was neglected approximately for high horizontal speeds of the plate.
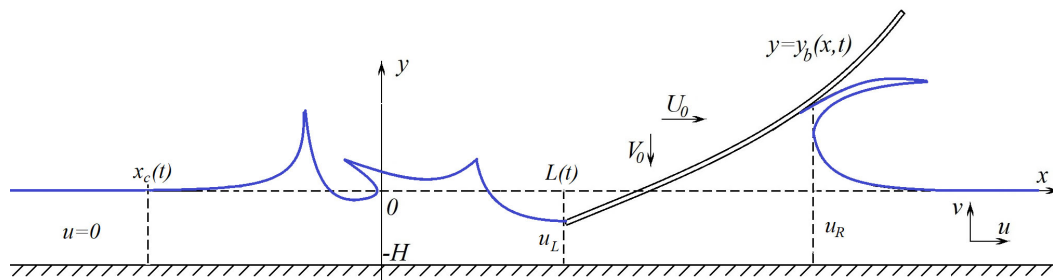
Figure 1. Sketch of the problem.

In the present paper, the study is focused on the flow behind the impacting plate. The flow is governed by both the motion of the separation point of the liquid from the plate surface and the liquid velocity under this point. Analytical formulae for the shape of the free surface behind the plate are derived. Major attention is paid to the possibility of the formation of jets arising from the wake perpendicular to the liquid layer. Similar problems were investigated in [6, 7].

2. Formulation of the problem

The problem of two-dimensional oblique impact of a thin elastic plate on a thin liquid layer is considered. The dynamics of the free surface of the liquid behind the plate after the impact is investigated. The liquid is inviscid and incompressible with constant density ρ_l. At equilibrium, the fluid is of constant depth h, $-h < y < 0$, see Figure 1. In the x-direction, the liquid is unbounded. Here (x, y) are the Cartesian coordinates. Initially, $t = 0$, the left edge of the plate touches the liquid surface at a single point $x = 0$, $y = 0$. Then the plate penetrates the liquid layer and moves along it at the same time. The position of the separation point of the liquid from the plate has coordinates $(L(t), -s(t))$, where $L(t)$ and $s(t)$ were calculated in [2]. The problem is formulated within the shallow water theory and is described by the corresponding equations with zero gravity, see [2] for the details of modelling:

$$\frac{\partial \widetilde{\eta}}{\partial t} + \frac{\partial (\widetilde{\eta} u)}{\partial x} = 0 \quad (x_c(t) < x < L(t)), \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0 \quad (x_c(t) < x < L(t)), \tag{2}$$

where $\widetilde{\eta}(x, t)$ is the liquid depth, $u(x, t)$ is a speed of the flow in the x-direction, $x_c(t)$ is the leftmost point of the perturbed part of the liquid layer. On the right boundary of the wake region, $x = L(t)$, the functions $u(x, t)$ and $\widetilde{\eta}(x, t)$ are given

$$u(L(t), t) = u_L(t), \quad \widetilde{\eta}(L(t), t) = h - s(t), \tag{3}$$

where $L(t)$, $u_L(t)$ and $s(t)$ were calculated in [2]. Initially, $\tilde{\eta}(x, 0) = h$, $u(x, 0) = 0$, $x_c(t) = 0$, $L(0) = 0$ at $t = 0$.

3. Method of the solution

The problem (1)–(3) is solved by using a Lagrangian variable τ ($0 < \tau < t$), which corresponds to the time instant when a liquid particle separates from the plate surface. Then, in the Lagrangian variables, the solution to the problem (2)–(3) is

$$x(\tau, t) = L(\tau) + u_L(\tau)(t - \tau), \quad u[x(\tau, t), t] = u_L(\tau). \tag{4}$$

To determine the shape of the free surface, we use equation (1),

$$\frac{\partial \eta}{\partial t} + \frac{\partial[(\eta + h)u]}{\partial x} = 0 \quad (x_c(t) < x < L(t)), \tag{5}$$

where $\eta(x,t) = \tilde{\eta}(x,t) - h$ and $\eta(x,t) = \eta[x(\tau,t),t] = H(\tau,t)$. Equation (5) in the Lagrangian variables has the form

$$\frac{dH}{dt} = -(h + H)\frac{u'_L(\tau)}{L'(\tau) - u_L + u'_L(\tau)(t - \tau)} \quad (t > \tau), \tag{6}$$

where the following equations were used

$$u'_L(\tau) = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \tau}, \quad \frac{\partial u}{\partial x} = \frac{u'_L(\tau)}{x'_\tau(\tau,t)}, \quad \frac{\partial x}{\partial \tau} = L'(\tau) + u'_L(\tau)(t - \tau) - u_L(\tau).$$

