Analysis of Power System State Estimation

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Dedication

To My Parents

Abstract

State Estimation plays an predominant role in modern power systems. The error in the measurements and the communication system will influence the estimated system states. The present work provides procedure to suppress the influence of these errors, which includes Weighted Least Square (WLS) state estimation, constrained state estimation. The algorithm for bad data detection is implemented and the results are discussed. Measured values and initially estimated values are obtained from data acquisition system that is used for performing Newton Raphson load flow/ power flow analysis. The error between the states is minimized by minimizing the objective function. Weighted Least Square method for state estimation is implemented on IEEE-14 and IEEE-30 bus system and constrained state estimation is performed on a 3-bus test system. The error function is minimized using optimization techniques in GAMS software. State estimation algorithm and power flow analysis are implemented using MATLAB.

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Nomenclature

List of symbols

n	Number of Sate Variables .
m	Number of Measurements.
r	Redundancy ratio of measurement.
e	Measurement error Vector.
p	Input random variable vector.
h	Non linear measurement function vector.
I	Current(p.u).
k	Iteration number.
X	Random number.
R	Measurement noise co-variance matrix.
W	Weight Matrix of the traditional.
Z	Measurement error.
Y	Magnitude of Y_{bus} element.
G	Conductance.
H	Jacobean matrix(m*n).
ρ	Waiting factor
δ	Bus angle.
η	Number of input random variables.
ϑ	Number of output random variables.
σ	Standard deviation of random variable.
μ	Mean valve.
Θ	voltage phase angle.
$\hat{x}$	Estimated state vector.
$G(x)$	The general state estimation.
$h(.)$	Nonlinear functional vector.
$G(x)$	The general state estimation.
$C(.)$	Nonlinear equality constraint.
$g(.)$	Nonlinear inequality constraint.
x^{true}	True state vector.
$x(.)$	State vector.
r_m	Normalized residual.
σ_i	Standard deviation of i^{th} measurement error..
V_i	Voltage measurement form bus i.
P_i	Active power injection measurement form bus i.
Q_i	Reactive Power injection measurement form bus i.
P_{ij}	Active Power flow measurement from i to bus j.
Q_{ij}	Reactive power flow measurement from i to bus j.

List of Abbreviations

NR	Newton Raphson Method.
SE	State Estimation.
PT	Potential Transformers.
CT	Current Transformers.
AC	Alternating Current.
MW	Mega Watt
WLS	Weighted Least Square Sate Estimation
CSE	Constrained State Estimation.
RTU	Real Terminal Unit.
PMU	Phasor Measurement Unit.
EMS	Energy Management System.
Pdf	Probability Density Function.
WLAV	Weight Least Absolute Values
SCADA	Supervisory Control And Data Acquisition.
IEEE	Institute of Electrical and Electronics Engineers

Chapter 1

Introduction

1.1 Introduction

Power system state estimation is the process carried out in the energy control centers in order to provide a best estimate of the system state based on the real-time system measurements and a pre-determined system model. A redundant set of real-time measurements, including bus voltage magnitudes, real and reactive power injections at the buses, real and reactive line power flows, and sometimes line current magnitudes, are collected from the entire network through the Supervisory Control And Data Acquisition (SCADA) system.These telemetered raw measurements are usually corrupted by different kinds of errors. State estimator is a digestive system that removes these impurities statistically to determine the state of the system. In formulating power system state estimation problem, the complex bus voltages (bus voltages magnitudes and phase angles) are commonly used as the state variables. Once system state is determined, the entire system quantities such as line power flows, line current magnitudes and bus power injections can be calculated.

Power-system State Estimation (PSE) was introduced by Fred Schweppes at MIT in 1969 [1], it has remained an extremely active and challenging area. At present, state estimation not only plays an essential role in modern Energy Management Systems (EMS) providing a complete, consistent, accurate and reliable database but also helps in execution of other key functions of the EMS system, such as security monitoring, optimal power flow, security analysis, on- line power flow studies, supervisory control, automatic voltage control and economic dispatch control [2,3].

The deregulation of the electric power industry has transformed state estimation from an important application into a critical one. Many critical commercial issues in the power market, such as congestion management, need to be founded and justified on a precise model of power system, which is derived from the state estimation process. Hence, the improvement of the state estimation to achieve a more accurate and more reliable system state is a timely task.

Although the role of a state estimator is clear, there is much freedom of choice in its practical implementation. One of the important options is that of the statistical methodology used to purify the measured data. Various methods for state estimation have been introduced [4,5] in the past decades. Among those methods, Weighted Least Squares (WLS) algorithm is the most popular one. The objective function to be minimized of this method is chosen as the weighted sum of squares of the measurement residuals. Since this kind of problem can be solved by efficient numerical techniques, state estimators based on WLS approach have been installed in almost all the EMS systems all over the world. However, WLS method is highly sensitive to bad data in the measurement set . In order to solve this problem, an alternative formulation of the state estimation problem, Weight Least Absolute Values (WLAV) [6,7], has been used. It defines the sum of the weighted absolute values of the measurement residuals as the objective function. Although this method is not widely used in the industry due to slower speed compared to WLS method, its capability of automatic bad data rejection makes it useful in some special issues such as topology error identification.

When a state estimation model fails to yield estimates with in a degree of accuracy compatible with the standard deviations of the quantities estimated, one must conclude either that the measured quantities contain spurious data or that the model is unfit to explain the measured quantities. The procedure to identify and solve the former problem is called bad data analysis [8] while for the later one is topology error detection/identification. There exist many bad data analysis techniques and they are successfully utilized. However, the conventional state estimators are still vulnerable to errors in the topology of the system, which show up when the assumed status of the circuit breakers and switches do not coincide with their true statuses.

Observability analysis is another important procedure closely related to state estimation. Sometimes state estimation is not possible if it is not given enough measurements. If all the state variables (bus voltage magnitudes and relative phase angles) can be estimated using the available measurements, a system is said to be observable. Various methods proposed for network Observability analysis have been well document in the literature [9].

1.1.1 State Estimation

State estimation is a mathematical algorithm that estimates the states (bus voltages and angles) from the network data and sensor information . It can also be used to calculate system quantities where sensors are not available. A state estimator generally acquires the measurements in real time and processes them to obtain a snapshot of the power system. The data to the state estimator may get updated every few seconds to minutes or whenever there is a change in status of the network. A static state estimator is a steady state estimator that calculates the unknown values based on the most recent measurements. A dynamic state estimator predicts the future states based on the present variations and forecasted loads.

1.1.2 Thesis Outline

Chapter 2 gives background information about the sensors present in a power system, the energy control centre role of state estimation in control centres and the Newton Raphson (NR) technique for linearization. Chapter 3 presents the work that has been done in the field of state estimation . Chapter 4 explains the problem solution.The problem statement and description. 5 provides approach results and discussion for state estimation algorithms. Chapter 6 provides conclusions and future work.

Chapter 2

Background

2.1 Introduction

A typical electric power system consists of generation, transmission and distribution systems. In Alternating Current (AC) power systems the power from generating units is transmitted to substations at higher voltages through transmission lines. The voltages are stepped down at the substation with the help of transformers and the power is then either sub-transmitted to other substations or distributed to the loads. The generating units at the generation facility, transmission lines, and transformers at the substation and loads at the power consumers are monitored by various sensors or measurement devices. They provide the operating conditions of the power system components.

The energy control center which coordinates the energy management function of the power system requires the measurements from the sensors to assess the operating condition of the system. It is necessary to have information regarding every state of the power system to determine the operating condition of the system. Any kind of corrective or preventive actions can be taken based on the operating condition of the power system [2]. It is not possible to have all the states of the power system as measurements from the sensors as it is uneconomical to place sensors at all parts of the power system on every line, transformer and load. State estimation is used at the control centers to filter the measurements and calculate every state of the power system given the available set of measurements.

2.2 Sensors Monitoring Used In Electric Power System

2.2.1 Sensors in power system

At present, power systems are monitored with numerous measuring and control devices. Potential Transformers (PT), Current Transformers (CT), relays, and Phasor Measurement Units (PMU) are the important sensors used in power system networks. PTs and CTs measure the high level voltages and currents and convert them to operating level signals. Relays and PMUs are fed by CTs and PTs. Fig 2.1 shows some of the sensors that are used in power system.

Using relays, voltages and currents are sampled and converted into a digital format for further processing. Relays analyze these signals to provide the necessary protection. A conventional

Figure 2.1: Sensors in power system

measuring device measures the quantities across the power system at different instances of time. The measurements have to be synchronized to get an accurate picture of the power system. Apart from the above discussed sensors, there are also other sensors which monitor the temperature and pressure of the transformer oil. The measurements from the devices may not be accurate due to improper calibration, a loose connection and/or aging of the measurement devices. The error might be introduced due to noise in communication networks when the data is transmitted from the field to control center.

2.2.2 Supervisory Control and Data Acquisition System

SCADA is a type of industrial control system that acquires data from different remote locations and monitors the systems at remote places using the acquired data [8]. In the electric power industry, the data in the form of analog and digital quantities, is obtained from different sensors (discussed in the previous section) located at the electric utility substations. The sensors transmit the data to a Remote Terminal Unit (RTU) at the substation. An RTU relays the data (such as voltage, current, circuit breaker status, etc.) from the substation to computers at the control room of the utility center through a communication network. The information received from the RTU is processed by a state estimator to get a better picture of the system. The operators monitor the system (such as opening or closing remote circuit breakers) with the help of the available information [10]. Fig. 2.2 shows state estimation in a SCADA system.

