Transient Stability Assessment Using Energy Function Method

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To My parents and Friends

Abstract

There are numerous assumptions made to define the stability of power system models. Step-by-Step numerical integrations of power system models are used to simulate the system dynamic behavior. This type of power system stability analysis is based on the time-domain approach. The typical simulation period of post-fault system is 10s and can go beyond 15s if multi-swing instability is of concern. Energy Function theory is a method to describe the stability of power system and it eliminates the post-fault integration which reduces the simulation time.

Mechanical power, electrical power, torque, rotor speed of generators, loads are the parameters which specify the characteristics of stability. An energy function defines the nature of the system, when system exhibits a highly non-linear behavior. An energy function gives more accurate prediction when there is a severe disturbance in the system.

This thesis describes the various parameters affecting transient stability and discussed an energy function for a power system. It works as a general principle model to read stability in a very accurate and effective way to calibrate the amount of energy in a system during different rotor angles when the system is in post-fault or after disturbance.

This Thesis provides a general procedure for constructing a classical based energy functions for single machine infinite bus system as well as multi-machine system (using COI reference and relative rotor angle reference). It gives a theoretical discussion and simulation results for different methods like closest unstable equilibrium point (u.e.p) method, potential boundary surface method and controlling unstable equilibrium point (u.e.p) method.

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Nomenclature

List of Symbols

n	Number of generators.
P_{ei}	Electrical power output of generator i.
P_{mi}	Mechanical power input to the generator i.
M_i	Inertia constant of i^{th} generator.
J	Jacobian matrix.
t_c	Fault clearing time.
t_{cr}	Critical clearing time.
V_{cr}	Critical transient energy.
$V(\delta, \omega)$	System transient energy.
Y_{ij}	Transfer admittance between i^{th} bus to j^{th} bus.
G_{ij}	Transfer conductance between i^{th} bus to j^{th} bus.
B_{ij}	Susceptance between i^{th} bus to j^{th} bus.
V_{KE}	Kinetic energy.
V_{PE}	Potential energy.
ω_i	Angular velocity of the rotor.

θ^s	Stable equilibrium point.
θ^u	Unstable equilibrium point.
H	Inertia coonstant.
D_i	Damping coefficient of generator i.
P_i	Active power of generator i.
Q_i	Reactive power of generator i.
V_i	Voltage at i^{th} bus.
PQ	Load bus.
PV	Generator bus.
x^{uc}	Unsatble equilibrium point.
p	Number of poles.
τ_m	Mechanical torque applied by the prime mover.
τ_e	Electrical torque output of the generator.
$\theta_{i,j}$	Angle of $Y_{i,j}$.
E_i	Voltage at i^{th} bus.
E_j	Voltage at j^{th} bus.
X^d	Faulted system reactance.
X^{pf}	Post-Faul system reactance.
X^{pre}	Pre-fault system reactance.
f^{pre}	Pre-fault system.
f^d	Faulted system.

f^{pf}	Post-fault system.
P_e^{max}	Maximum post-fault electrical power.
P_e^F	Maximum faulted electrical power.
P_e	Maximum pre-fault electrical power.
ω	Angular frequency of machine.

List of Acronyms

u.e.p	Unstable equilibrium point.
UEP	Unstable equilibriium point.
s.e.p	Stable equilibrium point.
PEBS	Potential energy boundary surface method.
CUEP	Controlling unstable equilibrium point.
LUEP	Lowest unstable equilibrium point.
COI	Center of inertia.
TEF	Transient energy function.
FB	Faulted bus.
FL	Faulted line.

Chapter 1

Introduction

1.1 Overview

Every engineering department has the impact of computer-aided analysis and design in the recent years. One of the fundamental subjects in engineering and science is the stability study of a system.

The stability of a system refers to the ability of a system to return back to its steady state when subjected to a disturbance. Power is generated by synchronous generators that operate in synchronism with the rest of the system. A generator is synchronized with a bus when both of them have same frequency, voltage and phase sequence. We can thus define the power system stability as the ability of the power system to return to steady state without losing synchronism [6].

The ability of a power system to maintain stability under continuous small disturbances is investigated under the name of dynamic stability (also known as small-signal stability). These small disturbances occur due to random fluctuations in loads and generation levels. In an interconnected power system, these random variations can lead catastrophic failure as this may force the rotor angle to increase steadily.

Transient instability is concerned with sudden and large changes in the network condition due to transmission faults. Sudden load changes, and loss of generating units. Dynamic and steady state stability is basically the ability of the power system under load condition to retain synchronism when subject to small disturbance such as the continual changes in load or incremental variations in the operating point [9].

By nature, a power system is continuously experiencing disturbances. These may be classified as event disturbances and load disturbance. Generator outages, short-circuits caused by lightning or other fault conditions, sudden large load changes, or a combination of such events do come under Event disturbances. Event disturbances usually lead to a change in the configuration of the power system. Load disturbances, on the other hand, are the small random fluctuations in load demands. The system configuration usually remains unchanged after load disturbances. Recent trends towards full utilization of existing generation and transmission systems have increased the effects of these disturbances on power system security [18].

Transient stability analysis is concerned with a power system's ability to reach an acceptable steady-state (operating condition) following an event disturbance. The power system under this circumstance can be considered as going through changes in configuration in three stages: from pre-fault, to fault-on, and then to post-fault systems. The pre-fault system is usually in a stable steady state. The fault occurs (e.g., a short circuit), and the system is then in the fault-on condition before it is cleared by the protective system operation. Stability analysis is the study of whether the post-fault trajectory will converge to an acceptable steady-state as time passes.

Direct methods have a long development history spanning over four decades but, until recently, were thought by many to be impractical for large-scale power systems analysis with detailed models. However, recent developments have made direct methods a more practical means of solving large-scale power systems with network-preserving models. As seen in those early applications, direct methods provide several key advantages in performing on-line stability assessment using the actual power system configuration and on-line state estimated data. One key advantage is their ability to assess the degree of stability (or instability). The second advantage is their ability to calculate sensitivities of the stability margin to power system parameters, allowing for efficient computation of operating limits [6].

This thesis presents the transient stability assessment for single machine infinite bus system as well as multi-machine systems. Different energy function methods are used to find the transient stability. Those methods are closest unstable equilibrium point (u.e.p) method, potential boundary surface method(PEBS), controlling unstable equilibrium point (u.e.p) method. Which comes with a simple mathematical model to predict its stability.

1.2 Literature Survey

At present, stability analysis programs routinely used in utilities around the world are based on step-by-step numerical integrations of power system stability models used to simulate system dynamic behaviors. The stability of the post-fault system is assessed based on simulated post-fault trajectories. The typical simulation period for the post-fault system is 10s and can go beyond 15s if multi-swing instability is of concern, making the conventional approach rather time-consuming.

T.Athay, R.Podmore and S.Virmani [1]: This paper presents the development and evaluation of an analytical method for the direct determination of transient stability. The method developed is based on the analysis of transient energy and accounts for the nature of the system disturbance as well as for the effects of transfer conductance on system behavior. The method predicts critical clearing times for first swing transient stability which agree very closely with the results of simulations.

A.A. Fouad and Vijay Vittal [2]: In this paper, a criterion is presented for determining the critical energy to be used in direct transient stability assessment of a multi-machine. The particular criterion provides a method by which the controlling unstable equilibrium point (u.e.p.), for the disturbance under investigation, can be identified. The methodology proposed was tested on two medium-size power networks under complex situations. In all the cases, the correct u.e.p. was identified and verified using detailed system trajectories.

Ahmed H. El-Abaid and K.Nagappan [3]: It gives a new approach to the quantitative study of the transient stability of large power systems, using the second method of Liapunov. A region of asymptotic stability for the post-fault system is obtained through Liapunov theorems. If the initial conditions of the post-fault system at the time of switching (to restore normal operation) lie within this region, the system will be stable. The moment at which the system is at the boundary of the region of asymptotic stability gives the critical switching time. Liapunov's direct (second) method for the study of stability of nonlinear systems has been successfully applied to multi-machine power systems. The advantages at the particular method include Automatic determination of stability or instability as well as Automatic determination of the critical switching time.

C.L. Gupta and A.H. EL-Abiad [4]: In this paper, an algorithm presented based on the physical behavior of the power system. The particular method computationally faster than the previous methods. The algorithm presented in this paper removes one of the difficulties in the practical application of Liapunov's direct method to power systems. This approach provides a systematic method of eliminating the unstable equilibrium points which are of no interest in the search for the critical region of stability. Thus the exact determination of the unstable equilibrium point closest to the post-fault stable equilibrium point and reduction in the search time are the main contribution of this work.

G.D.Irisarri and G.C.Ejebe and J.G.Waight [5]: This paper presents a new method for the determination of stable and unstable equilibrium points in the Transient Energy Function method. Computation of the unstable equilibrium point is one of the fundamental steps in determination of the energy margin. The basis of the new method is its formulation as a power flow problem and the use of well-known power flow algorithms for its solution.

Hsiao-Dong Chiang, Chia-Chi Chu and Gerry Cauley [6]: This paper presents a theoretical foundation of direct methods for both network-reduction and network-preserving power system models. In addition to an overview, new results are offered. A systematic procedure to construct energy functions for both network-reduction and network-preserving power system models is proposed. The major breakthroughs presented in this paper include a solid mathematical foundation for direct methods. Numerical solution algorithms capable of supporting on-line applications of direct methods are provided. Practical demonstrations of using direct methods and the BCU method for on-line transient stability assessments on two power systems are described. Further possible improvements, enhancements and other applications of direct methods are outlined.

M K Khedkar and G M Dhole and V G Neve [7]: In this paper, a study of transient stability assessment of a multi-machine power system is presented. The criteria for identifying the machines likely to separate from the system, by loss of synchronism and the energy associated with this separation is also developed. A theoretical foundation for Transient Energy Function (TEF) Method is presented. A complete topological characterization of the stability boundary of a stable equilibrium point is derived. Determination of critical energy by using Closest UEP and Controlling UEP methods is presented.

T.S.Chung, Fang Da-Zhong [8]: In this paper, a new fast algorithm to perform online transient stability assessment (TSA) in a power system is proposed. The TSA is concerned with finding the critical clearing time (CCT), the stability limit, for a specified fault. The method possesses the merits of the fast speed of the potential energy boundary surface method and the accuracy of the conventional time domain simulation technique. In addition, an efficient stopping criterion for the post-fault trajectory simulation is also suggested in order to speed up the computation.

M A Pai, M Laufenberg and P W Sauer [9]: This paper presents to clarify the evaluation of path dependent integrals in the energy function method for stability analysis. Determination of path dependent integrals using trapezoidal method is presented. Straight line approximation is compared with the trapezoidal method. The equivalent of the energy function to the equal-area criterion is presented for the single machine system. For the multi-machine case, the PEBS method explained.

Pravin Varaiya, Felix F. Wu and Rong-Liang Chen [10]: This paper presents a critical review of research on direct methods carried out since 1970. Considerable theoretical properties of energy functions and the limitations of the classical model and structure preserving model is discussed. The stability boundary approximation is explained.

1.3 Structure of Thesis

Chapter Two: An overview of power system stability of a small-scale system. The discussion on stability and instability. This chapter also gives a basic mathematical approach that required for transient stability assessment. It also describes the single machine infinite bus system and equal area criterion which provides theory and techniques to discuss the stability of system.

Chapter Three: It provides a discussion on non-linear equation solving and load flow solutions. Energy function derivation for multi-machine system using relative rotor angle formation, center of inertia formation.

Chapter Four: This chapter gives a energy function methods that are used for finding transient stability are discussed. Properties of energy function methods and Results are discussed based on Matlab database.

Chapter Five: This chapter concludes the thesis, by providing summary on the completion of the thesis.

Chapter 2

State of The Art of Power System Stability

2.1 Introduction

Stable operation of a power system depends on the ability to continuously match the electrical output of generating units to the electrical load in the system. Power system stability becomes an important issue to power industry. This chapter gives a detailed description on power system stability, the swing equation and analytical tool for the study of power system stability.

2.2 Power System Stability

Power systems are vast and heavily interconnected systems with a wide array of devices. The stability of this complex power system may be broadly defined as its ability to return to a steady operating condition or thee ability to regain to an acceptable state of equilibrium after having been subjected to some form of disturbance [9].

