CP Violation and Sterile Neutrino

Animesh Adhikari

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Department of Physics

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(Signature)

Animesh Adhikari

(Animesh Adhikari)

PH13M1003

(Roll No.)

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-Name and affiliation-

Examiner

-Name and affiliation-

Examiner

Jyoti Mohanty

-Name and affiliation-

Examiner

-Name and affiliation-

Examiner

-Name and affiliation-

Advisor

Vandana Sharma

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(V. SHARMA

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Dedication

My project work is dedicated to my beloved parents, I am here today for their blessing and inspiration.

Abstract

Neutrinos are the most mysterious and interesting among all elementary particles. It does not have charge and it is massless according to S.M. Recent experiments indicate that neutrinos can oscillate from one flavor to another flavor which indicates the fact that neutrinos have tiny mass. The determination of mixing parameter and mass of neutrino is an interesting question in high energy physics. Furthermore, neutrinos could also be related to the matter-antimatter asymmetry of the universe, understanding of which requires determination of CP violation in neutrino sector. In this project, I have discussed about CP violation in neutrino oscillation in 3+1 scheme and compared it with ordinary three flavour oscillation. The difference between oscillation probabilities of neutrinos and anti-neutrinos leads to CP violation. I investigated CP violation in neutrino sector in vacuum and matter using 1. three ordinary neutrinos(ν_e, ν_μ and ν_τ) and with 2. one sterile neutrino with three ordinary neutrinos (3+1 scheme).

Contents

Chapter 1

Introduction

In the universe neutrinos are the most surprising and mysterious known particle. Besides, neutrinos are second most abundant particle in the universe (first one is photon). Numbers of photon and neutrinos in Cosmic microwave background is 400 cm^3 and 300 cm^3 and Neutrinos can't feel strong interaction, only it feels weak interaction but extremely weakly. Trillions of neutrinos pass through our bodies each second without leaving a trace due to there interacting character. We immerse in a sea of neutrino and neutrino density inside and outside of our body is same but we can't feel neutrino because of their small interaction. For 1 MEV neutrino has the cross section of order 10^{-44} cm^{2} this can be as ,for one meter solid detector probability will be 10^{-44} or a probability 10^{-11} while travelling through the diameter of the earth. If we want to stop the neutrinos from sun then the barrier will be few light years. The sun glows due to fusion reactions which produces trillions of neutrino. These reaction produce one type of neutrino but surprisingly on their way to earth we get two other kinds of neutrino. The mass of neutrino is million times lighter than the lightest charged particle. The non-zero and small mass of neutrino arises the probability that they get mass from unknown physics. The character of neutrino and there small non-zero mass and how they change from one kind to another, leads us towards a new kind of phenomena in physics. This phenomena opens a window on physics beyond the S.M. The crucial question about neutrinos involves how many different kinds there are.

The discovery of neutrino oscillation was announced in '98 conference in Takayama. After this declaration there were full of excitements in this field, the era of sturm und drang(German: Storm and Stress). From LSND experiment, we know there may be more than the canonical three families. The Mini-BooNE experiment already hints that multiple kinds of neutrinos are likely to exist and says neutrino can transform one type(flavor) to another type and back again. The SNO experiment in Canada and the Super-Kamiokande experiment in Japan gave us about the evidence for neutrino masses and mixing. The NuMI/MINOS program and the CERN-to-Gran Sasso long-baseline neutrino program are analysing about neutrino mixing. Experiments such as Daya Bay in China, Double CHOOZ in France, T2K in Japan and the NuMI Off-Axis Electron Neutrino Appearance Experiment (NOvA) at Fermilab tell about CP violation in neutrino physics.

Like electrons, neutrinos are also a elementary particle but it is a neutral particle. In the Standard Model of particle physics, matter is made up of two types of elementary particles: hadrons, which feel the strong force that holds protons together in the nucleus, and quarks together in the proton, and leptons, which don't feel the strong force. Neutrinos, like electrons, are leptons. Neutrinos should not be confused with neutrons, a constituent of the atomic nucleus, or with neutralinos, hypothetical particles that may explain the dark matter content of the Universe.

The beta decay experiment prove the existence of neutrinos. An electron or a positron is emitted. This decay is mediated by the weak force. In β^- decay, an electron and electron anti-neutrino produce and in β^+ decay i.e. positron emission emits a positron and electron neutrino. In β decay the electron did not carry away all the energy that had been lost by the decaying nucleus. In 1930, Wolfgang Pauli, an Austrian theoretical physicist, suggested that the missing energy must be accounted for by an undetected neutral particle also produced in the decay. A few years later, the Italian-American theoretical physicist Enrico Fermi called Paulis particle a neutrino, and the name has stuck.

Neutrinos are spin half particle(intrinsic angular momentum is half).Neutrinos spin are anti-parallel to their momentum & for anti neutrinos it is parallel i.e. all ν are left-handed & all $\bar{\nu}$ are right-handed .

According to SM neutrinos are massless particle. But Pontecorvo and others many scientists proposed about "neutrino flavor oscillations". Before understand this phenomena, we consider mixing in quark sector. We know about the decay of a W boson(virtual) into a quark sector. It is as follow

$$W \longrightarrow \bar{u} + d', W \longrightarrow \bar{c} + s', W \longrightarrow \bar{t} + b' \tag{1.1}$$

This d' quark is linear superposition of another three quark states with charge $-1/3$ and have well defined mass. This prime state is connected with unprime state by the CabibboKobayashiMaskawa matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V^{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.2}$$

left matrix is Weak eigenstates(participate in decay process) & right matrix is Mass eigenstates we can say it is energy levels of the system.
This mixing phenomenon also occurred in lepton sector.

$$\begin{pmatrix} e' \\ \mu' \\ \tau' \end{pmatrix} = U^{PMNS} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \tag{1.3}$$

where U^{PMNS} is the PontecorvoMakiNakagawaSakata matrix. left matrix is flavor eigenstates)& right matrix is Mass eigenstates with definite mass.

In Homestake experiment Ray Davis want to detect solar neutrinos using a tank of cleaning fluid(C_2C_{14}) but he saw only 1/3 of the number he expected. This expected number calculated by John Bahcall and co-workers(the Standard Solar Model (SSM)). The other experiments i.e. Kamiokande (/Super-Kamiokande),SAGE and GALLEX/GNO also saw low number of solar neutrinos than expected. This is the Solar Neutrino Problem. The Sudbury Neutrino Observatory (SNO) said that the total number of neutrinos of all types was consistent with the SSM. Before that, all experiments could see only electron neutrinos . Basically, SNO used D_2O instead of H_2O. In a paper(2002), SNO wrote that the total number of νs of all types was consistent with the Standard Solar Model. But they could not tell if the other type was muon-type, tau-type or a mixture.

Weak interaction violates charge conjugation and parity separately. Therefore CP violation occurs in weak interaction. Therefore, if CPT is conserved and CP violation implies violation of T. In quantum field theory, CPT theorem states that all interactions should be invariant under the combined application of parity(P),charge conjugation(C) and time reversal (T). In all fundamental interactions, CPT symmetry is an exact symmetry.

Chapter 2

The Standard Model

The basic building blocks of the Universe are called fundamental particles, governed by four fundamental forces. In The standard Model we basically describe strong, electromagnetic & weak interaction in the framework of quantum field theory (gauge theory). SM describes the interaction between three leptons and three quarks through gauge fields (bosons $W^{\pm}$,Z^0 and γ in electroweak sector & Gluons for quarks sectors) It is local symmetry gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$ where the subscripts C, L and Y denote color, left-handed chirality and weak hypercharge, respectively. .In standard model we relate these three forces with the fundamental particles.

Neutrinos interact with two different ways one is neutral current interaction (neutrinos exchange Z^0 boson & neutrinos change their four momentum but their identity will be same) & another is charge current interaction (neutrinos exchange $W\pm$ boson).The interaction Lagrangian which describes coupling between the gauge bosons & the fermions.

The charged-current (CC) Lagrangian is

$$\mathcal{L}_{I,L}^{CC} = -\frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^{\mu}(1-\gamma^5)eW_{\mu} + H.c = -\frac{g}{2\sqrt{2}}J^{\mu}_{W,L}W_{\mu} + H.c \tag{2.1}$$

Where $J^{\mu}_{W,L} = \bar{\nu}_e\gamma^{\mu}(1-\gamma^5)e = 2\bar{\nu}_{eL}\nu^{\mu}e_L$ is the leptonic charged current &$W_{\mu} \equiv \dfrac{A_1^{\mu} - iA_2^{\mu}}{\sqrt{2}}$ The W^{μ} field annihilates W^+ bosons and creates W^- bosons.

$$\mathcal{L}_{I,L}^{\mathrm{NC}} = \mathcal{L}_{I,L}^{(z)} + \mathcal{L}_{I,L}^{(\gamma)} \tag{2.2}$$

Where

$$\mathcal{L}_{I,L}^{(\gamma)} = -ej^{\mu}_{\gamma,L}A_{\mu}j^{\mu}_{\gamma,L} = -\bar{e}\gamma^{\mu}e. \tag{2.3}$$

And

$$\mathcal{L}_{I,L}^{(z)} = -\frac{g}{2\sin\theta_W}j^{\mu}_{z,L}Z_{\mu} \tag{2.4}$$

Where

$$j^{\mu}_{z,L} = 2g_L^{\nu}\bar{\nu}_{eL}\gamma^{\mu}\nu_{eL} + 2g_L^{\nu}\bar{e}_L\gamma^{\mu}e_L + 2g_L^{\nu}\bar{e}_R\gamma^{\mu}e_R \tag{2.5}$$

In the SM, the masses of the W and Z gauge bosons, as well as those of the fermions, are generated through the Higgs mechanism. Higgs doublet can be written as

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^-(x) \end{pmatrix} \tag{2.6}$$

$\phi^+(x)$and $\phi^-(x)$ is charged & neutral complex scalar field.

The Higgs part of the Standard Model Lagrangian is

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \tag{2.7}$$

The potential

$$V(\Phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \tag{2.8}$$

will be bounded when λ will be positive. To understand the spontaneous breaking of the symmetry $SU(2)_L \times U(1)_Y \longrightarrow U(1)_Q, \mu^2$ should be negative. The potential will be minimum for $\phi^\dagger\phi = v^2/2$ where

$$v = \sqrt{(-\mu^2/2)} \tag{2.9}$$

Higgs field have VEV for neutral complex scalar field but In charged scalar field, fermion & vector boson field have zero value in the vacuum .

In the unitary gauge transformation, the Higgs doublet can be written as

$$\Phi(x) = 1/\sqrt{2}\begin{pmatrix} 0 \\ (v + H(x)) \end{pmatrix} \tag{2.10}$$

Using this we get a mass term in Higgs part of the Standard Model Lagrangian $m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$. Since we don't know the value of μ^2, that's why SM can not predict the value of Higgs mass.

