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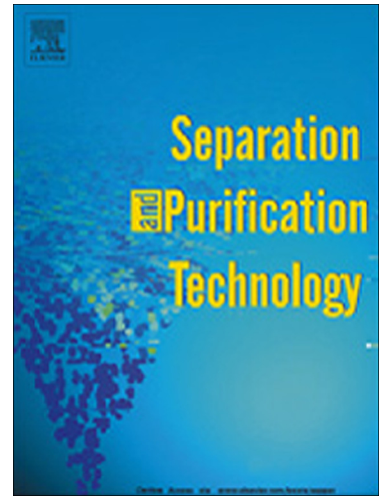

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Fish hook in classifier efficiency curves: An update

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Abstract

Fish hook in classifier efficiency curves has been receiving attention in the last three decades, more so with the advent of laser diffractometry. In the first part of this paper, we analyse two occurrences of fish hook reported recently in Separation and Purification Technology. It is shown that in both the cases, inaccuracies in measured particle size distributions could be the likely cause of the observed fish hook. In the second part, we re-examine the present state of knowledge on fish hook including the limitations of experimental observations reported so far and the drawbacks of theoretical explanations. Finally, we provide a basis on why it is to be considered nothing more than a scientifically insignificant *placebo*.

Keywords: Hydrocyclone; Fish hook effect; Efficiency curve; Laser diffractometry; Optical parameters

1. Introduction

Typically, in any classifier, recovery of particles to underflow, the actual efficiency, can be expected to increase monotonously with size. However, an inflexion in the efficiency curve showing a dip at sub sieve sizes, now commonly referred to as 'fish hook', was reported in early 1980s [1]. Since then, a considerable number of occurrences of fish hook and theories to explain this phenomenon have appeared in literature. In the first part of this paper, we discuss the reliability of two recent occurrences of fish hook [2, 3] and show that these could be due to erroneous particle size distributions.

In the second part, we re-examine the present state of state of knowledge on fish hooks and show that experimental observations of the phenomenon reported so far are not based on robust data. We then explain why it cannot be regarded as a scientifically significant physical effect. This is followed by an elucidation of why theoretical explanations proposed so far need considerable improvement. Finally,

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we show why exclusion of fish hook in simulation models is of little consequence for all practical purposes.

2. Discussion

The precision and accuracy of the efficiency curve in classifiers are dependent on particle size distributions (PSDs) from which they are derived. If the mode of particle size analyses is not specified, the reliability of the PSDs and consequentially the accuracy of the efficiency curve cannot be ascertained. The efficiency curve reported by Lv et al. [2] is subject to this limitation of PSDs of unknown precision and accuracy as they have not disclosed their method of size analysis. However, we can take note that Yang et al. [4] and Yang et al. [5], who are members of the same group and affiliated to the same institution, used Mastersizer 2000. As such, it can be reasonably inferred that Lv et al. too used the same instrument for size analysis.

Noticeably, Lv et al. [2] reported near zero efficiency of ultra fines (near zero sized particles) as shown in figure 1. The curve they obtained is remarkably similar to the efficiency curves (figure 2) reported by Majumder et al. [6] and Bourgeois and Majumder [7].

In a second report [3] discussed herein, the authors used Microtrac S3500 for determining the PSDs. The efficiency curve reported by them shows a gradual decrease in efficiency with size reaching a minimum, followed by a monotonous increase, a shape most common in fish hook literature.

While Laser diffractometry (LD) is a fast and reliable method for determining PSDs over a broad range of sizes, it could give highly misleading results if the technique is not properly applied. We discus briefly the problems with LD which could be a source of erroneous PSDs and which significantly influence the results and conclusions of Lv et al. [2] and Vakamalla et al. [3]

2.1. Size analysis by Laser diffractometry

ISO 13320:1999 for particle size analysis by Laser diffraction methods recommends application of Mie theory for all < 50 μm particles. Key inputs required for generating theoretical scatter pattern by Mie theory are the refractive index RI, the extinction coefficient (the imaginary refractive index), IRI of the test material and the refractive index of the dispersing medium. During the early years of LD, when computing power was a constraint, Fraunhofer approximation (of Mie theory) was applied for generation of theoretical scatter pattern. This does not require optical properties of

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test material and as the name implies it generates only an approximate scatter pattern in the sub sieve range. Consequently, whenever Fraunhofer approximation is the optical mode, PSD results are subject to errors in that range.

The software of recent LD instruments includes a database of optical properties of many common materials and dispersants. The data are available as standalone documents as well (for example, [8]). Obviously, where data are “sourced” from these databases, any inaccuracies in the optical parameters become a root cause of errors in the PSDs obtained.

We illustrate this with an example of sourcing RI value for SiO_2 present in the form of crystobalite. From the database issued by Malvern Instruments Ltd [8], we can observe that, for different forms of quartz namely, chalcedony, crystobalite, flint silica, silicon dioxide and tridymite, RI varies from 1.544-1.553. Elsewhere, in the same document RI of silica is listed as 1.487 for crystobalite; 1.544 for quartz and 1.468 for tridymite. Clearly, sourcing RI value from this database leads to ambiguity about the true value when SiO_2 is in the form of crystobalite (or tridymite).