When the rotor of a synchronous generator advances beyond a certain critical angle the magnetic coupling between the rotor and the stator fails. This creates imbalance between the input and output power. Which make the rotor to be unstable and accelerate without control. This loss of synchronism between the rotor and the stator magnetic fields results

in a large fluctuations in power output voltage and current. Which is a common form of power system instability.
The studies of power instability problems can be generally separated into two main types:

1. Transient stability

2. Dynamic and steady state stability, depending on the magnitude and nature of disturbance.

Transient instability is concerned with sudden and large changes in the network condition due to transmission faults. Sudden load changes and loss of generating units. Dynamic and steady state stability is basically the ability of the power system under load condition to retain synchronism when subject to small disturbance such as the continual changes in load or incremental variations in the operating point.
Transient instability problems can be studied in the first swing or multi-swing equation basis. First swing equation uses a simple generator model without control system and it is assumed to be stable if machines remain in synchronism within the first second following a system fault (single machine infinite bus system). Multi-swing equation involves a sophisticated machine model extending over a long period and the dynamic performance is influenced by the generator control system(multi-machine system).

2.2.1 Transient stability

The state of transient instability refers to the ability of the system to remain in synchronism when subjected to large disturbances. Transient stability may be applied to a synchronous machine and multi-machine [10].
When it is applied to a synchronous machine, it can be defined as follows:
If a synchronous machine operating in steady state equilibrium is subjected to a disturbance of any kind which results in speed deviations (oscillations) of the machine rotor from the (synchronously rotating) reference axis. Then the machine is called transient stable if the rotor ultimately reaches a new stable equilibrium position.
On the other hand, for a multi machine system, it can be defined as:

If the individual machines in a multi-machine system are operating in steady state equilibrium and a disturbance of any kind is imposed on the system, then the system is called transiently stable if each machine oscillates around and ultimately comes to rest at a new stable equilibrium point.

The disturbances usually considered in transient stability are caused by the more severe ones such as short circuit on transmission lines, buses or at transformer terminals. These faults may be symmetrical, three phase to ground, single phase to ground, or two phase to ground.

When a fault occurs, it is necessary to determine when to clear the fault without any of the rotating machines losing their synchronism. The maximum time through which the fault can be left on the system and yet have the system after its clearance is called the 'critical clearing time'. Determination of critical clearing time therefore constitute an important facet of transient stability study since such information enables the setting of relays to clear the fault, and et cera. Other disturbances considered are sudden disconnection of generating units or loads.

2.2.2 Steady state stability

Assuming that a system is operating in a certain steady state condition. For instance, if small changes occur in the system, the system returns to steady state as $t \to \infty$, this particular condition is called steady state stable [1]. The main aim of the power system is, whether the system is in stable condition or unstable condition. The rotor angle changes with respect to time. If the rotor angle δ continuously increases with respect to time then the system is unstable. The rotor angle does't change with time with respect to reference then the system is stable [17].

Figure 1.1a and Figure 1.1b shows the rotor stability/instability for multi-machine system. Figure 1.1a shows that all relative rotor angles are finite, as $t \to \infty$, so system is stable. Figure 1.1b shows a two machines are going to be unstable. Remaining machines are stable.

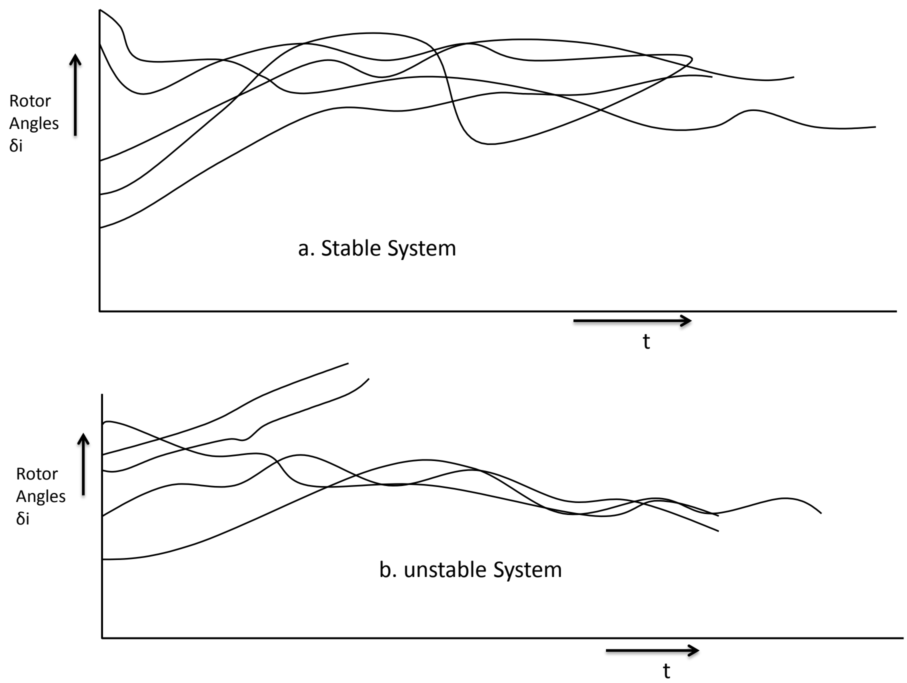

Figure 2.1: A.Stable and B. Unstable system

2.3 Mathematical Background

Dynamic systems are generally described by a set of nonlinear differential equations of the form

$$\dot{x} = F(x)$$

Power system can be described by the following equations, when system subjected to disturbance [9]:

$$\dot{\boldsymbol{x}}(t) = f^{pre}(x(t)) \qquad -\infty < t \leq 0 \tag{2.1}$$

$$\dot{\boldsymbol{x}}(t) = f^{d}(x(t)) \qquad 0 < t \leq t_{cr} \tag{2.2}$$

$$\dot{\boldsymbol{x}}(t) = f^{pf}(x(t)) \qquad t_{cr} < t \leq \infty \tag{2.3}$$

$x(t)$ is the vector of state variables of the system at time t. At the time $t = 0$ the system is experienced a fault the system state will change from pre-fault state to faulted state. Which is from f^{pre} to f^{d}. During $0 < t \leq t_{cr}$, the system under faulted period, before fault is clear (t_c) in the system there is a several changes. During fault period there is a number of switchings in the system. After fault clearing the system enter into the post fault f^{pf} region. In the pre-fault period $-\infty < t \leq 0$, system is under steady state. So, now the initial condition $x(0) = x_0$ is known [17]. Therefore, we then have only

$$\dot{\boldsymbol{x}}(t) = f^d(x(t)) \qquad 0 < t \leq t_{cr} \tag{2.4}$$

$$x(0) = x_0$$

and

$$\dot{x} = f^{pf}(x(t)) \qquad t > t_{cr} \tag{2.5}$$

The solution of equation(2.4) provides at each instant of time the possible initial conditions for equation(2.5). Let us assume the equation(2.5) has a stable equilibrium point x_s. The question is whether the trajectory $x(t)$ for equation(2.5) with the initial condition $x(t_{cr})$ will converge to x_s as $t \to \infty$.The largest value of t_{cr} for which this holds true is called the critical clearing time t_{cr} [17].
It is clear that if we have an accurate estimate of the region of attraction of the post-fault stable equilibrium point (s.e.p) x_s, then t_{cr} is obtained when the trajectory of equation(2.4) exits the region of attraction of equation(2.5). figure 2.2 illustrates this concept for a two-dimensional system. The computation of the region of attraction for a general nonlinear dynamical system is far from easy. It is not, in general, a closed region. In the case of power systems with simple machine models, the characterization of this region has been discussed theoretically in the literature [9].
The stability region consists of surface passing through the unstable equilibrium points (u.e.p's) of equation(2.5). For each fault, the mode of instability may be different if the fault is not cleared in time. We may describe the interior of the region of attraction of the post-fault system (equation(2.5)) through an inequality of the type $V(\delta, \omega) < V_{cr}$, where $V(\delta, \omega)$ is the Liapunov or energy function is shown in equation(2.5). $V(\delta, \omega)$ is generally the sum of the kinetic and potential energies of the post-fault system. The computation of V_{cr}, called the critical energy, is different for each fault and is a difficult step.

2.4 Swing Equation

Let us consider a three-phase synchronous alternator that is driven by a prime mover. The conversion of mechanical energy to electrical energy of generator depends entirely on the relative motion of the conductors with respect to the field lines. The dynamics torque produced in this electromechanical system can be expressed as a function of the electrical variables and the mechanical displacement. Therefore first swing equation is an important equation that describes the effect of imbalance between the electromagnetic torque and mechanical torque of the individual machine [16].
This electromechanical equation is derived from equating the inertia torque to the net torque causing acceleration (or deceleration) as shown below.

Now we can write

$$J\frac{d\omega_i}{dt} + D\omega_i = \tau_m - \tau_e \tag{2.6}$$

Includes some mechanical rotational loss due to windage and friction

$$\tau_t = \tau_e + D\omega_m$$

or

$$\tau_m = \tau_t - D\omega_m = \tau_e \tag{2.7}$$

where τ_m is the net mechanical shaft torque. If, due to some disturbance,$\tau_m > \tau_e$then the rotor accelerates. if $\tau_m < \tau_e$ then it decelerates.
Then the rotor speed expressed as

$$\omega_m = \omega_{sm} + \triangle\omega_m = \omega_{sm} + \frac{d\delta_m}{dt} \tag{2.8}$$

Where δ_m is the rotor angle expressed in mechanical radians. $\triangle\omega_m = \frac{d\delta_m}{dt}$ is the speed deviation in mechanical radians per second.
Substituting equation(2.8) into equation(2.6) gives

$$J\frac{d^2\delta_i}{dt^2} + D_d\frac{d\delta_i}{dt} = \tau_m - \tau_e \tag{2.9}$$

Multiplying through by the rotor synchronous speed ω_{sm} gives

$$J\omega_{sm}\frac{d^2\delta_i}{dt^2} + D_d\omega_{sm}\frac{d\delta_i}{dt} = \omega_{sm}\tau_m - \omega_{sm}\tau_e \tag{2.10}$$

As power is the product of angular velocity and torque, the terms on the right hand side of this equation can be expressed in power to give

$$J\omega_{sm}\frac{d^2\delta_i}{dt^2} + D_d\omega_{sm}\frac{d\delta_i}{dt} = \frac{\omega_{sm}}{\omega_m}P_m - \frac{\omega_{sm}}{\omega_m}P_e \tag{2.11}$$

where P_m is the net shaft power input to the generator and P_e is the electrical air-gap power both expressed in watts. During a disturbance the speed of a synchronous machine is normally quite close to synchronous speed so that $\omega_m \simeq \omega_{sm}$and equation(2.11) becomes

$$J\omega_{sm}\frac{d^2\delta_i}{dt^2} + D_d\omega_{sm}\frac{d\delta_i}{dt} = P_m - P_e \tag{2.12}$$

The coefficient $J\omega_{sm}$ is the angular momentum of the rotor at synchronous speed and, when given the symbol M_m, now equation(2.12) to be written as

$$M_m\frac{d^2\delta_i}{dt^2} = P_m - P_e - D_m\frac{d\delta_i}{dt} \tag{2.13}$$

Where $D_m = \omega_{sm}D_d$ is the damping coefficient. Equation(2.13) is called the swing equation and is the fundamental equation governing the rotor dynamics.
It is commonly practice to express the angular momentum of the rotor in terms of a normalized inertia constant when all generators of particular type will have similar 'inertia' values regardless of their rating. The inertia constant is given the symbol H defined as the stored kinetic energy in mega-joules at synchronous speed divided by the machine rating S_n in megavolt-amperes so that

$$H = \frac{0.5J{\omega_{sm}}^2}{S_n} \qquad and \qquad M_m = \frac{2HS_n}{\omega_{sm}} \tag{2.14}$$

The power angle and angular speed can be expressed in electrical radians and electrical radians per second respectively, rather than their mechanical equivalent, by substituting

$$\delta = \frac{\delta_m}{p/2} \qquad and \qquad \omega_s = \frac{\omega_{sm}}{p/2} \tag{2.15}$$

Where p is the number of poles. Introducing the inertia constant and substituting equation(2.15) in equation(2.13) allows the swing equation to be written as

$$\frac{2HS_n}{\omega_s}\frac{d^2\delta_i}{dt^2} + D\frac{d\delta_i}{dt} = P_m - P_e \tag{2.16}$$

Where D is the damping coefficient ($D = 2D_m/p$). The equation(2.16) can be rationalized by defining an inertia coefficient M and damping power P_D such that

$$M = \frac{2HS_n}{\omega_{sm}} \qquad and \qquad P_D = D\frac{d\delta}{dt} \tag{2.17}$$