Fermions get mass through this Higgs mechanism in the presence of Yukawa couplings with the Higgs doublet. For this mass term, left & right handed is coupled. Since in SM neutrinos don't have right handed component neutrinos are massless.

The HiggsleptonYukawa Lagrangian is

$$\mathcal{L} = -\sum_{\alpha,\beta=e,\mu,\tau} Y'^{l}_{\alpha\beta} L'_{\alpha L}\bar{\Phi} l'_{\beta R} + H.c$$

The Higgs doublet in the unitary gauge transformation is

$$\mathcal{L} = -((v+H)/\sqrt{2}) \sum_{\alpha,\beta=e,\mu,\tau} Y'^{l}_{\alpha\beta} l'_{\alpha L} \bar{l}'_{\beta R} + H.c$$

The term which is proportional to VEV v gives mass term for charged fermion & the 2nd term gives the trilinear couplings between the charged leptons and the Higgs boson. But to get definite mass we should diagonalize the Y'^{l} matrix.

Then HiggsleptonYukawa Lagrangian will be

$$\mathcal{L} = -((v+H)/\sqrt{2}) l'_L \bar{Y'^{l}} l'_R + H.c$$

After diagonalization(biunitary transformation) we get

$$\mathcal{L} = -((v+H)/\sqrt{2}) \bar{l_L} Y^{l} l_R + H.c$$

l_R and l_L right-handed and left-handed components with definite masses.

Again $l_\alpha \equiv l_{\alpha L} + l_{\alpha R}$ where $(l = e, \mu, \tau)$ & $(l_e \equiv e, l_\mu \equiv \mu, l_\tau \equiv \tau)$.
We get

$$\mathcal{L} = - \sum_{\alpha=e,\mu,\tau} \frac{y^l_\alpha v}{\sqrt{2}} \bar{l_\alpha} l_\alpha - \sum_{\alpha=e,\mu,\tau} \frac{y^l_\alpha}{\sqrt{2}} \bar{l_\alpha} l_\alpha H$$

Here the mass term is $m_\alpha = \dfrac{y^l_\alpha v}{\sqrt{2}}$

But in SM, the coefficients y^l_e, y^l_μ and y^l_τ are unknown parameters, so we cannot be predicted the masses of the charged leptons from SM.

Chapter 3

MASSIVE NEUTRINOS

3.1 Dirac Masses

Like leptons & quarks, Dirac particles get their masses through Higgs mechanism. In SM we only take left handed neutrino field but in minimally extended Standard Model we take right handed field also .This model is asymmetry in SM between lepton and quark sector since here we take right handed field of neutrino. This right handed field is basically different from left handed field because this is singlets under of $SU(3)_C \times SU(2)_L$ and $Y = 0$ (Hypercharge).This right handed field is called sterile since in weak interaction(or strong and electromagnetic interaction) they do not participate . They interact through gravitational interaction. That's why we called left handed field as active

$$\mathcal{L}_{H,L} = -\sum_{\alpha,\beta=e,\mu,\tau} Y'^{l}_{\alpha\beta}\bar{L}_{\alpha L}\Phi l'_{\beta R} - \sum_{\alpha,\beta=e,\mu,\tau} Y'^{\nu}_{\alpha\beta}\bar{L}_{\alpha L}\Phi\nu'_{\beta R} + H.c \tag{3.1}$$

Y'^{ν} Yukawa couplings matrix. After unitary gauge transformation & diagonalization we get HiggsleptonYukawa Lagrangian

$$\mathcal{L}_{H,L} = -\frac{v+H}{\sqrt{2}}(\sum_{\alpha=e,\mu,\tau} \bar{l_L}Y^l l_R + \bar{n_L}Y^\nu n_R) + H.c = -\frac{v+H}{\sqrt{2}}(\sum_{\alpha=e,\mu,\tau} y^l_\alpha \bar{l}_{\alpha L} l_{\alpha R} + \sum_{\alpha=e,\mu,\tau} y^\nu_k \bar{\nu}_{kL}\nu_{kR}) + H.c \tag{3.2}$$

Again, using the expression of Dirac charged lepton fields($l_{\alpha L} + l_{\alpha R}$) and Dirac neutrino fields ($\nu_k = \nu_{kL} + \nu_{kR}$) we get,

$$\mathcal{L} = -\sum_{\alpha=e,\mu,\tau} \frac{y^l_\alpha v}{\sqrt{2}} - \sum_{k=1,2,3} \frac{y^\nu_k v}{\sqrt{2}}\bar{\nu}_k\nu_k - \sum_{\alpha=e,\mu,\tau} \frac{y^l_\alpha}{\sqrt{2}}\bar{l_\alpha} l_\alpha \sum_{k=1,2,3} \frac{y^\nu_k}{\sqrt{2}}\bar{\nu}_k\nu_k \tag{3.3}$$

Neutrino masses are

$$m_k = \frac{y^\nu_k v}{\sqrt{2}} \quad (k = 1, 2, 3) \tag{3.4}$$

Therefore, neutrino masses are proportional to v(HiggsVEV). From experiments,we know that the masses of neutrinos are very small than other elementary particles. Using this mechanism we can only describe, but we can not explain about the very small values of the eigenvalues y^ν_k. Therefore, using the framework of the SM we can not explain the origin of the mass of quark and lepton. Masses of quark and lepton help us to understand beyond the Standard Model physics.

3.1.1 Lepton numbers

In Klein-Gordon equation probability density is negative due to the time derivative. It can be avoid by taking the wave equation as first order derivative in time. In this time Dirac propose a wave equation which is linear in first order of time derivative ,so it should be 1st order in space coordinates . According to Dirac formalism the Euler-Lagrangian equation for neutrino is

$$\begin{aligned} \mathcal{L}_{H,L} &= -(\frac{v+H}{\sqrt{2}})[\bar{l}_L Y^l l_R + \bar{\nu}_L U Y^\nu n_R] + H.c \\ &= -(\frac{v+H}{\sqrt{2}}) \sum_{\alpha=e,\mu,\tau} [y^l_\alpha \bar{l}_{\alpha L} l_{\alpha R} + \bar{\nu}_{\alpha L} \sum_{k=1}^{3} U_{\alpha k} y^\nu_k \nu_{kR}] + H.c \end{aligned} \tag{3.5}$$

In HiggsleptonYukawa Lagrangian due to neutrino part, the flavor lepton number is violated .This non-conserved flavor lepton number leads to neutrino oscillation . In case of massive neutrino though flavor lepton number is not conserved, but total lepton number is conserved. The conserved charge (Lepton number) distinguishes neutrino (L=1) from anti-neutrino(L=1).Therefore Dirac character in case of massive neutrinos tells that neutrino and anti-neutrino are different particle. Dirac neutrino has positive helicity & Dirac anti-neutrino has negative helicity . But in case of Majorana particle neutrino & anti-neutrino are same particle , they have same lepton number, therefore they violate the lepton number conservation.

3.1.2 Mixing

The Dirac neutrino mixing matrix depends on four physical parameters(three mixing angles and one CP violating phase). In quark sector five of the six phases in mixing matrix can be omitted by re-phasing the corresponding fields. It happens because it is invariant under the global phase transformations

$$\nu_{kL} \longrightarrow e^{i\phi_k} \nu_{kL} \quad , \quad \nu_{kR} \longrightarrow e^{i\phi_k} \nu_{kR} \quad (k = 1, 2, 3) \tag{3.6}$$

$$l_{\alpha L} \longrightarrow e^{i\phi_k} l_{\alpha L} \quad , \quad l_{\alpha R} \longrightarrow e^{i\phi_k} l_{\alpha R} \quad (\alpha = e, \mu, \tau) \tag{3.7}$$

The Dirac three-neutrino mixing matrix contains similarity with the quark mixing matrix. Therefore CP violation can be written in terms of the Jarlskog invarient

$$J = \mathfrak{IM}[U_{\mu 3} U_{e2} U^*_{\mu 2} U^*_{e3}] \tag{3.8}$$

For three-neutrino mixing this Jarlskog invarient can be written as

$$\mathfrak{IM}[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} = s_{\alpha\beta;kj} J \tag{3.9}$$

because

$$\sum_{k\neq j}\mathfrak{I}\mathfrak{M}[U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j} = 0 \quad (\alpha \neq \beta) \tag{3.10}$$

$$\sum_{\alpha\neq\beta}\mathfrak{I}\mathfrak{M}[U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j} = 0 \quad (k \neq j) \tag{3.11}$$

The coefficients $s_{\alpha\beta;kj} = \pm 1$ and

$$s_{\alpha\beta;kj} = -s_{\beta\alpha;kj} = -s_{\beta\alpha;jk} \tag{3.12}$$

Therefore the Dirac neutrino mixing matrix is

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{3.13}$$

Where $c_{ab} \equiv \cos\theta_{ab}$ and $s_{ab} \equiv \sin\theta_{ab}$.The range of mixing matrix(θ_{ab}) and the CP-violating phase(δ_{13})are $0 \leq \theta_{ab} \leq \pi/2$ and $0 \leq \delta_{13} < 2\pi$

Using this parametrization, the Jarlskog invariant is

$$\begin{aligned} J &= c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13}\sin\delta_{13} \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\cos\theta_{13}\sin 2\theta_{13}\sin\delta_{13} \end{aligned} \tag{3.14}$$

3.2 One-generation DiracMajorana mass term

The Dirac equation is

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 \tag{3.15}$$

Where $\psi = \psi_L + \psi_R$. For massless fermions m=0

$$i\gamma^\mu\partial_\mu\phi_L = 0 \tag{3.16}$$

$$i\gamma^\mu\partial_\mu\phi_R = 0 \tag{3.17}$$

These equations are called Weyl equation &ψ_L&ψ_R are Weylspinors. We know ν_L exists because it comes into SM. So the neutrino Lagrangian for ν_L is

$$\mathcal{L}^L_{mass} = \frac{1}{2}m_L\nu_L^\dagger\mathcal{C}^\dagger\nu_L + H.c \tag{3.18}$$

If ν_R exists then the neutrino Lagrangian is

$$\mathcal{L}^R_{mass} = -m_D\bar{\nu}_R\nu_L + H.c \tag{3.19}$$

Majorana mass term for ν_R is

$$\mathcal{L}^R_{mass} = \frac{1}{2} m_R \nu_R^\dagger \mathcal{C}^\dagger \nu_R + H.c \tag{3.20}$$

Hence the total DiracMajorana neutrino mass term is

$$\mathcal{L}^{L+R}_{mass} = \mathcal{L}^L_{mass} + \mathcal{L}^D_{mass} + \mathcal{L}^R_{mass} \tag{3.21}$$

We define the column matrix of left-handed chiral fields

$$N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \begin{pmatrix} \nu_L \\ \mathcal{C}\bar{\nu_R}^c \end{pmatrix} \tag{3.22}$$

$\mathcal{L}^{L+R}_{mass} = \frac{1}{2} N_L^T \mathcal{C}^\dagger M N_L + H.c$,where M is the symmetric mass matrix

