# On the Sparsest Representation of Electrocardiograms

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## Abstract

*In recent years, telecardiology has been growing in significance, due to the shortage of local caregivers in various parts of the world. As the cardiac data volume grows, compact representation becomes imperative in view of bandwidth, storage, power and other constraints. In this backdrop, we present empirical studies on electrocardiogram (ECG) signal representation using a wide variety of wavelet bases. Specifically, we arrange the transform coefficients in decreasing order of magnitude, and count the number of coefficients accounting for $99\%$ of the signal energy (a sparser representation requires less number). We observe that 'Symlet' and 'Daubechies' families generally offer more compact representation compared to Meyer wavelet as well as biorthogonal and reverse biorthogonal families. In particular, the sparsest representation is provided by the 'sym4' (closely followed by the 'db4') wavelet basis for a broad class of ECG signals. Interestingly, this behavior is observed quite consistently across all fifteen (twelve standard and three Frank) leads. Our study assumes significance in the context of basis selection for various ECG signal processing applications, including compression, denoising and compressive sensing.*

## 1. Introduction

Telecardiology involves acquisition and transmission of electrocardiogram (ECG) signals to a remote facility, where diagnosis is performed towards possible intervention. Unfortunately, the far flung population, who are perhaps in the biggest need of telecardiology, are often underprivileged and have limited access even to nominal infrastructure such as bandwidth and power. In such circumstances, it becomes imperative to design healthcare systems taking infrastructural constraints into account. Note that one can reduce storage and transmission requirements by compressing ECG signals. In dire circumstances, where inadequate power makes intensive compression algorithms infeasible, one may adopt the low-power alternative of compressive sampling, albeit sacrificing some compression efficiency.

ECG signal compression has been studied over several decades [1]. A variety of compression algorithms represent ECG signals in suitable orthogonal basis and exploit signal redundancy in the transformed domain [2–4]. Indeed, success of a compression algorithm depends on how compactly the signal is represented upon transformation. In this context, various researchers have reported ECG signals to be sparse in wavelet bases [5]. In other words, only a few wavelet coefficients pack most of the signal energy. Signal sparsity also plays a crucial role in recent compressive sensing (CS) based telecardiology solutions [6,7]. Other applications, such as denoising, are facilitated by sparsity. For example, a sparser representation allows more coefficients to fall below a threshold thus allowing more noise to be removed [8].

Adaptive/data-dependent basis selection for ECG signals based on machine learning has also been suggested. However, the simplicity of a fixed basis is attractive in various applications. Indeed, empirical studies on wavelet basis selection have been carried out for compression as well as denoising [9, 10]. However, prior studies either narrowly target one wavelet family, or consider limited number of ECG signals, or only one lead, thereby limiting their utility. In contrast, we consider a large collection of wavelet bases including those from the Daubechies, Symlet, biorthogonal, and reverse biorthogonal families. Further, we use all ECG signals with fifteen lead data from The PTB Diagnostic ECG Database from Physionet [11]. This database offers $549$ signals from $290$ subjects recorded at $1000$ Hz. Each record contains information from standard twelve leads and three Frank leads making available over $8200$ signals for analysis.

We observe that 'Symlet' and 'Daubechies' families generally offer more compact representation compared to Meyer wavelet as well as biorthogonal and reverse biorthogonal families. In particular, the sparsest representation is provided by the 'sym4' (closely followed by the 'db4') wavelet basis for a broad class of ECG signals. Interestingly, this behavior is observed quite consistently across all fifteen (twelve standard and three Frank) leads. Our study assumes significance in selecting basis in various ECG signal processing applications, including compression, denoising and compressive sensing.

The rest of the paper is organized as follows. Sec. 2 provides the application context. Description of data and methodology are explained in Sec. 3. Sec. 4 presents the results, the concluding discussion appears in Sec. 5.

## 2. Application context

In this section, we shall elaborate on three application contexts, namely, compression, denoising and compressive sensing, where sparse representation of signals plays a critical role.