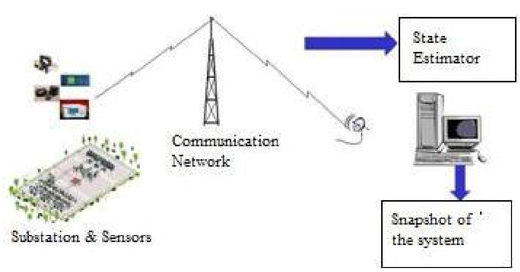

Figure 2.2: State Estimator in SCADA system

2.3 Energy Control Centers

An energy control centre is a place where operators of the electric utility grid use computer-aided tools to monitor, control and optimize the generation, transmission and distribution processes of a power system.

2.4 Newton Raphson Method

The non-linear equations are generally represented by Taylor series expansion. The Newton Raphson (NR) algorithm is derived from Taylor series expansion by neglecting the higher order terms in Taylor series. The higher order terms (higher order derivatives/partial derivatives of the equation with respect to variables) are neglected assuming that the initial guess for the iterative process is closer to the solution. Hence the equation for Newton Raphson (NR) method is one of the reduced forms of Taylor series expansion.

The NR method is an iterative process for solving non-linear equations. In the iterative process, the Jacobean (first order derivative of non-linear equation with respect to variables) and the input vector (input vector is obtained by substituting the values for variables in non- linear equation) are calculated at an initial guess of the variables for the first iteration. The change in variables is then calculated by solving linear equations that contains the Jacobean and input vector. The change in variable is used for updating the variables in each iteration. The updated variables are used in successive iteration. The iterative process is stopped when the error (difference of the function values of non-linear equations calculated at the updated variable) reaches a pre-specified tolerance. NR method is useful to linearize the power flow equations which are non-linear. The LU factorization is used to solve the linear system of equations obtained from NR method. State Estimation uses the same equations as power flow for representing the measurements such as line flows and power injections. Hence NR method is very important for this research work.

2.5 State Estimation in Energy Control Centers

2.5.1 State Estimation and its Functions

State estimation is a algorithm that calculates the values of all the states and other relevant system data from the available set of measurements with error. Voltage magnitudes and bus angles are the states in the state estimation program. A state estimator acts as a filter between the measurements and various energy management functions like contingency analysis, economic dispatch and automatic generation control. The network data and raw measurements are the inputs to the state estimator. Generally, estimated measurements along with voltage magnitudes and angles at every bus of the power system are the outputs of the state estimator in an energy control center.

2.5.2 Major Functions of State Estimation

Bad data processing Needed to detect, identify, and eliminate or process the bad data in the measurement set

Topology processing Based on the telemetered values of the circuit breaker and switch statuses, most likely states of them are determined, and a single line diagram of the system is prepared

Observability analysis state estimation is that the system states should be uniquely determined based on the available measurements. Numerical or topological observability analysis algorithm can be used

Parameter estimation Estimation of the parameters (such as the line impedances) is also a part of the state estimation process

2.5.3 Measurements Available for State Estimation

- Voltages at buses.
- Real power line flows.
- Reactive power line flows.
- Real power injections.
- Reactive power injections.
- Current measurements.
- Voltage and current angles from PMUs

2.6 Summary

This chapter briefly explains background information related to present sensors used in the power system, state estimation and its functions in an energy control centre. The concepts of energy control centre and a state estimation tool with its inputs and outputs are also presented. The application of different numerical methods that are used for solving complex problems in power system analysis is also provided here.

Chapter 3

Literature Review

3.1 Introduction

This chapter reviews the research work related to state estimation algorithms, constrained state estimation, bad data processing techniques developed to identify and eliminate bad data from early research to most recent.

3.2 Evolution of State Estimation Technique

Power system state estimation refers to the collection of a redundant set of measurements from around the system and computing a state vector of the voltage magnitude and load angle at each observed bus. While technology has improved state estimators and other control center applications over many decades, the fundamental concepts and algorithms behind these proven techniques remain much the same. Measurements which are non-linear functions of the system state are collected and load-flow-like calculations are performed to iteratively determine the most probable system state from the known information. This chapter presents the mathematical basis for traditional state estimation techniques and investigates several reformulations of these algorithms in the estimator to improve the quality of the estimate.

The state of the power system is a function of several parameters. These include system level variables such as real and reactive power flows, power injections and voltages which are unknown but measured, network topology, and parameters such as resistance, reactance, and susceptance of transmission lines which is assumed to be known[3].

3.3 State Estimation Methods

For this research work it is important to review the literature related to different types of state estimation methods that are developed. The parts of the state estimation such as bad data processing and observability analysis are also explained in this section.

3.3.1 Weighted Least Squares Method

Fred C. Schweppe and J. Wildes [1] first developed the mathematical model for the state estimation problem. They used the weighted least squares method to solve the state estimation problem.The weighted least squares method developed by them minimized the weighted sum of squares of the difference between the measured and estimated values.The weight for each measurement is obtained from the accuracy of the device which is termed as the standard deviation of the measurement. Greater accurate measurements will be given more weight so that the estimation procedure can influence the solution based on the strong confidence in measurements of greater accuracy. Full Newton Raphson(NR) method was used for linearization and solving the states iteratively. Fast decoupled formulation of weighted least squares method was later developed which utilized less memory and was computationally faster than full NR weighted least square method. H.M. Merrill et.al [11] proposed a bad data suppression method based on quadratic constant and weighted least square method. The bad data is suppressed in a post processing step after initial weighted least square iterations.A.Monticelli et.al[12]investigated an improved bad data processing method for the weighted least square state estimation considering the coherence between measurement with largest normalized residuals and the remaining measurements. S.Y.Lin [13]proposed a method to perform distributed state estimation using weighted least square method.The state estimation is performed by distributing the network on different computers.A.Monticelli[12]developed an innovative method to include status of breakers into weighted least square method along with measurements. The gross errors introduced due to status of the breakers and measurements are handled by the developed method by imposing constraints on status and analog errors.Shan Zhong[4]et.al developed a method to update the weights in weighted least square method based on variances obtained from recorded measurement residuals.It was extended to tune the weights automatically for online process.J.H.Teng[15] proposed a method to include current measurements in the weighted least square method for both distribution and transmission systems. The comparison was made on different test cases for the weighted least square approach with and without a fast decoupled formulation.

3.3.2 Least Absolute Value Method

R.Irving,R.C.Owen and M.J.H.Sterling[16]investigated the Least Absolute Value(LAV)method for solving the state estimation problem.The objective function for minimization in this method is the sum of the absolute values of the difference between measured and estimated quantities with constraints on equations for measurements. They used linear programming techniques to formulate and solve the problem as a linear programming problem.W.W.Kotiuga and M.Vidya Sagar [17] developed the Weighted Least Absolute Value (WLAV) state estimator which is more robust than the WLS method. It has the inherent bad data identification and rejection properties. A.Abur and M.K.Celik[18],developed a faster and efficient version of WLAV method. Their method consumed less time of simulation and was able to solve the problem of leverage points present in the method described in [19].A.Abur and also introduced equality and inequality constraints on the measurement residual(difference between the measured and estimated value) to solve the LAV problem using Simplex method of linear programming. Their efforts increased the reliability of LAV state estimation by increasing the performance and computational efficiency of the algorithm.H.Singh and F.L.Alvarado [19] extended the interior point method of solving the linear programming problem to the least absolute value state estimation problem. They proved to hat the method is more efficient than the Simplex method in terms of convergence and computational time. R.A.Jabr and B.C.Pal used the Newton Raphson(NR)method to solve the least absolute value problem.They applied the least squares implementation method to weighted least absolute value problem without using the linear programming techniques.

3.3.3 M-Estimators

M-Estimators are maximum likelihood estimators which minimize an objective function expressed as the function of difference between measured and estimated values with constraints imposed on the equation for measurements [20].E.Handschin,F.Schweppe,J.Kohlas and A.Fitcher[21]introduced Quadratic Constant (QC), Quadratic Linear (QL),and Square Root(SR) based objective functions for the M type state estimators.These methods are designed to suppress the bad data within the iterative process of solving the state estimation problem.

All the state estimation methods work to minimize the difference between measured and estimated quantities. They produce an output which has the voltage magnitudes,bus angles and estimated values of the measurements.Additional information about the system can be calculated from these values.

