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Neighbour Balanced Designs for Diallel Cross Experiments

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SUMMARY

This paper deals with obtaining series of block designs for complete and partial diallel cross experiments balanced for neighbour effects. Two series of Neighbour Balanced Block (NBB) designs for diallel cross in complete blocks and two series of NBB for diallel cross in incomplete blocks have been obtained. A catalogue of designs for number of lines ≤ 20 has been prepared listing the parameters of the designs.

Keywords: Diallel cross, Neighbour effects, General combining ability, Specific combining ability, Variance balanced.

1. INTRODUCTION

The main purpose of plant breeding programmes is to sttudy genetic properties of inbred lines involved in the crosses in order to develop improved crop varieties. When there are a number of strains of a crop, one type of investigation consists of crossing between such strains to evolve new varieties. Through such type of investigation, combining abilities of the strains can be studied. Complete Diallel Cross (CDC) is a set of all possible matings between genotypes which may be individuals, clones, homozygous lines, etc. When there are large numbers of lines, the number of crosses becomes large and sometimes unmanageable. In such situations, it is necessary to choose a suitable fraction of all possible crosses so that the effect of all strains can be estimated with this incomplete number of crosses, this fraction of crosses are said to constitute Partial Diallel Crosses (PDC).

A comparison of the performances of different lines for hybridization work can be best made on the basis of the concepts of the two combining abilities *i.e.* general combining ability (gca) and specific combining ability (sca). The gca of an inbreed line is defined as the average performance of the hybrids which this line produces with other lines chosen from a random mating population and sca refers to the effects due to a pair of inbred lines involved in the cross. Diallel crossing as a means of comparing the breeding values of parents are used in estimating gca and sca of inbred lines involved in the crosses. A lot of work is available in literature on the theory and analysis of diallel crosses [Hayman (1954a, 1954b, 1958, 1960), Griffing (1956a, b) and Kempthorne (1956)]. Experimental design issues in diallel cross experiments has received considerable attention in the literature [Gupta and Kageyama (1994), Parsad *et al*. (1999) and Parsad *et al*. (2005), Varghese *et al.* (2005), Varghese *et al.* (2015) and Varghese and Varghese (2017)].

Excluding reciprocal crosses and parental inbreeds, there are $N = n(n-1)/2$ possible single crosses among the set of n lines which are usually tested in a suitably replicated randomized design. It is also assumed in such trials that there is no overlap in the response between varieties (crosses) planted in nearby plots. This may not be case in practice. In the case of block designs, where the blocks are made up of plots, plots within a block cannot be sufficiently isolated from each other. For example, the branches of a tree may form plots while the tree serves as a block. In a fertilizer trial where plants in an unfertilized plot may

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rob a share of the plants in a nearby heavily fertilized plot. Inter-varietal competition may be an important factor in long and narrow plots without guards where yield of a variety may be depressed by more aggressive neighbouring varieties (Kempton, 1982). Neighbour effects may be caused by the differences in height, root vigour or germination date of the varieties.

Competition or interference between neighbouring units in field experiments can contribute to variability in experimental results and lead to substantial losses in efficiency. If the neighbour effect is significant, there will be substantial reduction in the residual sum of squares which will ensure more precise estimation of gca effects. Neighbour balanced designs, wherein the allocation of genotypes (crosses) is such that every genotype occurs equally often with every other genotype as neighbours, are used for these situations. Please see for reference Azais *et al.* (1993), Bailey (2003), Bailey and Druilhet (2004), Tomar *et al.* (2005), Jaggi *et al.* (2006), Pateria *et al.* (2007), Jaggi *et al.* (2007), Pateria *et al.* (2009), Abeynayake and Jaggi (2009), Varghese *et al.* (2011), Varghese *et al.* (2014), Jaggi *et al.* (2015) and Jaggi *et al.* (2017).

In this paper, an attempt has been made to obtain methods of constructing designs for diallel cross experiments balanced for neighbour effects resulting in more precise estimation of gca effects besides estimating the effects from the neighbouring crosses.