When the swing equation takes the common form

$$M\frac{d^2\delta}{dt^2} = P_m - P_e - P_D \tag{2.18}$$

2.5 Energy Function For Single Machine Infinite Bus System

The Energy function is always constructed for post-fault system [17]. In the case of Single Machine Infinite Bus system(Figure 2.2), the post-fault equations are

$$M\frac{d^2\delta}{dt^2} = P_m - P_e - P_D \tag{2.19}$$

Assume there is no damping in the system. Consider classical model.The equation(2.19) has two equilibrium points ($\delta^s; \omega = 0$) and ($\delta; \omega = 0$)

$$M\frac{d\omega}{dt} = P_m - P_e{}^{max} sin\delta = f(\delta) \tag{2.20}$$

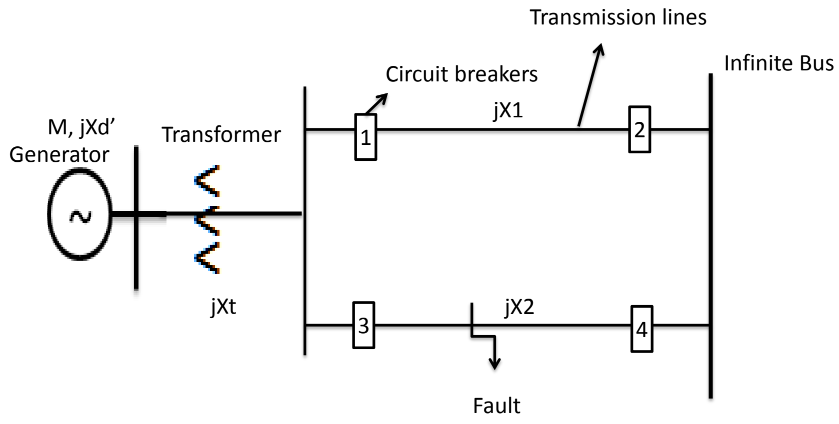

Figure 2.2: Single machine infinite bus system

Where $P_e{}^{max} = \frac{E_1 E_2}{X}$, δ is the rotor relative to the infinite bus, and $\frac{d\delta}{dt} = \omega$ is the relative rotor-angle velocity. Now we can write

$$dt = \frac{Md\omega}{f(\delta)} \qquad and \qquad dt = \frac{d\delta}{\omega} \tag{2.21}$$

By equating two terms in equation(2.22). Now we have

$$\frac{Md\omega}{f(\delta)} = \frac{d\delta}{\omega} \tag{2.22}$$

Multiply both sides by ω, and integrate both sides.

$$V(\delta, \omega) = \int_0^{\omega} M\omega d\omega - \int_{\delta^s}^{\delta} (P_m - P_e{}^{max} sin\delta) d\delta = constant \tag{2.23}$$

Evaluating the integrals gives the Energy function:

$$V(\delta, \omega) = \frac{1}{2} M\omega^2 - [P_m(\delta - \delta^s) + P_e{}^{max}(cos\delta - cos\delta^s)] \tag{2.24}$$

The quantity $V(\delta,\omega)$ is sum of Kinetic and Potential energies, and remains constant once the fault is cleared.

$$V(\delta,\omega) = V_{KE} + V_{PE} \tag{2.25}$$

Where

$$V_{KE}(\omega) = \frac{1}{2}M\omega^2 \qquad and \qquad V_{PE}(\delta) = -P_m(\delta - \delta^s) - P_e{}^{max}(cos\delta - cos\delta^s)$$

Where $\delta^s = \sin^{-1}(\frac{P_m}{P_e^{max}})$, This is a stable equilibrium point surrounded by two unstable equilibrium points $\delta^u = \pi - \delta^s$ and $\delta^u = -\pi - \delta^s$.

The critical value of the Lyapunov function V at the nearest stationary point which, for the system considered here, is equal to the second equilibrium point $(\pi - \delta^s, \omega = 0)$. Substituting these values into equation(2.24), now the energy function will be:

$$V_{cr} = 2cos\delta^s P_e^{max} - P_m(\pi - 2\delta^s) \tag{2.26}$$

The generator-infinite busbar system is stable for all initial conditions (δ_0, ω_0) satisfying the condition

$$V(\delta_0, \omega_0) < V_{cr} \tag{2.27}$$

2.6 Energy Function For Two Machines

A system having two finite machines may be replaced by an equivalent system having one finite machine and an infinite bus, so that the swing equation and swing curves of angular displacement between the two machines are same for both the systems. It is necessary to use an equivalent inertia constant, equivalent input, equivalent output for the equivalent finite machine. The equivalent inertia constant is a function of the inertia constants of the two actual machines, and the equivalent input and output are functions of inertia constants, input, and output of the two actual machines [7] [12].

Let the swing equation of the two equivalent machines be:

$$M_1 \frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \tag{2.28}$$

$$M_2 \frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \tag{2.29}$$

Where

$P_{mi}(i = 1, 2)$ Mechanical inputs

$P_{ei}(i = 1, 2)$ Electrical inputs

Let relative rotor angle be $\delta = \delta 1 - \delta 2$

Then the equivalent circuit swing equation is:

$$\frac{d^2\delta}{dt^2} = \frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \frac{P_{m1} - P_{e1}}{M1} - \frac{P_{m1} - P_{e2}}{M2} \tag{2.30}$$

Multiplying both sides by $M_1M_2/M_1 + M_2$ gives

$$\frac{M_1M_2}{M_1 + M_2}\frac{d^2\delta}{dt^2} = \frac{1}{(M_1 + M_2)}[M_2(P_{m1} - P_{e1}) - M_1(P_{m2} - P_{e2})] \tag{2.31}$$

$$= \frac{1}{(M_1 + M_2)}[M_2P_{m1} - M_1P_{m2} - (M_2P_e1 - M_1P_e2)] \tag{2.32}$$

Where

$$P_{meq} = \frac{1}{(M_1 + M_2)}(M_2P_{m1} - M_1P_{m2})$$

$$P_{eeq} = \frac{1}{(M_1 + M_2)}(M_2P_{e1} - M_1P_{e2})$$

$$M_{eq} = \frac{M_1M_2}{(M_1 + M_2)}$$

Therefore the final equivalent Energy function for two machine system is

$$M_{eq}\frac{d^2\delta}{dt^2} = P_{meq} - P_{eeq} \tag{2.33}$$

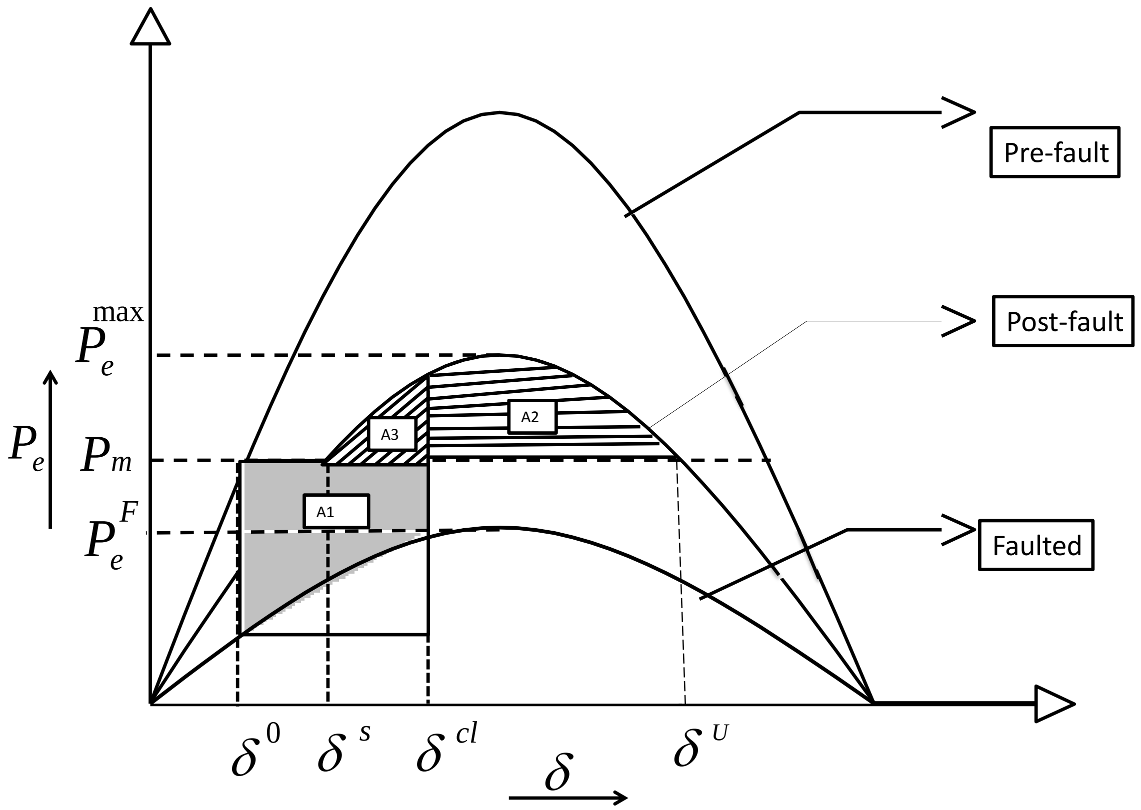

Figure 2.3: Equal area criterion for SMIB case

2.7 Equal Area Criterion and The Energy Function

The pre-fault, faulted, and post-fault power angle curves for the single machine infinite-bus system are shown in below figure 2.3. The system is initially at $\delta = \delta_0$. We shall now show that are A_1 represents the kinetic energy injected into the system during the fault.A_2 represents the ability of the post-fault system to absorb this energy. By the equal- area criterion, the system is stable if $A_1 < A_2$ [17].

Let the faulted and post-fault equations, respectively, be

$$M\frac{d^2\delta}{dt^2} = P_m - {P_e}^F sin\delta \tag{2.34}$$

$$M\frac{d^2\delta}{dt^2} = P_m - {P_e}^{max} sin\delta \tag{2.35}$$

Where

$$P_e{}^F = \frac{E_1 E_2}{X^F}$$

and

$$P_e{}^{max} = \frac{E_1 E_2}{X}$$

The area A_1 is given by

$$A_1 = \int_{\delta^0}^{\delta^{cl}} (P_m - P_e{}^F sin\delta) d\delta$$

$$= \int_{\delta^0}^{\delta^{cl}} M \frac{d\omega}{dt} d\delta$$

$$= \int_{\delta^0}^{\delta^{cl}} M \frac{d\omega}{dt} \omega dt$$

$$= \int_{0}^{\omega^{cl}} M\omega d\omega = \frac{1}{2} M {\omega_{cl}}^2 \tag{2.36}$$

Hence, A_1 is the Kinetic energy injected into the system due to the fault. Area A_2 is given by

$$A_2 = \int_{\delta^{cl}}^{\delta^u} (P_e{}^{max} sin\delta - P_m) d\delta = -P_e{}^{max}(cos\delta^u - cos\delta^{cl}) - P_m(\delta^u - \delta^{cl})$$

$$= V_{PE}(\delta^u) - V_{PE}(\delta^{cl})$$

From equation(2.25) add area A_3 to both sides of the criterion $A_1 < A_2$, the result is

$$A_1 + A_3 < A_2 + A_3 \tag{2.37}$$

Now

$$A_3 = \int_{\delta^s}^{\delta^{cl}} (P_e{}^{max} sin\delta - P_m) d\delta = -P_e{}^{max}(cos\delta^{cl} - cos\delta^s) - P_m(\delta^{cl} - \delta^s) \tag{2.38}$$

Changing δ^{cl}, ω^{cl} to any δ, ω and adding A_1 to A_3, gives

$$A_1 + A_3 = \frac{1}{2} M\omega^2 - [P_m(\delta - \delta^s) + P_e{}^{max}(cos\delta - cos\delta^s)] \tag{2.39}$$

This is the same as $V(\delta, \omega)$ as in equation(2.24). Now from Figure 2.3

$$A_2 + A_3 = \int_{\delta^s}^{\delta^{\pi - \delta^s}} ({P_e}^{max} sin\delta - P_m) d\delta$$

$$= 2cos\delta^s P_e^{max} - P_m(\pi - 2\delta^s) \quad (2.40)$$

The right hand side of the equation(2.40) is also verified to be the sum of the areas A_2 and A_3. Equation(2.40) is verified from equation(2.26). Substitute $\delta = \delta^u$ and $\omega = 0$ in energy function we will get

$$V(\delta^u, 0) = -P_m(\pi - 2\delta^s) + 2cos\delta^s P_e^{max} = A_2 + A_3$$

$$= V_{PE}(\delta^u)$$

$$\cong V_{cr} \quad (2.41)$$

So, the energy function for single machine infinite bus bar system is

$$V(\delta, \omega) = \frac{1}{2} M\omega^2 - [P_m(\delta - \delta^s) + {P_e}^{max}(cos\delta - cos\delta^s)] \quad (2.42)$$

Thus, the equal-area criterion $A_1 < A_2$ is equivalent to $A_1 + A_3 < A_2 + A_3$, this system is stable if

$$V(\delta, \omega) < V_{cr} \quad (2.43)$$

Where $V_{cr} = V_{PE}(\delta^u)$. Note that δ, ω are obtained from the faulted equation
V_{cr} is the critical energy of the synchronous motor closest to the unstable equivalent point along the faulted trajectory at certain clearing time.
Thus it can be seen that the Single Machine Infinite Bus energy function can be obtained using the equal area criterion.