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \tag{3.23}$$

The unitary matrix U, for the diagonalization of the mass matrix must be such that

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \tag{3.24}$$

and $m_k \geq 0$. Therefore the DiracMajorana mass term is

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \sum_{k=1,2} m_k \nu_{kL}^L \mathcal{C}^\dagger \nu_{kL} + H.c \\ &= -\frac{1}{2} \sum_{k=1,2} m_k \bar{\nu_k} \nu_k \end{aligned} \tag{3.25}$$

Where the Majorana massive neutrino field ν_k is

$$\nu_k = \nu_{kL} + \nu_{kL}^C = \nu_{kL} + \mathcal{C}\bar{\nu}_{kL}^\dagger \tag{3.26}$$

The oscillation between active and sterile neutrino is possible because active and sterile neutrino can be written as linear combination of massive neutrino field ν_{1L} and ν_{2L} .

$$\begin{aligned} \nu_L &= U_{11}\nu_{1L} + U_{12}\nu_{2L} \\ \nu_R^C &= U_{21}\nu_{1L} + U_{22}\nu_{2L} \end{aligned} \tag{3.27}$$

3.2.1 CP invariance

The real and symmetric mass matrix can be diagonalized through the unitary matrix transformation.

$$U = \mathcal{O}\rho \tag{3.28}$$

Where $\mathcal{O}$ is an 2×2 orthogonal matrix

$$\mathcal{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{3.29}$$

and ρ is diagonal matrix

$$\rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \tag{3.30}$$

and $\rho_K^2 = \pm 1$. For diagonalization we need the orthogonal matrix $\mathcal{O}$

$$\mathcal{O}^T M \mathcal{O} = \begin{pmatrix} m'_1 & 0 \\ 0 & m'_2 \end{pmatrix} \tag{3.31}$$

Where the eigenvalues of the mass matrix are m'_1 and m'_2.

$$m'_{2,1} = \frac{1}{2}[m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}] \tag{3.32}$$

and

$$\tan 2\theta = \frac{2m_D}{m_R - m_L} \tag{3.33}$$

ρ_k^2 is chosen in such way that,if m'_1 then ρ_1^2 is 1.

$$m_k = \rho_k^2 m'_k \tag{3.34}$$

Then we have

$$\begin{aligned} m_2 &= \frac{1}{2}[m_L + m_R + \sqrt{(m_L - m_R)^2 + 4m_D^2}] \\ m_1 &= \frac{1}{2}[m_L + m_R - \sqrt{(m_L - m_R)^2 + 4m_D^2}] \end{aligned} \tag{3.35}$$

CP invariance of the one-generation DiracMajorana mass term implies that the charged lepton has an imaginary CP parity.

The mixing is maximal when $\theta = \pi/4$ i.e. $m_L = m_R$ and the mass matrix reduce to

$$m'_{2,1} = m_L \pm m_R \tag{3.36}$$

If

$$m_L = m_R = 0 \tag{3.37}$$

then $m'_{2,1} = \pm m_D$ In this case a Dirac field is equivalent to two Majorana fields with the same mass and opposite CP parities and ν i.e. the Dirac field has the definite CP parity $\xi_\nu^{CP} = i$

If $|m_L|, m_R << m_D$ then

$$m'_{2,1} \simeq \frac{1}{2}[m_L + m_R] \pm m_D \quad (3.38)$$

Sincem'_1 is negative, we have $\rho_1^2 = -1$ and then

$$m'_1 \simeq m_D \pm \frac{1}{2}[m_L + m_R] \quad (3.39)$$

These two almost degenerate Majorana neutrinos are usually called pseudo-Dirac neutrinos because it is very difficult to distinguish them from a Dirac neutrino, which corresponds to a pair of degenerate Majorana neutrinos.The best way to reveal pseudo-Dirac neutrinos are activesterile neutrino oscillations due to the small squared-mass difference

$$\Delta m^2 \simeq m_D(m_L + m_R) \quad (3.40)$$

The oscillations occur with practically maximal mixing:

$$\tan 2\theta = \frac{2m_D}{m_R - m_L} \quad (3.41)$$

3.2.2 See-saw mechanism

If $m_D << m_R$, $m_L = 0$ then,

$$m'_1 \simeq -\frac{m_D^2}{m_R} \quad , \quad m'_2 \simeq m_R \quad (3.42)$$

Since m'_1 is negative therefore $\rho_1^2 = -1$ and

$$m_1 \simeq \frac{m_D^2}{m_R} \quad (3.43)$$

$$m_2 \simeq m_R \quad (3.44)$$

From this equation we get mass of $\nu_2(m_R)$ is greater than mass of ν_1 because mass of ν_1 is suppressed with respect to m_D by the small ratio $\frac{m_D^2}{m_R}$. This is the famous see-saw mechanism.

The small mixing angle implies that

$$\tan 2\theta = 2\frac{m_D}{m_R} << 1 \quad (3.45)$$

which implies that ν_1 is composed mainly of active ν_L and ν_2 is composed mainly of sterile ν_R

$$\nu_{1L} \simeq -i\nu_L \quad , \quad \nu_{2L} \simeq \nu_R^C \quad (3.46)$$

This mechanism gives us the explanation of the smallness of neutrino masses with respect to the masses of the other fermions in the SM, i.e. charged leptons and quarks. The Higgs mechanism generates The Dirac mass m_D which should be of the order of the charged lepton mass of the same generation or of the order of the up-like quark mass of the same generation. In type-II see-saw mechanism the left-handed Majorana mass is small but non-zero. Let us consider a general possibility

$$m_L << m_D << m_R \tag{3.47}$$

with

$$m_L = g\frac{m_D^2}{\mathcal{M}} \tag{3.48}$$

Where g means numerical coefficient and $\mathcal{M}$ means high-energy scale of new physics beyond the SM. This is mixed see-saw mechanism. So, one can say $|m_L| >> \frac{m_D^2}{m_R}$ is for type-I see-saw mechanism and for type-II see-saw mechanism $|m_L| << \frac{m_D^2}{m_R}$

3.3 Three-generation DiracMajorana mixing

We can add sterile right-handed neutrino fields N_s with three active left-handed neutrino fields ($\nu'_{eL}, \nu'_{\mu L}$ and $\nu'_{\tau L}$), with $s = s_1, s_2, ...s_{N_s}$. Therefore the DiracMajorana mass term is

$$\mathcal{L}_{mass}^{L+R} = \mathcal{L}_{mass}^{L} + \mathcal{L}_{mass}^{R} + \mathcal{L}_{mass}^{D} \tag{3.49}$$

with the symmetric mass matrix

$$M^{D+M} = \begin{pmatrix} m_L & m_D^T \\ m_D & m_R \end{pmatrix} \tag{3.50}$$

After the diagonalization of the DiracMajorana mass term, we get

$$\begin{aligned} \mathcal{L}_{mass}^{D+M} &= -\frac{1}{2}n_L^T\mathcal{C}^\dagger M n_L + H.c \\ &= \frac{1}{2}\sum_{k=1}^{N} m_K \nu_{kL}^T \mathcal{C} \nu_{kL} + H.c \end{aligned} \tag{3.51}$$

where

$$n = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_N \end{pmatrix} \quad and \quad \nu_k = \nu_{kL} + \nu_{kL}^C \tag{3.52}$$

3.4 See-saw mechanism

The DiracMajorana mass matrix with $M_L = 0$,

$$M^{D+M} = \begin{pmatrix} 0 & m_D^T \\ m_D & m_R \end{pmatrix} \tag{3.53}$$

and a right-handed Majorana mass matrix M_R ¿¿ the Dirac mass matrix M_D. If all the eigenvalues of M_R are greater than all the elements of M_D then the mass matrix can be diagonalized by blocks

$$W^T M^{D+M} W \simeq \begin{pmatrix} M_{light} & 0 \\ 0 & M_{heavy} \end{pmatrix} \tag{3.54}$$

where, $M_{light} \simeq -M^{D^T}(M^R)^{-1}M^D$ and $M_{heavy} \simeq M^R$

3.4.1 Quadratic See-Saw

When,

$$M^R = \mathcal{M} I \tag{3.55}$$

we get,

$$M_{light} \simeq \frac{-M^{D^T} M^D}{\mathcal{M}} \tag{3.56}$$

Where I is identity matrix($N_s \times N_s$) and $\mathcal{M}$ is the high-energy scale of new physics beyond the SM in which the total lepton number is violated. Therefore the light neutrino masses are

$$m_k = \frac{(m_k^D)^2}{\mathcal{M}} \quad (k = 1, 2, 3) \tag{3.57}$$

This is quadratic see-saw because

$$m_1 : m_2 : m_3 = (m_1^D)^2 : (m_2^D)^2 : (m_3^D)^2 \tag{3.58}$$

3.4.2 Linear See-Saw

If $N_s = 3$, then

$$M^R = \frac{\mathcal{M}}{\mathcal{M}^D} m_k^D \tag{3.59}$$

Therefore we get

$$M_{light} \simeq -\frac{\mathcal{M}^D}{\mathcal{M}} m_k^D \tag{3.60}$$

The light neutrino masses are

$$m_k = \frac{\mathcal{M}^D}{\mathcal{M}} m_k^D \tag{3.61}$$

This linear See-Saw because,

$$m_1 : m_2 : m_3 = (m_1^D) : (m_2^D) : (m_3^D) \tag{3.62}$$

Chapter 4

NEUTRINO OSCILLATIONS IN VACUUM

Due to very small mass of neutrino, there exist a quantum-mechanical phenomenon, i.e. flavor oscillation in neutrino sector. For quarks we can determine the mass in final state, but in case of neutrino it is practically impossible, so amplitude for the different neutrino should be added coherently. The amplitude for different mass component is a periodical function of time i.e. it evolve space and time. Suppose ν_e produced at a point A, B & C are two points which are near & far to the point A. We get more ν_e in B point than C point.We get more ν_μ or ν_τ at the point C. Mass of neutrino is about 1 eV, but we can detect neutrino with minimum energy 100 keV.

Neutrinos are detected in:

1. In neutral-current(NC) or Charged-current(CC) scattering process there exist an energy threshold. We get the lowest threshold value(0.23 MeV) for Charged-current(CC) scattering process in gallium solar neutrino experiments

$$^{71}Ga \longrightarrow ^{71} Ge + e^- \tag{4.1}$$

2. In elastic scattering process energy threshold is some MeVs. In the Super-Kamiokande solar neutrino experiment this threshold value is about 5 MeV.

A neutrino with flavor α& momentum $\vec{p}$ can be described by the flavor state

$$\mid \nu_\alpha >= \sum_k U^*_{\alpha k} \mid \nu_k > \tag{4.2}$$

Where,

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{4.3}$$

This is standard parametrization. $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and δ is the CP violating phase where $\delta \equiv \phi_{12} + \phi_{23} - \phi_{13}$. If $\phi_{12} = \phi_{23} = 0$, then δ corresponds to the Diracphase relevant for neutrino oscillations.