Also, it is highly desirable to recheck the data supplied by manufactures. Rawle [9], reports that for a sample of SiO_2 powder supplied as crystobalite by the manufacturer, the RI was stated as 1.486. The sample actually turned out to be in the form of quartz for which the RI determined experimentally was found to be 1.543.

The imaginary refractive index (IRI) depends on physical properties, such as, colour, surface roughness etc. in addition to chemical composition.   Unfortunately, there are no methods by which IRI can be directly measured for use in laser diffractometry. Malvern Instruments Ltd [10] outlines a method for its estimation which relies on the volume concentration (C_v) of particles, a parameter calculated by the instrument. The value of IRI is needed as input for this calculation. The basic principle for this trial and error method involves taking a sample(s) of known C_v and comparing it with the value calculated by the instrument for different assumed values of IRI. That value of IRI for which the agreement between calculated and actual C_v is closest is inferred as the IRI of the test material. It should be noted that to prepare a sample with known C_v, the density of the material needs to be measured.

The influence of optical parameters (RI, IRI) of the test material on the size analysis results from laser techniques has been a subject of thorough investigation. It has been established conclusively [9, 11-17] that their influence on the particle size distribution results is significant, more so when the material tested contains < 10 μm

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particles. It is relevant to mention here that one of the objectives of the comprehensive study by Keck and Muller [17] was to clarify whether or not inclusion of optical parameters is necessary as suggested by ISO 13320. Based on a detailed investigation on the influence of RI and IRI on PSDs from LD, they report that depending upon the optical parameters used, the mean size of latex particles 'as measured' varied from:

- 330 nm to 905 nm for a tetramodal mixture;
- 284 nm to 1005 nm for a trimodal mixture and
- 79 nm to 465 nm for a bimodal mixture.

Similarly, for a bimodal mixture, the distributions as obtained from LD were monomodal, bimodal, trimodal, tetramodal and even pentamodal depending upon the RI and IRI values used. Their thorough investigation establishes *conclusively that* ***laser diffractometry for characterisation of sub micron particles gives meaningful results only when correct optical parameters are applied.***

They conclude categorically that any laser diffraction data without information of the optical parameters and also those using guessed parameters must be doubted. They estimate that probably 90% of all published PSD data in sub sieve range obtained from laser diffractometry is false.

It is apparent from the above that by simply selecting the RI and IRI values form databases of the instrument software, or literature or from data provided by material suppliers could cause erroneous inputs for calculation of scatter pattern and hence the resulting size distribution. The only option for getting accurate RI and IRI values of the test materials is to determine them experimentally.

Apart from the necessity to pick up a representative sample [18], for robustness of size analysis results Rawle [9] recommends actual determination of the refractive index (RI) up to two decimal places by Becke line method. Although, PSDs from LD are less sensitive to the value of IRI, determining it experimentally using volume concentration method [10] is recommended. While its value accurate up to a factor of three is acceptable according to Beckman Coulter [19], the tolerance limit suggested by Rawle [9] is an order of magnitude. Additionally, determining the density using helium gas pycnometer (or otherwise) is recommended. This density is required for preparing samples of known volume concentration for the volume concentration test.

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It is also useful for getting an initial estimate or confirmation of the refractive index using Gladstone- Dale relationship [20] if the test material is a crystalline mineral.

It is reasonable to state at this juncture, that the reliability of PSDs particularly in the sub sieve range is largely dependent on the accuracy of the optical properties of the test material. Consequentially, inaccurate optical properties cause errors in size distributions from LD. It is our contention that the so called 'fish hook phenomenon' observed in the sub sieve range is an outcome of such errors in measured PSDs. For research applications for which standard operating procedures are not established, it appears to be a good practice to verify at least some results by alternate methods (Ray et al. [21], Santos et al. [22]).

The procedure for size analyses followed by Lv et al. [2] and Vakamalla et al. [3] are to be discussed in the above background.

Earlier we [23] explained that the efficiency curves reported by Majumder and colleagues [6, 7] showing perfect separation of near zero sized particles were inconsistent with the experience and knowledge base obtained over years of hydrocyclone practice. We conjectured [24] that the most plausible reason for such inconsistent and irreproducible efficiency curves could be usage of inaccurate optical parameters as inputs to Mastersizer 2000. A remarkable similarity between the efficiency curves of Lv et al. [2] and Majumder et al, [6] can be seen from figures 1 and 2. It is reasonable to conclude then that irrespective of the method used for size analyses (LD or otherwise), such anomalous efficiency curves could only be due to inaccuracies in PSDs. If Lv et al. had used Mastersizer 2000, then usage of incorrect optical parameters is the most likely cause.

Vakamalla et al. [3] have neither disclosed the values of optical parameters nor how they obtained them. No explicit data have been presented or shown to indicate that they have experimentally determined these optical parameters. Consequently, the parameters that they have used for determining PSDs are to be deemed as of unknown precision and accuracy. Consequently, the accuracy of the efficiency curve including the occurrence of fish hook reported by them is uncertain.