Chapter 3

Energy Function For Multi-machine System

3.1 Introduction

The classical model for the generator is used to describe the stability of power systems. To analyze the stability of these power system, normally the direct method is employed. Because this classical model has the advantage of close relationship between the mechanical rotor angle deviation and the electrical equivalent circuit, power engineers are described that this model is sufficient in predicting the stability of power system [3].
To analyze any Multi-machine system, this classical model with the reference bus can be used to develop the energy function. This can be accomplished by treating the loads as constant impedance and include non-linear loads.

3.2 Load Flow Studies

Load flow studies are used to ensure that electrical power transfer from generators to consumers through the grid system is stable, reliable and economic. Conventional techniques for solving the load flow problem are iterative, using the Newton-Raphson or the Gauss-Seidel methods.It is used to find the magnitude and phase angle of the voltage at each bus and the real and reactive power flows in the system components.

3.2.1 Bus classification

In power system each bus having four variables: voltage magnitude, voltage angle, real power and reactive power. During the operation of power system each bus has two unknown variable and two known variables. Generally each bus in system classified as one of the following bus type [16]:

1. Slack or Swing or Float Bus:

   This bus is considered as the reference bus. It is connected to the generator whose rating is high relative to the other generators. During the operation, the voltage of this bus is always specified and remains constant in magnitude and angle. It provides if any additional power required to the other generators.

2. Generator or PV Bus:

   During the operation the voltage magnitude at this the bus is kept constant. Also, the active power supplied is kept constant at the value that satisfies the economic operation of the system. Most probably, this bus is connected to a generator where the voltage is controlled using the excitation and the power is controlled using the prime mover control.Sometimes, this bus is connected to a VAR device where the voltage can be controlled by varying the value of the injected VAR to the bus.

3. Load or PQ Bus:

   This bus is not connected to a generator so that neither its voltage nor its real power can be controlled. On the other hand, the load connected to this bus will change the active and reactive power at the bus in a random manner. To solve the load flow problem we have to assume the complex power value (real and reactive) at this bus.

3.3 Load Flow Using Newton Raphson Method

3.3.1 General approach

The Newton-Raphson method is an iterative technique for solving systems of simultaneous equations in the general form:

$$f_1(x_1, ..., x_n) = K_1$$

$$f_2(x_1, ..., x_n) = K_2$$

$$f_n(x_1, ..., x_n) = K_n \tag{3.1}$$

Where $f_1, f_2, .., f_n$ are differential functions of the variables $x_1, x_2, .., x_n$ and $k_1, k_2, .., k_n$ are constants. Applied to the load flow problem, the variables are the nodal voltage magnitudes and phase angles, the functions are the relationships between power, reactive power and node voltages, while the constants are the specified values of power and reactive power at the generator and load nodes.

Power and reactive power functions can be derived by starting from the general expression for injected current at node n:

$$I_n = \Sigma_{k=1}^{n} Y_{nk} V_k \tag{3.2}$$

so the complex power input to the system at node n is:

$$S_n = V_n I_n^* \tag{3.3}$$

where the superscript $*$ denotes the complex conjugate. Substituting I_n into S_n with all complex variables written in polar form:

$$S_n = V_n \Sigma_{k=1}^{n} Y_{nk}^* V_k^* = \Sigma_{k=1}^{n} |V_n||V_k||Y_{nk}| \angle(\delta_n - \delta_k - \theta_{nk}) \tag{3.4}$$

The power and reactive power inputs at node n are derived by taking the real and imaginary parts of the complex power:

$$P_n = Real(S_n) = \Sigma_{k=1}^{n} |V_n||V_k||Y_{nk}| cos(\delta_n - \delta_k - \theta_{nk}) \tag{3.5}$$

$$Q_n = imag(S_n) = \Sigma_{k=1}^{n} |V_n||V_k||Y_{nk}| sin(\delta_n - \delta_k - \theta_{nk}) \tag{3.6}$$

The load flow problem is to find values of voltage magnitude and phase angle, which,when substituted into above two equations, produce values of real power and reactive power equal to the specified set values at that node, P_{ns} and Q_{ns}.

The first step in the solution is to make initial estimates of all the variables: $|V_n^0|, \delta_n^0$ where the superscript ^{0} indicates the number of iterative cycles completed. Using these estimates, the power and reactive power input at each node can be calculated from above equations. These values are compared with the specified values to give a real power and reactive power error. For node n:

$$\Delta P_n^0 = P_{ns} - \Sigma_{k=1}^{n} |V_n^0||V_k^0||Y_{nk}| cos(\delta_n^0 - \delta_k^0 - \theta_{nk}) \tag{3.7}$$

$$\Delta Q_n^0 = Q_{ns} - \Sigma_{k=1}^{n} |V_n^0||V_k^0||Y_{nk}| sin(\delta_n^0 - \delta_k^0 - \theta_{nk}) \tag{3.8}$$

The power and reactive power errors at each node are related to the errors in the voltage magnitudes and phase angles, e.g.$\Delta|V_n^0|, \Delta\delta_n^0$, by the first order approximation:

$$\begin{bmatrix} . \\ . \\ . \\ \Delta P_n^{(0)} \\ --- \\ . \\ . \\ . \\ \Delta Q_n^{(0)} \end{bmatrix} = \begin{bmatrix} . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ \frac{\partial P_n}{\partial \delta_2} & \frac{\partial P_n}{\partial \delta_3} & . & \frac{\partial P_n}{\partial \delta_n} & . & . & \frac{\partial P_n}{\partial |V_{n-1}|} & \frac{\partial P_n}{\partial |V_n|} \\ --- & --- & - & --- & - & - & - & --- \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ \frac{\partial Q_n}{\partial \delta_2} & \frac{\partial Q_n}{\partial \delta_3} & . & \frac{\partial Q_n}{\partial \delta_n} & . & . & \frac{\partial Q_n}{\partial |V_{n-1}|} & \frac{\partial Q_n}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ . \\ \Delta\delta_n^{(0)} \\ --- \\ . \\ . \\ \Delta|V_{n-1}^{(0)}| \\ \Delta|V_n^{(0)}| \end{bmatrix}$$

where the matrix of partial differentials is called the Jacobian matrix, $[J]$. The elements of the Jacobian are calculated by differentiating the power and reactive power expressions and substituting the estimated values of voltage magnitude and phase angle.
At the next stage of the Newton-Raphson solution, the Jacobian is inverted. Matrix inver-

sion is a computationally-complex task with the resources of time and storage increasing rapidly with the order of $[J]$. This requirement for matrix inversion is a major drawback of the Newton-Raphson method of load flow analysis for large-scale power systems.However, with the inversion completed, the approximate errors in voltage magnitudes and phase angles can be calculated by pre-multiplying both sides of above expression.

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ . \\ \Delta\delta_n^{(0)} \\ --- \\ . \\ . \\ \Delta|V_{n-1}^{(0)}| \\ \Delta|V_n^{(0)}| \end{bmatrix} = \begin{bmatrix} J^{(0)} \end{bmatrix}^{-1} \begin{bmatrix} . \\ . \\ . \\ \Delta P_n^{(0)} \\ --- \\ . \\ . \\ . \\ \Delta Q_n^{(0)} \end{bmatrix}$$

The approximate errors from above expression are added to the initial estimates to produce new estimated values of node voltage magnitude and angle. For node n:

$$|V_n^1| = |V_n^0| + \Delta|V_n^0| \qquad and \qquad \delta_n^1 = \delta V_n^0 + \Delta\delta_n^0 \tag{3.9}$$

The new estimates are not exact solutions to the problem. However, they can be used in another iterative cycle. The process is repeated until the differences between successive estimates are within an acceptable tolerance band.

The description above relates specifically to a load node, where there are two unknowns (the voltage magnitude and angle) and two equations relating to the specified real power and reactive power. For a generator node the voltage magnitude V_n and real power P_n are specified, but the reactive power Q_n is not specified. The order of the calculation can be reduced by 1. There is no need to ensure that the reactive power is at a set value and only the angle of the node voltage needs to be calculated, so one row and column are removed from the Jacobian. For the floating bus, both voltage magnitude and angle are specified, so there is no need to calculate these quantities.

3.4 Energy Function For Multi-Machine System

The swing equations for n machines system:

$$\frac{H_i}{\pi f}\frac{d^2\delta_i}{dt^2} + D_i\frac{d\delta_i}{dt} = P_{mi} - P_{ei}, \qquad i = 1, 2, .., n \tag{3.10}$$

where

H_i inertia constants of the i^{th} machine

D_i damping constant in sec/radian of the i^{th} machine

P_{mi} mechanical input to the i^{th} machine(assumed constant) in per unit

P_{ei} electrical power output of the i^{th} machine in per unit

δ_i rotor angle of the i^{th} machine with respect to a synchronously rotating reference frame in radians

3.4.1 Classical model

The Figure 3.1 shows a classical model of a 3-machine 9-bus system. It represents 3 generators (machines) and 9 bus. The first generator is a swing generator while the $2^n d$ and $3^r d$ can be referred as the P.V bus which provides a real power and small amount of reactive power. The objective of this thesis will be fulfilled using this system and its energy function which will be discussed as follows [1].

3.4.2 Methods to obtain energy function for multi-machine system

Multi-Machine equation can be derived similarly to the single machine system [2]. The complexity of transient stability analysis can be reduced by similar simplifying assumptions made as that of the single machine as follows:

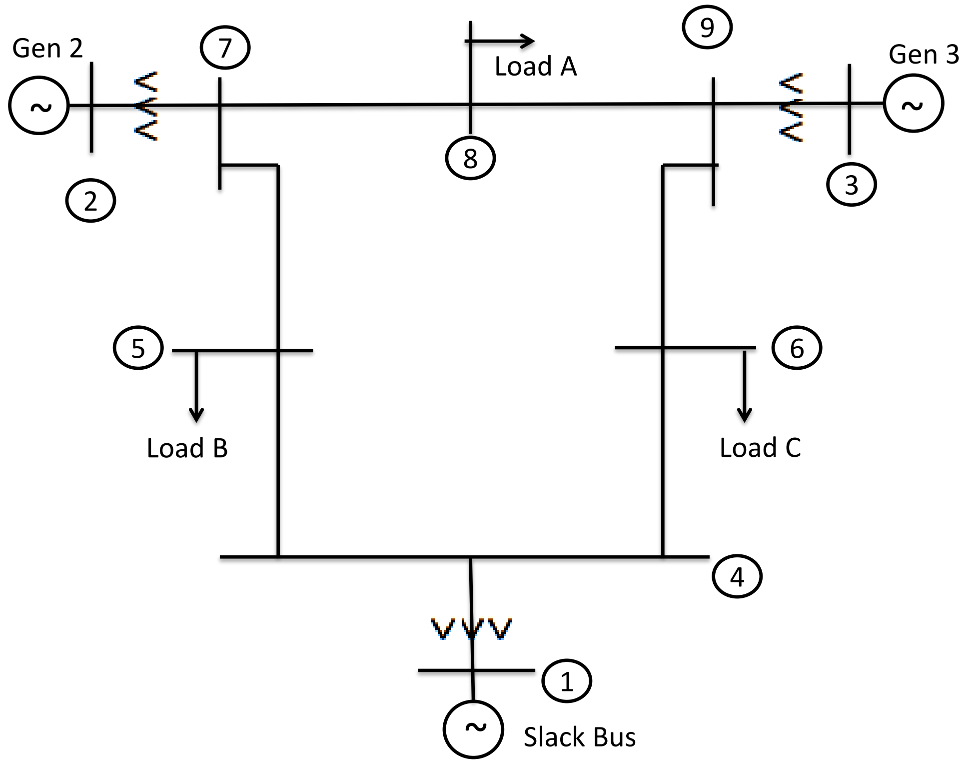

Figure 3.1: 3-Machine 9-Bus System

1. Each synchronous machine is represented by a constant voltage source behind the direct axis transient reactance. This representation neglects the effect of saliency and assumes constant flux linkages.

2. Damping or asynchronous power are ignored.

3. The governor's actions are neglected and the input powers are assumed to remain constant during the entire period of simulation.

4. Machines belonging to the same station swing together and are said to be coherent. This group of coherent machines can be grouped as one equivalent machine.