2. EXPERIMENTAL SETUP AND MODEL FOR DIALLEL CROSS INCORPORATING ONE-SIDED NEIGHBOUR EFFECT

For p lines with $v=\frac{p(p-1)}{2}$ distinct (F_1) crosses (Griffing IV model), following is the general linear model considered for diallel cross with one-sided neighbour effect:

$$Y_{ij} = \mu + \tau_{[i,j]} + \tau_{[i-1,j]} + \beta_j + e_{ij} \quad \ldots(1)$$

where Y_{ij} is the response from the i^{th} {i= 1,2, …, k_j (size of j^{th} block)} plot in the j^{th} {j = 1,2, …, b (number of blocks)} block, μ is the grand mean, $\tau_{[i,j]}$ is the effect of cross in the $(i, j)^{th}$ plot, $\tau_{[i-1,j]}$ is the effect of the cross appearing in the left neighbouring plot on cross in the $(i, j)^{th}$ plot assuming the plots within j^{th} block to be placed adjacent linearly with no gaps, β_j is the effect of j^{th} block and e_{kl} is iid $N(0,\sigma^2)$.

Consider $\tau_{[i,j]} = g_s + g_t$, $1 \le s, t \le p$, where g_s (g_t) denotes the gca effect of the s^{th} (t^{th}) line. Also it is assumed that

$$\sum_{s=1}^{p} g_s = 0$$

Further, sca effects are assumed to be negligible. The model can be written in matrix notation as:

$$\mathbf{Y} = \mu\mathbf{1} + \Delta'\boldsymbol{\tau} + \Delta_1'\boldsymbol{\tau}_1 + D'\beta + \mathbf{e},$$

where **Y** is the n × 1 vector of observations, Δ' is the design matrix of observations versus lines of order n × p, Δ_1' is the design matrix of observations versus crosses of order n × v, D' is the design matrix of observations versus blocks of order n × b, μ is the general mean, $\boldsymbol{\tau}$ is the vector of gca effects, $\boldsymbol{\tau}_1$ is the vector of neighbour effects, β is the vector of block effects and **e** is the vector of random errors.

The joint information matrix for direct effects of lines and neighbour effects of crosses is obtained as follows:

$$\mathbf{C} = \begin{bmatrix} \mathbf{R}_{\boldsymbol{\tau}} - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{M}_1 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_2' \\ \mathbf{M}_1' - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{R}_{\boldsymbol{\tau}_1} - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_2' \end{bmatrix} \quad \ldots(2)$$

Here, $\mathbf{R}_{\boldsymbol{\tau}} = \Delta\Delta'$ is the p×p diagonal matrix with diagonal entries as replications of lines, $\mathbf{R}_{\boldsymbol{\tau}_1} = \Delta_1\Delta_1'$ is the v×v diagonal matrix of replications of crosses, $\mathbf{M}_1 = \Delta\Delta_1'$ is p×v incidence matrix of lines vs. left neighbouring crosses, $\mathbf{N}_1 = \Delta D'$ is p×b incidence matrix of lines vs. blocks, $\mathbf{N}_2 = \Delta_1 D'$ is v×b incidence matrix of left neighbouring crosses vs. blocks. The (p+v) × (p+v) matrix **C** in equation (2) is symmetric, non-negative definite with zero row and column sums. From this joint information matrix, the information matrix for estimating gca effects can be obtained.

3. EXPERIMENTAL SETUP AND MODEL FOR DIALLEL CROSS INCORPORATING TWO-SIDED NEIGHBOUR EFFECTS

Following is the general block model considered for diallel cross with two-sided neighbour effects:

$$Y_{ij} = \mu + \tau_{[i,j]} + \tau_{[i-1,j]} + \tau_{[i+1,j]} + \beta_j + e_{ij} \quad \ldots(3)$$

where $\tau_{[i+1,j]}$ is the effect of the cross appearing in the right neighbouring plot on cross in the $(i, j)^{th}$ plot and all other terms are same as explained in previous section.