### 2.1. Compression

Consider set $\mathcal{E}$ of ECG signals of length $N$. Intuitively, $\mathcal{E}$ should be a relatively small subset of the set $\mathcal{R}^{\mathcal{N}}$ of all $N$-length signals. First, let us state the linear signal approximation problem, where one assumes the existence of subspace $\mathcal{S}$ of dimension $K << N$ such that projection $\hat{x}$ of any $x \in \mathcal{E}$ onto $\mathcal{S}$ provides an $\epsilon$-accurate linear approximation, i.e., $\|x - \hat{x}\| < \epsilon\|x\|$ for small $\epsilon > 0$. Here $\|\cdot\|$ indicates norm generally, and the 2-norm specifically. One then seeks that subspace which achieves $\epsilon$-accuracy with minimum $K$. In transform coding parlance, the above translates to the problem of identifying the optimal unitary transform $\Psi$ such that the minimum number $K$ of transform coefficients provides $\epsilon$-accuracy. In other words, $x$ becomes sparse when represented in $\Psi$. One seeks $\Psi$ that offers 99% approximation accuracy ($R^2$ score as described later) with least value of $K$.

### 2.2. Denoising

An ECG signal often gets corrupted by measurement and other noises, which are often modeled as an additive white Gaussian (AWGN) process. Earlier research has demonstrated effectiveness of wavelet based multiresolution analysis over conventional filtering based (linear/nonlinear) denoising techniques [9]. Indeed, sparser the representation, higher is the number of wavelet coefficients that can be thresholded to zero without hampering clinical diagnostic quality, leading to superior denosing. Even further, modern approaches such as Basis Pursuit Denoising (BPDN) rely on signal sparsity [12].

### 2.3. Compressive sensing

Motivated by CS theory, which essentially states that a sufficiently sparse signal can be reconstructed from few measurements, various low-power telecardiology systems have been proposed [6, 7]. Consider a vector $x \in \mathbb{R}^N$, where $N$ is large. A vector $x$ is called $K$-sparse if $||x||_0 \leq k$, where $||.||_0$ denotes the $l_0$ norm. Now given a matrix $\Phi \in \mathbb{R}^{m \times N}$, obtain reduced set of measurements $y \in \mathbb{R}^m$ by performing $y = \Phi x$, where $m << N$. The measurement process is then, $y = \Phi x = \Phi\Psi\tilde{x} = \Theta\tilde{x}$. Our task is to solve $y = \Phi x$ for K-sparse $x$. This is a combinatorial problem known to be NP-Complete for practical signals. However, if $m = O(k \log n)$ and matrix $\Theta$ satisfies Restricted Isometry Property (RIP), it has been shown that solving the $\ell_1$ minimization problem gives the desired solution [12]. Note that the CS-based recovery is possible from the fewest number of measurements, if one selects the basis in which the signal exhibits the maximum degree of sparsity.

Table 1. List of wavelets used

| Wavelet Family | Biorthogonal and reverse biorthogonal ($biorM.N$, $rbioM.N$) | Symlets $symN$ | Daubechies $dbN$ | Meyer |
|---|---|---|---|---|
| Wavelets used | 1.1, 1.3, 1.5, 2.2, 2.4, 2.6, 2.8, 3.1, 3.3, 3.5, 3.7, 3.9, 4.4, 5.5, 6.8 | 2 to 8 | 1 to 10 | Meyer |

## 3. Experimental setup and procedure

In this paper, all ECG signals from The PTB Diagnostic ECG Database from Physionet have been used without any pre-processing [11]. The database offers 549 signals from 290 subjects recorded at 1000 Hz. Each record contains information from standard 12 leads and 3 Frank leads making available over 8200 signals for analysis. Without loss of generality, all possible consecutive segments of length 4096 samples were chosen from each signal, with overlap of few samples between them. Normally, such a segment (called 'signal' henceforth) contains 5 cardiac cycles. Table 1 lists the wavelet bases used in this study.

