# Implementation of MAD and Mean Absolute Deviation based Smoothing Algorithm for Displacement Data in Digital Image Correlation Technique

Kenneth Joseph PAUL[1], Harilal R[1] and Ramji M[1]

[1]Engineering Optics Lab, Department of Mechanical and Aerospace Engineering, IIT Hyderabad, India

**Abstract**
This study aims at obtaining smoothed strain fields from digital image correlation technique (DIC), by smoothing the displacement fields prior to strain computation. Both median absolute deviation (MAD) and mean absolute deviation based smoothing algorithms are implemented for data smoothing. On implementation of these algorithms, the noise levels in the strain fields are found to be reduced to a greater extent. For executing the whole field smoothing algorithm, an accurate boundary encoding algorithm is also developed.

**Key words**
Digital Image Correlation, Mean Absolute Deviation, Median Absolute Deviation, Noise, Smoothing.

## 1. Introduction

Digital Image Correlation (DIC) is an optical technique which estimates the displacement field by comparing the speckle patterns in deformed and undeformed configurations [1]. The low cost, simple optics and the easier specimen preparation has made DIC an important non-contact, optical method in the field of experimental mechanics. Several researchers have focused on the influence of various parameters of DIC namely the subset size, step size and corresponding errors in measurements. These studies have contributed greatly in improvisation of algorithms for displacement and strain estimation. Error analysis in DIC still remains as one of the major issue to overcome. One of the major factors for the cause of error in DIC is the noise that gets incorporated into the displacement field data. These noises occur due to the interference from the electronic circuits. The calculations of strains involve numerical differentiation of displacement field leading to amplification of noise and hence the error in computation. Smoothing the image prior to displacement measurement can reduce the error but it is not advised. Smoothing algorithms like adaptive median filtering, moving average filter and Savitzky-Golay filter have been employed for a long time towards displacement and strain field smoothing. One approach to reduce the error is to use the Savitzky Golay digital differentiator to estimate strains from displacement data as suggested by Bing Pan et al. [2]. In 2000, Kuendong Ha [3] used two dimensional curve fitting to reduce the noise in the displacement data. Another study was conducted by Na Chen et al. [4] in which a smooth surface was constructed from a discrete displacement field and the strain was derived from it. In 2012 Jiaqing zhao et al. [5] developed a thin-plate spline smoothing method that was implemented for the smoothing of displacement data prior to the calculation of strain estimation.

Smoothing is dependent on several parameters including the type of algorithm used. One of the main factors that influence smoothing is the window size. The size of a window determines the extent to which the actual details of the data are retained. Studies were conducted by J Huang et al. [6], regarding implementation of a self-adaptive Gaussian window so as to reduce the effect of window size. A smaller window size helps in retaining the details of the data but the noise is not reduced, whereas a larger window size reduces the noise but along with it removes the minute details. A median filter can also be used to smooth the displacement data. In 2007, Ramji and Ramesh [7] implemented a median absolute deviation (MAD) filter to smooth the digital photoelastic data. They concluded that the MAD based whole field algorithm was very effective and could be extended to any other technique. The main objective of this study is to implement the same MAD and mean absolute deviation based smoothing algorithm to smooth the displacement data. The smoothing is done in two parts: horizontal and vertical. A set of neighbouring points or span is chosen and the smoothing is done on the data points contained within the span. The smoothing algorithm is implemented in MATLAB [8] and the displacement data is obtained from an open source 2D DIC software Ncorr [9] developed at Georgia Institute of Technology, USA. A comparison of the smoothed and unsmoothed data is also presented for quantitative comparison.