We recapitulate that the same considerations, namely probable inaccuracies in optical parameters of test materials prompted us [24, 25] to question the reliability of fish hook data reported by Zhu and Liow [26], Abdiollahzadeh et al. [27], Alves et al. [28] and Altun and Benzer [29].

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3 Present state of knowledge

In this section we discuss and critically analyse the current state of knowledge on fish hook with regards to the experimental observations. Based on this analysis, we examine whether or not it is to be treated as a scientifically significant phenomenon. We later re-examine the theoretical explanations to explain this phenomenon. Finally, we discuss the practical utility of fish hook in simulation models

3.1 Experimental observations

We begin our discussion with experimentally reported fish hook phenomena. Noticeably, all reported occurrences of fish hook are in the sub sieve range, that is, < 30 μm. Also, for particle size analysis, laser diffractometry is followed in almost all reports [30]. Based on literature, it may be more appropriate to state that it is only with the advent of LD that reports of fish hook increased considerably. It is equally interesting to note that, when such alternate methods as Coulter counter [31], Dynamic Light scattering [32], Andresen pipette [33], Disc centrifuge [21], Ladeq equipment [33] etc. are used for size analysis, fish hook is not reported. Also, when two or more techniques are used, fish hook is reported only when LD is used for size analyses [21]. Significantly, when two different Laser instruments, one which applies full Mie theory (LS 13320) and the other its Fraunhofer approximation (HELOS) are used, fish hook was not detected with LS 13320 by Margraf [34]. He rechecked the accuracy of LD results from Mie theory by Scanning Electron Microscopy and reported good agreement in the < 10 μm range.

Eswaraiah et al. [35] recently reported fish hook in the 10-25 μm range with sieving as the method for size analysis. However, their base data cannot be relied upon as discussed in detail elsewhere [25]. Suffice it to say that they have neither mentioned the apertures of < 30 μm sieves used nor the procedure followed and precautions taken etc in sieving < 30 μm particles, thereby raising a serious concern on the reliability of their results.

Further, there are a number of reports in which the method of particle size analysis is not mentioned, for example, Aydogan and Ergun [36]. However, since these authors mention using laser techniques for size analysis in other publications [29, 37, 38], we may gather that probably LD was the method used. The exhaustive work of Minkov, Dueck and colleagues also suffers from this serious drawback as they have not mentioned the mode of size analysis in most of their publications (for example, [39-47]). However, Dueck et al. [48, 49] mention that they used a Malvern Mastersizer X,

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Farghaly [50] used Malvern Mastersizer 2000. In their latest paper, Neesse et al. [51] confirm using laser techniques. Accordingly, we may realistically infer that laser diffractometry was probably used in their other studies [39-47] also.

Noting that fish hook is observed in the sub sieve range (< 30 μm) where application of Mie theory is essential we can conclude that:

a) Those reports of fish hook which are based on PSDs obtained from using Fraunhofer approximation may be considered to be unreliable.
b) The reliability of those occurrences which are based on interpretation of scatter data with full Mie theory, but with guessed optical parameters is uncertain. They carry errors in PSDs which are unpredictable *a priori* and which range from nil (in case the optical parameters are close to the true values) to an unknown magnitude.
   Additional information is necessary to assess the quality of PSDs and the calculations based on them. For example, we could gather that Bourgeois and Majumder [7] probably used incorrect optical parameters from the inconsistent efficiency curves. Similarly, agreement of LD results with those from SEM for <10 μm particles indicates that probably Margraf [34] used correct optical parameters. Obviously, in the absence of any additional information, we cannot arrive at any conclusion with regards to the occurrence of fish hook or otherwise
c) It is only when optical properties, experimentally determined to the required accuracy are used as inputs to instruments which use Mie theory can the PSDs be robust [9, 17].

When the experimental conditions reporting fish hook are scrutinised taking the above into consideration, we arrive at the following conclusion. ***There is not even a single report of fish hook which is based on robust size distribution data with LD or otherwise.*** This confirms our earlier conjecture that reports of fish hook are due to errors in size analysis [52].

3.1.1 Reproducibility

There is absolutely no doubt regarding the repeatability of experimental observations of every occurrence of fish hook. By repeatability we mean here that successive measurements agree within acceptable limits of experimental error. This is different from reproducibility [53] which is: ‘closeness of agreement between the results of measurements of the same measurand under changed conditions of measurement

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which may include: the principle / method of measurement; the observer; the measuring instrument; reference standard; location; conditions of use and time'. The definition for reproducibility in ISO 13320 is: '...closeness of agreement between multiple measurement results of a given property in different aliquots of a sample, prepared and executed by different operators in similar instruments according to the same method ...'.

Clearly, repeatability of results does not imply that systematic errors are not present, in particular those which are due to usage of incorrect optical parameters in LD. The recent' proof of existence of fish hook', based on data from closely controlled experiments [7] is a set of repeatable efficiency curves. However, they are irreproducible as they are inconsistent with the experience and knowledge base obtained over years of hydrocyclone practice. A perfect separation of near zero sized particles is inconceivable in hydrocyclone practice, the most conspicuous feature in those efficiency curves.

