Material Characterization of Carbon Fiber Reinforced Polymer Laminate Using Virtual Fields Method

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ABSTRACT

In this study, virtual fields method (VFM) has been implemented to extract the constitutive parameters of carbon fiber reinforced composite laminates and the results are compared with experimental observations. The extracted parameters are longitudinal modulus (E_{11}), transverse modulus (E_{22}) and in-plane Poisson's ratio (v_{12}). Digital image correlation (DIC), a full field non contact optical surface displacement and strain measurement technique is used for obtaining surface strain map over the tensile specimens at various load levels. Using the principle of virtual work (PVW), which is nothing but the weak form of equilibrium equation and by selecting kinematically admissible virtual displacement fields, a system of linear equations is constructed. Sought parameters are obtained by solving these linear equations. Comparison of the results shows that the parameters predicted using VFM is in very good agreement with experiment results. It is also observed that VFM is fast and easy to implement in comparison with traditional material characterization methods.

Keywords: Carbon fiber reinforced plastic, Constitutive parameters, Digital image correlation, Virtual fields method

1. INTRODUCTION

Carbon fiber reinforced polymers (CFRP) are now extensively used in design of structural components especially in aerospace applications because of their high specific strength, easy formability and improved fatigue life. The properties of a composite material depend on various parameters such as, the properties of the constituent components (matrix, reinforcing fibers), the processing technique used etc. It is important to optimize these parameters to enhance the properties of a composite material. In this view, the characterization of a composite laminate is essential to understand the relation between the processing technique and properties of the composite [1]. In recent years, the development of whole field strain measurement techniques such as digital image correlation (DIC) has opened up newer methods for estimating mechanical properties of materials. Kashfuddoja et. al [2] have successfully implemented DIC to characterize CFRP panel and the results are compared with finite element analysis (FEA) predictions. Grédiac and Pierron have proposed virtual fields

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method (VFM) [3, 4] which utilizes the surface displacement and strain field to identify unknown constitutive parameters. VFM uses principle of virtual work (PVW) and whole field surface strain distributions from DIC technique to get a weak form of equilibrium equations. In the elastic region, kinematically admissible (KA) virtual displacement fields equal to the number of unknown parameters are used to generate a linear system of equations and the solution of this linear system of equations give rise to material parameters. Grédiac et al. [5] have published a review paper summarizing the basics and capabilities of VFM. Pierron et. al [6] have used virtual fields method (VFM) and surface strain measurements of a thick composite coupon subjected to a shear-bending load and estimated the four through-thickness moduli. In their study, they have conducted a sensitivity analysis of results with respect to noise by adding simulated noise into the input data and concluded that proper selection of virtual fields gives consistent results except for the Poisson's ratio estimate. Toussaint et al. [7] have introduced piecewise virtual fields for identification of the constitutive parameters of a heterogeneous material. Avril et al. [8] have examined the sensitivity of VFM to the noise in surface strain measurement and proposed an algorithm to systematically select the virtual displacement fields to minimize the sensitivity to random noise. Grédiac et al. [9] have implemented VFM for identifying elasto-plastic constitutive parameters from in-plane strain measurements. Further, Palmieri et al. [10] have utilized VFM for characterizing hyper elastic materials.

In this study virtual fields method (VFM) has been implemented for estimating the elastic properties of unidirectional (UD) Carbon/Epoxy composite laminate and results are compared with the results outlined in reference [2]. Tensile test is performed as per ASTM D-3039 standards on $[0°]_3$ specimen at a rate of 2 mm/min for obtaining the longitudinal elastic parameters and $[90˚]_6$ specimen is used to obtained the transverse properties. A 3D DIC system is employed for measuring the surface strain distribution. Two kinematically admissible and independent virtual displacement fields are selected. A MATLAB® program is developed to generate a system of equations using PVW and the surface strains measured using DIC to solve for the unknowns. The extracted parameters are longitudinal modulus (E_{11}), transverse modulus (E_{22}) and in-plane Poisson's ratio (v_{12}).