The solution of the ordinary differential equation (6) subject to the initial conditions $H(\tau,\tau) = -s(\tau)$ is

$$H(\tau,t) = \frac{(h - s(\tau))|L'(\tau) - u_L(\tau)|}{|L'(\tau) - u_L(\tau) + u'_L(\tau)(t - \tau)|} - h. \tag{7}$$

The derivatives $L'(t)$ and $u'_L(t)$ in (7) are calculated numerically. The solution (4) and (7) describes smooth flow in the wake of the impacting plate if the functions $L(t)$, $s(t)$, $u_L(t)$ are smooth and the denominator in (7) is not equal to zero. Here we are interested in cases where the denominator is equal to zero,

$$L'(\tau) - u_L(\tau) + u'_L(\tau)(t - \tau) = 0, \tag{8}$$

which implies unbounded growth of the wake thickness within the thin liquid layer approximation. Let τ_*, t_* be values of the Lagrangian variable and time at which the liquid layer thickness H is unbounded. At $x_* = x(\tau_*, t_*)$ this approximation does not work and more complex models of the flow should be used near $x = x_*$ and close to $t = t_*$. A possible way of resolving this singularity is to introduce a jet at $x = x_*$, which is perpendicular to the liquid layer [8, 9]. We shall determine first the conditions when the flow singularity in the wake occurs.

We consider motion of the plate in the positive x-direction, $L'(\tau) > 0$, while the speed of liquid outflow from under the plate is negative, $u_L(\tau) < 0$. At the initial moment, the liquid velocity is zero, $u_L(0) = 0$. First, we shall find the conditions, under which the singularity occurs at the impact point, $x = 0$. In this case $\tau_* = 0$ and $L'(0) + u'_L(0)t_* = 0$. There are following subcases:

1.1. $L'(0) = 0$, $u'_L(0) \neq 0$, then the singularity occurs at the initial time of the impact, $\tau_* = 0$, $t_* = 0$. This is the case of vertical impact with non-zero speed.

1.2. $L'(0) = 0$, $u'_L(0) = 0$, then both the numerator and the denominator in (7) equal to 0 and further study is required. This is the case of vertical impact with zero initial speed of the plate.

1.3. $L'(0) \cdot u'_L(0) < 0$, then the singularity occurs at $\tau_* = 0$, $t_* = -L'(0)/u'_L(0)$. This is the case of oblique impact.

Now we consider the liquid particles which flow out from under the plate after the impact instant, $\tau > 0$. There are following subcases for these particles:

2.1. $u'_L(\tau) > 0$, this is the decelerating outflow. In this case $L'(\tau) - u_L(\tau) + u'_L(\tau)(t - \tau) > 0$ and the flow in the wake is smooth.

2.2. $u'_L(\tau) = 0$, which means the liquid flows out at a constant speed. Then $L'(\tau) - u_L(\tau) > 0$ and wake is smooth.

2.3. $u'_L(\tau) < 0$, this is the liquid outflow accelerates. Such flow occurs, in particular, at the initial stage of the impact [2]. In this case, the equation (8) provides

$$t_*(\tau) = -\frac{L'(\tau) - u_L(\tau)}{u'_L(\tau)} + \tau. \qquad (9)$$

We need to find τ_*, at which $t_*(\tau)$ is minimum. Then $t = t_*(\tau_*)$ is the time of the first jet formation from the wake. The initial position of the jet, $x_{*0} = x(\tau_*, t_*)$, is obtained by the method of characteristics. The characteristics, $x = L(t) + u_L(\tau)(t - \tau)$, are straight lines in the (x, t) plane, where τ is a parameter, $0 < \tau < t$. The characteristics of the model problem with the following specified functions, which mimic the functions computed in [2],

$$L(t) = Ut, \qquad u_L(t) = U[e^{(-1/[1-t^2])} - e^{-1}] \quad (t < 1), \qquad u_L(t) = -Ue^{-1} \quad (t \ge 1), \qquad (10)$$

are shown in Figure 2 for $U = 1\text{m/s}$ and $t = 2$ s (a), $t = 3$ s (b). The thick line corresponds to the motion of the separation point. The characteristics are shown by thin lines. In this case $t_* \approx 2.3$ s, $\tau_* \approx 0.7$ s. The red dot shows the position of the formation of the jet. Then the flow velocity is discontinuous starting from $t = t_*$ and $x = x(\tau_*, t_*)$ with the discontinuity, $x = x_*(t)$, moving in such a way that the relative flow velocity on the right of the discontinuity, $x'_*(t) - u[x_*(t) + 0, t]$, is equal to the relative flow velocity on the left of the discontinuity, $x'_*(t) - u[x_*(t) - 0, t]$, but with opposite sign [9]. In this case, the liquid enters the jet from both sides with equal speeds. There are two characteristics coming at the discontinuity with $\tau = \tau^+$ and $\tau = \tau^-$ from the right and from the left correspondingly. The functions $\tau^\pm(t)$ are defined by