## 2. Digital Image Correlation

Digital image correlation is one of the most widely used non-contact whole field surface displacement and strain measurement techniques. The technique uses artificial or natural speckle pattern to track the motion of the reference subsets of the un-deformed configuration in the deformed configuration. For estimating displacements, the method utilizes a correlation criterion in evaluating the similarity between the reference and deformed subsets. With the help of the correlation criterion coupled with optimization algorithms, the location of the subset is identified in the deformed image by comparing the intensity pattern of the reference subset. A widely used correlation criterion is ZNCC according to Eq.(1) [10]

$$C_{ZNCC}(p) = \frac{\sum_{\Omega}[f(x_i, y_i) - \bar{f}][g(x_i^{'}, y_i^{'}) - \bar{g}]}{\sqrt{\sum_{\Omega}[f(x_i, y_i) - \bar{f}]^2}\sqrt{\sum_{\Omega}[g(x_i^{'}, y_i^{'}) - \bar{g}]^2}} \quad (1)$$

where $\Omega$ is the selected subset, $f(x,y)$ and $g(x',y')$ are the intensity values in the reference and deformed images. Here $\bar{f}$ and $\bar{g}$ are the mean intensity values of the subsets in un-deformed and deformed configurations. The coordinates $x'$ and $y'$ can be expanded according to Eq.(2) and Eq.(3).

$$x' = x + u + u_x(x - x_0) + u_y(y - y_0) \tag{2}$$

$$y' = y + v + v_x(x - x_0) + v_y(y - y_0) \tag{3}$$

where the deformation parameter $p = (u, v, u_x, u_y, v_x, v_y)$, $x'$ and $y'$ are the coordinates of the point which is being tracked in the current image, $x_o$ and $y_o$ are the coordinates of the center of the reference subset, $x$ and $y$ are the coordinates of the point which is being tracked in the reference image, $u$ is the displacement along the $x$ direction, $u_x$ and $u_y$ are the partial derivatives of $u$ w.r.t *'x'* and *'y'* respectively, $v$ is the displacement along the $y$ direction, $v_x$ and $v_y$ are the partial derivatives w.r.t *'x'* and *'y'* respectively.

There are several methods for estimating the correlation of a particular subset in the deformed image with a subset in the reference image. The principle behind the matching is relating the greyscale value of the subset points in the reference and the deformed image. The $C_{ZNCC}$ criterion in Eq. (1) is used to track the degree of correlation and has a range from -1 to 1. As the similarity between the two regions increase, the value of $C_{ZNCC}$ gets closer to one. Several nonlinear optimization algorithms can be used to improve the correlation coefficient. The most common among them is the Gauss-Newton (GN) iterative optimization scheme. In this method an initial guess is provided and the subsequent iterations are conducted to reduce the error. Forward additive Gauss-Newton (FA-GN) and Inverse Compositional Gauss-Newton (IC-GN) are two methods that are used in calculating the displacements [11]. The strain is then calculated from the displacement field with the help of numerical differentiation. The differentiation process makes strain estimation highly sensitive to noise. Hence it is advised to smooth the displacement data prior to the estimation of strain so that the noise in the strain could be reduced to greater extent.

**3 Algorithms adopted for whole field smoothing**

Smoothing a given data is to implement a function that can lower the noise and identify the important patterns in the actual data without losing the original trend of it. Various algorithms for smoothing have been implemented throughout the years. Savitzky-Golay filter, Moving Average filter, Median Absolute Deviation filter are some of the filters that have been used to smooth data. In Savitzky-Golay filter a low degree polynomial is fitted into adjacent subsets and in moving average filter the values in the subsets are averaged and the subsets are shifted after each calculation. In both of these algorithms outliers affect the final result. The algorithms implemented in this study are based on median absolute deviation and mean absolute deviation principle to smooth the displacement data directly obtained from DIC technique. Generally to smooth data, regression analysis is used to establish a relationship between the dependent and the independent variables in order to fit a polynomial curve of degree $n$ [12]. There are many techniques available for doing regression analysis and they can be broadly classified into two categories namely, parametric and non-parametric. Linear regression and ordinary least squares regression are included under parametric methods. In these methods, the data given is used to find parameters of a predefined regression function. The non-parametric methods include methods like loess and lowess, where the form of regression function is not specified in advance but is estimated from a set of functions which are smooth and continuous involving the given data. 'Loess' is a non-parametric regression method in which a $p^{th}$- order weighted-least-squares polynomial regression is performed [13]. Small subsets of $n$ points are considered around the point to be smoothed and a polynomial fit is performed. In a local regression method, weights are calculated for each point in the data set so as to determine the closeness of that point to the data set base value as a whole. Thus the points that are lying far away will have less weight and thereby will have only less influence when the curve fitting is done. A linear window or span is specified to select the number of nearest points to be chosen for each iteration. Then a regression weight is assigned to each point in the span using a tri-cube function according to Eq.(4).