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2. DIGITAL IMAGE CORRELATION (DIC)

DIC is a powerful non contact type optical technique widely used for obtaining the surface displacement and strain distribution. This technique is based on image comparison where the undeformed configuration of the specimen (reference image) is compared with the deformed configuration by utilizing pattern matching algorithms (Refer Fig.1).
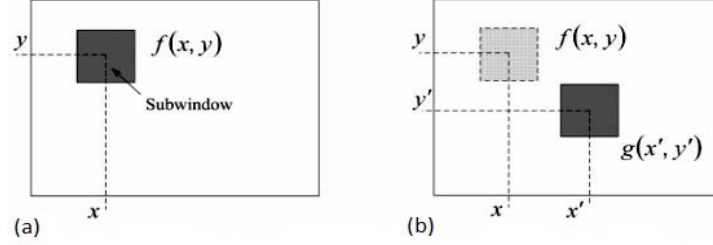

Figure 1. Subset matching in DIC (a) undeformed (b) Deformed.

For pattern matching, a correlation function as given in Eq. (1) is used. It is not practical to compare pixel to pixel so, a group of adjacent pixels known as pixel subsets are compared here based on their gray scale values by searching for the best cross correlation using the correlation function. In order to get distinct gray scale values for the pixel subsets, the specimen surface is coated with white paint and random black dots are produced (speckle pattern) over it by spraying black paint. Once the displacement map is obtained then the corresponding strain components are obtained by numerically differentiating the displacements.

$$C(u,v) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m}\left[f(x_i,y_j)-\overline{f}\right]\left[g(x'_i,y'_j)-\overline{g}\right]}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{m}\left[f(x_i,y_j)-\overline{f}\right]^2}\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{m}\left[g(x'_i,y'_j)-\overline{g}\right]^2}}$$

(1)

Where

$\overline{f}$ = mean intensity value of reference subset
$\overline{g}$ = mean intensity value of deformed subset

$$x' = x + u_0 + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

$$y' = y + v_0 + \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$$

m is the width of subset in pixels, u_0 and v_0 are the translation of the centre of the subset in x and y directions.

In a 2D DIC system, a single camera is used for image grabbing then the DIC system is capable of measuring only the in-plane displacements and strains. By using two cameras for

simultaneous image capturing, one can calculate the out-of-plane displacements in addition to the in-plane displacements and strain. This configuration is known as 3D DIC. A schematic diagram of 3D DIC is shown in Fig. 2. In the present study, a 3D DIC configuration is used for the whole field surface strain measurements during the tensile testing. Vic 3D purchased from Correlated Solutions[®] Inc., USA is used for Post processing the images.
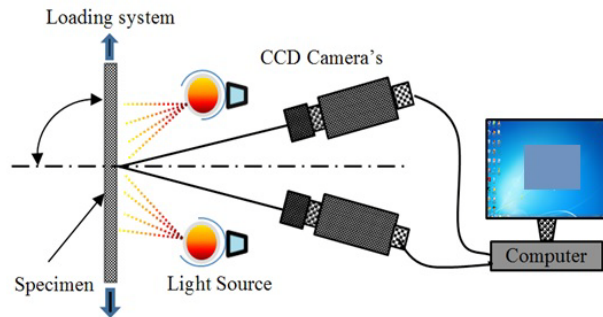