Following the same procedure as in Section 2, the joint information matrix (**C**) for estimating line effects and neighbours effects from left and right neighbouring crosses is obtained as follows:

$$\mathbf{C}=\begin{bmatrix}\mathbf{R}_{\tau}-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{M}_1-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_2-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_3' \\ \mathbf{M}_1'-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{R}_{\tau_1}-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_3-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_3' \\ \mathbf{M}_2'-\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{M}_3'-\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{R}_{\tau_2}-\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_3'\end{bmatrix} \quad \ldots(4)$$

where, besides the terms defined in Section 2, $\mathbf{M}_2$ is a p × v incidence matrix of lines vs. right neighbouring crosses, $\mathbf{M}_3$ is v × v incidence matrix of left versus right neighbouring crosses, $\mathbf{N}_3$ is v × b incidence matrix of right neighbouring crosses versus blocks, $\mathbf{R}_{\tau_2}$ is the v × v diagonal matrix of replications of right neighbouring crosses, The (p+2v) × (p+2v) matrix **C** is symmetric, non-negative definite with zero row and column sums. The information matrix for estimating the direct effects ($\mathbf{C}_\tau$) of lines is obtained from equation (4) as given below:

$$\mathbf{C}_{\tau}=\mathbf{C}_{11}-\mathbf{C}_{12}\mathbf{C}_{22}^{-}\mathbf{C}_{21}, \quad \ldots (5)$$

where

$\mathbf{C}_{11}=\mathbf{R}_{\tau}-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_1'$,

$\mathbf{C}_{12}=[\,\mathbf{M}_1-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_2' \quad \mathbf{M}_2-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_3'\,]$,

$\mathbf{C}_{21}=[\,\mathbf{M}_1'-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_1' \quad \mathbf{M}_2'-\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_1'\,]'$

and

$$\mathbf{C}_{22}=\begin{bmatrix}\mathbf{R}_{\tau_1}-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_3-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_3' \\ \mathbf{M}_3'-\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{R}_{\tau_2}-\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_3'\end{bmatrix}.$$

Neighbour effects in an experiment could be from one side or from both, left and right, sides of the block. Under block design setup, following are some of the definitions:

Definition 3.1: A block design for diallel crosses is said to be neighbour balanced if each line has all the crosses appearing as left and/or right neighbour equal (say λ) number of times.

Definition 3.2: A block design for diallel crosses incorporating neighbour effects is said to be variance balanced in terms of gca if all the elementary contrast pertaining to gca effects are estimated with same variance.

Definition 3.3: A block design for diallel crosses incorporating neighbour effects is said to be partially variance balanced in terms of gca if the variance of elementary contrast pertaining to gca effects are estimated with different variances. The variances depend on the association between the lines in the design.

4. NEIGHBOUR BALANCED COMPLETE BLOCK DESIGNS FOR DIALLEL CROSSES

Two series of Neighbour balanced block (NBB) designs for diallel cross in complete blocks are presented here in which the first series is for CDC plans named as NBB-CDC that results in a variance balanced design whereas second one is for PDC plans named as NBB-PDC that results in a partially variance balanced design.

Method 4.1: This method is based on the method of constructing cross over design given by Williams (1949). Let p be the number of lines and v = $\frac{p(p-1)}{2}$ are the number of crosses. Consider the number of crosses v to be even and represented as 1, 2, …, v. Construct an initial row as follows:

$$1, v, 2, v-1, \ldots, \frac{v}{2}, \frac{v}{2}+1$$

Generate this row to form v-1 more rows cyclically mod v as given by Williams (1949). Then arrange the v-1 rows according to the numbers in the first row such that the v × v array so obtained is symmetric. Replace the v symbols of the array by the corresponding v number of crosses and make the rows as circular by adding the border crosses. This will result in a NBB-CDC design with parameters p (number of lines), v = $\frac{p(p-1)}{2}$ (number of crosses), b = v (number of blocks), r = v (replications of the crosses), k = v (block size), λ = p – 1 (number of times each line has every cross appearing as left and right neighbour).