5. Using pre-fault bus voltages, all loads are converted to equivalent admittances to ground and assumed to be constant.

6. the angle of voltage behind the machine reactance.

The swing equation shown below which is already derived. Energy function for multi-machine bus system based on the

1. Network Reduction Technique
   a) Relative rotor angle formation
   b) Center of inertia formation

$$\frac{H_i}{\pi f}\frac{d^2\delta_i}{dt^2} + D_i\frac{d\delta_i}{dt} = P_{mi} - P_{ei}, \qquad i = 1, 2, .., n \tag{3.11}$$

Let $\frac{H_i}{\pi f} = M_i$, $P_i = P_{mi} - |E_i^2|G_{ii}$

Then

$$M_i\frac{d^2\delta_i}{dt^2} + D_i\frac{d\delta_i}{dt} = P_i - \Sigma_{j=1_{\neq i}}^{n}(C_{ij}sin\delta_{ij} + D_{ij}cos\delta_{ij}) \tag{3.12}$$

It can be write in another form as:

$$M_i\frac{d^2\delta_i}{dt^2} + D_i\frac{d\delta_i}{dt} = P_i - P_{ei}(\delta_1, ..., \delta_n), \qquad i = 1, 2, .., n \tag{3.13}$$

3.5 Network Reduction Technique

3.5.1 Center of inertia formation

The alternative method uses the center of inertia (COI) as the reference angle since it is a representation of the 'mean motion' of the system. The resulting energy function $V(\delta, \omega)$ obtain from this method is similar that of the relative rotor angle formation [3] [9] [19]. Assuming the transfer conductance (D_{ij}) to be zero. The center of inertia for whole system is:

$$\delta_0 = \frac{1}{M_T}\Sigma_{i=1}^{n}M_i\delta_i \tag{3.14}$$

with the center of speed as:

$$\omega_0 = \frac{1}{M_T}\Sigma_{i=1}^{n}M_i\omega_i \tag{3.15}$$

where

$$M_T = \Sigma_{i=1}^{n}M_i$$

Next, transform the variable δ_i and ω_i to the COI variables as:

$$\theta_i = \delta_i - \delta_0 \tag{3.16}$$

$$\tilde{\omega_i} = \omega_i - \omega_0 \tag{3.17}$$

Then the swing equation becomes with $D_i = 0$ can be depicted as:

$$M_i \frac{d^2\theta_i}{dt^2} = P_i - \Sigma_{j=1_{\neq i}}^{n}[C_{ij}sin\theta_{ij} + D_{ij}cos\theta_{ij}] - \frac{M_i}{M_T}P_{COI} \tag{3.18}$$

$$\cong f_i(\theta) \qquad i = 1, 2, .., n \tag{3.19}$$

where

$$P_i = P_m i - |E_i^2|G_{ii}$$

$$P_{COI} = \Sigma_{i=1}^{n} P_i - 2\Sigma_{i=1}^{n-1}\Sigma_{j=i+1}^{n} D_{ij}cos\theta_{ij}$$

Corresponding to the post-fault which is the area of our interest, we have the differential equation:

$$M_i \frac{d\tilde{\omega}_i}{dt} = f_i(\theta) \tag{3.20}$$

$$\frac{d\theta_i}{dt} = \tilde{\omega}_i, \qquad t \geq t_{cl} \qquad i = 1, 2, ..., n \tag{3.21}$$

Let the post-fault system have the stable equilibrium point at $\theta = \theta^s$, $\tilde{\omega} = 0$. Which can be obtained by solving the non linear algebraic equations

$$f_i(\theta) = 0, \qquad i = 1, 2, ..., n \tag{3.22}$$

Since $\Sigma_{i=1}nM_i\theta_i = 0$, θ_n can be expressed in terms of other θ_i's and used in above equation. Therefore the above equation becomes

$$f_i(\theta_1, ..., \theta_{n-1}) = 0, \qquad i = 1, 2, ..., n \tag{3.23}$$

There is general agreement that the first integral of motion[13]. The proper energy function is derived as follows.

We have, for i=1,2,..,n

$$dt = \frac{M_1 d\tilde{\omega_1}}{f_1(\theta)} = \frac{d\theta_1}{\tilde{\omega_1}} = \frac{M_2 d\tilde{\omega_2}}{f_2(\theta)} = \frac{d\theta_2}{\tilde{\omega_2}} = ...\frac{M_n d\tilde{\omega_1}}{f_n(\theta)} = \frac{d\theta_n}{\tilde{\omega_n}} \tag{3.24}$$

Integrating the pairs of equation for each machine between $(\theta_i^s, 0)$, the post-fault s.e.p to $(\theta_i, \tilde{\omega_i})$ result in

$$V(\theta, \tilde{\omega}) = \frac{1}{2}\Sigma_{i=1}^{n} M_i \tilde{\omega_i^2} - \Sigma_{i=1}^{n} \int_{\delta_i^s}^{\delta_i} f_i(\theta) d\theta_i \tag{3.25}$$

Adding these function for all the machines, we obtain the first integral of motion for the system as(omitting the algebra):

$$V(\theta, \tilde{\omega}) = \frac{1}{2}\Sigma_{i=1}^{n} M_i \tilde{\omega_i^2} - \Sigma_{i=1}^{n} P_i(\theta_i - \theta_i^s) - \Sigma_{i=1}^{n-1}\Sigma_{j=i+1}^{m}[C_{ij}(cos\theta_{ij} - cos\theta_{ij}^s)$$

$$- \int_{\delta_i^s+\delta_j^s}^{\delta_i+\delta_j} D_{ij} cos\theta_{ij} d(\theta_i + \theta_j)] \tag{3.26}$$

$$= V_{KE}(\tilde{\omega}) + V_{PE}(\theta) \tag{3.27}$$

The above equations contains path-dependent integral terms. In view of this, we cannot assert that V_i and V are positive-definite. If $D_{ij} \equiv 0$, it can be shown that $V(\theta, \tilde{\omega})$ constitutes a proper Lyapunov function.

3.5.2 Relative rotor angle formation

The method employs the idea of choosing the machine with heavy inertia to be the reference machine, for instance in the case of a 3 machine 9 bus system. Choose Generator 1 as the reference bus [18].

We know

$$\frac{d\delta_i}{dt} = \omega_i \tag{3.28}$$

$$\frac{d\omega_i}{dt} = -\frac{D_i}{M_i}\omega_i + \frac{P_i}{M_i} - \frac{P_{ei}}{M_i} \tag{3.29}$$

Let us take generator 1 as the reference bus, now the above equation becomes:

$$\frac{d(\delta_i - \delta_1)}{dt} = (\omega_i - \omega_1) \tag{3.30}$$

$$\frac{d(\omega_i - \omega_1)}{dt} = -\frac{D}{M}(\omega_i - \omega_1) + \frac{P_i}{M_i} - \frac{P_1}{M_1} - \frac{P_{ei}}{M_i} + \frac{P_{e1}}{M_1} \tag{3.31}$$

The electrical power output will be

$$P_{ei} = \Sigma_{j=1_{\neq i}}^{n} |V_i||V_j||Y_{ij}|cos(\theta_{ij}+\delta_j-\delta_i) = \Sigma_{j=1_{\neq i}}^{n} |V_i||V_j||Y_{ij}|cos(\theta_{ij}+\delta_j-\delta_1-\delta_i+\delta_1) \tag{3.32}$$

Now

$$P_{e1} = \Sigma_{j=1}^{n} |V_1||V_j||Y_{1j}|cos(\theta_{1j} + \delta_j - \delta_1) = \Sigma_{j=1}^{n} |V_1||V_j||Y_{1j}|cos(\theta_{1j} + \delta_j - \delta_1) \tag{3.33}$$

Let us take

$$\delta_i - \delta_1 = \tilde{\delta_i} \tag{3.34}$$

$$\omega_i - \omega_1 = \tilde{\omega_i} \tag{3.35}$$

Now the first order differential equations by using δ_i with respect to a synchronous rotating frame δ_1 and introducing the state variables $\tilde{\delta_i}$ and $\tilde{\omega_i}$. Now the dynamic equation becomes:

$$\frac{d\tilde{\delta_i}}{dt} = \tilde{\omega_i} \qquad i = 2, 3, .., n \tag{3.36}$$

$$\frac{d\tilde{\omega_i}}{dt} = -\frac{D}{M}\tilde{\omega_i} + \frac{P_{mi}}{M_i} - \frac{P_{m1}}{M_1} - \frac{1}{M_i}\Sigma_{j=1_{\neq i}}^{n} |V_i||V_j|B_{ij}sin(\tilde{\delta_i} - \tilde{\delta_j}) + \frac{1}{M_1}\Sigma_{j=1}^{n} |V_1||V_j|B_{1j}sin(\tilde{\delta_j}) \tag{3.37}$$

Now the above equation we write as:

$$\frac{d\tilde{\omega_i}}{dt} = f_i(\theta) \tag{3.38}$$

$$\frac{d\theta_i}{dt} = \tilde{\omega_i}, \qquad t \geq t_{cl} \qquad i = 2, ..., n \tag{3.39}$$

Let the post-fault system have the stable equilibrium point at $\theta = \theta^s$, $\tilde{\omega} = 0$. Which can be obtained by solving the non linear algebraic equations

$$f_i(\theta) = 0, \qquad i = 2, ..., n \tag{3.40}$$

There is general agreement that the first integral of motion. The proper energy function is derived as follows.

We have, for i=2,..,n

$$dt = \frac{M_i d\tilde{\omega}_i}{f_i(\theta)} = \frac{d\delta_i}{\tilde{\omega}_i} \tag{3.41}$$

Integrating the pairs of equation for each machine between $(\theta_i^s, 0)$, the post-fault s.e.p to $(\theta_i, \tilde{\omega}_i)$ result in

$$V(\delta, \tilde{\omega}) = \frac{1}{2} M_i \tilde{\omega_i^2} - \int_{\tilde{\delta}_i^s}^{\tilde{\delta}_i} f_i(\theta) d\delta_i \tag{3.42}$$

Now

$$V(\delta, \tilde{\omega}) = \frac{1}{2} M_i \tilde{\omega_i^2} - \int_{\tilde{\delta}_i^s}^{\tilde{\delta}_i} (P_i - P_1 \frac{M_i}{M_1}) d\tilde{\delta}_i - \int_{\tilde{\delta}_i^s}^{\tilde{\delta}_i} \Sigma_{j=1_{\neq i}}^n |V_i||V_j| B_{ij} sin(\tilde{\delta}_i - \tilde{\delta}_j) d\tilde{\delta}_i$$

$$-\frac{M_i}{M_1} \int_{\tilde{\delta}_i^s}^{\tilde{\delta}_i} \Sigma_{j=1}^n |V_1||V_j| B_{1j} sin(\tilde{\delta}_j) d\tilde{\delta}_i \tag{3.43}$$

Now the energy function is

$$V(\delta, \tilde{\omega}) = \frac{1}{2} \Sigma_{i=2}^n M_i \tilde{\omega_i^2} - \Sigma_{i=2}^n \tilde{P}_i (\tilde{\delta}_i - \tilde{\delta}_i^s) - \Sigma_{i=1}^{n-1} \Sigma_{j=i+1}^n |V_i||V_j| B_{ij} [cos(\tilde{\delta}_i - \tilde{\delta}_j) - cos(\tilde{\delta}_i^s - \tilde{\delta}_j^s)]$$

$$-\Sigma_{i=1}^n \frac{M_i}{M_1} |V_1||V_i| sin(\tilde{\delta}_i) \tag{3.44}$$

where

$\tilde{P}_i$ $= (P_i - \frac{M_i}{M_1} P_1)$

n = number of machines

ω_i = angular velocity of the rotor

ω_s = synchronous speed in radians per second

δ_{in} = rotor angle of $i^t h$ machine expressed with respect to a synchronous rotating reference time in radians

δ_{jn} = rotor angle of $j^t h$ machine expressed with respect to a synchronous rotating reference time in radians

P_{mi} = mechanical power input to the i^th machine

$P_i = P_{mi} - |E_i^2|G_{ii}$, power injected into i^th machine

$P_j = P_{mj} - |E_j^2|G_{jj}$, power injected into j^th machine

To obtain an energy function, D_i must be equal to zero so that it can be represent as a sum of a path independent integral between $V_{KE}(\tilde{\omega}) + V_{PE}(\delta)$, together with a path dependent integral $V_d(\delta)$.
Therefore the energy function using the relative rotor angle formation is:

$$V(\delta, \tilde{\omega}) = V_{KE}(\tilde{\omega}) + V_{PE}(\delta) \tag{3.45}$$

$$V(\delta, \tilde{\omega}) = V_{KE}(\tilde{\omega}) + V_P(\delta) + V_d(\delta) \tag{3.46}$$

where

$V_{KE}(\tilde{\omega})$ kinetic energy of machine $V_{PE}(\delta)$ overall potential energy of machine

$V_P(\delta)$ potential energy of machine at initial state

$V_d(\delta)$ change in potential energy of machines due to path dependent of D_{ij} terms

Chapter 4

Energy Function Techniques

4.1 Introduction

The theoretical basis of direct methods for power system transient stability assessment is the knowledge of the stability region. We are presenting analytical results of stability region of a non-linear systems can be completely characterized. For this non-linear systems, which have energy functions, a complete characterization of the stability boundary will be derived and optimal schemes to estimate its stability regions via an energy function will be presented [6].