Figure 2. Schematic view of 3D DIC setup

3. EXPERIMENTAL CONSIDERATIONS

Unidirectional properties are estimated using series of tensile tests as recommended by ASTM standard D-3039. A servo hydraulic tensile testing machine of 100kN capacity (MTS make) is used for loading the specimens in displacement control mode. Displacement rate is kept at a rate of 2 mm/min. Specimen dimensions are as depicted in Fig.3 and experimental setup is shown in Fig.4. 3D DIC system used in this study consists of two Grasshopper[®] CCD cameras having a spatial resolution of 2448 x 2048 pixel^2 coupled with Schneider Xenoplan lenses of 35mm focal length. Image capturing is automated using Vic-Snap software from Correlated solutions USA and the image grabbing rate is set at 10 images per second. A subset size of 35 x 35 pixel^2 with a step size of 7 pixels is considered for post processing the images. Readers are requested to refer [2] for detailed testing procedure and surface strain plots because the experiment data used in the current study is from the results and data published in reference [2].
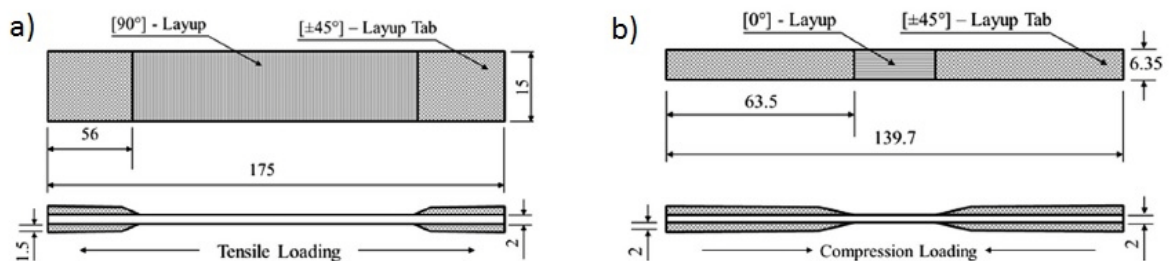

Figure 3. Specimen dimensions of CFRP laminate [2] (a) [90°] layup (b) [0°] layup

4. VIRTUAL FIELDS METHOD

VFM is based on principle of virtual work and the fundamental equation of VFM is given in Eq (2). A schematic view of tensile testing is given in Fig.5..

$$-\int_V \sigma_{ij} : \varepsilon_{ij}^* dv + \int_{s_f} \overline{T}.u_i^* ds_f + \int_V B.u_i^* dv = \int_V \rho a.u_i^* dv \quad \forall u_i^* \, KA \qquad (2)$$

Where
a acceleration [mm/s^2]
B body force per unit volume [N/mm^3]
S_f surface area where traction is applied [mm^2]
$\overline{T}$ applied traction [MPa]
u_i^* virtual displacement in i $(x$ or $y)$ direction [mm]
V volume of the specimen [mm^3]
ε_{ij}^* virtual strain tensor derived from u_i^*
ρ density [kg/m^3]
σ_{ij} stress tensor
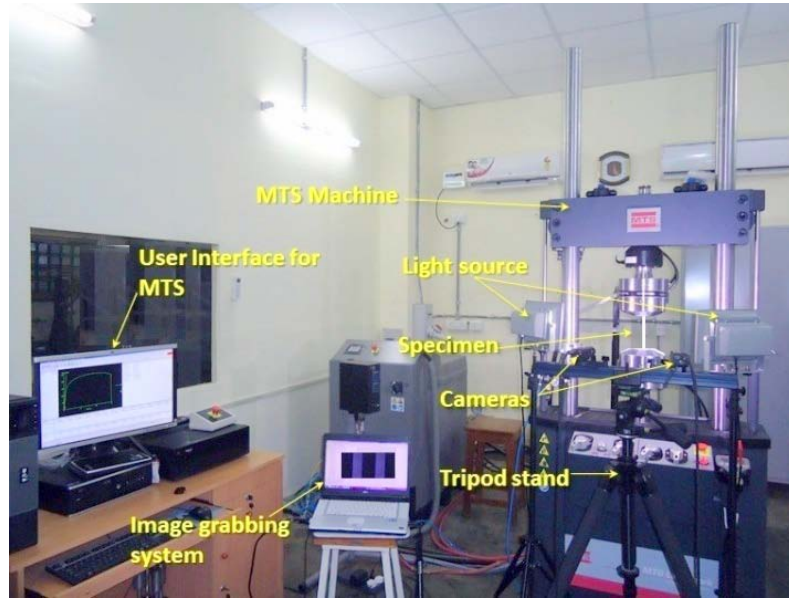KA kinematically admissible

Figure 4. Experimental setup

Above equation is nothing but the local equilibrium equation derived using principle of virtual work. Also note that the variables super scripted using * are virtual quantities. Since the contribution of body forces and inertia forces have negligible contribution to the total work done, one can write Eq. (2) as given below. Where S_f is the surface area where traction is applied in mm^2.

$$\int_V \sigma_{ij} : \varepsilon_{ij}^* dv = \int_{S_f} \overline{T}.u_i^* ds_f \tag{3}$$

A planes stress condition is considered so that it is assumed that there is no variation of stresses in the thickness direction, one can convert the volume integrals given in Eq. (3) to surface integrals as given in Eq. (4). Where L_f is the width of the area where traction is applied in mm, b is the thickness of the specimen in mm.