3.3.4 Bad Data Processing Method

Bad data in state estimation problem can cause wrong results so we need to identify presence of bad data.Large measurement errors, measuring the values by reversing the terminals of the devices,large interference due to communication systems and faulty measurement devices are some of the sources of bad data [20,6].State estimation algorithms employ different bad data processing techniques. Weighted least squares method has the bad data processing as a post processing step. The chi-square test [20,6] is employed after the estimation of states by the WLS method to detect the presence of bad data. The test uses the weighted sum of the squared residuals and an error probability against a threshold to detect the presence of bad data. The largest normalized residual testing and hypothesis testing identification methods [22] were developed to identify the bad data. In normalized residual testing the normalized residuals are calculated by dividing the absolute value of each residual (difference between measured and estimated value) with the corresponding diagonal element of the covariance matrix. The largest among the normalized residuals is chosen and compared against an identification threshold. The measurement is removed if its corresponding normalized residual exceeds the identification threshold and the weighted least squares state estimation is re-run to find better estimates. Hypothesis testing identification [23] is an improved method of bad data identification which can handle multiple bad data based on estimating the measurement errors. It uses two types of hypotheses to make decisions whether to accept or reject a rule. The two hypotheses are complements to each other. For example, if two out of six measurements have normalized residuals greater than a threshold, then the rules can be as:(i)Measurement 1 is bad and measurement 2 is good. (ii) Measurement 2 is bad and measurement 1 is good.(iii) Both the measurements are bad.Each rule is implemented by removing the bad measurement and re-running the state estimator. After implementing every rule, the chi-square test can be employed to detect the presence of bad data. If the chi-square test fails, then the hypothesis is true; otherwise it is false [20]. The M-estimators and least absolute value methods have inherent bad data processing capabilities which make them more robust than traditional weighted least squares method.

3.3.5 Observability analysis

A power system is said to be observable if the states of the system can be solved from an available set of measurements. The state estimator breaks down if the numbers of measurements are less than the number of states. The observability analysis helps the state estimator to avoid such problems.The general condition to meet the observability criteria is the rank of the measurement Jacobian must be equal to the number of states.Pseudo-measurements can be added to the system to make the system observable [20,24].

Chapter 4

Problem Formulation

4.1 Introduction

This chapter narrates the formulation of a method [25,26] to compare the performance of algorithms in terms of norms and error indices. The performance indices obtained from the norms can also be used to identify a better state estimation algorithm at different data redundancy levels. This chapter also describes the method of introducing error in measurements, norms, error indices and the flow chart of different state estimation algorithms.

4.2 Addition of error to measurements

The measurements obtained from the sensors are not exact values.They are erroneous due to accuracy limits,bad calibration,of the sensors and also communication noise. The exact values of measurements are obtained from a power flow program [18].A random error is introduced into the measurements in this work to represent an erroneous measurement.The random error is introduced into the actual values. This is generally used in iterative techniques to solve non-linear equations. The errors obtained as a result of one iteration when added to the measured value has a chance to make the measured value accurate. The accuracy of the measured values lack the error portion and from the iteration we can determine the rough error to be added. This is mainly done to reach the final accurate state estimation is done faster. $Z_i = A_i * (1 + RAN * \sigma_i)$
Z_i=Measured valve i=1,2,3...m

4.2.1 Norms and error indices

A norm is a quantity that describes length, size or extent of an object [27].Different types of norms are used in this thesis to compare the performance of the state estimation algorithms.The norms are used to represent the difference between the measured and estimated values. Smaller the value of a norm, the better is the performance of the algorithm with loss of data.The norms are described in the following sections.

4.2.2 Infinity norm

Infinity norm is the maximum of the absolute value of the difference between measured and estimated values. The expression for the infinity norm is given by the infinity norm is applied on the array of measurement residuals[28]. Infinity norm= Maximum $(|Z_m - \hat{Z}|)$

Where,Z_m= measured value $\hat{Z}$ = estimated value of the measurement from state estimator

4.3 Maximum Likelihood Estimation

The mathematical stochastic theory is applied in state estimation to estimate system states is method called maximum likelihood estimation. It begins by creating the likelihood function of the measurement vector.The likelihood function is simply the product of each of the probability density functions of each measurement. Maximum likelihood estimation aims to estimate the unknown parameters of each of the measurements probability density functions through an optimization[20].

It is assumed that the probability density function for power system measurement errors is the normal (or Gaussian) probability density function.

$$f(Z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}(\frac{z-\mu}{\sigma})^2} \tag{4.1}$$

Where, Z is the random variable of the probability density function,μ is the expected value, and σ is the standard deviation. This function would yield the probability of a measurement being a particular value. Therefore, the probability of measuring a particular set of m measurements each with the same probability density function is the product of each of the measurements probability density functions, or the likelihood function for that particular measurement vector.

$$f_m(Z) = \prod_{i=1}^{m} f(Z_i) \tag{4.2}$$

Where Z_i is the i^{th} measurement and

$$\begin{bmatrix} Z \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ . \\ . \\ z_m \end{bmatrix} \tag{4.3}$$

Maximum likelihood estimation aims to maximize this function to determine the unknown parameters of the probability density function of each of the measurements

This can be done by maximizing the logarithm of the likelihood function,$f_m(Z)$ or minimizing the weighted sum of squares of the residuals[3].This can be written as

Minimize

$$\sum_{i=1}^{m} w_{ii} e_i^2 \tag{4.4}$$

Subject to

$$Z_i = h_i(x) + e_i \tag{4.5}$$

The solution to this problem is referred to as the Weighted Least Squares estimator for x.

4.4 Formation of Measurement function

Most commonly used measurement types are bus power injections, line power flows and bus voltage magnitudes in power system state estimation.These measurements can be expressed in form of equations using the state variables.

Consider a system having N buses; the state vector will have $(2N-1)$ components which are composed of N bus voltage magnitudes and $(N-1)$ phase angles. The state vector is equal to $(|v_1|, |v_2|, |v_n|\Theta_1, \Theta_2, \Theta_n)$ such as 0 is set to be the phase angle of one reference bus. If we define as the admittance of the series branch line connecting buses i and j ,and as the admittance of the shunt branch connected at bus i ,the equivalent Π model[20,28]. The nominal Π circuit of a transmission line

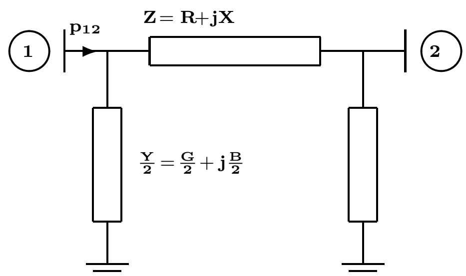

Figure 2.1 Nominalπ-network of a transmission line.

Real and reactive power injection at bus i can be expressed by,

$$P_i = |V_j|\Sigma_{j\epsilon N_i}^{N}|V_j|(G_{ij}cos\theta_{ij} + B_{ij}sin\theta_{ij}) \tag{4.6}$$

$$Q_i = |V_i|\Sigma_{j\epsilon N_i}^{N}|V_j|(G_{ij}sin\theta_{ij} + B_{ij}cos\theta_{ij}) \tag{4.7}$$

Real and reactive power flow from bus i to bus j are

$$P_{ij} = |V_i|^2(g_{si} + g_{ij}) - |V_i||V_j|(g_{ij}cos\theta_{ij} + b_{ij}sin\theta_{ij}) \tag{4.8}$$

$$Q_{ij} = -|V_i|^2(b_{si} + b_{ij}) - |V_i||V_j|(g_{ij}sin\theta_{ij} + b_{ij}cos\theta_{ij}) \tag{4.9}$$

Jacobian matrix H components for real power injection measurement are

$$\frac{\partial P_i}{\partial|V_i|} = \Sigma_{j=1}^{N}|V_j|(G_{ij}cos\theta_{ij} + B_{ij}sin\theta_{ij}) - |V_i|^2G_{ii} \tag{4.10}$$

$$\frac{\partial P_i}{\partial|V_j|} = |V_i|(G_{ij}cos\theta_{ij} + B_{ij}sin\theta_{ij}) \tag{4.11}$$

$$\frac{\partial P_i}{\partial \theta_i} = \Sigma_{j=1}^{N} |V_i||V_j|(-G_{ij}cos\theta_{ij} + B_{ij}sin\theta_{ij}) - |V_i|^2 B_{ii} \tag{4.12}$$

$$\frac{\partial P_i}{\partial \theta_i} = |V_i||V_j|(G_{ij}sin\theta_{ij} + B_{ij}cos\theta_{ij}) \tag{4.13}$$

Jacobian matrix H components for reactive power injection measurement are

$$\frac{\partial Q_i}{\partial |V_i|} = \Sigma_{j=1}^{N} |V_j|(-G_{ij}sin\theta_{ij} + B_{ij}cos\theta_{ij}) - |V_i|^2 B_{ii} \tag{4.14}$$

$$\frac{\partial Q_i}{\partial |V_i|} = |V_i|(G_{ij}sin\theta_{ij} - B_{ij}cos\theta_{ij}) \tag{4.15}$$

$$\frac{\partial Q_i}{\partial \theta_i} = \Sigma_{j=1}^{N} |V_i||V_j|(G_{ij}cos\theta_{ij} + B_{ij}sin\theta_{ij}) - |V_i|^2 G_{ii} \tag{4.16}$$

$$\frac{\partial Q_i}{\partial \theta_i} = |V_i||V_j|(-G_{ij}cos\theta_{ij} - B_{ij}sin\theta_{ij}) \tag{4.17}$$

Jacobian matrix H components for real power flow measurement are,

$$\frac{\partial P_{ij}}{\partial V_i} = -|V_i|(g_{ij}cos\theta_{ij} + b_{ij}sin\theta_{ij}) \tag{4.18}$$

$$\frac{\partial P_{ij}}{\partial \theta_j} = |V_i||V_j|(g_{ij}sin\theta_{ij} + b_{ij}cos\theta_{ij}) \tag{4.19}$$

$$\frac{\partial P_{ij}}{\partial \theta_j} = -|V_i||V_j|(g_{ij}sin\theta_{ij} + b_{ij}cos\theta_{ij}) \tag{4.20}$$

The H matrix has rows at each measurement and columns at each variable. If the System is large, the H matrix has more zero components. Therefore, usually the sparse matrix technique is used to build this matrix.