The information matrix of this design for estimating the gca effects of the p lines is obtained as

$$\mathbf{C} = \frac{p(p-1)(p-2)}{2}[\mathbf{I} - \frac{\mathbf{J}}{p}]$$

Example 4.1.1: Let p = 4, therefore v = $\frac{p(p-1)}{2}$ = 6. Obtain a William square of order 6

1	6	2	5	3	4
2	1	3	6	4	5
3	2	4	1	5	6
4	3	5	2	6	1
5	4	6	3	1	2
6	5	1	4	2	3

Leaving the first row, rearranging the remaining 5 rows in the order given in the first row, the following symmetric arrangement is obtained:

1	6	2	5	3	4
6	5	1	4	2	3
2	1	3	6	4	5
5	4	6	3	1	2
3	2	4	1	5	6
4	3	5	2	6	1

Replacing the 6 symbols of the array by the 6 crosses as follows:

1 × 2 – (1) 1 × 4 – (3) 2 × 4 – (5)

1 × 3 – (2) 2 × 3 – (4) 3 × 4 – (6)

Taking the border crosses, the resulting design is NBB-CDC design for 4 lines, 6 complete crosses in 6 complete blocks, each cross replicated 6 times and the number of times each line has every cross appearing as left and right neighbor is 3. The design obtained is as follows:

1× 2	1× 2	3× 4	1× 3	2× 4	1× 4	2× 3	2× 3
3× 4	3× 4	2× 4	1× 2	2× 3	1× 3	1× 4	1× 4
1× 3	1× 3	1× 2	1× 4	3× 4	2× 3	2× 4	2× 4
2× 4	2× 4	2× 3	3× 4	1× 4	1× 2	1× 3	1× 3
1× 4	1× 4	1× 3	2× 3	1× 2	2× 4	3× 4	3× 4
2× 3	2× 3	1× 4	2× 4	1× 3	3× 4	1× 2	1× 2

The information matrix **C** pertaining to gca effects in the presence of two-sided neighbour effects is of the form $\mathbf{C} = 12\mathbf{I} - 3\mathbf{J}$.

Method 4.2: Consider a Group Divisible (GD) association scheme for p = ml lines arranged in m rows of size l each. Obtain the distinct PDC plan from this association scheme by taking the crosses of a line with all the lines in the other rows i.e. the crosses between the second associates of GD scheme resulting in v = $\frac{\text{ml(ml}-\text{l)}}{2}$ crosses. Only even number of crosses are considered here.

Construct an initial row as given in Method 4.1 and follow the same procedure of obtaining the v × v array as in Method 4.1. Replace the v symbols of the array by the $\frac{\text{ml(ml}-\text{l)}}{2}$ crosses and make the rows as circular by adding the border crosses. This will result in a NBB-PDC design with parameters p = ml, v = $\frac{\text{ml(ml}-\text{l)}}{2}$, b = r = k = v. The resultant design will be partially variance balanced for gca effects following the GD association scheme with λ = l(m-1) (number of times each line has crosses formed from the second associates appearing as left and right neighbour).

Example 4.2.1: Let m = 3, l = 2 then p = 6 lines. The GD association scheme for p = 6 and the v = $\frac{\text{ml(ml}-\text{l)}}{2}$ = 12 crosses obtained using the above procedure is given as follows:

GD association scheme:

1	2
3	4
5	6

Crosses obtained :

1 × 3 - (1)	2 × 5 - (7)
1 × 4 - (2)	2 × 6 - (8)
1 × 5 - (3)	3 × 5 - (9)
1 × 6 - (4)	3 × 6 - (10)
2 × 3 - (5)	4 × 5 - (11)
2 × 4 - (6)	4 × 6 - (12)

The 12 × 12 array obtained using the initial row given in Method 4.1 and developing it mod 12 and rearranging is as follows:

1	12	2	11	3	10	4	9	5	8	6	7
12	11	1	10	2	9	3	8	4	7	5	6
2	1	3	12	4	11	5	10	6	9	7	8
11	10	12	9	1	8	2	7	3	6	4	5
3	2	4	1	5	12	6	11	7	10	8	9
10	9	11	8	12	7	1	6	2	5	3	4
4	3	5	2	6	1	7	12	8	11	9	10
9	8	10	7	11	6	12	5	1	4	2	3
5	4	6	3	7	2	8	1	9	12	10	11
8	7	9	6	10	5	11	4	12	3	1	2
6	5	7	4	8	3	9	2	10	1	11	12
7	6	8	5	9	4	10	3	11	2	12	1

Replacing the 12 symbols of the array by the 12 crosses and taking the border crosses, the resulting design is NBB-PDC design for 6 lines, 12 partial crosses in 12 complete blocks, each cross replicated 12 times with the number of times each line has crosses formed from the second associates appearing as left and right neighbour is 4.