$$w_i = \left[1 - \left|\frac{x - x_i}{d(x)}\right|^3\right]^3 \tag{4}$$

where $d(x)$ is the distance to the most distant predictor in the span along the abscissa, $x$ is the predictor value and $x_i$ are the nearest neighbors of $x$ in the span

After the calculation of weights, a weighted linear least-square regression is implemented over subset of the data. A second degree polynomial is used for 'loess'. The polynomial which is obtained from the weighted linear least-square regression is used to find the regression value at the predictor value of interest. This is repeated for each point in the span. The main disadvantage of the 'loess' is that even the outliers can have an influence in the curve fitting. In this 'rloess' which is a modified form of 'loess' is implemented. In 'rloess' a robust local regression method is used in which the influence of the outliers can be eliminated. In this method after the loess smoothing is done, the residuals are calculated at every point. Then the median of those values are calculated. Later each of them is assigned a robust weight and outliers are eliminated according to Eq.(5).

$$w_i = \left[1 - \left(\frac{r_i}{6MAD}\right)^2\right]^2, |r_i| < 6M,$$

$$0, |r_i| \geq 6M \tag{5}$$

where $w_i$ is the weight given to the data point $i$, $r_i$ is the residual of the $i^{th}$ data point produced by the regression smoothing procedure, and M is the median absolute deviation or mean absolute deviation of the residuals. The data is again smoothed using the robust weights. Thus the final result is obtained by using both local regression and robust weight. In this study the main focus is to smooth the displacement data before strain computation using median absolute deviation and mean absolute deviation algorithms.

## 4. Boundary Encoding

The experiment was conducted on a disc under diametrical compression which is a simply connected surface. The ncorr uses a region of interest polygon method to detect the boundary and calculate the displacements. The displacement matrix returned has a zero value outside the boundary. Horizontal smoothing and vertical smoothing is done separately. The boundary for the horizontal smoothing is detected by storing the coordinates of the first non-zero digit from left and right. A similar process is used for the vertical smoothing as well.

## 5. Experimental Data Acquisition

The smoothing is performed on the whole field displacement data estimated for an epoxy disc under diametral compression using the 2D DIC technique. The 2D DIC experimental setup consists of a CCD camera of spatial resolution 2448 x 2048 pixels at a frame rate of 15 fps. The CCD camera is mounted with a Tamron lens of 180 *mm* focal length. Surface of the epoxy disc is coated with a thin layer of white acrylic paint. Using an airbrush carbon black paint is sprayed on the white surface creating random black and white artificial speckle pattern. The epoxy disc is subjected to a diametral load of 1kN using a computer controlled MTS Landmark servo hydraulic system. Images are collected as the loading progresses and are further post processed using Ncorr software for estimating the whole field inplane displacement fields. An additional smoothing module comprising of MAD and mean absolute deviation based algorithm is developed in Ncorr. In each case, smoothing is done in two steps, horizontal smoothing followed by vertical. The results are analyzed in section 6.