4.4.1 The Measurement Jacobian

the measurement Jacobean is simply the derivative of the measurement function with respect to the state vector, for application purposes it is simpler to construct this matrix from a symbolic representation and derivative of the measurement function. The measurement Jacobian has the following general structure[20].

$$[H] = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial v} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial v} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial v} \\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial v} \\ \frac{\partial I_{imag}}{\partial \theta} & \frac{\partial I_{imag}}{\partial v} \\ 0 & \frac{\partial V_{mag}}{\partial v} \end{bmatrix} \tag{4.21}$$

The order of the measurement vector will correspond to the order of the rows in the measurement function, and therefore, the measurement Jacobian. While the above partitioning is not required, consistency between the measurement vector and these two matrices is important. Similarly, the columns will correspond to the order of the state vector. Once constructed, the Jacobian matrix elements are each non-linear functions of the state variable and are re-evaluated for each iteration of the estimation solution.

4.5 Description of steps involved in state estimation methods

The steps described in this section are the same for state estimation methods.

4.5.1 Solving for the states of the system

The states are solved with the help of equation obtained after minimization of the objective function from the specific state estimation method. The following subsections explain solving for the states with different state estimation methods.

4.5.2 Weighted Least Squares Method

In weighted least square method, the objective function,f,to be minimized is given by equation The state estimator takes the measurements received from the power system and uses them to estimate the system states. As it is an estimate,there will be some nominal errors associated with each measurement. This mathematical relationship is expressed below[6,31].

Consider the nonlinear measurement model

$$Z_i = h_i(x) + e_i \tag{4.22}$$

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ . \\ . \\ Z_m \end{bmatrix} = \begin{bmatrix} h_1 & (x_1 & x_2 & x_3 & . & . & x_n) \\ h_2 & (x_1 & x_2 & x_3 & . & . & x_n) \\ h_3 & (x_1 & x_2 & x_3 & . & . & x_n) \\ & & . & . & . & . & \\ & . & . & . & . & . & \\ h_m & (x_1 & x_2 & x_3 & . & . & x_n) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ . \\ . \\ e_n \end{bmatrix} \tag{4.23}$$

Consider a measurement vector denoted by containing 'm' number of measurements and a state vector denoted by containing n number of state variables.The Measurement sets which are non-linear functions of the system state vector.These functions are denoted by and can be assembled in vector form as well.These functions, evaluated at the true system state would yield a measurement set containing the true measurement values. However, all of these measurements each have their own unknown error associated with them denoted by in vector form.

The errors are assumed to be independent and uncorrelated with a zero mean. Furthermore, they are assumed to have a Gaussian (Normal) distribution. The covariance matrix associated with the errors will be a diagonal matrix R with the variance of the measurements as its entries.

$$E(e_i) = 0 \qquad (i = 1, 2, 3, ...m) \tag{4.24}$$

$$E(e_i e_j) = 0 \qquad (i = 1, 2, 3, ...m)$$
$$(j = 1, 2, 3, ...m)$$

$$cov(e) = E(ee^T) = R = diag(\sigma_1^2, \sigma_2^2....\sigma_m^2) \tag{4.25}$$

WLS method estimation will be applied to the above equations in order to extract the state quantities, bus voltages and angles, from the measured values, power flows, injections and PMU measurements.WLS estimation to minimize the weighted sum of the squares of the measurement errors.

This minimization will occur when the following objective function is minimized

$$J(x) = \Sigma_{i=1}^{m}(z_i - h_i(x)^2)/R_{ii} \tag{4.26}$$

$$= [z - h_i(x)]^T R^{-1}[z - h_i(x)] \tag{4.27}$$

To minimize the above function,we simply set its first derivative with respect to x equal to zero, as shown below.

$$G(x) = \frac{\partial J(x)}{\partial x} = -[H(x)]^T.R^{-1}[z - h(x)] = 0 \tag{4.28}$$

$$H(x) = \frac{\partial hx)}{\partial x} \tag{4.29}$$

We can apply the Gauss-Newton method to solve the above equation as shown below.

$$x_{k+1} = x_k - [G(x_k)]^{-1}g(x_k) \tag{4.30}$$

Above, is the iteration index and k

$$G(x_k) = \frac{\partial g(x_k)}{\partial x} = -[H(x)]^T.R^{-1}H(x_k) = 0 \tag{4.31}$$

$$g(x_k) = [H(x_k)]^T.R^{-1}[z - h(x_k)] \tag{4.32}$$

The gain matrix is typically rather sparse and decomposed into its triangular factors. For every iteration,forward and backward substitutions are used to solve the following linear equations.

$$[G[(x_k)]\Delta x_{k+1} = [h(x)]^T.R^{-1}[z - h(x_k)] \tag{4.33}$$

$$= [H(x_k)]^T.R^{-1}.\Delta z_k \tag{4.34}$$

$$\Delta x_{k+1} = x_{k+1} - x_k \tag{4.35}$$

These iterations will continue until one of the two following conditions is satisfied. The first condition would be the maximum number of allowable iterations is exceeded while the second condition would be that the change in state variables within an acceptable range.

It is clear that the only information required to iteratively solve this optimization is the covariance matrix of measurement errors, R, and the measurement function h(x). The measurement Jacobian, H(x) is simply the derivative of the measurement function with respect to the state vector. The measurement function and measurement Jacobian can be constructed using the known system model including branch parameters, network topology, and measurement locations and type. The error covariance matrix should also be constructed prior to the iterations with the accuracy information of the meters installed in the system.

For the first iteration of the optimization the measurement function and measurement Jacobian should be evaluated at flat voltage profile, or flat start. A flat start refers to a state vector where all of the voltage magnitudes are 1.0 per unit and all of the voltage angles are 0 degrees. In conjunction with the measurements, the next iteration of the state vector can be calculated again and again until a desired tolerance is reached.

4.5.3 WLS Algorithm

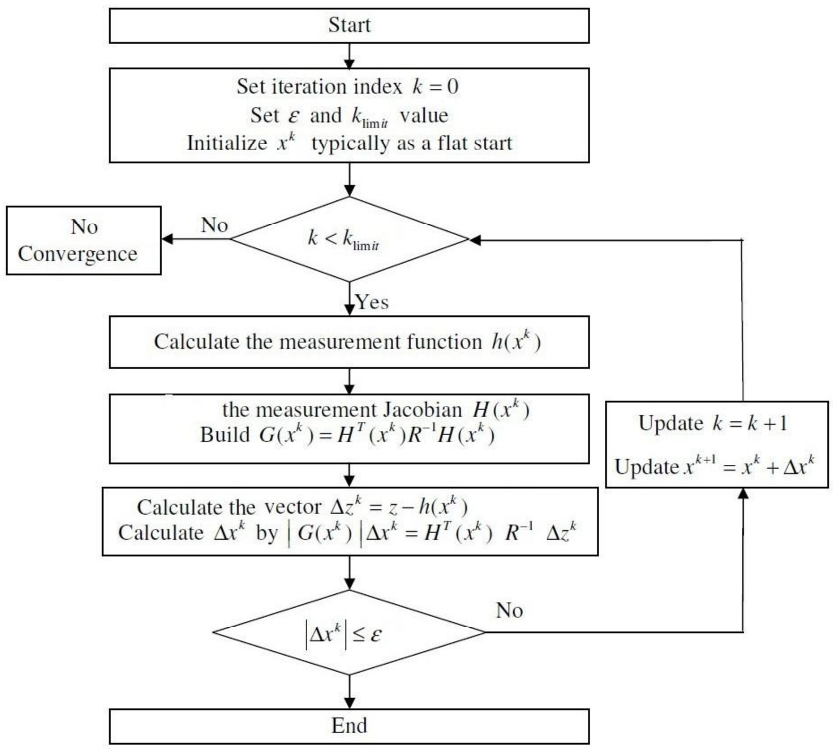

Figure 4.1: Flow chart WLS

WLS state estimation algorithm is started with an initial guess,which is typically chosen as the flat start, i.e. all bus voltages are assumed to be 1.0 per unit and in phase with each other.The flow-chart of the iterative algorithm for WLS state estimation problem can be outlined.

1. Initially set the iteration counter $k = 0$, define the convergence tolerance ε and the iteration limit k_{limit} values

2. If $K > k_{limit}$, then terminate the iterations

3. Calculate the measurement function,$h(x^k)$ the measurement Jacobian $H(x^k)$, and The gain matrix $G(x^k) = H[(x^k)]^T.R^-1.H(x^k)$

4. Solve Δx^k using gain matrix
5. if $|\Delta x^k| < \varepsilon$ then go to step2

   Else, Stop Algorithm Converged

4.5.4 Constrained State Estimation

State estimation can be implemented by considering system constraints to minimize error function minimization problem. Some buses in the network may have neither load nor generation. They are cases with zero power injection at buses, called virtual measurements. The idea is to use this very accurate information in order to enhance the accuracy of these estimates. These measurements are treated separately from the telemetered measurements and imposed them as additional constraints to the WLS problem[20,31].