We begin with an ECG signal $x$ of length $N$. Its $N$-point discrete wavelet transform (DWT) is computed using wavelet $\Psi$ to obtain wavelet coefficients $\tilde{x}$. Next, let $\tilde{y}$ contain first $K$ significant coefficients from $\tilde{x}$, without altering their original locations. The remaining $N$-$K$ locations in $\tilde{y}$ are set to zero. Then, a linear approximation $x_r$ of the ECG signal $x$ is obtained by performing Inverse DWT on $\tilde{y}$. The value of $K$ is incremented in steps of 1 until the $R^2$ score,

$$R^2 = 1 - \frac{||x - x_r||_2{}^2}{||x - \bar{x}||_2{}^2} \quad (1)$$

(where $\bar{x}$ is the mean value of $x$), reaches 99%. This procedure is depicted in Fig. 1.

We repeat the aforementioned steps for all the ECG signals in the database, where our aim is to find $K$ for different leads and wavelet bases. Once a wavelet or a set of few wavelets that offers sparse representation is identified, we examine reconstruction performance of a CS re-

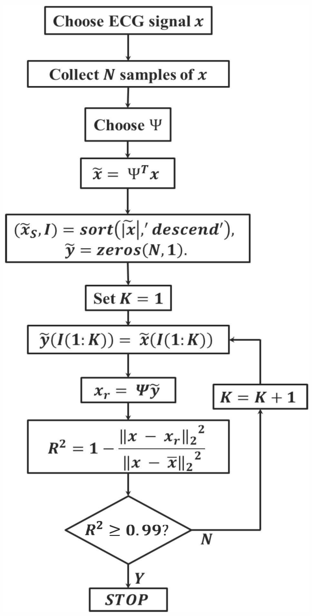


Figure 1. ECG approximation method

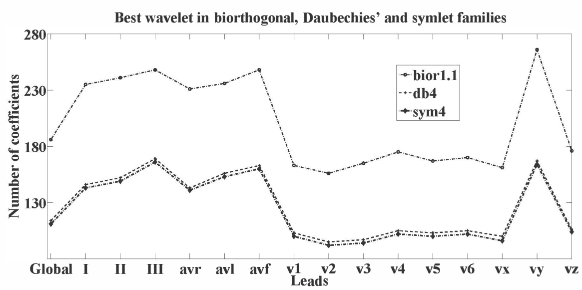


Figure 2. Averaged performance of selected wavelets across all leads.

covery scheme called Targeted Orthogonal Matching Pursuit (TOMP) using these wavelets [13] . The measurement matrix Φ was chosen to be sparse random matrix whose entries are sampled from normal distribution. The matrix Φ is $m \times N$, where $m = N/d$ (rounded to integer) and $d = 2, 3, .., 16$. For each $m$, 1000 random matrices were obtained and TOMP was employed to solve the CS recovery problem in each such case.

## 4. Results

Figure 2 compares average performance of best wavelet bases from the 'Symlet', 'Daubechies' and 'Biorthogonal' families, respectively, for fifteen leads. Notice that the highest sparsity is exhibited by 'sym4' (Symlet), which is closely followed by 'db4' (Daubechies), but far ahead of 'bior1.1' (biorthogonal). Interstingly, the aforementioned behavior remains more or less consistent across all fifteen leads. Even more remarkably, similar behavior is seen

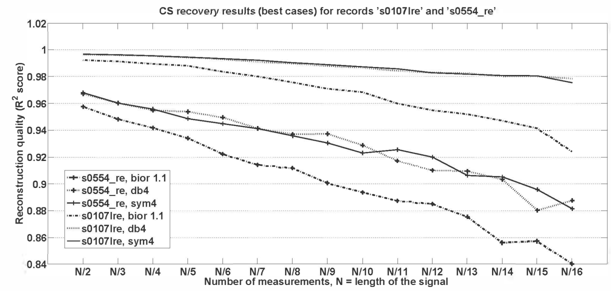


Figure 3. Signal recovery from compressively sensed data

even across disease classes (not shown due to space constraints). A more exhaustive study is furnished in Table 2. Wavelet bases which do not pack 99% of the signal energy (equivalent to achieving 99% $R^2$ score) within about $N/8$ coefficients are not shown (except Meyer wavelet).