As illustrated above, repeatability and reproducibility are distinctly different. Nevertheless, they have been arbitrarily used to mean the same in literature on this topic (for example, Zhu and Liow [26], Papp [54] etc.). This probably led to a wide spread belief that fish hook is an established physical phenomenon. The following extracts from recent publications illustrate this trend. Wang et al. [55] remark: '...the future study of cyclonic separation should pay sufficient attention to how to make use of the fish-hook phenomenon and how to reinforce the mechanism of fish-hook effect to improve the efficiency for the separation of ions, molecules, nanoparticles and sub-micron particles with hydrocyclones ...'. Elsewhere, Minkov et al. [40] observe: '... This phenomenon is often a hindrance for engineers using hydrocyclones because the sharpness of fractionation often deteriorates due to the fine particles (in practice, particles smaller than 10microns) contrary to expectations falling into the coarse product'. Vakamalla et al. [3] assert it as an accepted and established fact among hydrocyclone practitioners: '... A fish-hook phenomenon in hydrocyclone is defined as the increase of fine particles recovery to the underflow up to a critical size and there after it decreases. It is believed that the fishhook phenomenon is due to turbulent dispersion, boundary layer water recovery to underflow, entrapment of fine particles in the clusters of coarse particles'.

3.1.2 *Scientific significance of fish hook*

As noted earlier, no reported occurrence of fish hook is based on robust PSD data, which implies that they are not reproducible. It is thus not surprising that the

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conditions under which it is reproducible are not specified categorically in literature. It is also important to emphasise here that not even a single reported occurrence is corroborated by independent investigators.

In view of the above, we can conclude that fish hook does not meet the criteria proposed by Popper [56] to be classified as a scientifically significant physical effect, 'which can be regularly reproduced by anyone who follows the prescribed instructions'. In fact Popper rejects any effect such as this for whose reproduction no instructions are given as an occult effect.

3.2 Theoretical explanations

Evidently, any theoretical model to explain fish hook should really be an adaptation of the normal efficiency curve model. Since in any discussion on efficiency curve of hydrocyclones, the notion of 'bypass' proposed by Kelsall [57] is vital, we re-examine the concept of bypass before further analysis of models that explain fish hook.

3.2.1 Variable bypass models

To explain the regular observation that recovery of near zero size particles is equal to that of water (R_f) Kelsall postulated that a fraction of particles of all sizes equal to R_f 'bypass' to underflow without undergoing classification [57]. This bypass concept served its purpose well, namely, transforming the actual efficiency curve which varies from R_f to 1 to a 'hypothetical' corrected / centrifugal efficiency for which the range is from zero to 1. Indeed, it is the foundation for all hydrocyclone models [58] used in practice.

Over the years, presumably due to its highly successful application in modelling of hydrocyclones, the concept of bypass started gaining acceptance as the true physical representation of the classification process itself. The 'mechanistic' model proposed by Lynch and Rao [59] is an excellent example supporting this belief. Flintoff et al. [60] too expressed similar views while suggesting fundamental studies on this issue. In fact, these days a 'physical' meaning to 'bypass' is so widespread that it is part of standard text books in mineral processing (Wills and Napier-Munn [61]).

Even if a physical meaning could be attributed, it is certainly an oversimplification to conceptualise that particles of all sizes 'bypass' to underflow in the same proportion as water. As Napier-Munn and Lynch [62] observe, this seems intuitively unlikely. Nevertheless, its simplicity and familiarity through long use are the major factors for

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the continued use of constant bypass of Kelsall as rightly noted by Wills and Napier-Munn [61].

As reports of occurrences of fish hook started appearing, the need for an alternative to the constant bypass model of Kelsall became inevitable. It is at this juncture Finch [1] proposed that the fraction of 'bypass' to underflow is size dependent. This notion of variable bypass could give a tangible explanation to the occurrence of fish hook. Over the years, Finch's model was further refined by Del Villar and Finch [63], Roldan-Villisana et al. [64] and Kraipech et al. [65]. These size dependent bypass models too gained acceptance as representing the physical processes actually taking place.

Significantly, Napier-Munn and Lynch [62] observed that there was no 'experimental consensus' on this issue of variable bypass advocated by Finch [1]. The crucial issue is can we ever achieve an experimental consensus? The answer without any doubt is in the negative. It is simply impossible to have an experimental consensus on the fraction of bypass to underflow. We cannot conceive any experiment by which we can distinguish whether a particle (or group of particles) bypassed to underflow or reported there due to classification action. That is, the 'bypass' theory does not meet the criterion of 'testability' proposed by Popper [56] for it to be classified as belonging to 'empirical sciences' to distinguish it from mathematics (logic and metaphysics as well).

Accordingly, we cannot attribute any physical meaning to 'bypass'. It is simply a mathematical object, used for transforming the actual efficiency to corrected efficiency with a numerical value equal to R_f.

Earlier, we [66] demonstrated that there are an infinite number of ways in which any actual efficiency curve for which the efficiency varies from R_f to 1 can be transformed into a 'normalised curve' in which the 'normalised efficiency' varies from 0 to 1. The method to generate normalised efficiency curves is explained in Appendix A. A few typical normalising functions are listed in Table A1. Evidently, Kelsall's method of normalising is the simplest of all, in which the normalising function is constant and equal to R_f.