## 6. Results and Discussion

### 6.1 Smoothing Using Mean Absolute Deviation

Using the input speckle images, the whole field displacement data is computed using Ncorr. A mean absolute deviation based smoothing algorithm is used to smooth the displacement field prior to strain calculation. Fig.4 (a) and Fig.4 (b) shows the raw and smoothed $u$ displacement field using mean absolute deviation for the problem of disc under diametrical compression. The same for $v$ displacement is shown in Fig.5 (a) and Fig.5 (b). The line plot variation of raw and smoothed displacement data $u$ and $v$, along the horizontal diameter AB are shown in Fig.1 and Fig.2 respectively. Fig.6 (a) shows the $\mathcal{E}_{xx}$ strain field directly computed from unsmoothed displacement field and Fig.6 (b) shows the strain field computed from smoothed displacement field using mean absolute deviation. The same for $\mathcal{E}_{yy}$ are shown in Fig.7 (a) and Fig.7 (b). For a deeper understanding, $\mathcal{E}_{xx}$ and $\mathcal{E}_{yy}$ strain values along horizontal diametral line AB are plotted in Fig.3 and Fig.8 respectively for both raw and smoothed data for quantitative comparison.

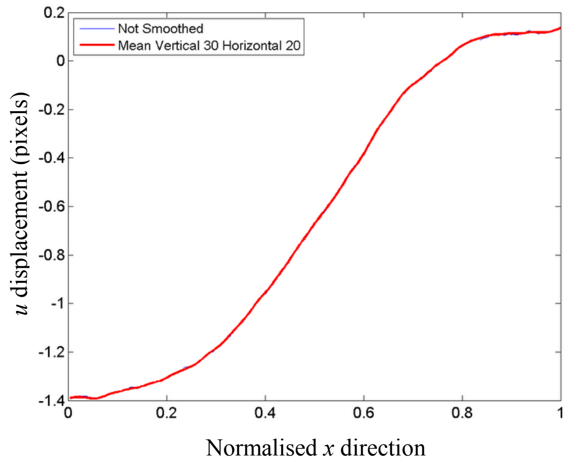



Fig.1 comparison of $u$ displacement before and after smoothing using mean absolute deviation smoothing

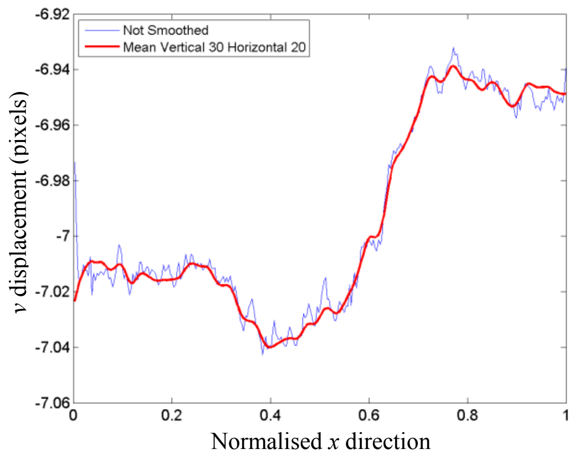



Fig.2 comparison of $v$ displacement before and after smoothing using mean absolute deviation smoothing