Given a wavelet Ψ which offers sparse representation of the signal at hand, one may encode the signal coefficients in a lossy or lossless manner. Apart from well studied compression problems, signal recovery from compressively sensed samples has attracted significant attention. Accordingly, we next examine the CS recovery performance. As an illustration, segments from two signal records, namely 's0107lre' (highly compressible) and 's0554_re' (less compressible) have been chosen. Figure 3 shows the results for the experimental setup described in Sec. 3. Notice that the 'sym4' basis leads to the best $R^2$ reconstruction from the same number of measurements, closely followed by 'db4', and distantly followed by 'bior1.1'. This observation corroborates the findings from Fig. 2.

## 5. Discussion

Whether the application at hand is compression, denoising or compressive sampling, the efficacy depends on signal sparsity. In compression, the sparsest representation provides the least dimension in which the signal space could be embedded. At the same time, such representation allows for the most efficient denoising, as well as perfect reconstruction from the least number of compressive samples. In this backdrop, we consider a large class of wavelet bases, drawn from various families including symlets and Daubechies' wavelets, and compare the sparsity of a broad collection of ECG signals in those bases. In the process, two interesting observations are made: (1) Symlets tend to outperform Daubechies' wavelets, with the fourth order symlet ('sym4') proving the most efficient overall; (2) When considering all fifteen leads, 'v1' through 'v6' and 'vx' (Frank X) appear to admit sparser representation (Fig. 2). The latter observation could also guide efficient subset selection while reconstructing the full set of leads from a subset, a key to portable design.

Table 2. Averaged performance of various wavelets across all leads. Number of coefficients ($K$) is rounded to integer.

| | Leads | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Wavelet** | **I** | **II** | **III** | **avr** | **avl** | **avf** | **v1** | **v2** | **v3** | **v4** | **v5** | **v6** | **vx** | **vy** | **vz** | **Overall** |
| **sym4** | 143 | 149 | 166 | 141 | 153 | 160 | 100 | 92 | 94 | 102 | 100 | 102 | 96 | 164 | 104 | 111 |
| **sym6** | 144 | 148 | 165 | 141 | 153 | 159 | 100 | 93 | 95 | 104 | 101 | 103 | 97 | 162 | 105 | 112 |
| **sym5** | 144 | 149 | 166 | 141 | 154 | 160 | 101 | 93 | 96 | 104 | 101 | 103 | 97 | 164 | 106 | 113 |
| **db4** | 146 | 152 | 169 | 143 | 156 | 163 | 103 | 95 | 97 | 105 | 103 | 105 | 100 | 167 | 106 | 114 |
| **sym8** | 145 | 149 | 165 | 142 | 154 | 159 | 103 | 95 | 97 | 105 | 103 | 105 | 100 | 163 | 107 | 114 |
| **db3** | 148 | 154 | 171 | 145 | 157 | 166 | 102 | 94 | 97 | 105 | 104 | 106 | 100 | 170 | 106 | 115 |
| **sym3** | 148 | 154 | 171 | 145 | 157 | 166 | 102 | 94 | 97 | 106 | 104 | 106 | 100 | 170 | 107 | 115 |
| **sym7** | 146 | 150 | 166 | 143 | 155 | 160 | 103 | 95 | 98 | 106 | 104 | 106 | 101 | 164 | 107 | 115 |
| **db5** | 149 | 154 | 171 | 146 | 159 | 165 | 106 | 98 | 101 | 109 | 107 | 109 | 104 | 169 | 109 | 118 |
| **db6** | 153 | 157 | 174 | 149 | 162 | 168 | 110 | 102 | 105 | 113 | 112 | 113 | 108 | 171 | 113 | 121 |
| **db2** | 157 | 164 | 181 | 155 | 166 | 176 | 107 | 99 | 103 | 112 | 109 | 111 | 104 | 183 | 113 | 122 |
| **sym2** | 157 | 164 | 181 | 155 | 166 | 176 | 107 | 99 | 103 | 112 | 109 | 111 | 104 | 183 | 113 | 122 |
| **db7** | 155 | 159 | 175 | 152 | 164 | 169 | 112 | 105 | 108 | 116 | 115 | 116 | 111 | 172 | 115 | 124 |
| **db8** | 158 | 161 | 177 | 155 | 167 | 171 | 115 | 108 | 111 | 119 | 119 | 119 | 115 | 175 | 118 | 127 |
| **db9** | 162 | 165 | 181 | 158 | 170 | 174 | 118 | 111 | 114 | 123 | 123 | 123 | 118 | 177 | 121 | 131 |
| **db10** | 165 | 168 | 184 | 161 | 173 | 177 | 122 | 114 | 117 | 127 | 126 | 127 | 122 | 180 | 124 | 134 |
| **bior1.1** | 235 | 241 | 248 | 231 | 236 | 248 | 163 | 156 | 165 | 175 | 167 | 170 | 161 | 266 | 176 | 186 |
| **rbio1.5** | 236 | 243 | 249 | 233 | 237 | 250 | 164 | 158 | 166 | 176 | 169 | 172 | 163 | 267 | 178 | 187 |
| **rbio1.3** | 236 | 243 | 250 | 233 | 238 | 250 | 164 | 158 | 166 | 176 | 168 | 172 | 163 | 267 | 178 | 188 |
| **bior4.4** | 363 | 322 | 337 | 320 | 351 | 324 | 212 | 233 | 244 | 231 | 201 | 206 | 189 | 371 | 246 | 241 |
| **rbio4.4** | 492 | 416 | 425 | 429 | 462 | 408 | 317 | 379 | 390 | 361 | 307 | 307 | 299 | 470 | 380 | 354 |
| **Meyer** | 977 | 987 | 990 | 982 | 985 | 991 | 964 | 961 | 964 | 965 | 964 | 971 | 957 | 986 | 967 | 968 |