The objective function:

$$J(x) = 1/2[z - h(x)]R^{-1}[z - h(x)] \tag{4.36}$$

sub $c(x)$=0

The constrained

$$\text{Ł}(r, x, \lambda) = 1/2[z - h(x)]R^{-1}[z - h(x)] + \lambda^T c(x) \tag{4.37}$$

$$\frac{\partial L_{x,y}}{\partial \lambda} = c(x) = 0 \tag{4.38}$$

The optimal solution may be obtained by an iterative solution method for the non-linear equations at each iteration the following linearized equation is solved.

$$\begin{bmatrix} H^T(x^k)R^{-1}H(x^k) & c^T(x^k) \\ c(x^k) & 0 \end{bmatrix} \begin{bmatrix} s^{k+1} \\ \lambda^{k+1} \end{bmatrix} = \begin{bmatrix} H^T(x^k)R^{-1} & r(x^k) \\ -c(x^k) & 0 \end{bmatrix} \tag{4.39}$$

the coefficient of matrix in equation (4.39)is no longer positive definite.care must be exercised in the triangular factorization of the matrix $c(x)$ is the constraint equation Jacobian matrix

$$Where H(x) = \frac{\partial h}{\partial x} and c(x) = \frac{\partial c}{\partial x} \tag{4.40}$$

The coefficient matrix above is indefinite, therefore row ordering must be employed in order to preserve numerical stability. Constrained weighted least-squares problem can be solved by using GAMS program and MATLAB to do that we have to use the formulation as mention below using GAMS we can minimize this function. we can give all input parameters values through MATLAB and variable declarations in GAMS and calling through MATLAB interfacing.

All measured values are taken from data acquisition system.

$$f = \sum_{i=1}^{n} (Z_{measered,i} - Z_{estimated,i})^2 \tag{4.41}$$

$$Z_{estimated,i} = f_i(v_{estimated}, \delta_{estimated})$$

$$Z_{measered,i} = Z$$

$$Z = [P_i, Qi, P_{ij}, Q_{ij}, v_i, \theta_i] \tag{4.42}$$

$$x = [v, \delta]$$

$$Z_{estimated,i}(\hat{Z}) = constraints$$

$$z = [\hat{P}_i, \hat{Q}i, \hat{P}_{ij}, \hat{Q}_{ij}, \hat{v}_i, \hat{\theta}_i] \tag{4.43}$$

$$\hat{x} = [\hat{v}, \hat{\delta}] \tag{4.44}$$

$$e = \sum_{i=1}^{m} W_i(V_i - V_i{}^m)^2 + \sum_{i=1,j=1}^{m} W_{ij}{}^p[P_{ij}(\hat{X}) - P_{ij}^m]^2 + \sum_{i=1,j=1}^{m} W_{ij}{}^q[Q_{ij}(\hat{X}) - Q_{ij}{}^m]^2 \tag{4.45}$$

Equality constraints: Power balance at each nodes

$$p_k^G - p_k^L = \sum_{i=1}^{N} v_k v_i [G_{ki} \cos\theta_{ki} + B_{ki} \sin\theta_{ki}] \tag{4.46}$$

$$Q_k^G - Q_k^L = \sum_{i=1}^{N} v_k v_i [G_{ki} \sin\theta_{ki} + B_{ki} \cos\theta_{ki}] \tag{4.47}$$

Estimated Parameters

Assuming transmission line parameters as PI-Model network

Active and Reactive power flow

$$\hat{P}_i = |V_j| \Sigma_{j\epsilon N_i}^N |V_j| (G_{ij} cos\theta_{ij} + B_{ij} sin\theta_{ij}) \tag{4.48}$$

$$\hat{Q}_i = |V_i| \Sigma_{j\epsilon N_i}^N |V_j| (G_{ij} sin\theta_{ij} + B_{ij} cos\theta_{ij}) \tag{4.49}$$

Active and Reactive power injections

$$\hat{P_{ij}} = |V_i|^2 (g_{si} + g_{ij}) - |V_i||V_j| (g_{ij} cos\theta_{ij} + b_{ij} sin\theta_{ij}) \tag{4.50}$$

$$\hat{Q_{ij}} = -|V_i|^2 (b_{si} + b_{ij}) - |V_i||V_j| (g_{ij} sin\theta_{ij} + b_{ij} cos\theta_{ij}) \tag{4.51}$$

4.6 Bad data Detection and Identification

4.6.1 Introduction

Bad data due to various reasons such as random errors and telecommunication medium errors always exists in the measurements set. This data can affect the estimation result heavily so bad data detection and identification is an essential function of the state estimator.When using WLS estimation method, this function algorithm can be done by processing the measurement residuals. The performance of this function program also depends on the redundancy of the measurement set and the number of bad data. Besides, bad data also appear in several different ways depending upon the type, location and number of them.

They can be classified as: (1) single bad data which means only one of the measurements in the entire system has a large error; (2) multiple bad data which means more than one measurement have errors.[20,32]

The chi-square test is employed after the estimation of states by the WLS method to detect the presence of bad data. The test uses the weighted sum of the squared residuals and an error probability against a threshold to detect the presence of bad data. The largest normalized residual testing and hypothesis testing identification methods were developed to identify the bad data. In normalized residual testing the normalized residuals are calculated by dividing the absolute value of each residual(difference between measured and estimated value) with the corresponding diagonal element of the covariance matrix. The largest among the normalized residuals is chosen and compared against an identification threshold. The measurement is removed if its corresponding normalized residual exceeds the identification threshold and the weighted least squares state estimation(WLS) is rerun to find better estimates. From random theory, we know that if a set of N independent random variables $(X_1, X_2 \, . \, . \, X_N, \, X_i)$ where each has the standard normal distribution$X_i \sim N(0,1)$ then the random variables[20]. The first step in any bad-data filtering algorithm is to detect the presence of bad data. This is commonly accomplished using the Chi squares test. The minimization function which minimizes the sum of the squared residuals, $J(x)$, over the system state, x. $J(x)$.

4.6.2 Chi-square Test

Chi-square-$\chi^2(N)$-distribution with N degrees of freedom, assuming X_k follows a standard normal distribution $N(0,1)$. In the case of the objective function $J(x)$ N equals $(Nm - Ns)$, which is the number of degrees of freedom (DOF) in the system. This is because, with $Nm > Ns$ in the power system, at most $(N_m - N_s)$ of the measurement errors will be linearly independent. A plot of the $\chi^2(N)$ probability density function (p.d.f.), represents the probability of finding $J(x)$ in the corresponding region. The mean value of $\chi^2(N)$ is $\sqrt{DOF} = (N_m - N_s)$ with standard deviation of In the Chi-squares testing method, if the value of $J(x)$ is above a set threshold, we say there is a presence of bad data in the SE. The thresholdx_t designated as the dashed line in is often chosen to constitute a 5% probability of error, or false alarms. For reference in choosing a threshold, tables exist in statistical literature giving Chi-square distribution function values for different degrees of freedom[20,32].

$$Y = \sum_{i=1}^{N} {X_i}^2 \tag{4.52}$$

χ^2_Ndistribution with N degrees of freedom for power system sate estimation the objective function can be written as,

$$J(X) = \sum_{i=1}^{m} \frac{e_1^2}{R_{ii}} = \sum_{i=1}^{m} (\frac{e_i}{\sqrt{R_{ii}}})^2 = \sum_{i=1}^{m} (e_i^N)^2 \tag{4.53}$$

Where $e_i^N(0,1)$ Then $J(x)$ has a Chi-squares distribution with mn degrees of freedom. mn is the number of redundant measurements in the power system m-n being the number of measurements and states respectively

4.6.3 Test for Bad Data

The mathematical model of WLS sate estimation. as shown So the WLS state estimation function can be used the test for bad data. The steps are as follows: Solve the WLS estimation problem and compute the objective function:

$$f = \Sigma_{j=1}^{N_m} (Z_i - h_i(x))^2 / R_{ii} \tag{4.54}$$

$\hat{X}$ is the estimated state vector with dimension n Look up the value from the Chi-squares distribution table corresponding to a detection confidence with probability p(e.g.0.95)and m n degrees of freedom. Say this value is.

Test if $J(X) \geq \chi^2_{(m-n,p)}$ If so, then bad data is detected, otherwise there is no bad data in the measurement set.Once the bad data is detected in the measurement set, it should be identified and eliminated from the measurement set. Largest Normalized Residue (LNR) method which uses the properties of the residue is widely used present single bad data identification program. (At the same time, LNR can also be used as bad data detection).