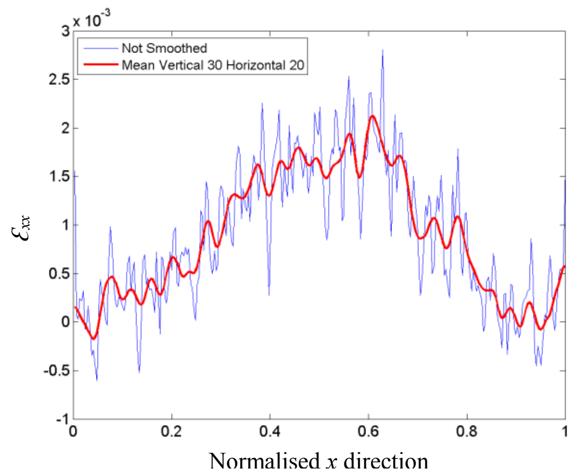
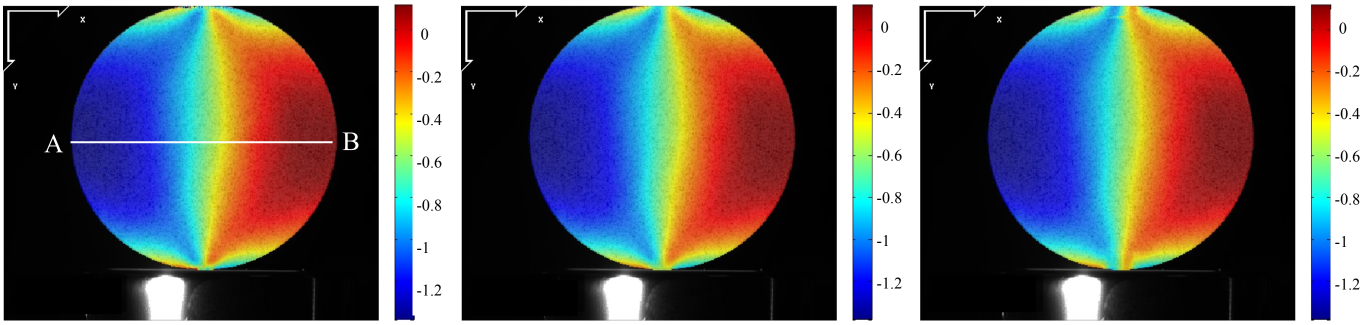


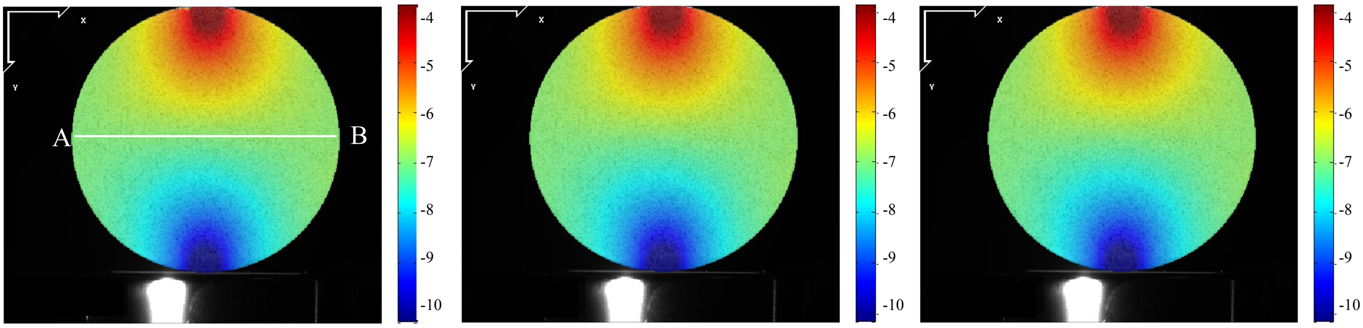

Fig.3 comparison of $\mathcal{E}_{xx}$ values obtained from the un-smoothed and smoothed displacement data involving mean absolute deviation smoothing