Power system stability analysis is concerned whether the post-fault system is settle to an acceptable considerable operating conditions. The direct methods for stability analysis use algorithms procedure to asses, without performing post-fault integration, System post-fault stability property by comparing the system energy at the initial state of the post-fault trajectory to a critical energy value. Direct methods not only eliminate time consuming procedure of numerical integration of post-fault system but also provide a quantitative measure of the degree of system stability.

The following steps are composed for transient stability using direct methods.

1. Simulate the fault on trajectory.

2. Find the initial point of the post-fault system.

3. Compute an energy function for the post-fault power system.

4. Compute the energy function value at the initial point of the post-fault system.

5. Compute the critical energy for the fault on trajectory.

6. Perform stability analysis by comparing the system energy $V(\delta, \omega)$ at the initial state of post fault system with the critical energy V_{cr}. If system energy is less than the critical energy then the post-fault trajectory will be stable; otherwise,it is unstable.

4.1.1 Properties of energy function

We consider a general non-linear autonomous dynamical system described by following equation

$$\dot{x(t)} = f(x(t)) \tag{4.1}$$

We say a function $f(x(t))$ is an energy function for the system. if the following three conditions are satisfied:

1. $\frac{dV}{dt} \leq 0$ across any trajectory

2. $\dot{V(x)} = 0$ only if x is an equilibrium point

3. $V(x(t))$ is bounded means $x(t)$ is also bounded

Three methods are methods are explained in these thesis. These methods are:

4.1.2 Closest u.e.p method

$V_{cr} = V(x^{uc})$, where x^{uc} is the unstable equilibrium point (u.e.p) resulting in the lowest value of V_{cr} among the u.e.p's inside in the stability boundary. This requires the computation of many u.e.p's of the post-fault system and, hence, is not computationally attractive. It gives conservative results, it is explained in controlling u.e.p method [4].
If a post-fault system trajectory at point (δ_3, ω_3), whose energy function is $V(\delta_3, \omega_3)$. If this is less than V_{cr}, then the system state is inside the stability region $(\delta_s, 0)$. The figure 4.2 shows the both stable and unstable system. The closest u.e.p method gives some conservative results. The figure 4.2(a) shows the stable system, whose Post-fault trajectory

of this system is inside the stability boundary. Figure 4.2(b) shows the unstable system, whose post-fault trajectory of this system is outside the boundary [18].

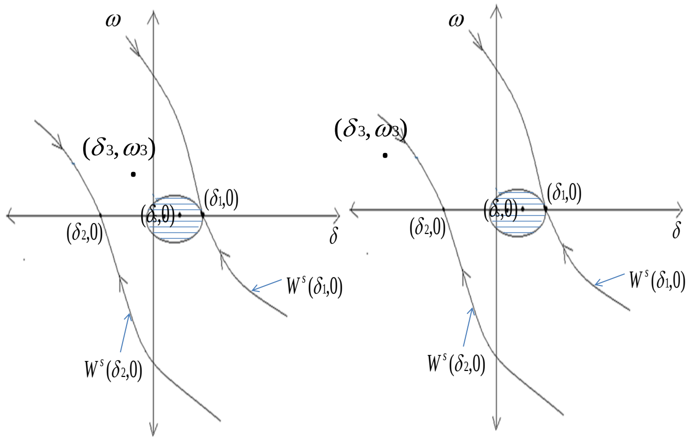

Figure 4.1: (a).Stable system using closest u.e.p method (b).Unstable system using closest u.e.p method

4.1.3 Determination of the critical energy

1. Find the all equilibrium points [5].
2. Arrange these type of equilibrium points according to the values of energy function.
3. The equilibrium point with lowest energy function, ie, inside the stability boundary is the closest u.e.p [12].
4. The critical energy V_{cr} is the value of $V(\delta, \omega)$ at the closest u.e.p, ie, $V_{cr} = V(x_{uc})$.

4.1.4 Determination of stability

1. Calculate the value of $V(\delta, \omega)$ at the time of fault cleared.
2. If $V(\delta, \omega) < V_{cr}$ then the post-fault system is stable, otherwise it is unstable.

4.1.5 Controlling u.e.p method

$V_{cr} = V(x^u)$, at which x^u is the relevant or controlling u.e.p, i.e., the u.e.p closest to the point where the fault on trajectory exits that is the region of attraction. This is called the controlling u.e.p method. If the fault on trajectory $(\delta(t), \omega(t))$ moves towards δ_1, the controlling u.e.p method uses the constant energy surface passing through the u.e.p $(\delta_1, 0)$, which is $\{(\delta, \omega) : E(\delta, \omega) = V_{cr}(\delta_1)\}$, if the fault on trajectory $(\delta(t), \omega(t))$ moves towards δ_2, the controlling u.e.p method uses the constant energy surface passing through the u.e.p $(\delta_2, 0)$, which is $\{(\delta, \omega) : E(\delta, \omega) = V_{cr}(\delta_2)\}$ is the local approximation [1].
The figure 4.4 shows that the comparison between the both closest u.e.p method and controlling u.e.p method. The post-fault trajectory $(\delta(t), \omega(t))$ starts at point P which is inside the boundary according to controlling u.e.p method but according to closest u.e.p method it is outside the boundary. So, the system is stable if the method is used is controlling u.e.p method. The system is unstable if the method is used is closest u.e.p method [18].

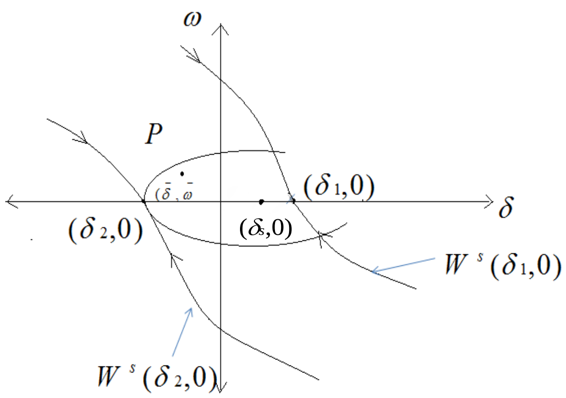

Figure 4.2: The approximated controlling u.e.p. method

4.1.6 Determination of critical energy

1. Find all the equilibrium points [5].

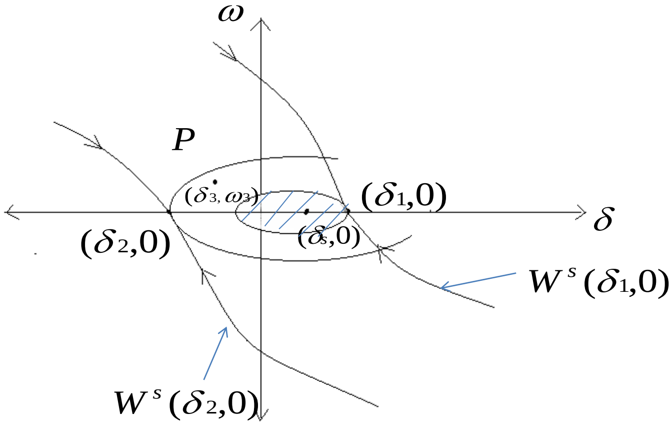

Figure 4.3: The pre-fault trajectory ends at point P. which is lies inside the controlling u.e.p boundary. So, this is stable

2. The equilibrium point with highest energy function, ie, inside the stability boundary is the controlling u.e.p.

3. The critical energy V_{cr} is the value of $V(\delta, \omega)$ at the closest u.e.p, ie, $V_{cr} = V(x_{co})$.

4.1.7 Determination of stability

1. Calculate the value of $V(\delta, \omega)$ at the time of fault cleared.

2. If $V(\delta, \omega) < V_{cr}$ then the post-fault system is stable, otherwise it is unstable.

4.1.8 Potential energy boundary surface method

The controlling u.e.p method has one problem. That is, it is very difficult to find the u.e.p(controlling) in the controlling u.e.p method. V_{cr} is the value of potentia energy along the fault-on trajectory (δ, ω). The PEBS method gives the approximation of the boundary. No need to find unstable equilibrium points. This is the one of the advantage of the PEBS method compared to controlling u.e.p method. It takes very less computational time [17].

4.1.9 Determination of the critical energy

1. Computing all the stable equilibrium points [5].

2. Formulating the energy function $V(\delta, \omega)$. Generally, $V(\delta, \omega)$ is the sum of kinetic and potential energies of the post-fault system, i.e., $V(\delta, \omega) = V_{KE}(\omega) + V_{PE}(\delta)$.

3. Computing of V_{cr}.
   The fault-on trajectory $(\delta(t), \omega(t))$ integrates until the potential energy $V_{PE}(\delta)$ reaches a maximum potential energy. This maximum potential energy (V_{PE}^{max}) is called the critical energy of the PEBS method.

4. Calculating the time instant t_{cr} when $V(\delta, \omega) = V_{cr}$ on the faulted trajectory. The faulted trajectory has to be integrated for all the three methods to obtain t_{cr}. In the PEBS method, the faulted trajectory is already available while computing V_{cr}. The computation time is least for the PEBS method.

4.1.10 Determination of stability

1. Compare the energy obtained from solving the power flow of the system with V_{cr}.
   a. If V_{cr} is smaller than the energy function value $V(\delta, \omega)$ using the energy function, the system is stable.
   b. If V_{cr} is larger than the energy function value $V(\delta, \omega)$ using the energy function, the system is unstable.

4.2 Results and Discussion

4.2.1 Introduction

Based on the stability criterion energy function already discussed in previous chapters, a mathematical database was created using the Matlab software. This database defines the energy function. It can also standard if any small-scale power system is stable in accordance to the energy function of that particular system examined.

4.2.2 Input parameters

The input parameters for the energy function are the electrical power, mechanical power, rotor angle, speed of machines, real power, reactive power. These parameters are used to find the critical clearing time and stability of power system. Figure 2.2 shows the single machine infinite bus system and Figure 3.1 shows the 3-machines 9-bus system.

4.3 Single Machine Infinite Bus System

For transient stability assessment consider single machine infinite bus system is shown in figure 2.2. Fault occur on one of the transmission line. Critical clearing time using closest u.e.p method is found by comparing critical energy(V_{cr}) with system energy($V(\delta, \omega)$) continuously. When the critical energy(V_{cr}) and system energy($V(\delta, \omega)$) are equal($V_{cr} = V(\delta, \omega)$) the time at that instant is called critical clearing time. Whose critical clearing time for closest u.e.p method is 0.57s is shown in figure 4.4. Controlling u.e.p method is also same as closest u.e.p method. Whose critical clearing time is 0.59s is shown in figure 4.5.