This neutrino can be produced together with a anti-lepton l^+ or with a lepton l^- in CC weak interaction process. In this equation there are no limitation on the number of massive neutrinos. It can be three(because number of active flavour neutrino is three ν_e, ν_μ, ν_τ) or it can be greater than three(N). Here for the additional massive neutrinos(N-3), flavor neutrinos are sterile. Sterile neutrinos do not participate in weak interactions but interact only through gravitational interactions. For massive neutrinos $\mid \nu_k >$, $H \mid \nu_k >= E_k \mid \nu_k >$. Where $E_k = \sqrt{\vec{p}^2 + m_k^2}$. This definite momentum $\vec{p}$ is same for neutrinos of all flavor . This is called the equal momentum assumption.

Then the flavour state at time t can be written as

$$|\nu_\alpha(t)\rangle = \sum_k U^*_{\alpha k} \exp -iE_k t \mid \nu_k > \tag{4.4}$$

The amplitude $\nu_{\alpha \longrightarrow \beta}$ at time t=0 is

$$A_{\nu_\alpha \longrightarrow \nu_\beta}(t) \equiv < \nu_\beta | \nu_\alpha(t) >= \sum_k U^*_{\alpha k} U_{\beta k} \exp(-iE_k t) \tag{4.5}$$

Again $E_k - E_j \simeq \dfrac{\triangle m^2_{kj}}{2E}$.
Therefore transition probability is

$$\begin{aligned} P_{\nu_\alpha \longrightarrow \nu_\beta}(t) &= |A_{\nu_\alpha \longrightarrow \nu_\beta}(t)\ |^2 \\ &= \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta k} \exp(-i(E_k - E_j)t) \\ &= \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta k} \exp(-i\frac{\triangle m^2_{kj}}{2E} t) \end{aligned} \tag{4.6}$$

In neutrino oscillation experiments, we know the length between the source and the detector &

neutrinos are ultra-relativistic particles, so we can replace t as L(c=1).This is called the light-ray approximation.

$$P_{\nu_\alpha \longrightarrow \nu_\beta}(t) = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta k} \exp{(-i\frac{\triangle m^2_{kj}}{2E}L)} \tag{4.7}$$

Therefore phase of neutrino oscillation is

$$\Phi_{kj} = \frac{\Delta m^2_{kj} L}{2E} \tag{4.8}$$

Therefore the phases depends on the squared-mass differences $\Delta m^2_{kj}, L\&E$. The amplitude of the oscillations depends on the the mixing matrix U. But the quadratic product($U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta k}$) is invariant under this transformation

$$U_{\alpha k} \longrightarrow e^{i\psi_\alpha} U_{\alpha k} e^{i\phi_k} \tag{4.9}$$

& the quadratic product is independent on the specific parametrization of the mixing matrix.
Again

$$U_{\alpha k} = U^D_{\alpha k} e^{i\lambda_k} \tag{4.10}$$

$U^D_{\alpha k}$ is for Dirac case &$e^{i\lambda_k}$ is for Majorana case. Therefore this rephasing is free from the Majorana phases.
We can write oscillation probability as

$$P_{\nu_\alpha \longrightarrow \nu_\beta}(L,E) = \delta_{\alpha\beta} - 4\sum_{k>j} \mathcal{R}e[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}] \sin^2(\frac{\Delta m^2_{kj} L}{2E}) + 2\sum_{k>j} \mathcal{I}m[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}] \sin^2(\frac{\Delta m^2_{kj} L}{2E}) \tag{4.11}$$

The oscillation length is defined by $L^{osc}_{kj} = \frac{4\pi E}{\Delta m^2_{kj}}$. Δm^2_{kj} generates phase difference 2π after a distance L^{osc}.
For antineutrino case($\bar{\nu_\alpha} \longrightarrow \bar{\nu_\beta}$), the oscillation probability is

$$P_{\nu_\alpha \longrightarrow \nu_\beta}(L,E) = \delta_{\alpha\beta} - 4\sum_{k>j} \mathcal{R}e[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}] \sin^2(\frac{\Delta m^2_{kj} L}{2E}) - 2\sum_{k>j} \mathcal{I}m[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}] \sin^2(\frac{\Delta m^2_{kj} L}{2E}) \tag{4.12}$$

In case of two neutrino mixing we take two massive neutrino. This is a simplified case, here few parameters is used than three-neutrino mixing. Flavour neutrino $\nu_\alpha \& \nu_\beta$ can be (ν_e, ν_μ) or (ν_μ, ν_τ) or (ν_e, ν_τ). Two flavor neutrino states can be written as a linear superposition of two massive neutrino states $\nu_1 \& \nu_2$. The mixing matrix is

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{4.13}$$

Where $0 \leq \theta \leq \frac{\pi}{2}$Then

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{4.14}$$

The squared-mass difference

$$\Delta m^2 \equiv \Delta m_{21}^2 \equiv m_2^2 - m_1^2 \tag{4.15}$$

Therefore, when $\alpha \neq \beta$

$$P_{\nu_\alpha \longrightarrow \nu_\beta}(L, E) = \sin^2(2\theta)\sin^2(\frac{\Delta m^2 L}{4E}) \tag{4.16}$$

When, $\alpha = \beta$,

$$P_{\nu_\alpha \longrightarrow \nu_\alpha}(L, E) = 1 - \sin^2(2\theta)\sin^2(\frac{\Delta m^2 L}{4E}) \tag{4.17}$$

The oscillation length

$$L^{osc} = \frac{4\pi E}{\Delta m^2} \tag{4.18}$$

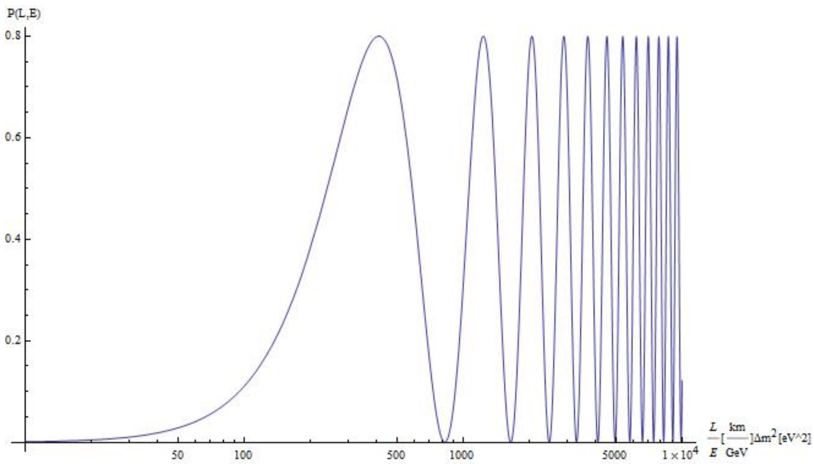


Figure 4.1: Unaveraged transition probability for $\nu_\alpha \longrightarrow \nu_\beta$ transition

If CPT is conserved then CP violation is equivalent to T violation and this CP violation is the difference between neutrinos and anti-neutrinos oscillation probabilities

$$\begin{aligned} \Delta P_{\nu\bar{\nu}\alpha\beta} &\equiv P_{\nu_\alpha \longrightarrow \nu_\beta} - P_{\bar{\nu_\alpha} \longrightarrow \bar{\nu_\beta}} \\ &= -16 J_{\alpha\beta} \sin\Delta_{12} \sin\Delta_{23} \sin\Delta_{31} \end{aligned} \tag{4.19}$$

Where $\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E}$ and

$$J_{\alpha\beta} = \Im(U_{\alpha 1} U_{\alpha 2}^* U_{\beta 1}^* U_{\beta 2}) = \pm J \tag{4.20}$$

$$
\begin{aligned}
J &= c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13}\sin\delta_{13} \\
&= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\cos\theta_{13}\sin 2\theta_{13}\sin\delta_{13}
\end{aligned} \tag{4.21}
$$

But we can not take precise value of L(source & detector distance) & the neutrino energy E to measure the oscillation probabilities for neutrinos because in experiments we have some spatial uncertainty, while measuring the distance between source & detector. The neutrino source has an energy spectrum & also detector has some limitation. So, it will be better to take an average value for the distance L and the energy E.

$$
< P_{\nu_\alpha \longrightarrow \nu_\beta}(L,E) >= \frac{1}{2}\sin^2(2\theta)[1- < \cos(\frac{\Delta m^2 L}{2E}) >] = \frac{1}{2}\sin^2(2\theta)[1-\cos(\frac{\Delta m^2}{2} < \frac{L}{E} >)\exp - \frac{1}{2}(\frac{\Delta m^2 \sigma_{L/E}}{2})^2] \tag{4.22}
$$

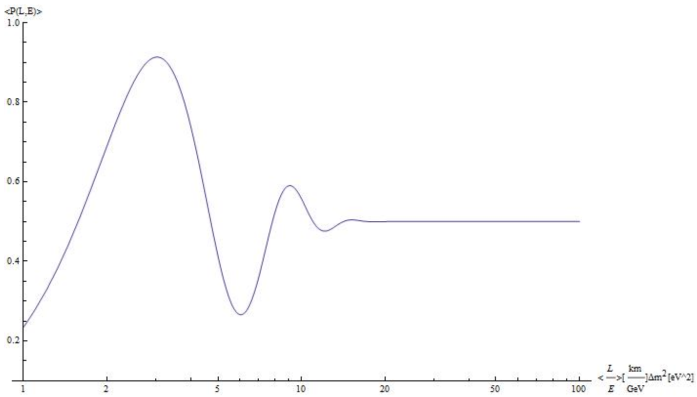


Figure 4.2: Average transition probability for $\nu_\alpha \longrightarrow \nu_\beta$ transition

In three-neutrino mixing, CP asymmetries equivalent to the T asymmetries. CP asymmetries are

$$
A_{\alpha\beta}^{CP} = 4J\sum_{k>j} s_{\alpha\beta;kj}\sin(\frac{\Delta m^2 L}{2E}) \tag{4.23}
$$

In trimaximal mixing(hypothetical case),all the elements of the mixing matrix have the same absolute value equal to $\frac{1}{\sqrt{3}}$ and $\theta_{13} = \theta_{23} = \pi/4$, $s_{13} = \frac{1}{\sqrt{3}}$, $\sin\delta_{13} = \pm 1$

Here the CP asymmetries are

$$
A_{e,\mu}^{CP}(L,E) = \pm\frac{2}{3\sqrt{3}}[\sin(\frac{\Delta m_{21}^2 L}{2E}) - \sin(\frac{\Delta m_{31}^2 L}{2E}) + \sin(\frac{\Delta m_{32}^2 L}{2E})] \tag{4.24}
$$

