Conjugacy Relation via Group Action on the set of Fuzzy Implications

VEMURI Nageswara Rao and JAYARAM Balasubramaniam

Department of Mathematics
Indian Institute of Technology Hyderabad-502 205.
INDIA
E-mail: ma10p001@iith.ac.in, jbala@iith.ac.in

Let Φ denote the set of all increasing bijections on $[0,1]$ and $\mathbb{I}$ the set of fuzzy implications. In [1], the authors proposed a new way of generating fuzzy implications from fuzzy implications using order automorphisms $\varphi \in \Phi$ on the unit interval via the following equation

$$I_\varphi(x,y) = \varphi^{-1}(I(\varphi(x), \varphi(y))), \qquad x, y \in [0,1] \ , \tag{1}$$

where $I \in \mathbb{I}$. Here I_φ is called the φ-**conjugate** of I.

Moreover, in the same work, they proposed also a conjugacy relation on $\mathbb{I}$, as follows:

$$I \sim J \ \text{ if and only if } J = I_\varphi \text{ for some } \varphi \in \Phi. \tag{2}$$

Clearly, $\sim$ is an equivalence relation on $\mathbb{I}$ and partitions $\mathbb{I}$ into conjugacy classes. Let us denote by $[I]$ the set of all φ-conjugates of I where φ varies over whole of Φ.

Recently, in [2], the authors proposed a novel method of generating fuzzy implications from fuzzy implications as follows:

Given $I, J \in \mathbb{I}$ we define

$$(I \circledast J)(x,y) = I(x, J(x,y)), \ x, y \in [0,1] \ . \tag{3}$$

Further, it was shown that the above binary operation imposes a monoid structure $(\mathbb{I}, \circledast)$ on the set of all fuzzy implications.

In this present work, we show that the conjugacy relation defined via Eq. (2) is nothing but a group action of $\mathbb{S}$ on the set $\mathbb{I}$, where $\mathbb{S}$ is the set of all invertible elements of the monoid $(\mathbb{I}, \circledast)$. In other words, we show that the conjugacy classes $[I]$ obtained from Eq. (2) are exactly the pieces in the partition obtained by the group action of $\mathbb{S}$ on the set $\mathbb{I}$.

References

[1] M. Baczyński, J. Drewniak: *Conjugacy classes of Fuzzy Implications.* Reusch.B (Ed.), Computational Intelligence, Theory and Applications, 6th Fuzzy Days, Springer. (1999) 287–298.

[2] N. R. Vemuri, B. Jayaram: *Fuzzy implications: Novel generation process and the consequent algebras.* Greco, S. *et al.* (Eds.), Advances on Computational Intelligence - 14th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU 2012, Springer. (2012) 365-374.