$$x_*(t) = L(\tau^\pm(t)) + u_L(\tau^\pm(t))(t - \tau^\pm(t)) \qquad (11)$$

which follows from (4). Correspondingly,

$$u(x_*(t) \pm 0, t) = u_L(\tau^\pm(t), t). \qquad (12)$$

The obtained relations provide the system of the three ordinary differential equations with respect to $x_*(t)$, $\tau^+(t)$ and $\tau^-(t)$,

$$\frac{dx_*}{dt} = \frac{u_L(\tau^+) + u_L(\tau^-)}{2}, \qquad \frac{d\tau^+}{dt} = \frac{u_L(\tau^-) - u_L(\tau^+)}{2D(\tau^+, t)}, \qquad \frac{d\tau^-}{dt} = \frac{u_L(\tau^+) - u_L(\tau^-)}{2D(\tau^-, t)}, \qquad (13)$$

where $t > t_*$, $D(\tau, t) = L'(\tau) + u'_L(\tau)(t - \tau) - u_L(\tau)$, and

$$x_*(t_*) = x_{*0}, \qquad \tau^+(t_*) = \tau_*, \qquad \tau^-(t_*) = \tau_*. \qquad (14)$$

The shape of the free surface is calculated using the formula (7) on the right and on the left from the discontinuity.

Equations (13) are integrated numerically using the Runge–Kutta method and the initial asymptotic behaviour of the solution for small $(t - t_*)$.

4. Numerical results and discussion
The evolution of the free surface of the wake is investigated numerically for the case when the initial velocity of outflow is zero, $u_L(0) = 0$, and the first jet occurs at the initial point of impact, $\tau_* = 0$. The functions $L(t)$, $u_L(t)$ and $s(t)$ are obtained numerically by the method of [2] for $t \in [0, 9.44 \cdot 10^{-4}]$ with the step $\Delta t = 8 \cdot 10^{-7}$ s for the following impact conditions: plate length is 10 cm, liquid thickness is 2 cm, horizontal speed of the plate is 5 m/s, initial vertical speed of the plate is 5 m/s, density of the plate material is 2670 kg/m^3, plate thickness is 2 mm, Young
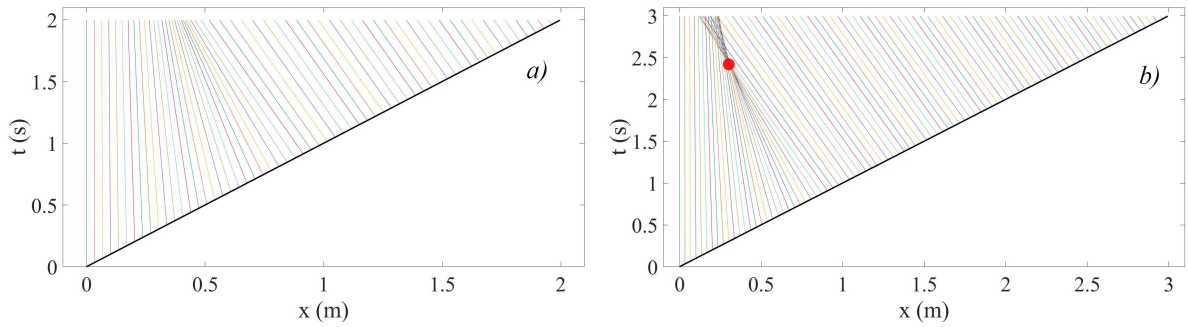
Figure 2. Characteristics of the equation (2) for the model case (10).
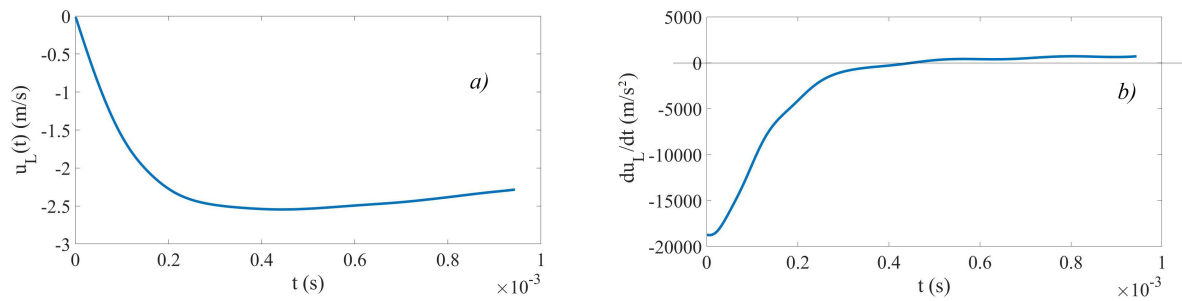

Figure 3. The velocity of the liquid outflow from under the plate $u_L(t)$ (a) and its time-derivative du_L/dt (b).

modulus of the plate material is $68 \cdot 10^6$ Pa and the initial angle of inclination of the plate is 3°. Functions $u_L(t)$ and $u'_L(t)$ are shown in Figure 4 in the dimensional variables.