Based on neutrino oscillation phenomena, we extend the SM. This extension is required in leptonic sector, so that we can say about the neutrino mass. But this model can not say why neutrino sector has large mixing angles & small mass compared to the quark sector. To know more about

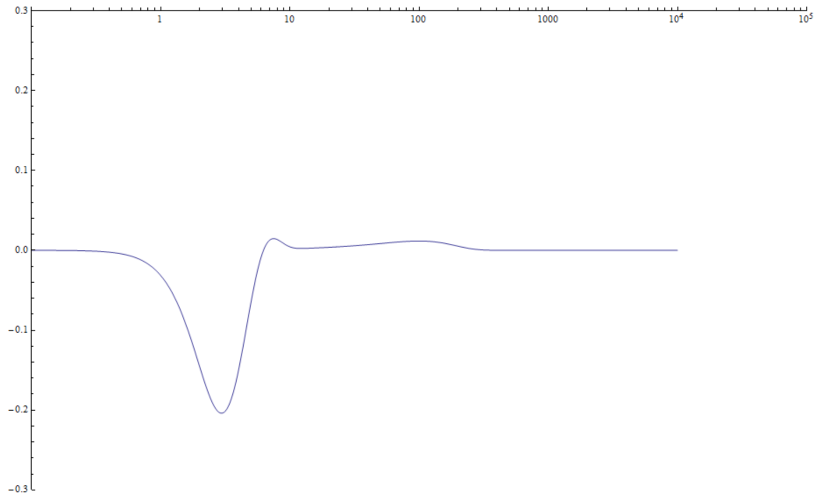
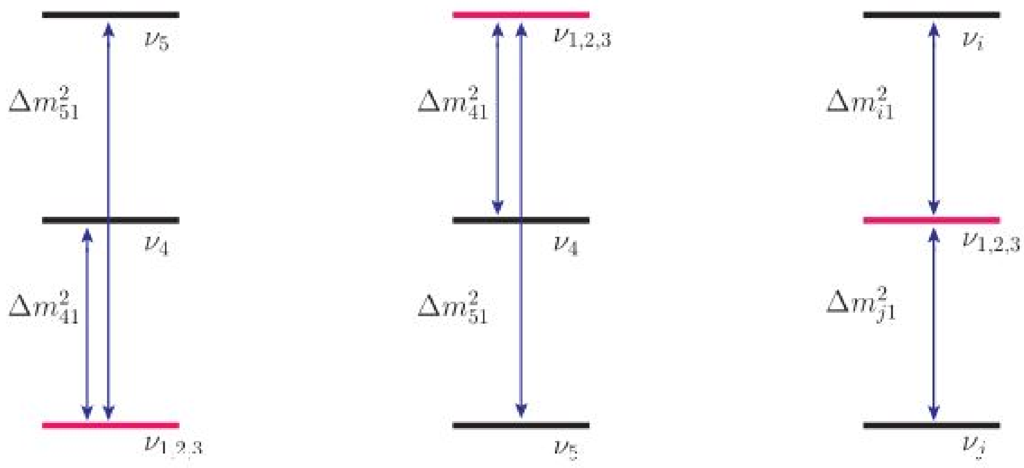


Figure 4.3: The pure CP violation effect $A^{CP}_{e,\mu}(L, E)$ with respect to L($E = 1$GeV)

neutrino phenomenology we should find a more complete theory which can tell the other unexpected properties of neutrinos. Sterile neutrinos do not participate in strong & weak interaction i.e they can not exchange $W^{\pm}\&Z^0$ boson while they interact, it interacts through gravitational interaction. Not only active to active, the neutrino oscillation can happen between active to sterile neutrinos. It is experimentally observed in the LSND , MiniBooNE and reactor experiments. Basically this sterile neutrino states are additional states beyond the standard known flavour states $(\nu_e, \nu_\mu, \nu_\tau)$.The existence of sterile neutrinos could open a powerful window in beyond the standard model. They belong many Beyond Standard Model theories. At this stage , we can not well predict the mass scale for sterile neutrinos.In many Beyond Standard Model they introduced as gauge singlets. These non-interacting flavor states (sterile neutrinos)are connected via extended mixing matrix with extra mixing angles and CP violating phases.In experiments when a active flavor neutrino state transform to a sterile neutrino state it disappears, this is the experimental evidence for the sterile neutrino state. But theory can not predict the no of neutrino. Depending on the no of states we say it as a (3 + N) model. We can implement sterile neutrinos in the measurements of the radiation density in the early universe & here mass of sterile neutrinos are smaller than about 1 eV. In cosmic microwave background contain neutrino mass much smaller than 1 eV. The model of sterile neutrinos are (3+1),(3+2) e.t.c. Here ν_e, ν_μ, ν_τ states have mass less then 1 eV& one or two additional massive neutrinos states have masses of the order of 1 eV. For sterile neutrinos the range of Δm^2 is 0.1 to $10eV^2$.In four neutrino model(3+1 model) we take 3 flavor state $(\nu_e, \nu_\mu, \nu_\tau)$, one sterile neutrinos state & with their four massive neutrino state. In this state we have to consider 3 different mass-squared difference i.e. $\Delta m^2_{solar}, \Delta m^2_{atm} \& \Delta m^2_{LSND}$. $\Delta m^2_{solar} = (10^{-11} - 10^{-5})eV^2, \Delta m^2_{atm} = (10^{-3} - 10^{-2})eV^2 \& \Delta m^2_{LSND} = (0.3 - 10)eV^2$.

The flavor eigenstates and mass eigenstates of neutrino are related by unitary matrix U.

$$\nu_\alpha = \sum_{i=1}^{4} U_{\alpha i} \nu_i \tag{4.25}$$



where $\alpha = e, \mu, \tau, s$

$$U_{\alpha i} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \tag{4.26}$$

The neutrino oscillation probability for $\nu_\alpha \longrightarrow \nu_\beta$ is

$$P_{\nu_\alpha \longrightarrow \nu_\beta}(L,E) = \delta_{\alpha\beta} - 4\sum_{k>j} \mathcal{R}e[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}] \sin^2(\frac{\Delta m^2_{kj} L}{2E}) + 2\sum_{k>j} \mathcal{I}m[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}] \sin(\frac{\Delta m^2_{kj} L}{2E}) \tag{4.27}$$

Exchange of U $\& U^* (U \longleftrightarrow U^*)$ we can get oscillation probability for the antineutrinos. The probability difference is

$$\Delta P_{\alpha\beta} \equiv P_{\nu_\alpha \longrightarrow \nu_\beta} - P_{\bar{\nu}_\alpha \longrightarrow \bar{\nu}_\beta} \equiv \sum_{k>j} \mathcal{I}m[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}] \sin(\frac{\Delta m^2_{kj} L}{2E}) \tag{4.28}$$

This is a direct measure of the genuine CP-violation effect in the neutrino oscillation in vacuum. In short baseline(SBL) experiments $(L/E \sim 1[km/GeV])$ the survival probability i.e. in the appearance channel in 3+1 experiments is

$$P^{3+1}_{CP}(\nu_\alpha \longrightarrow \nu_\beta) \simeq 4 \mid U_{\alpha 4} \mid^2 \mid U_{\beta 4} \mid^2 \sin^2(\frac{1.27 \Delta m^2_{41} L}{E}) \tag{4.29}$$

Since, Δ_{21} and $\Delta_{43} << 1$ and $\Delta_{41}, \Delta_{42}, \Delta_{31}, \Delta_{32} \simeq 1$. And the disappearance channel is

$$P^{3+1}_{CP}(\nu_\alpha \longrightarrow \nu_\alpha) \simeq 1 - 4(1 - \mid U_{\alpha 4} \mid^2) \mid U_{\alpha 4} \mid^2 \sin^2 \left(\frac{1.27 \sin^2 \left(\frac{1.27 \Delta m^2_{41} L}{E} \right)}{E} \right) \tag{4.30}$$

where $\alpha, \beta = e, \mu, \tau, s$. Therefore oscillation probability in SBL experiments depends on large square mass difference Δm^2_{41}

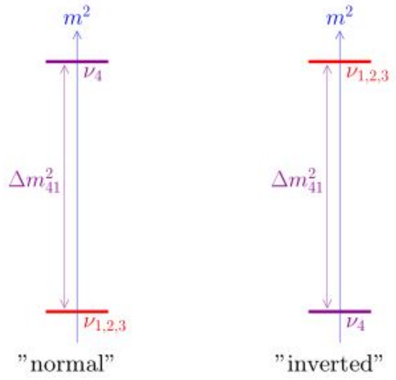


Figure 4.4: 3+1 schemes

The Bugey and CHOOZ experiments give strong upper limit to the $\nu_e \longrightarrow \nu_e$ disappearance two-family equivalent mixing angle. In two families,

$$P^{3+1}_{CP}(\nu_e \longrightarrow \nu_e) = 1 - \sin^2(2\theta)_{exp}\sin^2(\frac{\Delta m^2_{34}L}{4E}) \tag{4.31}$$

with $10^{-3} \leq \sin^2(2\theta)_{LSND} \leq 1$

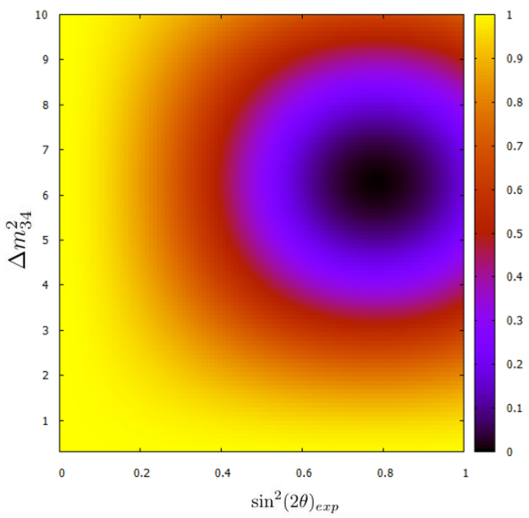


Figure 4.5: Heat map of $P^{3+1}_{CP}(\nu_e \longrightarrow \nu_e)$ equation

In the long-baseline experiment($L/E \sim 10-100[km/GeV]$) the probability difference in vacuum is

$$\Delta P_{\alpha\beta} \simeq 4Im(U^*_{\alpha 4}U_{\alpha 3}U^*_{\beta 3}U_{\beta 4})\sin(2\Delta_{43}) \tag{4.32}$$

The CP-conjugate channels for $\nu_\mu \longrightarrow \nu_e$ neutrino oscillations in the long-baseline experiments is

$$\Delta P(\nu_\mu \longrightarrow \nu_e) \simeq 4c_{02}s_{02}c_{03}^2 s_{03}s_{12}c_{13}s_{13}\sin\delta_1 \sin(\frac{\Delta m_{43}^2 L}{2E}) \tag{4.33}$$