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## References


[1] Ahmed N, Milne PJ, Harris SG. Electrocardiographic data compression via orthogonal transforms. IEEE Trans Biomed Eng 1975;BME-22(6):484–487.

[2] Bendifallah A, Benzid R, Boulemden M. Improved ECG compression method using discrete cosine transform. Electronics Lett 2011;47(2):87–89.

[3] Zhitao L, Youn KD, Pearlman WA. Wavelet compression of ECG signals by the set partitioning in hierarchical trees algorithm. IEEE Trans Biomed Eng 2000;47(7):849–856.

[4] Abo-Zahhad M, Rajoub BA. ECG compression algorithm based on coding and energy compaction of the wavelet coefficients. 8th IEEE International Conf on Electronics, Circuits and Systems 2001;1:441–444.

[5] Polania LF, Carrillo RE, Blanco-Velasco M, Barner KE. Compressed sensing based method for ECG compression. IEEE International Conf on Acoustics, Speech and Signal Processing 2011:761–764.

[6] Mamaghanian H, Khaled N, Atienza D, Vandergheynst P. Real-time compressed sensing-based electrocardiogram compression on energy-constrained wireless body sensors. IEEE International Symp on Circuits and Systems 2011;1744–1747.

[7] Chandra BS, Sastry CS, Jana S. Telecardiology: Hurst exponent based anomaly detection in compressively sampled ECG signals. IEEE International Conf on e-Health Networking, Applications and Services (Healthcom) 2013.

[8] Rabiul SMR, Xu H, Sharma D. Wavelet based denoising algorithm of the ECG signal corrupted by WGN and Poisson noise. International Symp on Communications and Information Technologies 2012;165–168.

[9] Singh BN, Tiwari AK. Optimal selection of wavelet basis function applied to ECG signal denoising. Digital Sig Proc 2006;16(3):275–287.

[10] Besar R, Eswaran C, Sahib S, Simpson RJ. On the choice of the wavelets for ECG data compression. IEEE International Conf on Acoustics, Speech, and Signal Processing 2000;6:3614–3617.

[11] Goldberger AL, Amaral LAN, Glass L, Hausdorff JM, Ivanov PCh, Mark RG, Mietus JE, Moody GB, Peng CK, Stanley HE. PhysioBank, PhysioToolkit, and PhysioNet: components of a new research resource for complex physiologic signals. Circulation 2000;101(23):e215-e220.

[12] M. Elad. *Sparse and Redundant Representations: From Theory to Applications in Signal and Image Processing.* Springer, 2010.

[13] Reddy DSS, Tamboli RR, Jana S. Universal nonuniform sampling of ECG signals: opportunities and obstacles. Biomed Eng International Conf 2012;1–5.



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