The important point to be noted is that ***no physical significance can be attributed to any of these normalising functions.*** They shed no light on the classification process. We can extend the same logic to all variable bypass based models to explain fish hook and conclude that they too are simple mathematical

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transformations. As such, their utility is limited to only fitting an equation for any efficiency curve showing fish hook or to generate one for use in simulation models. Clearly, they too do not shed any light on the physical processes taking place in the classification mechanism.

3.2.2 The entrainment model

The elaborate model proposed by Neesse, Dueck and colleagues [41, 46-49 etc.] takes into account hindered settling; counter flow of fluid due to settling particles and entrainment of fines by the coarser ones. Although a number of simplifying assumptions are made in its development, the model is useful to predict the complete efficiency curve. They showed that their model predictions agree closely with the experimentally determined efficiency curve.

As mentioned earlier, they did not specify the method of size analysis in most of their reports. However, in two reports [48, 49], they mentioned using Malvern Mastersizer X for size analysis The feed material for one of these studies [48] was fine feldspar of d_{10} = 3.1μm, d_{50}= 12.5 μm and d_{90}= 31.2 μm. For the other study [49], quartz of two different particle size distributions which could be approximated by Rosen-Rammler-Bennet-Sperling (RRSB) function was the feed material. The parameters for this function, namely, the characteristic particle size, d_m, and the steepness ***n*** are 7 μm and 1.23 respectively for the finer material and 12 μm and 1.3 respectively for the other.

It is important to note that the PSDs of test materials in both the studies [48, 49] are such that application of Mie theory is essential for getting accurate size analysis results from LD. However, Mastersizer X which was released prior to publication of ISO 13320:1999 does not implement full Mie theory [67, 68]. Consequently, errors in PSDs from LD and hence in the experimentally observed efficiency curves are inevitable in both the studies [48, 49]. That is, their model predictions closely agree with the experimentally determined efficiency curves which are themselves subject to errors. Obviously their model development has to go a long way so as to predict efficiency curves determined accurately.

Similarly, the mechanism proposed by Schubert [69] is validated with the experimental data of Gerhart [70]. The feed material was quartz; tests were carried out with powders of nine different size distributions; the median size (x_{50}) of the test materials varied from 3 to 16 μm. Schubert mentions that following preliminary tests

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with Sedigraph 5100 and Mastersizer X, Gerhart used laser scattering method for size analysis. That is, efficiency curves which are themselves subject to measurement errors are explained by his model. Obviously, the model requires significant refinements. Interestingly, Dueck et al. [47] too validated the experimental efficiency curves of Gerhart with their entrainment model.

For the sake of completeness, it is worthwhile to mention here that the 'entrainment model' is also beyond the experimental domain. We cannot conceive any experiment by which we can distinguish the particles reaching underflow due to 'entrainment' from those reporting to underflow due to centrifugal forces. A similarity between the entrainment model and the 'bypass mechanism' with regards to this criterion of 'testability' is noteworthy. A more elaborate discussion on this model is not useful as it is validated with efficiency curves derived from erroneous particle size distribution data. Similar reasoning applies for the mechanism proposed by Schubert as well.

3.2.3 The mechanistic explanation

The 'mechanistic' approach [71] to explain fish hook phenomenon is based on dubious assumptions and calculations which show that settling velocity falls sharply with increase in size in a centrifugal field when the flow changes from Stokesian to transient regime. To arrive at this conclusion, principles of physics applicable to the motion of particles in the Stokesian regime only were applied in the transient regime. Further, the relation between settling velocities under gravity and in a centrifugal force field was assumed to be a discontinuous function, using which calculations were done. Suffice it to say that their treatment of the problem is in total disagreement with known principles of physics including the basic dictum that everything in nature within the domain of classical mechanics is continuous. A detailed discussion on the inconsistencies and fallacies in the mechanistic explanation is available elsewhere [30].

Finally, we may take note that validation of any theoretical/ empirical models with efficiency curves determined from robust size analysis data is indispensable. At present, lack of such data is a critical drawback for validating future modelling efforts.

3.3 General remarks

From the foregoing, we can summarise that experimental observations on fish hook are not based on robust data. Theoretical explanations likewise need considerable improvement. However, the suggestion of Lyttleton [72] regarding the scientific

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attitude we need to adopt towards any theory or hypothesis is relevant at this juncture. He rightly advocates that our stance should never be of an absolute certainty or a complete disbelief and recommends flexibility to change our view as and when new evidence becomes available. Popper [56] too expresses similar views. In his words: 'The game of science is, in principle, without end. He, who decides one day that scientific statements do not call for any further test, and that they can be regarded as finally verified, retires from the game'.

Taking the above into consideration, we can anticipate that new experimental evidence for fish hook phenomenon should be based on 'reproducible' efficiency curves. Also, it is absolutely essential to verify the accuracy of optical parameters of test material (s) if laser diffractometry is the mode for size analysis. Further, ascertaining the accuracy of sizing analyses by one or more alternate methods is highly desirable.

We wish to emphasise here that to begin with, it is necessary to ascertain the 'authenticity of occurrence of fish hook' beyond any reasonable doubt. It is only then can a theory be developed to explain it.