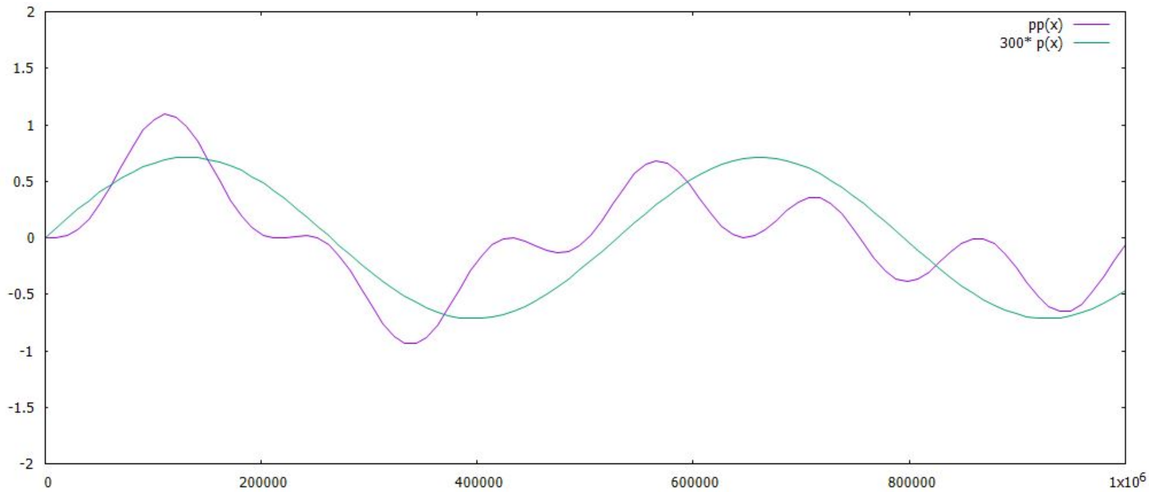


Figure 4.6: In this Fig. we show pure CP violation effect in vacuum in three neutrino mixing scheme(pp(x)) and the 3+1 scheme (p(x))as a function of baseline in $\Delta P(\nu_\mu \longrightarrow \nu_e)$ oscillation for the energy E = 1.2 GeV & for the typical parameter sets $s_{02} = s_{03} = 0.11, c_{02} = c_{03} = 0.994, s_{12} = 0.91, c_{12} = 0.415, s_{13} = 0.67, c_{13} = 0.742, s_{01} = s_{23} = \frac{1}{\sqrt{2}}, \delta_{01} = \delta_{02} = \delta_{03} = \delta_{12} = 0, \delta_2 = \pi/2 \Delta m_{21}^2 = 2.5 \times 10^{-3}, \Delta m_{32}^2 = 0.3, \Delta m_{43}^2 = 2.5 \times 10^{-3}$

Chapter 5

NEUTRINO OSCILLATIONS IN MATTER

5.1 Hamiltonian of neutrino in matter

When neutrinos propagate in matter they face a potential due to the coherent forward elastic scattering with electrons and nucleons i.e. with the particles in matter. This potential is equivalent to an index of refraction. This mixing changes the mixing angle in vacuum and therefore modifies the mixing of neutrinos. In 1978, L. Wolfenstein discovered about this mixing.

In 1985 S.P. Mikheev and A.Yu. Smirnov discovered about resonant flavor transitions when neutrinos propagate in a medium with varying density. When neutrino propagate through matter there is a region along the neutrino path in which the effective mixing angle passes through the maximal mixing value of $\frac{\pi}{4}$. This is MSW mechanism. This can explain the flavor conversion of solar neutrinos i.e. the vacuum mixing angle which is large but not maximal. Neutrinos in matter are affected not only by coherent forward elastic scattering, but it can also scattered by incoherent scatterings. But this effect is very small, so we can neglect it. The cross-section of weak interaction in case of neutrino in center of mass frame is

$$\sigma_{cm} \sim G_F.s \tag{5.1}$$

Here, s means Lorentz invariant Mandelstam variable which is square of the total energy in COM frames. $s = 2EM$, E means neutrino energy and mass of target particle is M.

$$\begin{aligned} \sigma_{lab} &\sim G_F EM \\ &\sim 10^{-38} cm^2 \frac{EM}{GeV^2} \end{aligned} \tag{5.2}$$

In matter (number density N) the mean free path of neutrino is

$$\begin{aligned} l &\sim \frac{1}{N\sigma} \\ &= \frac{10^{38} cm}{N cm^3 \frac{EM}{GeV^2}} \end{aligned} \tag{5.3}$$

For $E = 1GeV$ and number density $N \sim \frac{N_A}{cm^3} \sim 10^{24}/cm^3$ the mean free path will be

$$l_{matter} \sim \frac{10^{14} cm}{E/GeV} \tag{5.4}$$

Earth (diameter is $10^9 cm$) can be a opaque to neutrinos if there energy is more than 10^5GeV. In case of solar neutrino with energy about 1 MeV mean free path will be about 10^{17} cm which is nearly equal to 0.11 light years. But for nucleon density $N \sim 10^{12} \frac{N_A}{cm^3}$ with $E \sim 1MeV$ mean free path will be about 1 km. Due to coherent scattering evolution equation of active flavor neutrinos is affected.

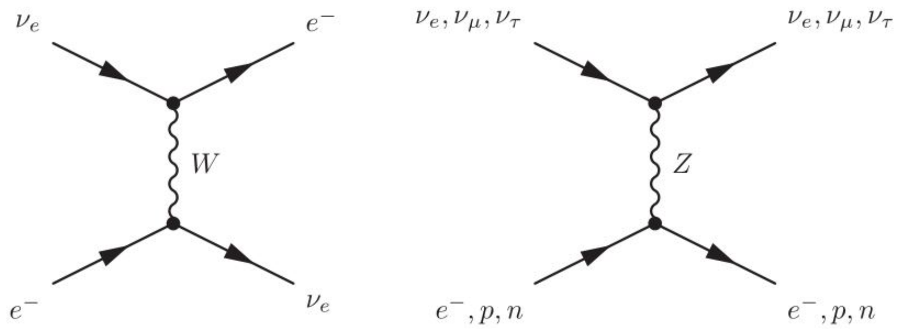


Figure 5.1: The Feynman diagrams of CC and NC scattering

The effective CC Hamiltonian for an electron neutrino with charge-current potential V_{CC} is

$$\mathcal{H}_{eff}^{CC} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_e(x)\gamma^\rho(1-\gamma^5)e(x)][\bar{e}(x)\gamma_\rho(1-\gamma^5)\nu_e(x)]$$

If we apply the Fierz transformation then

$$\mathcal{H}_{eff}^{CC} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_e(x)\gamma^\rho(1-\gamma^5)\nu_e(x)][\bar{e}(x)\gamma_\rho(1-\gamma^5)e(x)]$$

The average of the effective Hamiltonian over the electron background in the rest frame of the medium is given by

$$\bar{\mathcal{H}}_{eff}^{(CC)}(x) = V_{CC}\bar{\nu}_{eL}(x)\gamma^0\nu_{eL}(x) \tag{5.5}$$

Where,

$$V_{CC} = \sqrt{2} G_F N_e \tag{5.6}$$

The effective NC Hamiltonian is

$$\mathcal{H}_{eff}^{NC}(x) = \frac{G_F}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} [\bar{\nu}_\alpha(x)\gamma^\rho(1-\gamma^5)\nu_\alpha(x)] \sum_f [\bar{f}(x)\nu_\rho(g_V^f - g_A^f\gamma^5)f(x)] \tag{5.7}$$

Where N_f is density of medium of fermions f. The neutral-current potential due to coherent interaction is

$$V_{NC}^f = \sqrt{2}G_F N_f g_V^f \tag{5.8}$$

In low temperature and density and electrical neutrality implies, density of electrons and protons are equal, so in neutral current potential neutron's contribution exist only and others contribution will be eliminate.So,

$$V_{NC} = -\frac{1}{2}\sqrt{2}G_F N_n \tag{5.9}$$

So, the effective neutrino potential Hamiltonian in low temperature and density is

$$\mathcal{H}_{eff}(x) = \sum_{\alpha=e,\mu,\tau} V_\alpha \bar{\nu}_{\alpha L}(x)\gamma^0\gamma_{\alpha L}(x) \tag{5.10}$$

with the potentials

$$V_\alpha = V_{CC}\delta_{\alpha e} + V_{NC} = \sqrt{2}G_F(N_e\delta_{\alpha e} - \frac{1}{2}N_n) \tag{5.11}$$

Where,

$$\sqrt{2}G_F \sim 7.63 \times 10^{-14}\frac{eVcm^3}{N_A} \tag{5.12}$$

5.2 Evolution of neutrino flavors

For ultrarelativistic left-handed neutrino ($\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$) with momentum $\vec{p}$ the flavor state is

$$|\nu_\alpha >= \sum_k U_{\alpha k}^* |\nu_k > \tag{5.13}$$

The total Hamiltonian in matter is

$$H = H_0 + H_I \tag{5.14}$$

Where,

$$H_0|\nu_k >= E_k|\nu_k >$$
$$H_I|\nu_k >= V_\alpha|\nu_\alpha$$

The evolution equation of a neutrino state in the Schrdinger picture is

$$i\frac{d}{dt}|\nu_\alpha(t) >= H|\nu_\alpha(t) > \tag{5.15}$$

Where $|\nu_\alpha(0)>=|\nu_\alpha>$ The probability of finding neutrino with flavor α after some time t is

$$P_{\nu_\alpha \longrightarrow \nu_\beta} = |\psi_{\alpha\beta}(t)|^2 \tag{5.16}$$

Where $\psi_{\alpha\beta}(t) =<\nu_\beta|\nu_\alpha(t)>$ and $\psi_{\alpha\beta}(0) = \delta_{\alpha\beta}$ Now, the time evolution equation is

$$i\frac{d}{dt}|\psi_{\alpha\beta}(t) = \sum_\eta (\sum_k U_{\beta k} E_k U^*_{\eta k} + \delta_{\beta\eta} V_\beta)\psi_{\alpha\eta}(t)) \tag{5.17}$$

In case of ultra-relativistic neutrinos,

$$E_k \simeq E + \frac{m_k^2}{2E}, p \simeq E, t \simeq x \tag{5.18}$$

Where x is distance between source and detector. Using these approximations we get,

$$i\frac{d}{dt}\psi_{\alpha\beta}(x) = (p + \frac{m_1^2}{2E} + V_{NC})\psi_{\alpha\beta}(x) + \sum_\eta (\sum_k U_{\beta k}\frac{\Delta m^2_{k1}}{2E} U^*_{\eta k} + \delta_{\beta e}\delta_{\eta e} V_{CC})\psi_{\alpha\eta}(x)) \tag{5.19}$$

The 1st term in RHS is irrelevant for the flavor transitions, it can be eliminated by the phase shift. The evolution equation is

$$i\frac{d}{dt}\psi_{\alpha\beta}(x) = \sum_\eta (\sum_k U_{\beta k}\frac{\Delta m^2_{k1}}{2E} U^*_{\eta k} + \delta_{\beta e}\delta_{\eta e} V_{CC})\psi_{\alpha\eta}(x)) \tag{5.20}$$