Unsmoothed | Mean Absolute Deviation | Median Absolute Deviation



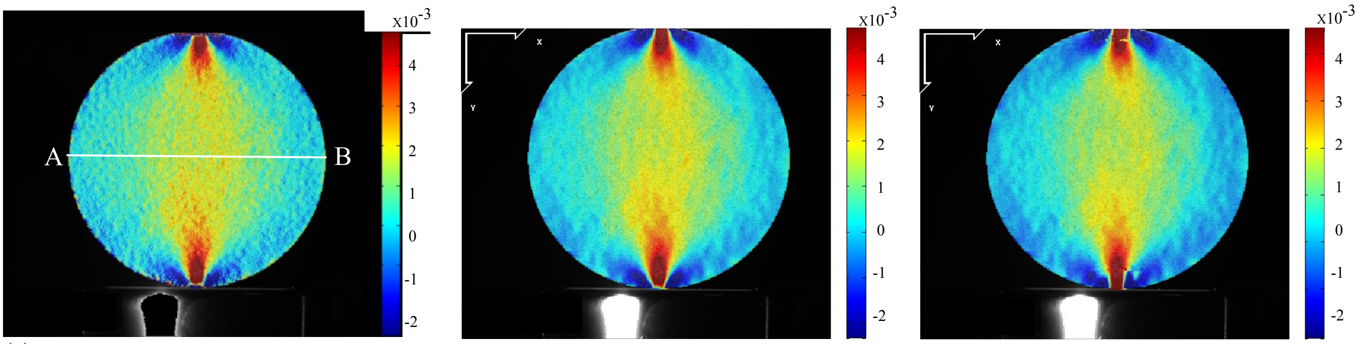

(a) (b) (c)

Fig.4 *u* displacement contours (a) unsmoothed (b) smoothed using mean absolute deviation (c) smoothed using median absolute deviation



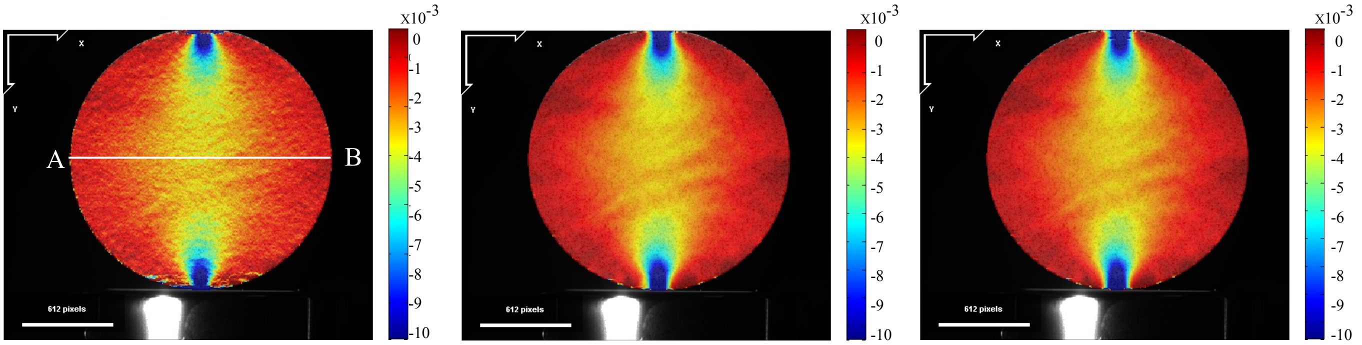

(a) (b) (c)

Fig.5 *v* displacement contours (a) unsmoothed (b) smoothed using mean absolute deviation (c) smoothed using median absolute deviation




(a) (b) (c)

Fig.6 $\varepsilon_{xx}$ contours (a) unsmoothed (b) calculated from displacement field smoothed using mean absolute deviation (c) calculated from displacement field smoothed using median absolute deviation




(a) (b) (c)

Fig.7 $\varepsilon_{yy}$ contours (a) unsmoothed (b) calculated from displacement field smoothed using mean absolute deviation (c) calculated from displacement field smoothed using median absolute deviation

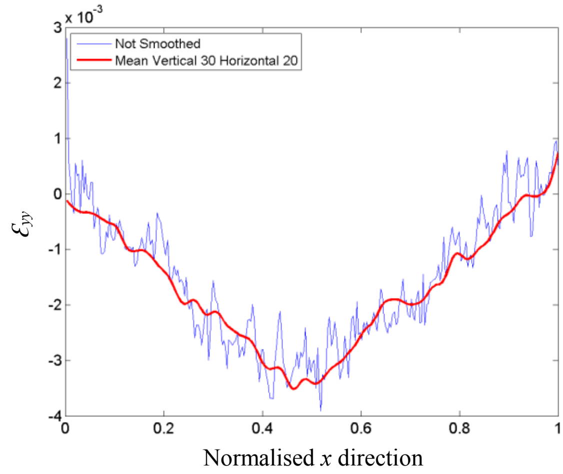



Fig.8 comparison of $\mathcal{E}_{yy}$ values obtained from the un-smoothed and smoothed displacement data involving mean absolute deviation smoothing.