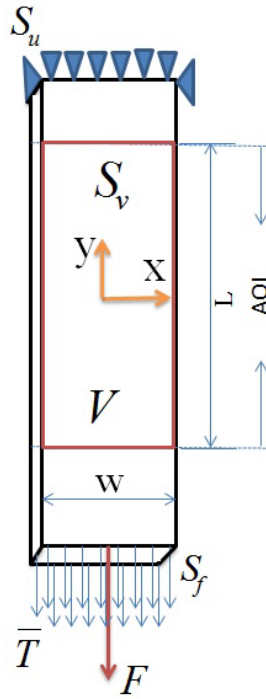$$b\int_{S_v} \sigma_{ij} : \varepsilon_{ij}^* ds_v = b\int_{L_f} \overline{T}.u_i^* dl_f \tag{4}$$

Figure 5. Schematic of specimen under tensile testing showing boundary conditions along with area of interest (AOI)

Also we have reaction force F as:

$$F(t_i) = b\int_{L_f} \overline{T} dl_f \tag{5}$$

On substituting Eq. (5) into Eq. (4) we will get

$$b\int_{S_v} \sigma_{ij} : \varepsilon_{ij}^* ds_v = F(t_i).u_i^* \tag{6}$$

This is the basic equation employed in this study for estimating the elastic properties using VFM. In a physical sense the above equation is nothing but the weak form of local equilibrium equations where the virtual work done by the external forces are equated to the internal virtual work done by actual stresses.

In the elastic region we have Young's modulus (E) and Poisson's ratio (v) as the unknowns. In order to introduce the unknown variables into Eq. (6), stress tensor is written in terms of the actual strain tensor by using the stress-strain relation given below.

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} \dfrac{E}{1-v^2} & \dfrac{vE}{1-v^2} & 0 \\ \dfrac{vE}{1-v^2} & \dfrac{E}{1-v^2} & 0 \\ 0 & 0 & \dfrac{E}{1+v} \end{pmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix} \tag{7}$$

On substitution of Eq. (7) into Eq. (6) and expanding one can write:

$$\frac{E}{1-v^2}\left(\int_{S_v} \varepsilon_{xx}\varepsilon_{xx}^* ds_v + \int_{S_v} \varepsilon_{yy}\varepsilon_{yy}^* ds_v \right) + \frac{vE}{1-v^2}\left(\int_{S_v} \varepsilon_{xx}\varepsilon_{yy}^* ds_v + \int_{S_v} \varepsilon_{yy}\varepsilon_{xx}^* ds_v \right) +$$

$$\frac{E}{1+v}\left(\int_{S_v} \varepsilon_{xy}\varepsilon_{xy}^* ds_v \right) = \frac{1}{b} F(t_i)u_i^* \tag{8}$$

Since the strain distributions obtained from DIC is not a continuous function of space coordinates but are discrete values at the centre of pixel subsets we can approximate the integration operation given in Eq. (8) with summation over the area as given in Eq. (9), Note that N, is the number of data points in the AOI and Δs_v is the area of the AOI.

$$\frac{E}{1-v^2}\left(\frac{1}{N}\sum_{i=1}^{N} \varepsilon_{xx}\varepsilon_{xx}^* \Delta s_v + \frac{1}{N}\sum_{i=1}^{N} \varepsilon_{yy}\varepsilon_{yy}^* \Delta s_v \right) + \frac{vE}{1-v^2}\left(\frac{1}{N}\sum_{i=1}^{N} \varepsilon_{xx}\varepsilon_{yy}^* \Delta s_v + \frac{1}{N}\sum_{i=1}^{N} \varepsilon_{yy}\varepsilon_{xx}^* \Delta s_v \right) +$$

$$\frac{E}{1+v}\left(\frac{1}{N}\sum_{i=1}^{N} \varepsilon_{xy}\varepsilon_{xy}^* \Delta s_v \right) = \frac{1}{b} F(t_i)u_i^* \tag{9}$$

Two independent virtual fields are selected for extracting E and v. It is to be noted that the chosen virtual field are kinematically admissible, which means both are continuous and satisfies the boundary conditions. The coordinate system used for this study is given in Fig.5.