Linearized measurement equation:

$$\Delta Z = H\Delta X + e \tag{4.55}$$

Then the WLS estimated result is given by:

$$\Delta\hat{X} = (H^T R^-1H)^T H^T R^-1\Delta Z = G^-1H^T R^-1\Delta Z \tag{4.56}$$

And the estimated value of measurement:

$$\Delta\hat{Z} = H\Delta\hat{X} = k\Delta Z \tag{4.57}$$

$$k = HG^-1H^T R^-1 calledhatmatrix \tag{4.58}$$

Then the measurement residue can be written as:

$$KH = HG^-1H^T R^-1H = H \tag{4.59}$$

Then the measurement residue can be written as:

$$r = \Delta Z - \Delta\hat{Z} \tag{4.60}$$

$$r = (1-k)\Delta Z$$

$$r = (1-k)(H\Delta X + e)$$

$$r = (1-k)e$$

$$r = Se \tag{4.61}$$

Where S is called as the residual sensitivity matrix which represents the sensitivity of the measurement residuals to the measurement errors. Also note that S has the properties:

$$SS...S = S$$

$$S.R.S^T = S.R \tag{4.62}$$

Then the covariance matrix Ω of the error term can be calculated as:

$$\Omega = cov(r) = [rr^T] \tag{4.63}$$

$$\Omega = SE[ee^T]S^T$$
$$\Omega = SRS^T$$
$$\Omega = SR \tag{4.64}$$
$$r_i^N = \frac{|r_i|}{\sqrt{\Omega ii}} = \frac{|r_i|}{\sqrt{R_{ii}S_{ii}}} \tag{4.65}$$

Therefore the residue has the distribution $r \sim N(0,\Omega)$ and normalized residual for the measurement i is $r_i^N = \frac{|r_i|}{\sqrt{\Omega ii}} = \frac{|r_i|}{\sqrt{R_{ii}S_{ii}}}$ which means $r_i^N \sim N(0,\Omega)$ thus the largest normalized residue can be compared against a statistical threshold decide it bad data.

4.6.4 Bad Data processing Algorithm

1. Determine $\hat{x}$ considering all the measurement
2. Determine $\hat{Z}$ by employing the relative $\hat{Z} = H\hat{x}$
3. Determine $\hat{e}$ through the relationship $\hat{e} = \tilde{Z} - \hat{Z}$
4. Calculate $\sum_{i=1}^{m} w_i \hat{e}_i^2$
5. Calculate k through the relationship $k = N_m - N_s$
6. Choose valve of α
7. check weather $f < f_{k,\alpha}^2$ if the condition should be satisfied there is no bad data other wise go to step7 Confidence level zero% always bad data
8. Calculate $\frac{\hat{e}}{\sqrt{R_{ii}}}$ and eliminate the measurement that corresponds $\frac{\hat{e}}{\sqrt{R_{ii}^-1}}$ and the redo the state estimation
9. go to step2,else,stop

Largest Normalized Residual

The steps of Largest Normalized Residual(LNR) method is as follows
Solve the WLS state estimation problem and obtain the measurement residual vector

- $r_i = Z_i - h_i\hat{(x)}\ i = 1, 2, 3...m$
- Calculate the normalized residues $r_i^N = \frac{|r_i|}{\sqrt{\Omega ii}}$
- Find K such that is the largest among all $r_i^N\ i = 1, 2, 3...m$
- if $r_i^N >$ threshold ,than the K^{th} measurement is treat as bad data,
- Else Stop there is no bad data in the measurement set
- Eliminate the K^{th} measurement and go to the first step

.

4.7 Summary

This chapter discussed state estimation techniques and presented the formulation of the WLS solution of a non-linear state estimation algorithm, constrained state estimation. This chapter provided an overview about the formulation of state estimation program using different state estimation methods. The LNR method is that it is based on the residuals which may be strongly correlated. Hence, in case of multiple bad data, this correlation may lead to comparable size residuals for good as well as bad data. Thus another way to distinguish good and bad data is by estimating the measurement errors directly. Hypothesis testing method [21], [28] is one of such approach. Although we did not implement this algorithm in this research, applying this approach can also be part of future work.

Chapter 5

Approach and Results

5.0.1 Introduction

Weighted Least Square State Estimation(WLS)and Constrained State Estimation(CSE),Bad data detection and Identification method, discussed in earlier chapter. the results of this methods are shown in this chapters.

5.0.2 Approach

The operating condition of a power system is always determined with the help of measurements obtained from the sensors monitoring the grid. The measurements for this research are obtained from the power flow program . The sensors are assumed to be distributed in such a way that the system is observable. The measurements are assumed to be lost due to failure of communication system or sensors. The power system is always assumed to be in steady state.

5.1 WLS IEEE 14 Bus System

Bus	Estimated Values		Actual Values	
No	Voltage(p.u)	Angle (deg)	Voltage(p.u)	Angle(deg)
1	1.0071	0	1.06	0
2	0.9902	-5.523	1.043	-3.1628
3	0.9522	-14.1944	1.0285	-5.6057
4	0.9583	-11.4067	1.0099	-7.4709
5	0.9618	-9.7523	1.01	-7.6213
6	1.0187	-16.0885	1.0785	-10.5268
7	0.9923	-14.7293	1.0224	-12.0301
8	1.0291	-14.7268	1.01	-14.9634
9	0.9768	-16.4851	1.0392	-11.2786
10	0.9752	-16.8683	1.0494	-11.4797
11	0.993	-16.5868	1.082	-11.4473
12	1.0011	-17.0283	1.0581	-11.1648
13	0.9941	-17.0662	1.071	-11.0619
14	0.9681	-17.5463	1.0473	-11.562

Table 5.1: Measured and Estimated Values for WLS method

5.2 WLS IEEE 30 Bus System

Bus	Estimated Values		Actual Values	
No	Voltage(p.u)	Angle (deg)	Voltage(p.u)	Angle(deg)
1	1.06	0	1.06	0
2	1.043	-5.3404	1.043	-5.3404
3	1.0246	-7.5846	1.0246	-7.5846
4	1.0165	-9.3449	1.0165	-9.3449
5	1.01	-14.1167	1.01	-14.1167
6	1.0176	-11.1558	1.0176	-11.1558
7	1.0068	-12.8957	1.0068	-12.8957
8	1.02	-11.9344	1.02	-11.9344
9	1.0537	-14.1747	1.0537	-14.1747
10	1.0469	-15.7609	1.0469	-15.7609
11	1.082	-14.1747	1.082	-14.1747
12	1.0596	-14.9474	1.0596	-14.9474
13	1.071	-14.9474	1.071	-14.9474
14	1.0451	-15.8287	1.0451	-15.8287
15	1.0409	-15.921	1.0409	-15.921
16	1.047	-15.5629	1.047	-15.5629
17	1.0416	-15.9078	1.0416	-15.9078
18	1.0309	-16.5553	1.0309	-16.5553
19	1.0281	-16.7422	1.0281	-16.7422
20	1.032	-16.554	1.032	-16.554
21	1.0324	-16.2675	1.0324	-16.2675
22	1.0387	-16.0988	1.0387	-16.0988
23	1.0323	-16.2732	1.0323	-16.2732
24	1.0274	-16.4657	1.0274	-16.4657
25	1.0251	-16.0949	1.0251	-16.0949
26	1.0075	-16.5081	1.0075	-16.5081
27	1.0321	-15.6082	1.0321	-15.6082
28	1.0173	-11.8197	1.0173	-11.8197
29	1.0125	-16.8167	1.0125	-16.8167
30	1.0011	-17.6836	1.0011	-17.6836

Table 5.2: Measured and Estimated values of WLS method.

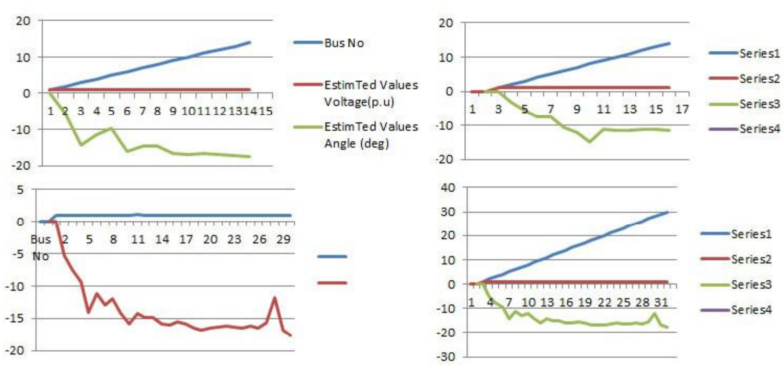

Figure 5.1: WLS

5.3 Constrained State Estimation Using GAMS

Variable	LOWER LEVEL	UPPER MARGINAL
—-VAR v1	-INF 1.022	+INF 8.2392E-9
—-VAR v2	-INF 0.980	+INF -3.839E-9
—-VAR v3	-INF 1.001	+INF -4.782E-9
—-VAR d2	-INF -0.100	+INF -2.068E-9
—-VAR d3	-INF -0.078	+INF -2.375E-9
—-VAR e	-INF 7.806	+INF

Table 5.3: Constrained State Estimation

Variable	MATLAB	GAMS
v1	1.0357	1.022
v2	0.0996	0.980
v3	0.9964	1.001
d1	-0.0977	-0.100
d2	-0.2024	-0.078

5.4 Test for Bad Data

Bus No	Voltage(p.u)	Angle (deg)
1	1.06	0
2	1.043	-3.1628
3	1.0285	-5.6057
4	1.0099	-7.4709
5	1.01	-7.6213
6	1.0785	-10.5268
7	1.0224	-12.0301
8	1.01	-14.9634
9	1.0392	-11.2786
10	1.0494	-11.4797
11	1.082	-11.4473
12	1.0581	-11.1648
13	1.071	-11.0619
14	1.0473	-11.562

Table 5.4: Bad Data Processing

Above result derived based on χ^2 test.The consideration as follows.
Number of Measurements(N_m)=41
Number of State variables(N_s)=28
Number of Degrees of freedom $(k) = (N_m - N_s)$ = 41-28=13
$\alpha = 0.05$ area under the curve between $\chi_{k\alpha}{}^2$ and χ^2
critical value f_t=22.36
the value derived from the program f_c=13.29
$f_c < f_t$ According to the χ^2 test statement we are confident that there is no bad data in measurements.