As shown in Fig.3 and Fig.8, after the application of smoothing algorithm the noisy peaks and valleys in the computed strain alleviates. Here a horizontal span width of 30 pixels and a vertical span width of 20 pixels is used. Less distortion is observed in the computed strain data as compared to the strain data derived from unsmoothed displacement data.

### 6.2 Smoothing Using Median Absolute Deviation

The unsmoothed displacement field *u* and *v* are smoothed using a median absolute deviation based algorithm and are shown in Fig.4 (c) and Fig.5 (c) respectively. Fig.9 and Fig.10 show the variation of raw and smoothed displacement data *u* and *v* along the horizontal diameter AB. The $\mathcal{E}_{xx}$ and $\mathcal{E}_{yy}$ strain fields computed from smoothed displacement field are shown in Fig.6 (c) and Fig.7 (c). The $\mathcal{E}_{xx}$ and $\mathcal{E}_{yy}$ strain values with prior displacement smoothing and without displacement smoothing along the line AB are compared in Fig.11 and Fig.12.

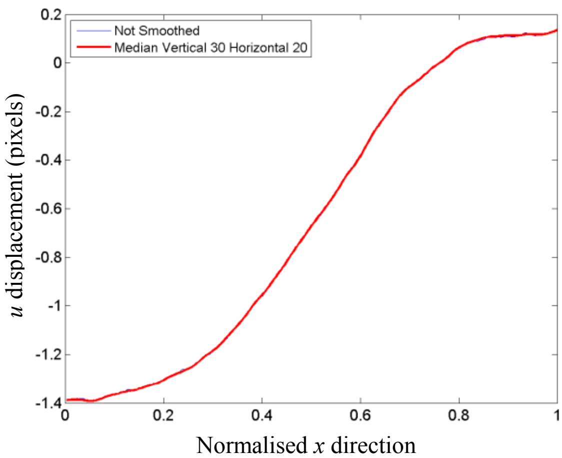



Fig.9 comparison of *u* displacement before and after smoothing using median absolute deviation smoothing

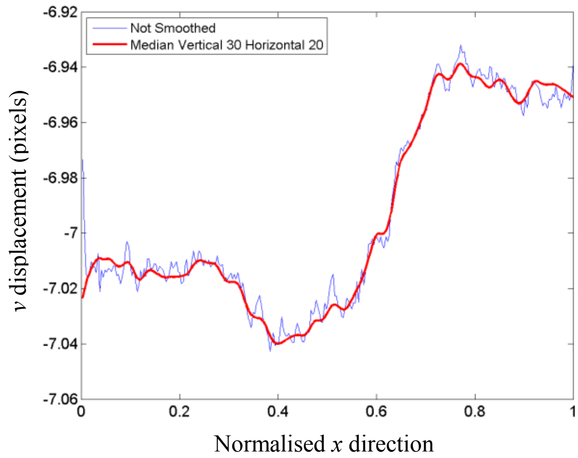



Fig.10 comparison of *v* displacement before and after smoothing using median absolute deviation smoothing