To find the critical value $V(\delta, \omega)$ the fault on trajectory is monitored until it crosses the PEBS at a point θ^*. In many cases, θ^u the controlling u.e.p is close to θ^*, so that $V_{PE}(\theta^u) \approx V_{PE}(\theta^*) \cong V_{cr}$. This crossing is also approximately the point at which $V_{PE}(\theta)$ is maximum along the faulted trajectory. Hence, V_{cr} can be taken as equal to $V_{PE}^{max}(\theta)$ along the faulted trajectory. The PEBS crossing is also the at which $f^T(\theta).(\theta - \theta^s) = 0$. $f(\theta)$ is the accelerating power in post-fault system. That this is the same point at which $V_{PE}(\theta)$ is maximum as shown in figure 4.6 and figure 4.7. The figure 4.7 is shown that the PEBS function crossing horizontal(zero axis) axis at the time 0.77s. The potential energy value at 0.77s as shown in figure 4.6(b) corresponding time value of system energy

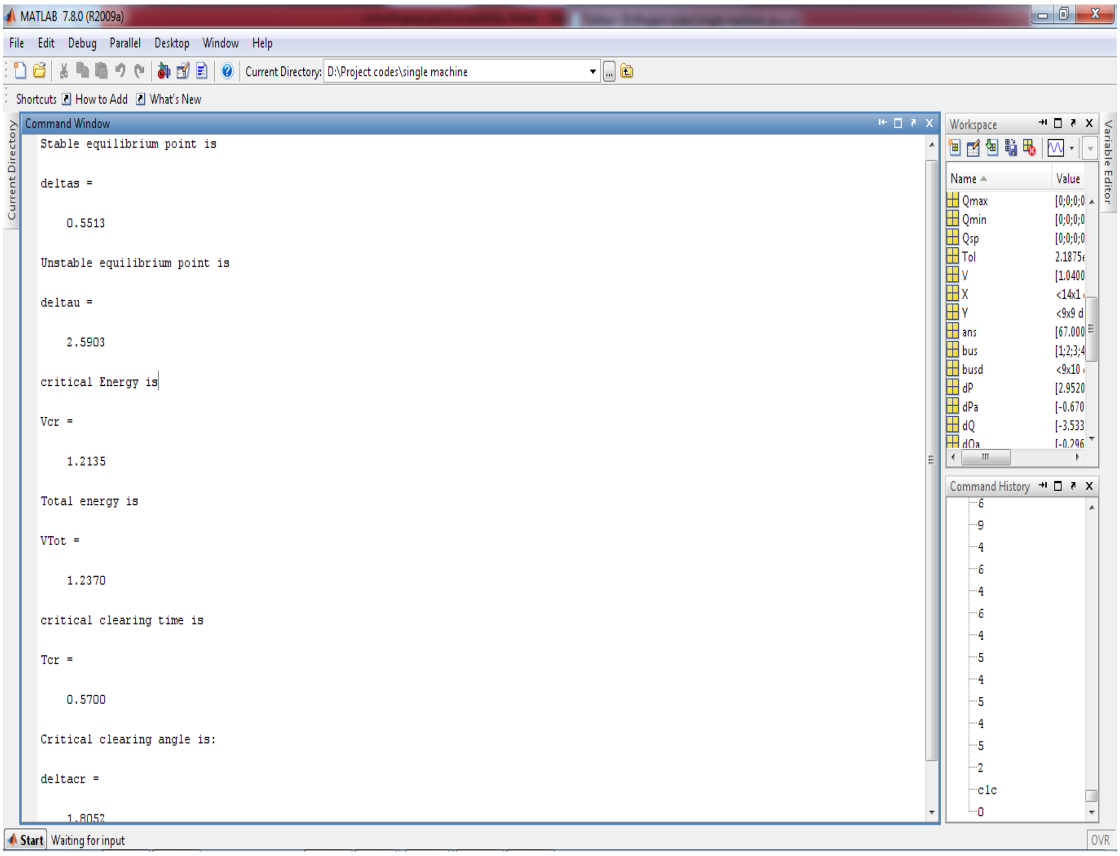

Figure 4.4: Single machine infinite bus system using closest u.e.p method whose critical clearing time is $T_{cr} = 0.57s$

is 0.58s is shown in figure 4.6(a). So, the critical clearing time for PEBS method is 0.58s.

The simulation results of proposed strategy, i.e., closest u.e.p, controlling u.e.p and potential energy boundary surface methods are given in table 4.1. Comparison of three methods is done with time domain simulation (given in table 4.2) whose critical clearing time is 0.61s. If fault clearing time is 0.57s then all the three methods are stable. If fault clearing time is 0.58s closest u.e.p method is unstable remaining two methods are in stable. If fault clearing time is 0.59s controlling u.e.p method is stable remaining two methods are unstable. From this comparison we can say that controlling u.e.p method is close to the time domain simulation.

After finding critical clearing time test the stability of the system. Clear the fault at $T_c = 0.2s$ whose transient energy is $V_{tot} = 0.2670$ and critical energy is $V_{cr} = 1.2135$.

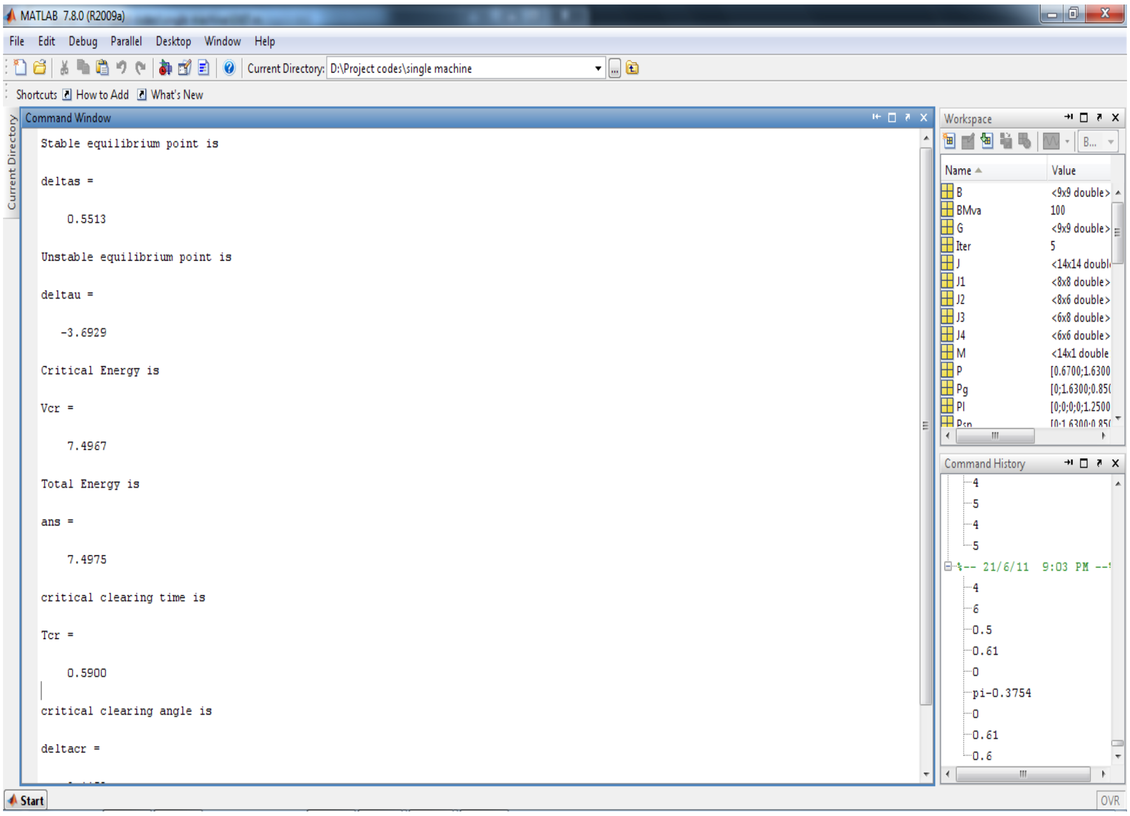

Figure 4.5: Single machine infinite bus system using controlling u.e.p method whose critical clearing time is $T_{cr} = 0.59s$

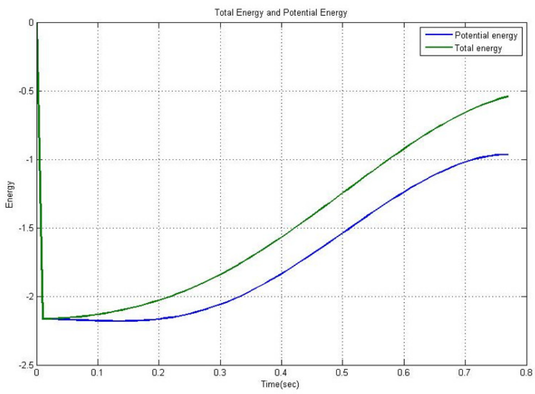

Figure 4.6: Single machine infinite bus system using PEBS method (a). $V(\delta, \omega)$ green line; (b). $V_{PE}(\theta)$ blue line

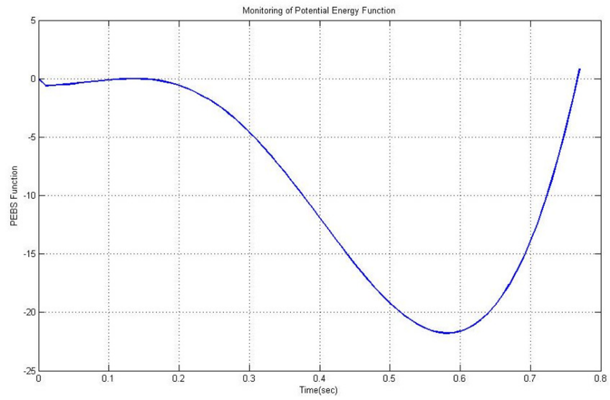

Figure 4.7: Single machine infinite bus system using PEBS method.The monitoring of the PEBS crossing by $f^T(\theta).(\theta - \theta^s)$

Table 4.1: Critical Clearing Time of Three Methods for Single Machine Infinite Bus System

S.No.	Method Name	δ_s	δ_u	V_{cr}	T_{cr}
1	Closest u.e.p	0.5513	2.5903	1.2135	0.57
2	Controlling u.e.p	0.5513	-3.6929	7.4967	0.59
3	PEBS method	0.5513	no need	-0.9642	0.58
4	Time domain	0.5513	no need	no need	0.61

δ_s: Stable equilibrium point; δ_u: Unstable equilibrium point;
T_{cr}: Critical clearing time; V_{cr}: Critical energy

Table 4.2: Comparison of Three Methods for Single Machine Infinite Bus System

S.No.	T_c	Closest u.e.p ($T_{cr} = 0.57s$)	Controlling u.e.p ($T_{cr} = 0.59s$)	PEBS Method ($T_{cr} = 0.58s$)
1	0.57s	stable	stable	stable
2	0.58s	unstable	stable	stable
3	0.59s	unstable	stable	unstable
4	0.60s	unstable	unstable	unstable

T_c: Fault clearing time; T_{cr}: Critical clearing time

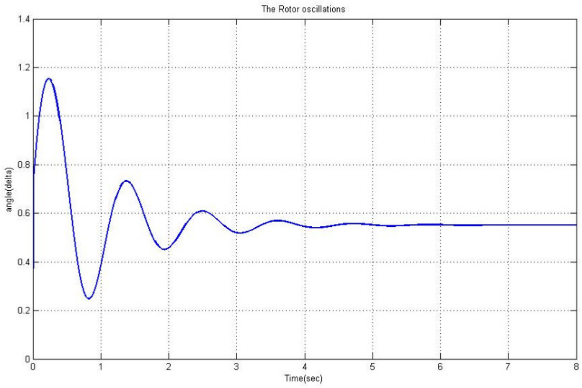

Figure 4.8: Single machine infinite bus system is stable for T_c = 0.2s, whose critical clearing time is T_{cr}=0.61s

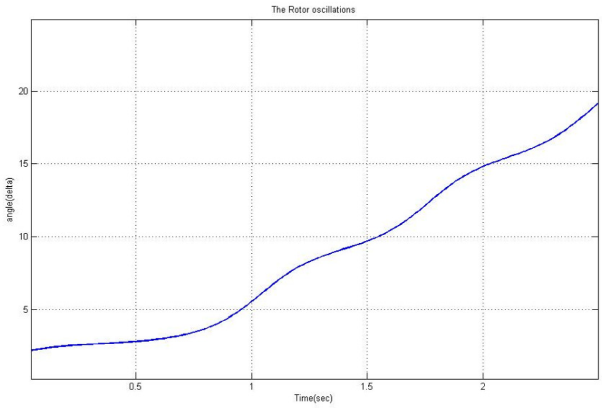

Figure 4.9: Single machine infinite bus system is unstable for T_c = 0.62s, whose critical clearing time is T_{cr}=0.61s

$V_{cr} > V_{tot}$ so the system is stable is shown in figure 4.8. Clear the fault at $T_c = 0.62s$ whose transient energy is $V_{tot} = 1.4053$ and critical energy is $V_{cr} = 1.2135$. $V_{cr} < V_{tot}$, so the system is unstable is shown in figure 4.9.

4.4 Multi-machine System

Consider 3-machine 9-bus multi-machine system shown in figure 3.1. Transient Energy Function method gives the results very fast; only it requires the calculation of critical transient energy V_{cr}, and system transient energy V_{tot} at the last instant of fault clearance. Considering the 3-phase to ground fault at different locations, the simulation results of proposed strategy, ie, closest u.e.p are given in Table 4.3. The critical energy calculated by Closest u.e.p approach is denoted by V_{cr}, which is determined by using the transient energy function (TEF) given in equation (3.51), while V_{tot} represents the system transient energy evaluated at the last instant of fault clearance.