Chapter 6

Conclusion and Future scope

State estimation in power system is an important task in security analysis to initiate control action at power control centres. We have used Weighted Least Square method(WLS) and Constrained State Estimation(CSE) algorithms to minimize error function. Since the sate estimation algorithm is to be completed in stipulated time we are interested to go with the advanced techniques. The required algorithm should minimize the error function in minimum iterations.

Weighted Least Square state estimation (WLS) and Constrained State Estimation (CSE) methods provide a way to a new hybrid method for quick and accurate estimation system.

The estimation based on available data is necessary for any kind of restorative or corrective action. The comparison of state estimation algorithms on different test cases based on error indices helps to indicate the best algorithm for getting an accurate picture of the power system.

This work has provided the groundwork for the situational awareness needed to move forward with the help of different state estimation algorithms, when uncertain data is available.

Appendix A

Test System

The typical power network models are

- **IEEE 14 Bus System**
- **IEEE 30 Bus System**
- **3 Bus System**

A.1 IEEE 14 Bus System

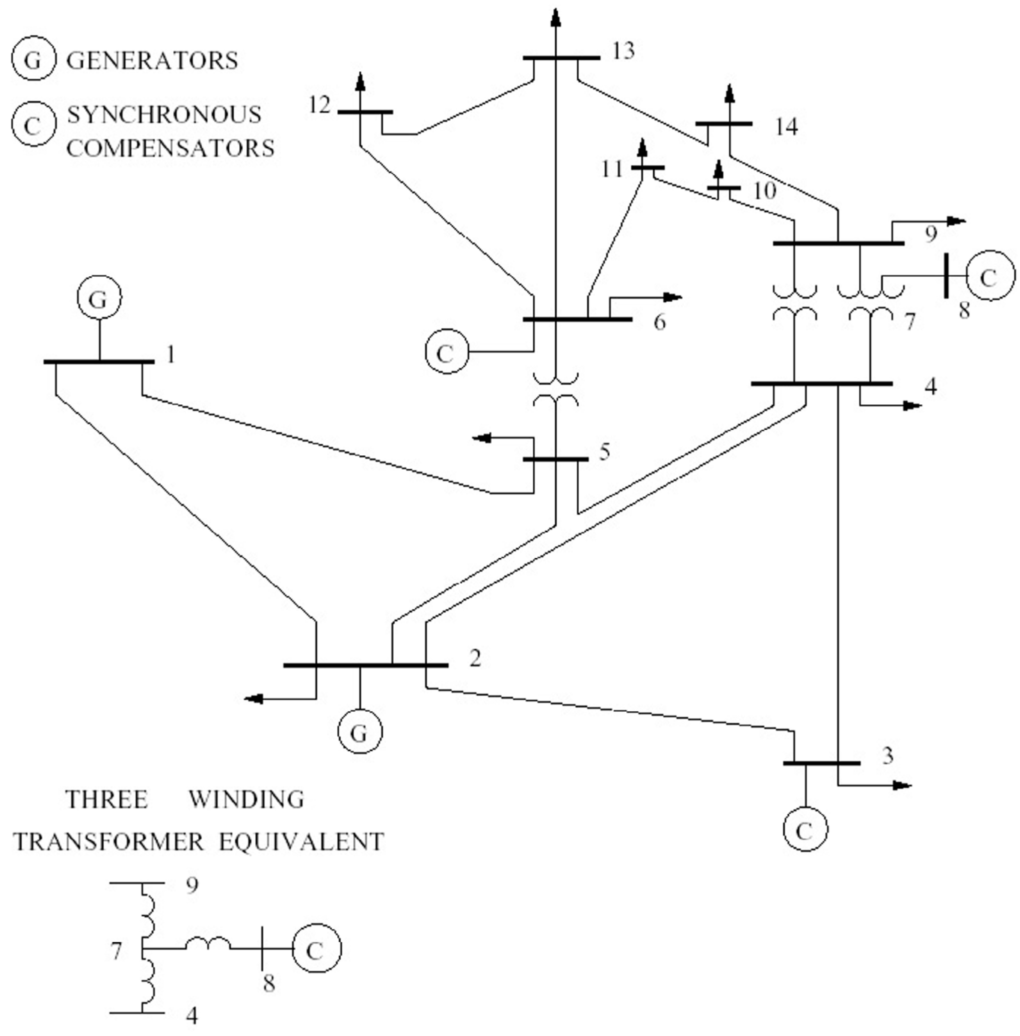

Figure A.1: IEEE 14 bus system

Line data of IEEE 14 bus system

Line No	Bus Code (p-q)	Resistance (p.u)	Reactance (p.u)	Half line shunt Conductance (p.u)	Half line shunt Susceptance (p.u)	Off-nominal ratio
1	1-2	0.01938	0.05917	0.00	0.0264	1.00
2	2-3	0.04699	0.19797	0.00	0.0219	1.00
3	2-4	0.05811	0.17632	0.00	0.0187	1.00
4	1-5	0.05403	0.22304	0.00	0.0246	1.00
5	2-5	0.05695	0.17388	0.00	0.1700	1.00
6	3-4	0.06701	0.17103	0.00	0.0173	1.00
7	4-5	0.01335	0.04211	0.00	0.0064	1.00
8	5-6	0.0000	0.25202	0.00	0.0000	0.932
9	4-7	0.0000	0.20912	0.00	0.0000	0.978
10	7-8	0.0000	0.17615	0.00	0.0000	1.00
11	4-9	0.0000	0.55618	0.00	0.0000	0.969
12	7-9	0.0000	0.11001	0.00	0.0000	1.00
13	9-10	0.03181	0.08450	0.00	0.0000	1.00
14	6-11	0.09498	0.19890	0.00	0.0000	1.00
15	6-12	0.12291	0.25581	0.00	0.0000	1.00
16	6-13	0.06615	0.13027	0.00	0.0000	1.00
17	9-14	0.12711	0.27038	0.00	0.0000	1.00
18	10-11	0.08205	0.19207	0.00	0.0000	1.00
19	12-13	0.22092	0.19988	0.00	0.0000	1.00
20	13-14	0.17093	0.34802	0.00	0.0000	1.00

Table A.1: Line data of IEEE 14 bus system

Bus data of IEEE 14 bus system

Bus No	Voltage magnitude	Load		Generation	
		MW	MVAR	MW	MVAR
1	1.06	0	0	0	0
2	1.045	21.7	12.7	40	42.4
3	1.01	94.2	19	0	23.4
4	1	47.8	-3.9	0	0
5	1	7.6	1.6	0	0
6	1.07	11.2	7.5	0	12.2
7	1	0	0	0	0
8	1.09	0	0	0	17.4
9	1	29.5	16.6	0	0
10	1	9	5.8	0	0
11	1	3.5	1.8	0	0
12	1	6.1	1.6	0	0
13	1	13.5	5.8	0	0
14	1	14.9	5	0	0

Table A.2: Bus data of IEEE-14 bus system

Regulated Bus Data of IEEE 14 bus system

Bus No	BUS Voltage(mag) p.u.	Minimum MVAR	Maximun MVAR
2	1.045	-40.00	50.00
3	1.010	0.00	40.00
6	1.070	-6.00	24.00
8	1.090	-6.00	24.00