It can be written as

$$i\frac{d}{dx}\Psi_\alpha = \mathcal{H}_F \Psi_\alpha \tag{5.21}$$

Where, the effective Hamiltonian matrix

$$\mathcal{H}_F = \frac{1}{2E}(U\mathcal{M}^2 U^\dagger + A) \tag{5.22}$$

For three-neutrino mixing case,

$$\Psi_\alpha = \begin{pmatrix} \psi_{\alpha e} \\ \psi_{\alpha\mu} \\ \psi_{\alpha\tau} \end{pmatrix} \quad \mathcal{M}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} \quad \mathcal{A} = \begin{pmatrix} A_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5.23}$$

Where,

$$A_{CC} \equiv 2EV_{CC} = 2\sqrt{2}EG_F N_e \tag{5.24}$$

5.3 The MSW effect

Due to same matter potential

$$i\frac{d}{dt}\begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\Delta m^2 \cos 2\theta + A_{CC} & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - A_{CC} \end{pmatrix} \tag{5.25}$$

The effective Hamiltonian matrix

$$\mathcal{H}_F = \frac{1}{4E}\begin{pmatrix} -\Delta m^2 \cos 2\theta + A_{CC} & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - A_{CC} \end{pmatrix} \tag{5.26}$$

This matrix can be diagonalized by the orthogonal transformation

$$U_M^T \mathcal{H} U_M = \mathcal{H}_M \tag{5.27}$$

where

$$\mathcal{H}_M \frac{1}{4E} \quad (-\Delta m_M^2, \Delta m_M^2) \quad , U_M = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \tag{5.28}$$

and

$$\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\theta - A_{CC})^2 + (\Delta m^2 \sin 2\theta)^2} \tag{5.29}$$

The effective mixing angle in matter is

$$\tan 2\theta_M \frac{\tan 2\theta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\theta}} \tag{5.30}$$

The interesting new phenomenon, discovered by Mikheev and Smirnov in 1985 [801, 802] (see also the lucid explanation in Ref. [222]), is that there is a resonance when A CC becomes equal to

There is a resonance when

$$A_{CC}^R = \Delta m^2 \cos 2\theta \tag{5.31}$$

which corresponds to the electron number density

$$N_e^R = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2} E G_F} \tag{5.32}$$

$$\cos 2\theta_m = \frac{\Delta m^2 \cos 2\theta - A_{CC}}{2 m_M^2} \tag{5.33}$$

For constant matter density i.e. $\frac{d\theta_M}{dx} = 0$ the transition probability will be

$$P_{\nu_\alpha \longrightarrow \nu_\beta}(x) = \sin^2(2\theta_M)\sin^2(\frac{\Delta m_M^2 L}{4E}) \tag{5.34}$$

Therefore in matter the oscillation length is

$$L_{osc}^M = \frac{4\pi E}{\Delta m_M^2} \tag{5.35}$$

In three neutrino mixing matrix case, nature has chosen two small parameters, $\sin^2\theta_{13} \leq 0.04$ and $\Delta m_{21}^2/\Delta m_{31}^2 \approx 0.03$ this allows us to factorize the three neutrino case into a product of two neutrino cases and therefore the individual Δm^2 in matter become

$$
\begin{aligned}
&\Delta m_{31}^2 \mid_N \approx \Delta m_{31}^2 \mid_N -2\sqrt{2} G_F N_e E \\
&\Delta m_{21}^2 \mid_N \approx -2\sqrt{2} G_F N_e E \\
&\Delta m_{32}^2 \mid_N \approx \Delta m_{32}^2
\end{aligned} \tag{5.36}
$$

Matter Effect on CP Violation

Still now, we can not identify CP violation in the leptonic sector. But we identified this in the hadronic sector through B & K meson decays. There ia a big opportunity to understand the origin of CP violation by observing CP violation in the neutrino oscillation. The atmospheric neutrino anomaly & the solar neutrino deficit directly prove the neutrino oscillation. The Liquid Scintillation Neutrino Detector (LSND) gives us a direct evidence of $\nu_\mu \longrightarrow \nu_e \& \bar{\nu}_\mu \longrightarrow \bar{\nu}_e$ oscillation. The recent experiments in the Sudbury Neutrino Observatory (SNO) with ν_e charged current process on deuteron disintegration indicate low mass & the large mixing angle in the MSW mechanism in the three-neutrino mixing scheme. The Super-Kamiokande Collaboration demands that the oscillation between active & sterile neutrino for both the solar neutrino and the atmospheric neutrino transitions is disfavored in the two-neutrino analysis. But recently various experiments (Sudbury Neutrino Observatory (SNO)) & analyses by Barger, Marfatia, and Whisnant and by Gonzalez-Garcia, Maltoni, and Pena-Garay prove that the oscillation between active & sterile neutrino is allowed in four-neutrino analysis. The matter effect affects on the CP violation about a few to 10 percent with the neutrino energy $E \backsim$ 1GeV, baseline L=250-730 km $\& \Delta m_{21}^2 \equiv Deltam_{solar}^2 \simeq 3 \times 10^{-5} eV^2, \Delta m_{31}^2 \equiv Deltam_{atm}^2 \simeq 3 \times 10^{-3} eV^2, \mid U_{e3} \mid \simeq 0.05$. Therefore matter effect on CP violation depending on the length of the baseline. But vacuum mimicking phenomena says for some cases neutrino oscillation probabilities are approximately independent of the presence of matter.

The flavor eigenstates $\nu_\alpha, (\alpha = e, \mu, \tau, s)$are related by mass eigenstates $\nu_i (i = 1, 2, 3, 4)$ by unitary mixing matrix U as follows:

$$\nu_\alpha = \sum_{i=1}^{4} U_{\alpha i}(0) \nu_i \tag{5.37}$$

The evolution equation in matter can be written as

$$i \frac{d\nu}{dx} = H\nu \tag{5.38}$$

Where x is propagation time for neutrino.

$$
\begin{aligned}
H &\equiv -U \; diag(p_1, p_2, p_3, p_4) U^\dagger \\
&\simeq \frac{U}{E} \; diag(\mu_1^2, \mu_2^2, \mu_3^2, \mu_4^2) U^\dagger
\end{aligned} \tag{5.39}
$$

The unitary matrix U & the masses μ_i^2 are related by

$$U\begin{pmatrix}\mu_1^2 & 0 & 0 & 0\\ 0 & \mu_2^2 & 0 & 0\\ 0 & 0 & \mu_3^2 & 0\\ 0 & 0 & 0 & \mu_4^2\end{pmatrix}U^\dagger = U^{(0)}\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & \Delta m_{21}^2 & 0 & 0\\ 0 & 0 & \Delta m_{31}^2 & 0\\ 0 & 0 & 0 & \Delta m_{41}^2\end{pmatrix}U^{(0)\dagger} + \begin{pmatrix}a & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & a'\end{pmatrix} \tag{5.40}$$

Where

$$\Delta m_{ij}^2 \equiv 2\sqrt{2}G_F N_e E = 7.60 \times 10^{-5}\frac{\rho}{[g(cm^{-3})]}\frac{E}{GeV}eV^2 \tag{5.41}$$

$$a' \equiv \sqrt{2}G_F N_n E \simeq \sqrt{2}G_F N_e E = \frac{a}{2} \tag{5.42}$$

$$H = H_0 + H_1 \tag{5.43}$$

$$H_0 = \frac{1}{2E}U^{(0)}\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & \Delta m_{31}^2 & 0\\ 0 & 0 & 0 & \Delta m_{41}^2\end{pmatrix}U^{(0)\dagger} \tag{5.44}$$

$$H_1 = \frac{1}{2E}U^{(0)}\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & \Delta m_{21}^2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}U^{(0)\dagger} + \frac{1}{2E}\begin{pmatrix}a & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & a'\end{pmatrix} \tag{5.45}$$

Where a &a' denotes matter effect on neutrino oscillation. Where a' comes from neutral current process of ν_e, ν_μ, ν_τ& comes from charged current process of ν_e.N_e is the electron density of the matter, ρ is the matter density &N_n is the neutron density which is approximately equal to N_e. From the evolution equation we can write

$$\nu(x) = S(x)\nu(0) \tag{5.46}$$

where,

$$S(x) = Te^{-i\int_0^x H(s)ds} \tag{5.47}$$

Where T is time ordering operator, the propagation time x is almost equal to the light velocity according to light-ray approximation. If we assume matter density is independent of space and time then we can write

$$S(x) = e^{-iHx} \tag{5.48}$$

The neutrino oscillation probability for $(\nu_\alpha \longrightarrow \nu_\beta)$ is

$$P(\nu_\alpha \longrightarrow \nu_\beta; L) = \mid S_{\beta\alpha}(L) \mid^2 \tag{5.49}$$

The oscillation probability $P(\bar{\nu}_\alpha \longrightarrow \bar{\nu}_\beta)$ i.e. for antineutrino cases obtained by replacing $U \longrightarrow U^*, a \longrightarrow -a, a' \longrightarrow -a'$. Again, probability difference between CP-conjugate channels gave the CP violation effect

$$\Delta P(\nu_\alpha \longrightarrow \nu_\beta) \equiv P(\nu_\alpha \longrightarrow \nu_\beta; L) - P(\bar{\nu}_\alpha \longrightarrow \bar{\nu}_\beta) \tag{5.50}$$

The CP violation quantity $\Delta P(\nu_\alpha \longrightarrow \nu_\beta)$ contains two things, one is the pure CP-violation effect which is due to $U(0)$ term & another is the fake CP-violation effect due to the matter effect. There exist two schemes in the four-neutrino model i.e. 3+1 and 2+2 schemes. In 2+2 scheme there exist the two pairs of close masses which is separated by the LSND mass gap of the order of 1 eV. In 3+1 scheme there exist a group of three masses which is separated by one mass by the gap of the order of 1 eV. In 2+2 scheme there exit two mass pattern;
(a).

$$\Delta m^2_{solar} \equiv \Delta m^2_{21} << \Delta m^2_{atm} \equiv \Delta m^2_{43} << \Delta m^2_{LSND} \equiv \Delta m^2_{32} \tag{5.51}$$

(b).

$$\Delta m^2_{solar} \equiv \Delta m^2_{43} << \Delta m^2_{atm} \equiv \Delta m^2_{21} << \Delta m^2_{LSND} \equiv \Delta m^2_{32} \tag{5.52}$$