3.4 Fish hook in Mineral processing software

At present, suitably modified Whiten function (equation 1) takes into account a possible fish hook in efficiency curves in popular commercial software packages, such as JKSimMet and Limn® [58, 73].

$$E_o(d/d_{50c}) = (1-R_f)\,\frac{(1+\beta\beta^{*}d/d_{50c})(e^{\alpha}-1)}{e^{\alpha\beta^{*}d/d_{50c}}+e^{\alpha}-2} \qquad \ldots \qquad (1)$$

In equation (1), d, d_{50c} and α have the same usual meaning, namely, particle size, corrected cut size and the material dependent sharpness index respectively. While $E_o(d/d_{50c})$ is the recovery of particles of size, d, to overflow, β determines the initial rise (since the efficiency is defined as recovery to overflow) of the curve at fine sizes. For a given α and β, the parameter β* is determined iteratively from the identity $E_o(1) = (1-R_f)/2$. Evidently, for normal efficiency curves, the fish hook parameters are β=0 and β* =1.

Of the three parameters in equation 1, the invariance of sharpness index, α with design and operating conditions is well established over many years of experience at JKMRC and elsewhere through the use of JKSimMet [58, 73, 74]. Also, a detailed

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mathematical analysis of the Whiten function and Plitt-Reid function shows that both the functions are fairly insensitive to variations in α and **m,** the sharpness index in Plitt-Reid function respectively. That is, the assumption of invariance of sharpness index with design and operating conditions is an excellent approximation for all practical purposes [73]. The experimental studies of Coelho and Medronho [33] also independently corroborate the invariance of reduced efficiency curve with design and operating conditions.

It is also relevant to mention that following Plitt's notion of variable sharpness index [75], Asomah and Napier-Munn [76] and recently Narasimha et al. [77] too developed equations for α in terms of design and operating variables However, the need for such equations to improve (if any) results of prediction is not established by either paper. Significantly, till now, no evidence is available in literature which shows conclusively that assuming a constant sharpness index could cause noticeable errors in model predictions. Moreover, as mentioned earlier, more than four decades of experience in hydrocyclone practice indicates that the current practice of assuming a constant material dependent sharpness index is an excellent approximation. Therefore, our earlier conclusion that the equations for α and **m** proposed by Asomah and Napier-Munn and Plitt are superfluous [73] can now be extended to the equation proposed by Narasimha et al. [77] as well. Also, it is reasonable to deduce that the notion of variable sharpness index is simply a remnant of early modelling efforts [75] and could be ignored.

More recently, Altun and Benzer [29] attempted to develop correlations for α, β and β^* in terms of design and operating variables of cement grinding circuits. However, the PSDs on which their calculations and results are derived cannot be relied upon as we had shown [25]. Accordingly, their usefulness is limited only to the extent that an attempt has been made in this direction.

Currently, there is no provision for assessing the influence (if any) of the operating and design variables on fish hook parameters β, β^* in the Whiten function. As such, the usefulness of these parameters is limited only to fitting an efficiency curve with a fish hook or for generating one through simulation. Furthermore, as shown in section 3.1, all the earlier reported occurrences of fish hook are based on PSDs of unknown precision and accuracy. This implies that the accuracy of the values of fish hook parameters from the earlier studies is dubious.

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In the light of above, any future modifications which attempt to show the necessity to include fish hook parameters in Whiten function (that is values of $\beta \neq 0$ and $\beta^* \neq 1$) need to eventually revalidate the extensive database and experience gained over years of hydrocyclone practice. Noting that fish hook effect is yet to be established as a scientifically significant physical phenomenon, it appears to be a long way before any modifications are incorporated in classification function in commercial software packages.

It is also relevant to recall that earlier we termed fish hook as a *placebo* and concluded that excluding it causes little difference to the results of prediction of classifier products [52]. So far, there are no reports to the contrary in the literature. As such, we maintain that till it is proven conclusively that excluding fish hook causes significant differences in simulation results, we can continue to ignore it for all practical purposes.

4 Summary and Conclusions

1. Although, there are numerous reports of occurrence of fish hook in efficiency curves, none of them is based on robust size distribution data. The studies by Lv et al. [2] and Vakamalla et al. [3] too are no exception.

2. So far, the conditions under which fish hook can be reproduced have not been specified by the proponents. None of the occurrences are corroborated by independent investigators. As such it cannot be regarded as a scientifically significant physical phenomenon as yet.

3. Early models to explain fish hook based on size dependent ‘bypass’ are beyond the reach of experimental domain. As such, they can be thought of as mere mathematical transformations. Their utility is limited only to fitting an equation for any efficiency curve showing fish hook or to generate one for use in simulation models. They do not shed any light on the physical processes taking place in the classification process

   The entrainment model developed by Dueck, Neesse, Minkov and colleagues validates experimentally determined efficiency curves which themselves are derived from erroneous particle size distribution data. Similarly, the mechanism proposed by Schubert explains fairly well the experimentally determined efficiency curve(s) with a fish hook which in turn was calculated from erroneous PSDs. The mechanistic explanation that settling velocity falls in a centrifugal field

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with change in flow regime is based on dubious assumptions in total disregard with known principles of physics.