The information matrix **C** pertaining to gca effects in the presence of two-sided neighbour effects is of the form

$$\begin{bmatrix} 32 & -16 & -4 & -4 & -4 & -4 \\ -16 & 32 & -4 & -4 & -4 & -4 \\ -4 & -4 & 32 & -16 & -4 & -4 \\ -4 & -4 & -16 & 32 & -4 & -4 \\ -4 & -4 & -4 & -4 & 32 & -16 \\ -4 & -4 & -4 & -4 & -16 & 32 \end{bmatrix}$$

It is clearly seen that all the diagonal entries are same and off-diagonal are of two types depending on the GD association scheme, hence the design obtained is partially variance balanced with respect to gca in the presence of neighbour effects.

NBB-PDC obtained by using GD association schemes has the flexibility in terms of the number of crosses. But the reduction in number of crosses for PDC is not very significant. This can be resolved by taking triangular association scheme.

Consider a triangular association scheme for $p = \frac{n(n-1)}{2}$, where $n \geq 5$. Obtain the distinct PDC plan from this association scheme by taking the crosses among rows and columns, i.e. with first associates resulting in $v = \frac{n(n-1)(n-2)}{2}$ crosses or PDC plan.

Follow the same procedure of obtaining the $v \times v$ array as in Method 4.1. Replace the v symbols of the array by the $\frac{n(n-1)(n-2)}{2}$ crosses and make the rows as circular by adding the border crosses. This will result in a NBB-PDC design with parameters $p = \frac{n(n-1)}{2}$, $v = \frac{n(n-1)(n-2)}{2}$ $b = r = k = v$. The resultant design will be partially variance balanced for gca effects following the triangular association scheme with $\lambda = 2(n-2)$ (number of times each line has crosses formed from the first associate appearing as left and right neighbour).

Example 4.2.2: Let n = 5, so p = 10 lines and the number of crosses formed will be $v = \frac{n(n-1)(n-2)}{2} = 30$. These 30 crosses can be obtained from the following arrangement of triangular association scheme:

$$\begin{bmatrix} * & 1 & 2 & 3 & 4 \\ 1 & * & 5 & 6 & 7 \\ 2 & 5 & * & 8 & 9 \\ 3 & 6 & 8 & * & 10 \\ 4 & 7 & 9 & 10 & * \end{bmatrix}$$

Since numbers in the same row and same column are first associates and remaining are second associates, so the PDC plan is obtained by crossing first associates and the crosses obtained are as follows:

1 × 2 - (1)	2 × 9 - (11)	5 × 8 - (21)
1 × 3 - (2)	3 × 4 - (12)	5 × 9 - (22)
1 × 4 - (3)	3 × 6 - (13)	6 × 7 - (23)
1 × 5 - (4)	3 × 8 - (14)	6 × 8 - (24)
1 × 6 - (5)	3 × 10 - (15)	6 × 10 - (25)
1 × 7 - (6)	4 × 7 - (16)	7 × 9 - (26)
2 × 3 - (7)	4 × 9 - (17)	7 × 10 - (27)
2 × 4 - (8)	4 × 10 - (18)	8 × 9 - (28)
2 × 5 - (9)	5 × 6 - (19)	8 × 10 - (29)
2 × 8 - (10)	5 × 7 - (20)	9 × 10 - (30)

These 30 PDC plans can be arranged in a 30 × 30 array obtained using the same procedure as in Method 4.1 which will result in a NBB-PDC design with parameters p = 10, v = 30, b = r = k = 30. The resultant

design will be partially variance balanced for gca effects following the triangular association scheme with $\lambda = 6$.