The origin is at the center of AOI. Virtual displacement fields and the corresponding virtual strain fields are given below. Here the virtual displacement field – 1 is a virtual elongation in the loading direction and virtual displacement field – 2 is a symmetrical bending about the center of the specimen.

Virtual displacement field - 1

$$u_x^* = 0$$
$$u_y^* = \left(y - \frac{L}{2} \right)$$

$$\Rightarrow$$

$$\varepsilon_{xx}^* = \frac{\partial u_1^*}{\partial x} = 0$$
$$\varepsilon_{yy}^* = \frac{\partial v_1^*}{\partial y} = 1$$
$$\varepsilon_{xy}^* = \frac{1}{2} \left(\frac{\partial u_1^*}{\partial y} + \frac{\partial v_1^*}{\partial x} \right) = 0$$

Virtual displacement field – 2

$$u_x^* = x \left[y^2 - \left(\frac{L}{2} \right)^2 \right]$$
$$u_y^* = 0$$

$$\Rightarrow$$

$$\varepsilon_{xx}^* = y^2 - \left(\frac{L}{2} \right)^2$$
$$\varepsilon_{yy}^* = 0$$
$$\varepsilon_{xy}^* = xy$$

Substitution of these virtual fields in Eq. (9) will give two equations containing unknowns. For any load level falling inside elastic limit, one can substitute actual strains obtained using DIC technique and reaction load obtained from the load cell. Thus, two equations are generated using VFM and can be readily solved using matrix inversion for extracting the unknown parameters.

5. RESULTS AND DISCUSSIONS

Even though one load value and corresponding strain distribution in the elastic deformation region is enough to extract the parameters using VFM, in this study the parameters are extracted at five different load values to check the precision of the methodology. So the results presented are the average of VFM analysis at five different load levels. The results obtained using VFM are tabulated and compared with the reference values in the Table 1 and they are in good agreement with the results presented in the reference [2]. Readers are advised to refer [2] for all the stress strain curves obtained from the experiments.

The advantages of using VFM are: it does not require any full range elastic stress strain curve to extract required parameters, no curve fitting and curve smoothing are required like in the case of conventional characterization using stress strain curves, a single load value and the corresponding whole field surface strain distribution is enough to solve for the constitutive parameters. Since the parameters are extracted by solving a system of linear equations, it is computationally fast and easy to implement.

Table1. Comparison of the properties estimated using VFM and reference values from [2]

Extracted Properties	Values using VFM		Reference Value	
	Avg.	Std. Dev	Avg.	Std. Dev
Longitudinal modulus , E_{11} (GPa)	81.84	3.08	81.9	1.96
Transverse modulus, E_{22} (GPa)	6.84	0.13	6.15	0.32
In-plane Poisson's ratio (v_{12})	0.366	0.032	0.34	0.018

6. CONCLUSIONS

In this study virtual fields method was implemented for extracting the in-plane properties of CFRP laminates. Results were compared with the experimental data and it was found out that they are in good agreement. Surface strain distribution of tensile test specimens was measured using 3D DIC technique. In VFM, the elastic constants were estimated by utilizing the principle of virtual work and surface strain distribution. The effect of DIC parameters such as subset size, step size etc on the parameters estimated using VFM is not considered in this study. Also the effect of noise in the calculated surface strain is also not attempted. It can be concluded from the present study that the VFM is as accurate as the traditional parameter estimation methodology in predicting the in-plane constitutive parameters of CFRP laminate. Also it was found that VFM is fast, easy and computationally efficient than conventional techniques.

ACKNOWLEDGMENTS

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