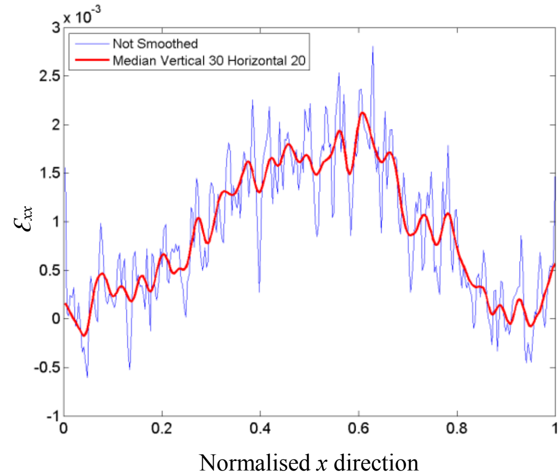



Fig.11 comparison of $\mathcal{E}_{xx}$ values obtained from the un-smoothed and smoothed displacement data involving median absolute deviation smoothing

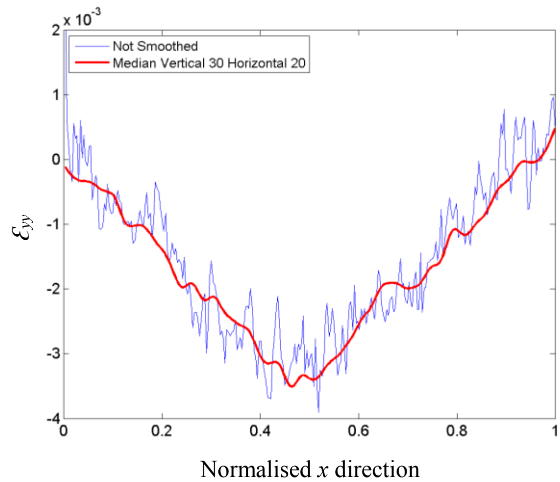



Fig.12 comparison of $\mathcal{E}_{yy}$ values obtained from the un-smoothed and smoothed displacement data involving median absolute deviation smoothing.

The computed strain field obtained from the displacement field smoothed using the median absolute deviation shows a similar reduction in noisy peaks and valleys as evident in Fig.11 and Fig.12. The same span width of 30 pixels along the horizontal direction and 20 pixels along the vertical direction is maintained here as well.

The range of the *u* displacement plot shown in Fig.1 and Fig.9 is -1.4 to 0.2 pixels while the range for *v* displacement plot shown in Fig.2 and Fig.10 is -7.06 to -6.92 pixels. Since the range is very small in *v* displacement plot, even minor variations created due to the experimental noise is evident unlike *u* displacement plot.

**7. Conclusion**

A whole field smoothing algorithm for the displacement field obtained from DIC has been developed. Both mean and median absolute deviation based algorithms are used to smooth the displacement data prior to strain calculation. Here, problem of disc under diametrical compression is considered. Boundary encoding algorithm is developed. Both mean and median absolute deviation algorithm tend to give similar results after the smoothing process. An elaborate study needs to be carried out involving the actual influence of span width of the smoothing algorithm on the actual raw experimental data. Currently, the authors are working in that direction.

**Nomenclature**

| | |
|---|---|
| $r_i$ | residuals obtained |
| $u$ | the displacement along $x$ direction [pixel distance] |
| $u_x$ | the partial derivative of the displacement along $x$ direction w.r.t '$x$' |
| $u_y$ | the partial derivative of the displacement along $x$ direction w.r.t '$y$' |
| $v$ | the displacement along y direction [pixel distance] |
| $v_x$ | the partial derivative of the displacement along $y$ direction w.r.t '$x$' |
| $v_y$ | the partial derivative of the displacement along $y$ direction w.r.t '$y$' |
| $w_i$ | weights given to a point |
| $x$ | abscissa of the point in the reference image |
| $y$ | ordinate of the point in the reference image |
| $x_o$ | abscissa of the center of the subset |
| $y_o$ | ordinate of the center of the subset |
| $x^{'}$ | abscissa of the point in the current image |
| $y^{'}$ | ordinate of the point in the current image |
| *ZNCC* | zero mean normalized cross-correlation |
| $\varepsilon_{yy}$ | strain along the $y$ direction |

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