The information matrix **C** pertaining to gca effects in the presence of two sided neighbour effects for this design is of the form

$$\begin{bmatrix} 144 & -6 & -6 & -6 & -6 & -6 & -6 & -36 & -36 & -36 \\ -6 & 144 & -6 & -6 & -6 & -36 & -36 & -6 & -6 & -36 \\ -6 & -6 & 144 & -6 & -36 & -36 & -36 & -6 & -36 & -6 \\ -6 & -6 & -6 & 144 & -36 & -36 & -6 & -36 & -6 & -6 \\ -6 & -6 & -36 & -36 & 144 & -6 & -6 & -6 & -6 & -36 \\ -6 & -36 & -6 & -36 & -6 & 144 & -6 & -6 & -36 & -6 \\ -6 & -36 & -36 & -6 & -6 & -6 & 144 & -36 & -6 & -6 \\ -36 & -6 & -6 & -36 & -6 & -6 & -36 & 144 & -6 & -6 \\ -36 & -6 & -36 & -6 & -6 & -36 & -6 & -6 & 144 & -6 \\ -36 & -36 & -6 & -6 & -36 & -6 & -6 & -6 & -6 & 144 \end{bmatrix}$$

It is seen that the design obtained is partially variance balanced with respect to gca in the presence of neighbour effects following triangular association scheme.

Remark 4.1.1: For n = 4, the procedure using triangular association scheme reduces to group divisible association scheme.

5. NEIGHBOUR BALANCED INCOMPLETE BLOCK DESIGNS FOR DIALLEL CROSSES

Two series of NBB designs for diallel cross in incomplete blocks are presented here in which the first series is for NBB-CDC design that results in a variance balanced design whereas second one is for NBB-PDC design that results in a partially variance balanced design.

Method 5.1: This method is based on the method given by Azais (1987). Let p be the number of lines represented by 1, 2,…,p (p being prime). Obtain a vector of ones of order (p-1). Then an array of (p-1) × p is obtained by cyclically adding the row number to the previous entry in each row by taking mod p.

Generate another set of (p-1) × p array by rearranging the first array with first element starting from the middle entry of each row and cyclically generating the remaining elements of the each row by the above mentioned method. Cross the entries of both the arrays cell-wise resulting in $v = \frac{p(p-1)}{2}$ number of crosses arranged in a (p-1) × p array. Make the rows as circular by adding the border crosses on left (one-sided directional neighbour) will result in a NBB-CDC design with parameters p, $v = \frac{p(p-1)}{2}$, $b = p - 1$, $r = \frac{(p-1)}{2}$ and k = p. The design is variance balanced with respect to gca effects.

Example 5.1.1: Let p = 5, the two arrays of size 4 × 5 are obtained as follows:

1	2	3	4	5	Addition of 1
1	3	5	2	4	Addition of 2
1	4	2	5	3	Addition of 3
1	5	4	3	2	Addition of 4

3	4	5	1	2
5	2	4	1	3
2	5	3	1	4
4	3	2	1	5

Superimposing the two arrays and crossing the respective entries will result in the following NBB-CDC design with parameters as p, $v = \frac{p(p-1)}{2} = 10$, $b = p - 1 = 4$, r = 2 and k = p = 5:

1 × 3	1 × 3	2 × 4	3 × 5	4 × 1	5 × 2
1 × 5	1 × 5	3 × 2	5 × 4	2 × 1	4 × 3
1 × 2	1 × 2	4 × 5	2 × 3	5 × 1	3 × 4
1 × 4	1 × 4	5 × 3	4 × 2	3 × 1	2 × 5

The information matrix **C** pertaining to gca effects in the presence of one-sided neighbour effects is of the form **C**= 2.5**I**-0.5**J**.

Method 5.2: Let p be an odd number, i.e. p = 2m + 1. Develop the following initial block mod 2m + 1 (Azais *et al.* 1993):

1, 2, -1, 3, -2, … , -(m-1), m+1

Developing this initial block cyclically results in neighbour balanced design. Now consider a circular association scheme by arranging p lines on the circumference of a circle. Here, two lines are first associate if they appear immediate neighbours on both sides, second associates if they appear as neighbours leaving one and so on till $\frac{(p-1)}{2}^{\text{th}}$ associate. Develop the v = p PDC plan by crossing each line with its $\frac{(p-1)}{2}^{\text{th}}$ associates. Replace the lines in the design developed above through the initial block by the crosses. The resultant design will be a NBB-PDC design with parameters p = v = 2m + 1, b = 2m + 1, r = 2m and k = 2m. The resultant design will be partially variance balanced for gca effects following the circular association scheme.