In short, theoretical explanations developed so far need considerable refinement, if fish hook phenomenon is proved to be scientifically significant in future. However, at present a major drawback for future efforts in this direction appears to be lack of reliable experimental data showing fish hook in efficiency curves for model validation.

4. The exclusion of fish hook in simulation models causes little difference to the mass flows and size distributions of the products. There are no reports to the contrary. As such it can be regarded as a *placebo* and its usefulness is nil for all practical applications.

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Appendix A

We [66] showed that actual efficiency $E_a(d)$ which varies from R_f to 1 can be normalised in an infinite number of ways using normalising functions, $N(\{E_a(d)\})$ such as those shown in Table A.1. The general notation for normalised efficiency is $E_c(d)$ or simply E_c. Simpler notation for $E_a(d)$ is E_a and $N(E_a)$ for the normalising function; additional subscripts and superscripts (column 3) are specific to the normalising function used. The standard form of normalisation is:

$$E_c = \frac{E_a - N(E_a)}{1 - N(E_a)} \quad \text{...} \quad \text{(A.1)}$$

One may note that for Kelsall transformation, $N(E_a)$ is constant for all values of $E_a(d)$ and is equal R_f, which is also the minimum value of $E_a(d)$. For all other normalising functions shown in column 2, the limiting values are:

$$N(E_a) = R_f \quad \text{when } E_a(d) = R_f \quad \text{...} \quad \text{(A.2)}$$

and

$$N(E_a) = 0 \quad \text{when } E_a = 1. \quad \text{...} \quad \text{(A.3)}$$

Table A.1. Typical functions for normalisation of actual efficiency

S No	Normalising function, $N(E_a)$	Normalised efficiency	Remarks
1	R_f	$E_c^{Kelsall}$	Kelsall transformation
2	$R_f(1-E_a) / (1-R_f)$	E_c^{lin}	-
3	$R_f \log E_a / \log R_f$	E_c^{log}	-
4	$R_f(1-E_a^{n+1}) / (1-R_f^{n+1})$	${}_nE_c^{I}$	For n ≠ -1
5	$R_f\ (1-E_a)^{n+1} / (1-R_f)^{n+1}$	${}_nE_c^{II}$	For n ≠ -1

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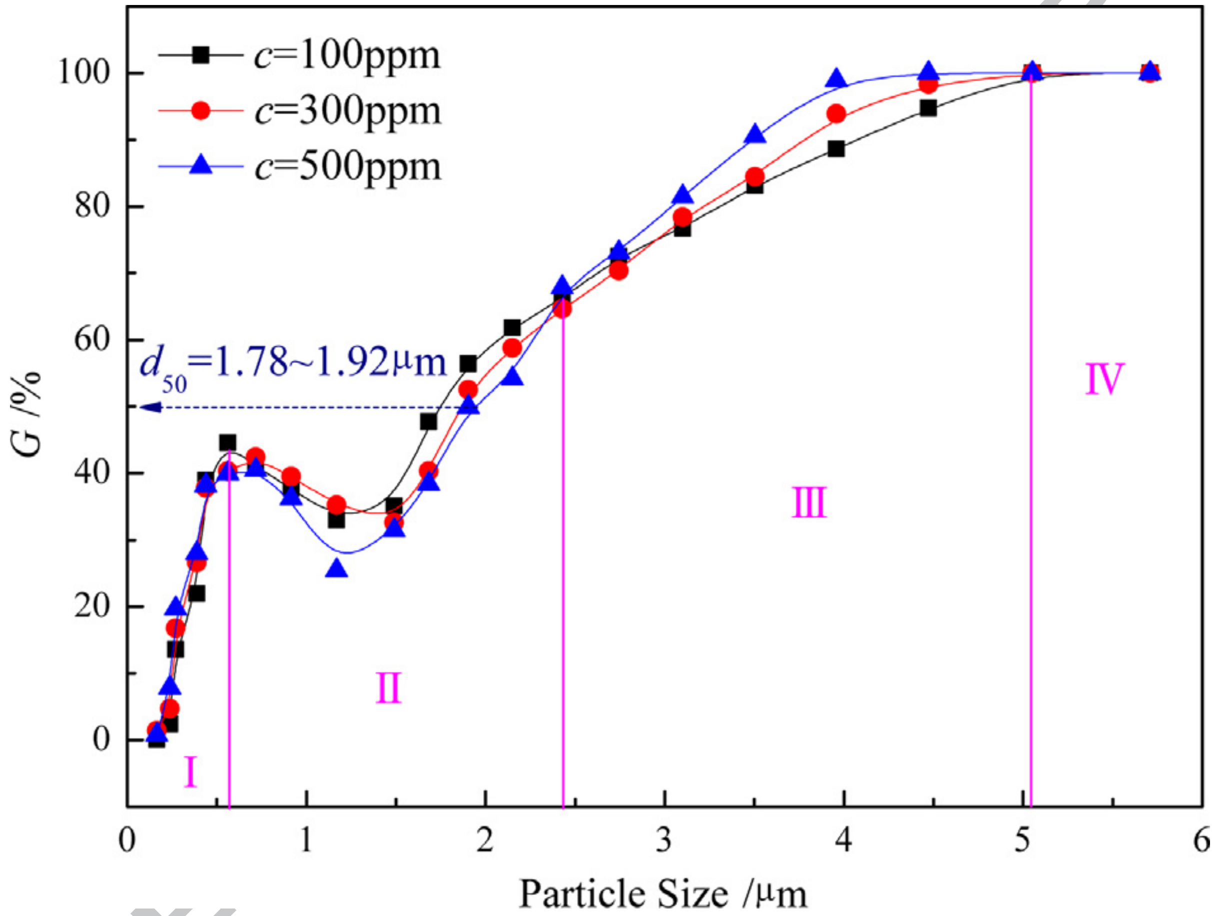