Table A.3: Regulated bus data of IEEE 14 bus system

Shunt capacitor Data of IEEE 14 Bus system

Bus No.	Susceptance(p.u.)
9	0.190

Table A.4: Shunt capacitor data of IEEE 14 bus system

A.2 IEEE 30 Bus System

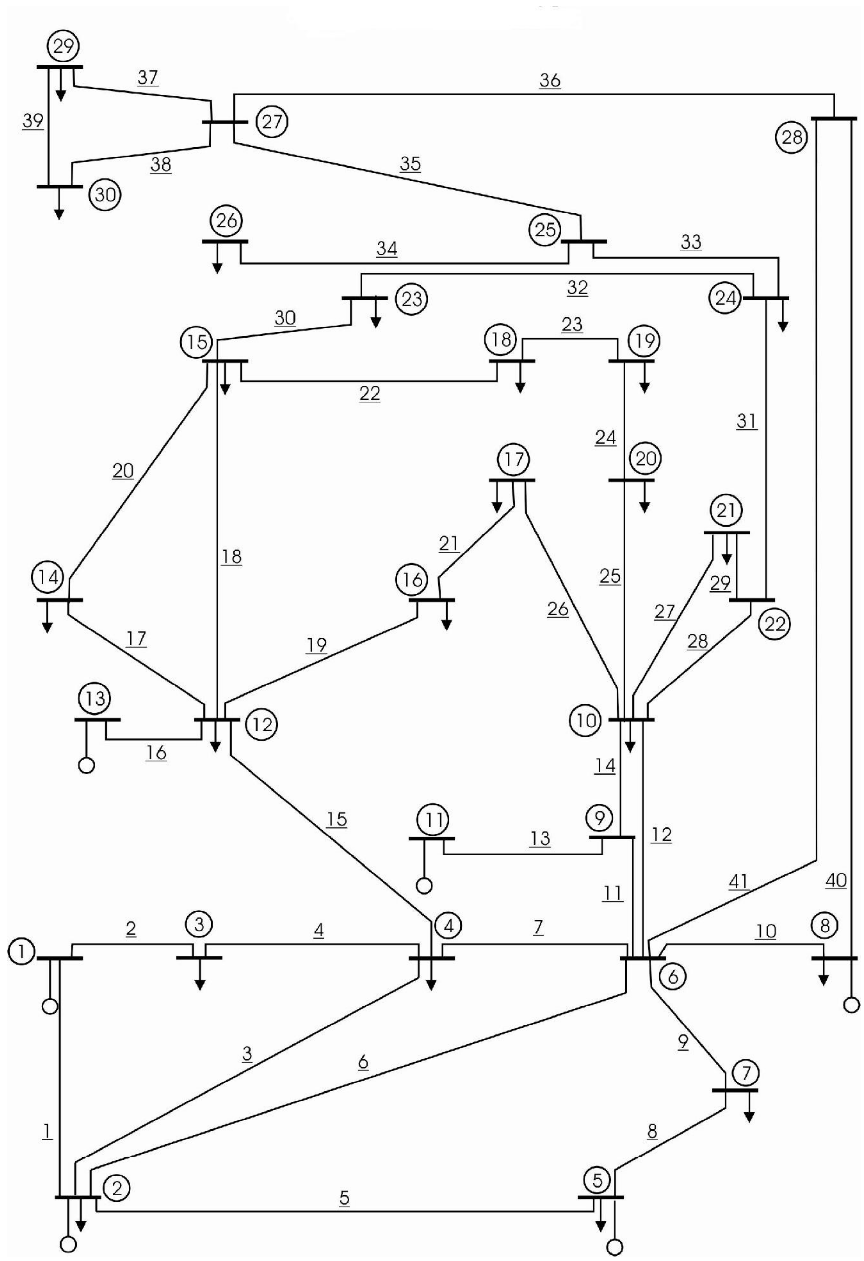

Figure A.2: IEEE-30 bus system

Line data of IEEE 30 bus system

Sl No.	Node Connected	Resistance (p.u)	Reactance (p.u)	Half line shunt Conductance (p.u)	Half line shunt Susceptance (p.u)	Off-nominal ratio
1	1-2	0.01927	0.0575	0.00	0.0264	1.0
2	1-3	0.04520	0.1852	0.00	0.0204	1.0
3	2-4	0.05700	0.1737	0.00	0.0184	1.0
4	3-4	0.01320	0.0379	0.00	0.0042	1.0
5	2-5	0.04720	0.1983	0.00	0.0209	1.0
6	2-6	0.05810	0.1763	0.00	0.0187	1.0
7	4-6	0.01190	0.0414	0.00	0.0045	1.0
8	5-7	0.04600	0.1160	0.00	0.0102	1.0
9	6-7	0.02670	0.0820	0.00	0.0085	1.0
10	6-8	0.01200	0.0420	0.00	0.0045	1.0
11	6-9	0.00000	0.2080	0.00	0.0000	1.0155
12	6-10	0.00000	0.5560	0.00	0.0000	0.9629
13	9-11	0.00000	0.2080	0.00	0.0000	1.0
14	9-10	0.00000	0.1100	0.00	0.0000	1.0
15	4-12	0.00000	0.2560	0.00	0.0000	1.0129
16	12-13	0.00000	0.1400	0.00	0.0000	1.0
17	12-14	0.12310	0.2559	0.00	0.0000	1.0
18	12-15	0.06620	0.1304	0.00	0.0000	1.0
19	12-16	0.09450	0.1987	0.00	0.0000	1.0
20	14-15	0.22100	0.1997	0.00	0.0000	1.0
21	16-17	0.08240	0.1932	0.00	0.0000	1.0
22	15-18	0.10700	0.2185	0.00	0.0000	1.0
23	18-19	0.06390	0.1292	0.00	0.0000	1.0
24	19-20	0.03400	0.0680	0.00	0.0000	1.0
25	10-20	0.09360	0.2090	0.00	0.0000	1.0
26	10-17	0.03240	0.0845	0.00	0.0000	1.0
27	10-21	0.03480	0.0749	0.00	0.0000	1.0
28	10-22	0.07270	0.1499	0.00	0.0000	1.0
29	21-22	0.01160	0.0236	0.00	0.0000	1.0
30	15-23	0.10000	0.2020	0.00	0.0000	1.0
31	22-24	0.11500	0.1790	0.00	0.0000	1.0
32	23-24	0.13200	0.2700	0.00	0.0000	1.0

(continued....)

Sl. No.	Node Connected	Resistance (p.u)	Reactance (p.u)	Conductance (p.u)	Susceptance (p.u)	Off-nominal ratio
33	24-25	0.18850	0.3292	0.00	0.0000	1.0
34	25-26	0.25440	0.3800	0.00	0.0000	1.0
35	25-27	0.10930	0.2087	0.00	0.0000	1.0
36	28-27	0.00000	0.3960	0.00	0.0000	0.9581
37	27-29	0.21980	0.4153	0.00	0.0000	1.0
38	27-30	0.32020	0.6027	0.00	0.0000	1.0
39	29-30	0.23990	0.4533	0.00	0.0000	1.0
40	8-28	0.06360	0.2000	0.00	0.0214	1.0
41	6-28	0.01690	0.0599	0.00	0.0065	1.0

Table A.5: Line data of IEEE-30 bus system.

Bus data of IEEE 30 bus system

Bus No	Voltage magnitude	Load		Generation	
		MW	MVAR	MW	MVAR
1	1.05	0	0	0	0
2	1.0338	21.7	12.7	57.56	2.47
3	1	2.4	1.2	0	0
4	1	7.6	1.6	0	0
5	1.0058	94.2	19	24.56	22.57
6	1	0	0	0	0
7	1	22.8	10.9	0	0
8	1.023	30	30	35	34.84
9	1	0	0	0	0
10	1	5.8	2	0	0
11	1.0913	0	0	17.93	30.78
12	1	11.2	7.5	0	0
13	1.0883	0	0	16.91	37.83
14	1	6.2	1.6	0	0
15	1	8.2	2.5	0	0
16	1	3.5	1.8	0	0
17	1	9	5.8	0	0
18	1	3.2	0.9	0	0
19	1	9.5	3.4	0	0

(continued..)

Bus No	Voltage magnitude	Load		Generation	
		MW	MVAR	MW	MVAR
20	1	2.2	0.7	0	0
21	1	17.5	11.2	0	0
22	1	0	0	0	0
23	1	3.2	1.6	0	0
24	1	8.7	6.7	0	0
25	1	0	0	0	0
26	1	3.5	2.3	0	0
27	1	0	0	0	0
28	1	0	0	0	0
29	1	2.4	0.9	0	0
30	1	10.6	1.9	0	0

Table A.6: Bus data of IEEE-30 bus system

Regulated Bus Data of IEEE 30 bus system

Bus No	BUS Voltage(mag) p.u.	Minimum MVAR	Maximun MVAR
2	1.0338	-20.0	60.0
5	1.0058	-15.0	62.0
8	1.0230	-15.0	50.0
11	1.0913	-10.0	40.0
13	1.0883	-15.0	45.0

Table A.7: Regulated bus data of IEEE 30 bus system

Shunt capacitor Data of IEEE 30 bus system

Bus No.	Susceptance(p.u.)
10	0.19
24	0.043

Table A.8: Shunt capacitor data of IEEE 30 bus system

A.3 3 Bus System

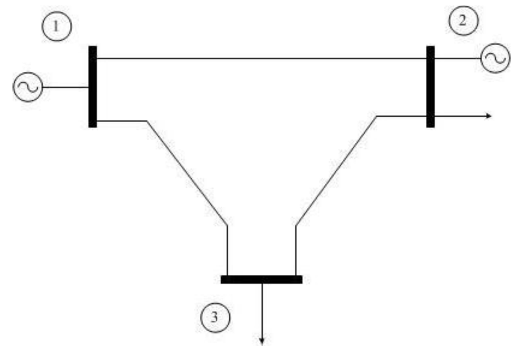

Figure A.3: **3 Bus System**

Line data of 3 bus system

Sl. No.	Bus Code (p-q)	Resistance (p.u)	Reactance (p.u)	Half line shunt Conductance (p.u.)	Half line shunt Susceptance (p.u.)	Off-nominal Ratio
1	1-2	0.020	0.010	0.000	0.0164	1.00
2	1-3	0.0025	0.15	0.000	0.0246	1.00
3	2-3	0.025	0.197	0.000	0.0219	1.00

Table A.9: Line data of 3 bus system

Bus data of 3 bus system

Bus No	Type	BUS Voltage (Magnitude)	Generation		Load	
			MW(P.u)	MVAR(P.u)	MW(P.u)	MVAR(P.u)
1	slack	1.0	0.50	0.00	0.00	0.00
2	PV	1.05	0.25	30.00	0.34	0.15
3	PQ	095	0.95	0.00	0.40	0.00

Table A.10: Bus data of 3 bus system

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