Example 5.2.1: Let m = 2, p = 5. The v = 5 crosses are obtained by crossing each line with its $\frac{(p-1)}{2}^{th}$ associate i.e. second associates. The 5 crosses so obtained are 1 × 3, 1 × 4, 2 × 4, 2 × 5 and 3 × 5.

The neighbour balanced design for 5 lines is

3	1	2	4	3	1
4	2	3	5	4	2
5	3	4	1	5	3
1	4	5	2	1	4
2	5	1	3	2	5

Replacing the lines with the crosses, i.e. 1 × 3 – (1), 1 × 4 – (2), 2 × 4 – (3), 2 × 5 – (4) and 3 × 5 – (5), the following NBB-PDC design is obtained with parameters p = v = 5, b = 5, r = 4 and k = 4:

2 × 4	1 × 3	1 × 4	2 × 5	2 × 4	1 × 3
2 × 5	1 × 4	2 × 4	3 × 5	2 × 5	1 × 4
3 × 5	2 × 4	2 × 5	1 × 3	3 × 5	2 × 4
1 × 3	2 × 5	3 × 5	1 × 4	1 × 3	2 × 5
1 × 4	3 × 5	1 × 3	2 × 4	1 × 4	3 × 5

The information matrix **C** pertaining to gca effects in the presence of neighbour effects is of the form

$$C = \begin{bmatrix} 3 & -2 & 0.5 & 0.5 & -2 \\ -2 & 3 & -2 & 0.5 & 0.5 \\ 0.5 & -2 & 3 & -2 & 0.5 \\ 0.5 & 0.5 & -2 & 3 & -2 \\ -2 & 0.5 & 0.5 & -2 & 3 \end{bmatrix}$$

This shows that the design is partially variance balance with respect to gca effects following 2 associate circular association scheme.

6. CATALOGUE OF NBB DESIGNS FOR DIALLEL CROSSES

A list of NBB-CDC and NBB-PDC designs obtained using methods given in Sections 4 and 5 for p ≤ 20 is given below containing the parameters of the resultant designs.

Table 6.1. NBB-CDC and NBB-PDC Designs in Complete Blocks

p	v	b	r	k	λ
NBB-CDC Designs					
4	6	6	6	6	3
5	10	10	10	10	4
8	28	28	28	28	7
9	36	36	36	36	8
12	66	66	66	66	11
13	78	78	78	78	12
16	120	120	120	120	15
17	136	136	136	136	16
20	190	190	190	190	19
NBB-PDC Designs (Using GD Association Scheme) where p = (m × l)					
6 = (3×2)	12	12	12	12	4
8 = (4×2)	24	24	24	24	6
8 = (2×4)	16	16	16	16	4
10 = (5×2)	40	40	40	40	8
12 = (6×2)	60	60	60	60	10
12 = (2×6)	36	36	36	36	6
14 = (7×2)	84	84	84	84	12
15 = (5×3)	90	90	90	90	12
16 = (8×2)	112	112	112	112	14
16 = (2×8)	64	64	64	64	8
16 = (4×4)	96	96	96	96	12
18 = (9×2)	144	144	144	144	16
18 = (3×6)	108	108	108	108	12
20 = (10×2)	180	180	180	180	18
20 = (2×10)	100	100	100	100	10
20 = (5×4)	160	160	160	160	16
20 = (4×5)	150	150	150	150	12

Table 6.2. NBB-CDC and NBB-PDC Designs in Incomplete Blocks

p	v	b	r	k
NBB-CDC Designs				
5	10	4	2	5
5	10	4	2	5
7	21	6	3	7
9	36	8	4	9
11	55	10	5	11
13	78	12	6	13
15	105	14	7	15
17	136	16	8	17
19	171	18	9	19
NBB-PDC Designs (Using Circular Association Scheme)				
5	5	5	5	4
7	7	7	7	6
9	9	9	9	8
11	11	11	11	10
13	13	13	13	12
15	15	15	15	14

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