Figure 1 Actual efficiency curves reported by Lv et al. [2]. Similarity with the curve reported by Majumder et al. [2] (Figure 2) may be noted. Both report near zero efficiency of near zero sized particles, followed by an increase in efficiency. This is followed by a decrease in efficiency till it reaches a minimum and then a monotonic increase with size..

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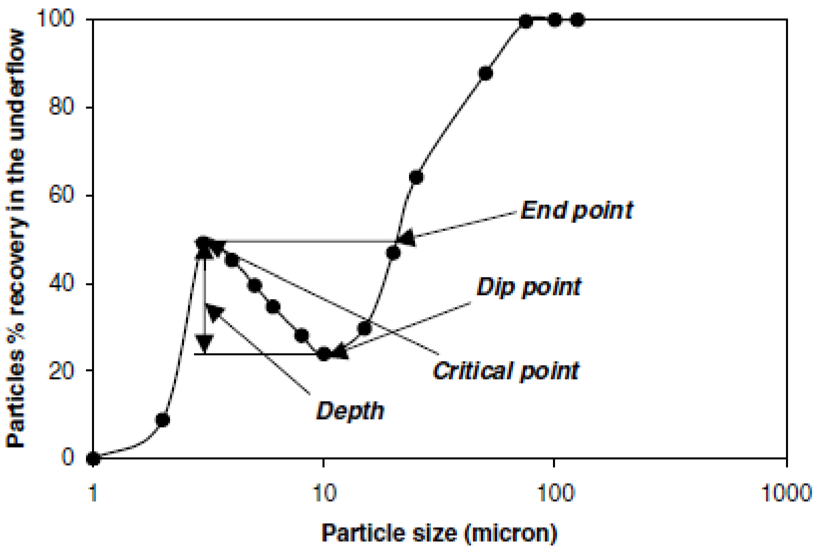

Figure 2. Efficiency curve showing a fish hook as reported by Majumder et al. [7]. The shape of this curve is distinctly different from all other efficiency curves reporting fish hook. None of them show the initial increase with size till 'critical point'.

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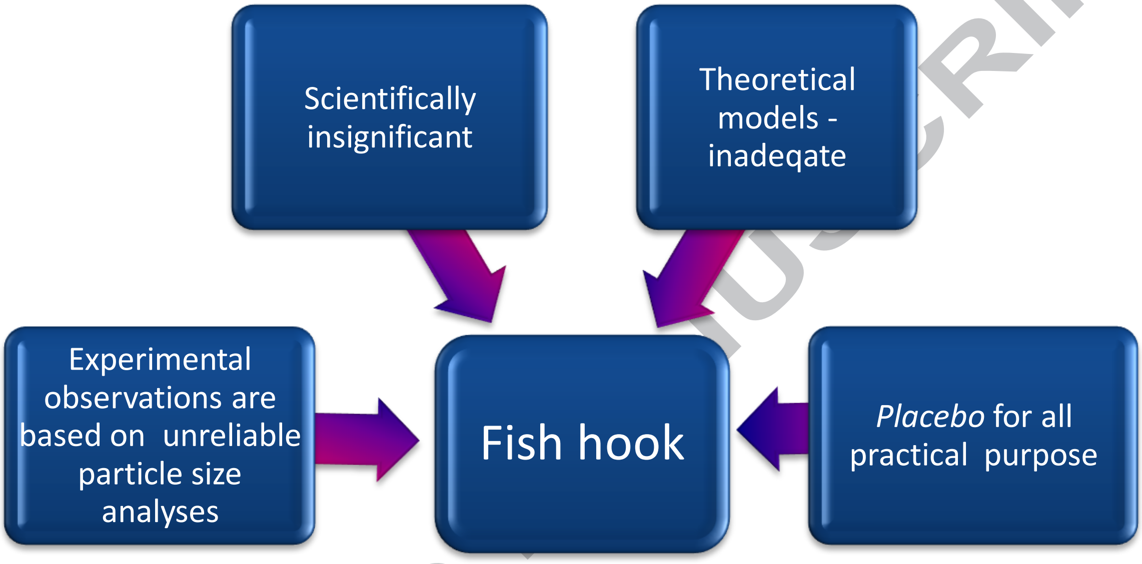
Graphical Abstract



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Highlights

- Optical parameters influence sizing of sub sieve particles by laser techniques
- Fish hook effect is a consequence of erroneous particle size analyses
- Theoretical models to explain fish hook adequately are not available
- Fish hook is a scientifically insignificant placebo

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