Here we will discuss about the first pattern and for second pattern if we exchange the indices $((1,2) \leftrightarrow (3,4))$ then we will get different expression such as the oscillation probabilities. Since $\Delta m^2_{21} << \Delta m^2_{31}, \Delta m^2_{41}$. We can decompose H in this way

$$H = H_0 + H_1 \tag{5.53}$$

with

$$H_0 = \frac{1}{2E} U^{(0)} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta m^2_{31} & 0 \\ 0 & 0 & 0 & \Delta m^2_{41} \end{pmatrix} U^{(0)\dagger} \tag{5.54}$$

$$H_1 = \frac{1}{2E} U^{(0)} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} U^{(0)\dagger} + \frac{1}{2E} \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a' \end{pmatrix} \tag{5.55}$$

Here, H_1 is just like a perturbation up to the first order in $a, a' \& \Delta m^2_{21}$. S(x) can be written as (using Arafune-Koike-Sato procedure)

$$S(x) \simeq e^{-iH_0 x} - i e^{-iH_0 x} \int_0^x H_1(s) ds \tag{5.56}$$

Where,

$$H_1(x) = e^{iH_0 x} H_1 e^{-iH_0 x} \tag{5.57}$$

Now I will discuss the CP violation effect in the long-baseline neutrino oscillations which depends on the rate of the active-sterile neutrino admixture & the matter effect. The expression of the mixing matrix U which depends on the mixing angles and phases, is too complicated, that's why I will show only the matrix elements which will be useful for the analysis.

$$U_{e1} = c_{01}c_{02}c_{03}, U_{e2} = c_{02}c_{03}s^*_{d02}, U_{e3} = c_{03}s^*_{d02}, U_{e4} = s^*_{d03} \tag{5.58}$$

$$U_{\mu 3} = -s^*_{do2}s_{d03}s^*_{d13} + c_{02}c_{13}s^*_{d12}, U_{\mu 3} = c_{03}s^*_{d13} \tag{5.59}$$

$$U_{\tau 3} = -c_{13}s^*_{d02}s_{d03}s^*_{d23} - c_{02}s^*_{d12}s_{d13}s^*_{d23} + c_{02}c_{12}c_{23}, U_{\tau 4} = c_{03}c_{13}s^*_{d23} \tag{5.60}$$

$$U_{s3} = -c_{13}s^*_{d02}s_{d03}c_{23} - c_{02}s^*_{d12}s_{d13}c_{23} - c_{02}c_{12}s_{d23} \tag{5.61}$$

Where

$$\begin{aligned} c_{ij} &\equiv \cos\theta_{ij} \\ s_{dij} &\equiv s_{ij}e^{i\delta_{ij}} = \sin\theta_{ij}e^{i\delta_{ij}} \end{aligned} \tag{5.62}$$

and the six angles are

$$\theta_{01}, \theta_{02}, \theta_{03}, \theta_{12}, \theta_{13}, \theta_{23} \tag{5.63}$$

and the six phases are

$$\delta_{01}, \delta_{02}, \delta_{03}, \delta_{12}, \delta_{13}, \delta_{23} \tag{5.64}$$

Out of six $(P_{12}, P_{13}, P_{14}, P_{23}, P_{24}, P_{34})$ oscillation probabilities, only three oscillation probabilities are independent. Therefore three of the six phases are determined by the measurements of the CP violation effects. The oscillation probabilities for $\nu_\mu \longrightarrow \nu_e$ and $\nu_\mu \longrightarrow \nu_\tau$ with respect to the mixing angles and phases in long-baseline are as follows

$$\Delta P(\nu_\mu \longrightarrow \nu_e) \simeq 4c_{02}s_{02}c^2_{03}s_{03}s_{12}c_{13}s_{13}\sin\delta_1\sin(\frac{\Delta m^2_{43}L}{2E}) \tag{5.65}$$

$$\Delta P(\nu_\mu \longrightarrow \nu_\tau) \simeq -4c^2_{02}c^2_{03}c_{12}s_{12}c^2_{13}s_{13}c_{23}s_{23}\sin\delta_2\sin(\frac{\Delta m^2_{43}L}{2E}) \tag{5.66}$$

Where $\delta_1 \equiv \delta_{02} - \delta_{03} - \delta_{12} + \delta_{13}$ and $\delta_2 \equiv \delta_{12} - \delta_{13} + \delta_{23}$

$\Delta P(\nu_\mu \longrightarrow \nu_e)$ depends on δ_1 and $\Delta P(\nu_\mu \longrightarrow \nu_e)$ depends on δ_2. The angle s_{23} determines the term $\Delta P(\nu_\mu \longrightarrow \nu_\tau)$ and the term $\Delta P(\nu_\mu \longrightarrow \nu_e)$ is unaffected. Again, s_{02} and s_{03} determine $\Delta P(\nu_\mu \longrightarrow \nu_e)$.

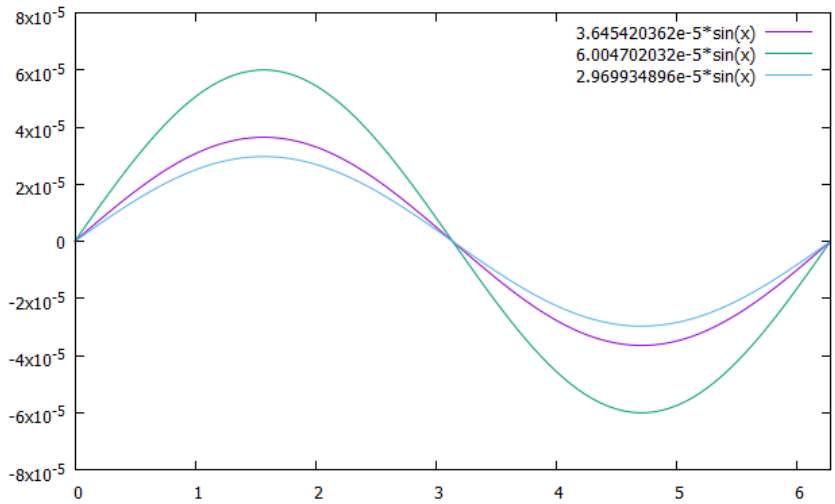


Figure 5.2: In this Fig. we show pure CP violation effect as a function of the phase δ_1 in $\Delta P(\nu_\mu \longrightarrow \nu_e)$ oscillation for the energy E = 1.2 GeV& for the baseline of L = 290 km for the typical three parameter sets $s_{02} = 0.12, s_{03} = 0.06, s_{12} = 0.93, s_{13} = 0.71,$ $s_{02} = 0.12, s_{03} = 0.05, s_{12} = 0.97, s_{13} = 0.71,$ $s_{02} = 0.15, s_{03} = 0.02, s_{12} = 0.95, s_{13} = 0.71$ and commonly taken as $s_{01} = s_{23} = \frac{1}{\sqrt{2}}, \delta_{01} = \delta_{02} = \delta_{03} = \delta_{12} = 0, \delta_2 = \pi/2 \Delta m^2_{21} = 2.5 \times 10^{-3}, \Delta m^2_{32} = 0.3, \Delta m^2_{43} = 2.5 \times 10^{-3}$

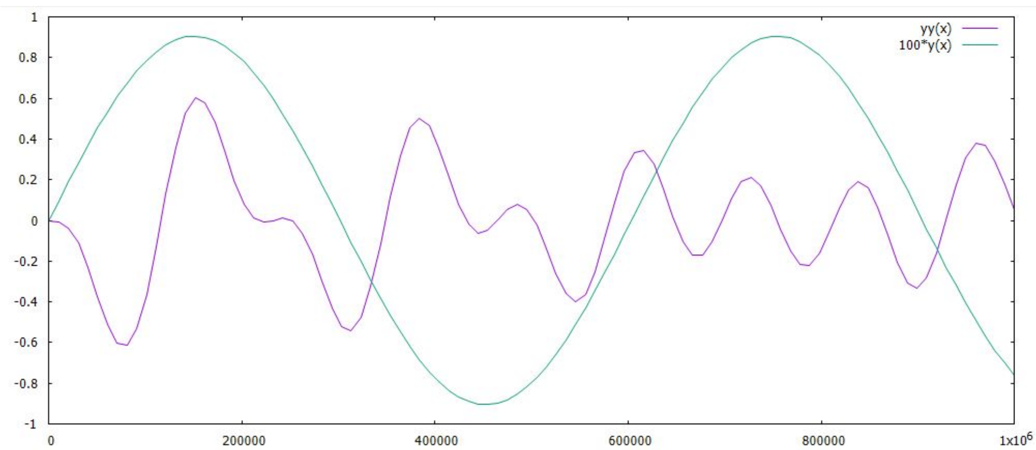


Figure 5.3: In this Fig. we show pure CP violation effect in matter in three neutrino mixing scheme(y(x)) and the 3+1 scheme (yy(x))as a function of baseline L in $\Delta P(\nu_\mu \longrightarrow \nu_e)$ oscillation for the energy E = 1.2 GeV& for the typical three parameter sets $s_{02} = 0.12, s_{03} = 0.06, s_{12} = 0.93, s_{13} = 0.71, s_{01} = s_{23} = \frac{1}{\sqrt{2}}, \delta_{01} = \delta_{02} = \delta_{03} = \delta_{12} = 0, \delta_2 = \pi/2 \Delta m^2_{21} = 2.5 \times 10^{-3}, \Delta m^2_{32} = 0.3, \Delta m^2_{43} = 2.5 \times 10^{-3}$

Chapter 6

Discussion and conclusion

In my project, I discussed about Standard Model, in which the neutrinos are massless as the right-handed neutrino does not exist. But we know neutrino oscillation does occur. Because neutrino oscillation can explain "The solar neutrino problem". Again neutrino oscillation depends on squared-mass difference. Therefore we can say neutrinos have mass. So, we should go from the Standard Model to the beyond Standard Model. we know equal amounts of anti-matter and matter were initially created in the universe(Big Bang theory). In annihilation process equal amounts of anti-matter and matter produce photons when they come into contact. The "Cosmic Microwave Background" is the relic of this primordial annihilation, the 2.7 Kelvin radiation that fills the entire Universe. But about one out of every billion quarks survived and they created today's universe. If matter and antimatter are created and destroyed together, it seems the universe should contain nothing but leftover energy. But we know about matter/antimatter asymmetry from particle physics experiments and we can not apply the laws of physics equally to matter and antimatter. So, we have some question i.e. are anti-matter and matter intrinsically different or not ? or are the "laws of physics" for anti-matter and matter different or not? If we will get the answer then we can say about the mystery of the matter-dominated Universe. The Russian physicist Andrei Sakharov in 1967 proposed a solution. But for this we need a violation in the fundamental symmetry of nature: the cp symmetry

If CPT is conserved and CP is violated then T is also violated. CP violation in neutrino sector means the difference between neutrino and anti-neutrino oscillation probability.When neutrino passes through vacuum we get CP violation which is different when neutrino passes through matter because matter introduces a fake CP violation. I discuss CP violation using scheme. When we take only three ordinary neutrino(ν_e, ν_μ and ν_τ) we get one type of CP violation(Figure 4.6) but when we introduce a fake CP violation(neutrino oscillation in matter) we get another type of CP violation(Figure 5.3).

Chapter 7

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