The jet starts at $x = 0$ and $t = 3.5 \cdot 10^{-4}$ s. Then the jet root moves in the negative x-direction. The jet-root position $x_*(t)$ in the case under consideration coincides with the left boundary of the perturbed area $x_c(t)$. Hence $u_L(\tau^-) = 0$. In general, a jet occurs only when $du_L/dt < 0$. While du_L/dt does not change sign, see Figure 4 (b), we can expect the formation of only one jet. If du_L/dt changes sign from $-$ to $+$ then the flow slows down and the solution is regular. The shape of the free surface in the wake behind the plate before the formation of the jet is shown in Figure 4 (a). The right points of the lines correspond to the separation points of the liquid from the plate. Jet shapes at different times after the formation of the jet are shown in Figure 4 (b). Here the left points of the lines correspond to the jet root.
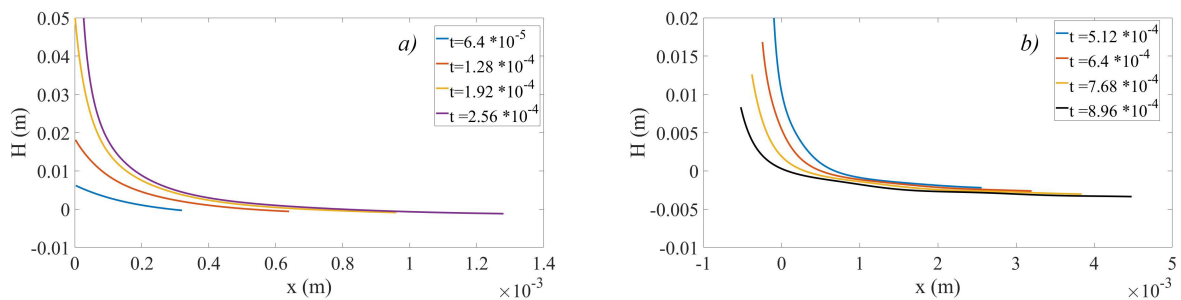

Figure 4. The shape of the free surface before (a) and after (b) the formation of the jet.

The resulting form of the jet is calculated. At each time instant, a liquid particle moves upward from the jet root at speed equal to the jet root speed. The particle moves inertially with gravity being the only force acting on it. The resulting jet makes a cavity, the shape of which at $t = 9.5 \cdot 10^{-4}$ s is shown in Figure 4. Note that, for the considered impact conditions, the formation of the jet and the shown elevation of the layer free surface occur in a short period of time. Therefore, the height of the outflow of liquid particles from the wake is relatively small. Jet thickness is not shown in this figure. These calculations explain a possible mechanism for the formation of small bubbles in the wake and aeration of a thin liquid layer during an oblique impact of an elastic body on it.
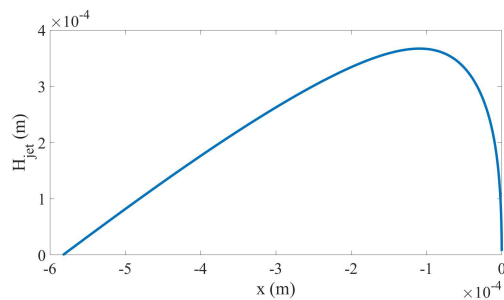

Figure 5. The cavity formed by the jet at $t = 9.5 \cdot 10^{-4}$ s.

5. Conclusion

The shape of the free surface in the wake behind an elastic plate and the conditions of the formation of jets from the wake surface are determined. It is shown that the formation of jets is primarily affected by the velocity of the liquid at the beginning of the wake, its time derivative and the motion of the separation point.

It is shown that a vertical jet occurs in the wake at the place of intersection of the characteristics of the equation for the flow velocity within the shallow water theory without gravity. The condition of the intersection is a negative value of the derivative of the liquid velocity at the separation point. The number of time segments with negatives value of this derivative is equal to the number of jets in the wake.

For all the considered parameters of the problem with a zero initial velocity of liquid outflow from under the plate it was found that the first jet always occurs at the initial point of the impact in a very short time after the impact.

Acknowledgments

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