From table 4.3, for the fault line number 7 and bus number 4; using closest u.e.p approach, the critical transient energy $V_{cr} = 2.2683$. Critical clearing time is $T_{cr} = 0.28s$. This is shown that the system is stable up to clearing time $T_c = 0.28s$. Controlling method is also same as closest u.e.p method, is shown in table 4.4, for the fault line number 8 and bus number 5;controlling u.e.p approach, the critical transient energy $V_{cr} = 3.9752$. Critical clearing time is $T_{cr} = 0.32s$. This is shown that the system is stable up to clearing time $T_c = 0.32$.

In multi-machine system also same as like single machine infinite bus system for finding

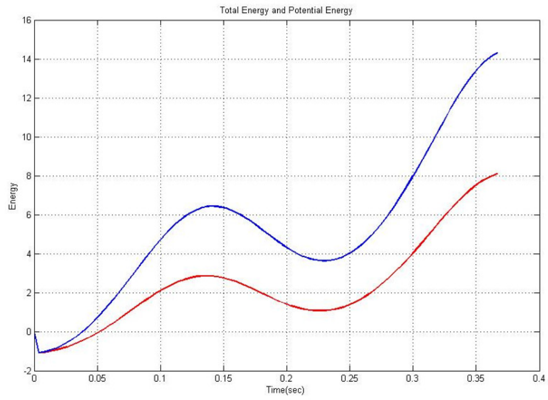

Figure 4.10: Multi-machine system using PEBS method (a). $V(\delta, \omega)$ blue line; (b). $V_{PE}(\theta)$ red line. Fault occur at 4-5

critical clearing time using PEBS method. To find the critical value $V(\delta, \omega)$ the fault on trajectory is monitored until it crosses the PEBS at a point θ^*. In many cases, θ^u the controlling u.e.p is close to θ^*, so that $V_{PE}(\theta^u) \approx V_{PE}(\theta^*) \cong V_{cr}$. This crossing is also approximately the point at which $V_{PE}(\theta)$ is maximum along the faulted trajectory. Hence, V_{cr} can be taken as equal to $V_{PE}^{max}(\theta)$ along the faulted trajectory. The PEBS crossing is also the at which $f^T(\theta).(\theta - \theta^s) = 0$. $f(\theta)$ is the accelerating power in post-fault system. The fault occur on line number 7 and bus number 4; The figure 4.11 is shown that the PEBS function crossing horizontal(zero axis) axis at the time 0.37s, which is the critical transient energy(V_{cr}). The potential energy value at 0.37s as shown in figure 4.10(b) corresponding time value of system transient energy($V(\delta, \omega)$) is 0.30s is shown in figure 4.10(a). So, the critical clearing time for PEBS method is 0.30s. Critical clearing times for different lines are shown in table 4.5.

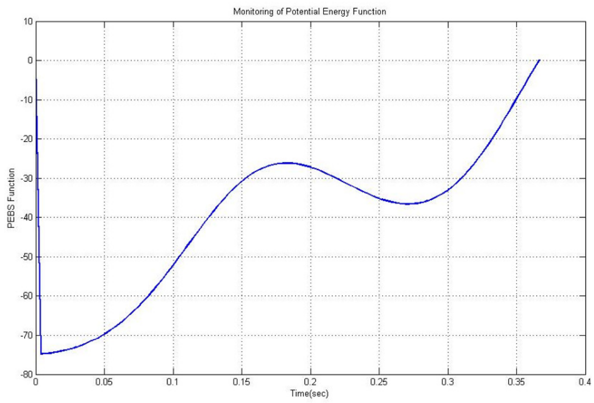

Figure 4.11: Multi-machine system using PEBS method. The monitoring of the PEBS crossing by $f^T(\theta).(\theta - \theta^s)$. Fault occur at 4-5

Table 4.3: Critical Clearing Time Using Closest u.e.p Method for Multi-machine System Using Relative Rotor Angle Reference Model

S.No.	FB	FL	Fault clearing type	T_{cr}	V_{cr}
1	4	7	Line trip	0.28s	2.2683
2	6	6	Line trip	0.24s	4.1447
3	5	8	Line trip	0.27s	2.3068
4	6	5	Line trip	0.26s	3.8710
5	7	9	Line trip	0.29s	3.6048
6	9	4	Line trip	0.28s	3.6629

FB:Fault bus; FL: Fault line; T_{cr}: Critical clearing time; V_{cr}: Critical energy

The fault occur on line number 6 and bus number 6; The figure 4.13 is shown that the PEBS function crossing horizontal(zero axis) axis at the time 0.41s, which is the critical transient energy(V_{cr}). The potential energy value at 0.41s as shown in figure 4.12(b) corresponding time value of system transient energy($V(\delta, \omega)$) is 0.30s is shown in figure 4.12(a). So, the critical clearing time for PEBS method is 0.30s. Critical clearing times for different lines are shown in table 4.5.

Table 4.4: Critical Clearing Time Using Controlling u.e.p Method for Multi-machine System Using Relative Rotor Angle Reference Model

S.No.	FB	FL	Fault clearing type	T_{cr}	V_{cr}
1	4	7	Line trip	0.32s	5.8996
2	6	6	Line trip	0.30s	3.9752
3	5	8	Line trip	0.32s	6.7811
4	6	5	Line trip	0.33s	5.0408
5	7	9	Line trip	0.32s	2.6064
6	9	4	Line trip	0.31s	2.6681

FB:Fault bus; FL: Fault line; T_{cr}: Critical clearing time; V_{cr}: Critical energy

Table 4.5: Critical Clearing Time Using PEBS Method for Multi-machine System Using Relative Rotor Angle Reference Model

S.No.	FB	FL	Fault clearing type	T_{cr}
1	4	7	Line trip	0.30s
2	6	6	Line trip	0.30s
3	5	8	Line trip	0.28s
4	6	5	Line trip	0.28s
5	7	9	Line trip	0.31s
6	9	4	Line trip	0.29s

FB:Fault bus; FL: Fault line; T_{cr}: Critical clearing time; V_{cr}: Critical energy

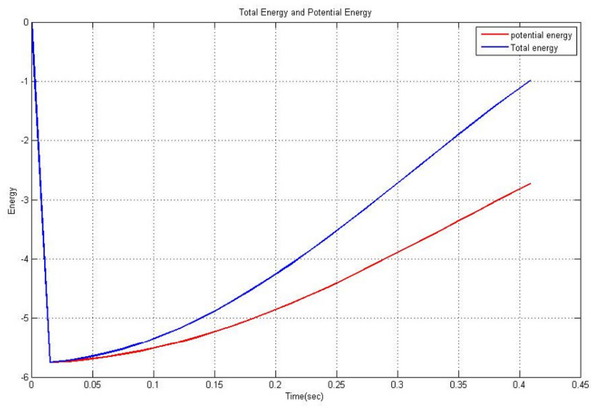

Figure 4.12: Multi-machine system using PEBS method (a). $V(\delta, \omega)$ blue line; (b). $V_{PE}(\theta)$ red line. Fault occur at 4-6

The fault occur on line number 8 and bus number 5; The figure 4.15 is shown that the PEBS function crossing horizontal(zero axis) axis at the time 0.35s, which is the critical transient energy(V_{cr}). The potential energy value at 0.35s as shown in figure 4.14(b)

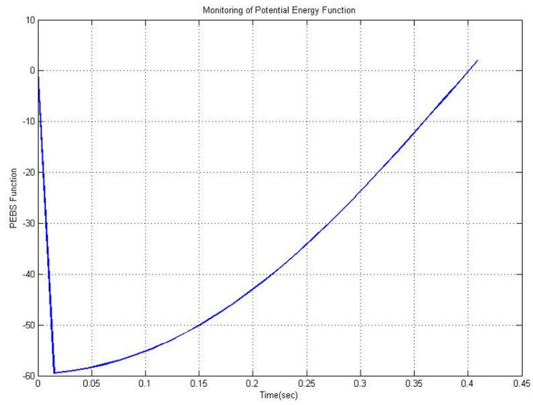

Figure 4.13: Multi-machine system using PEBS method. The monitoring of the PEBS crossing by $f^T(\theta).(\theta - \theta^s)$. Fault occur at 4-6

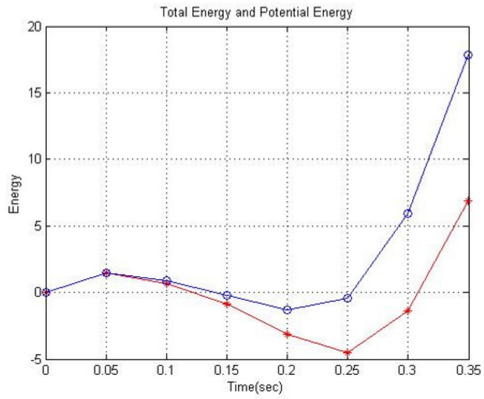

Figure 4.14: Multi-machine system using PEBS method (a). $V(\delta, \omega)$ blue line; (b). $V_{PE}(\theta)$ red line. Fault occur at 5-7

corresponding time value of system transient energy($V(\delta, \omega)$) is 0.31s is shown in figure 4.14(a). So, the critical clearing time for PEBS method is 0.31s.

Table 4.6: Critical Clearing Time Using PEBS Method by COI Reference Model

S.No.	FB	FL	Fault clearing type	T_{cr}
1	4	7	Line trip	0.28s
2	6	6	Line trip	0.31s
3	5	8	Line trip	0.28s
4	6	5	Line trip	0.31s
5	7	9	Line trip	0.27s
6	9	4	Line trip	0.29s

FB:Fault bus; FL: Fault line; T_{cr}: Critical clearing time; V_{cr}: Critical energy

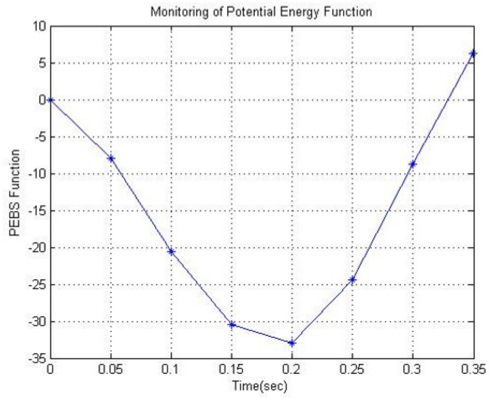

Figure 4.15: Multi-machine system using PEBS method. The monitoring of the PEBS crossing by $f^T(\theta).(\theta - \theta^s)$. Fault occur at 5-7

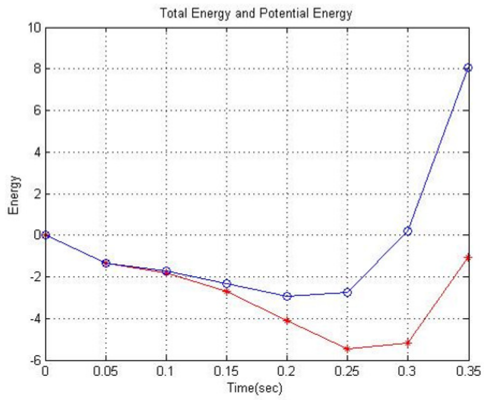

Figure 4.16: Multi-machine system using PEBS method (a). $V(\delta, \omega)$ blue line; (b). $V_{PE}(\theta)$ red line. Fault occur at 4-6

The fault occur on line number 6 and bus number 6; The figure 4.17 is shown that the PEBS function crossing horizontal(zero axis) axis at the time 0.35s, which is the critical transient energy(V_{cr}). The potential energy value at 0.35s as shown in figure 4.16(b) corresponding time value of system transient energy($V(\delta, \omega)$) is 0.28s is shown in figure 4.16(a). So, the critical clearing time for PEBS method is 0.28s. Critical clearing times for different lines are shown in table 4.5.

From table 4.7, for the fault line number 5 and bus number 6; using closest u.e.p approach, the critical transient energy $V_{cr} = 3.5133$ and Critical clearing time is $T_{cr} = 0.34s$. This is shown that the system is stable up to clearing time $T_c = 0.34s$.

From table 4.8, for the fault line number 4 and bus number 9; using closest u.e.p approach,

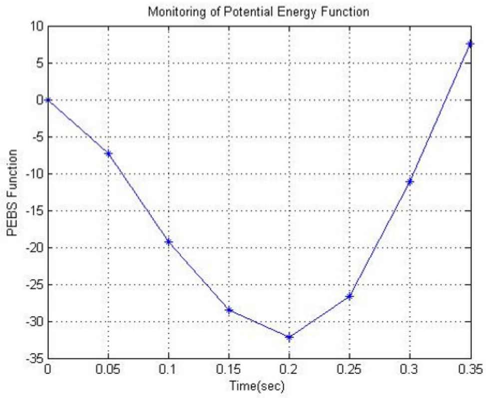

Figure 4.17: Multi-machine system using PEBS method. The monitoring of the PEBS crossing by $f^T(\theta).(\theta - \theta^s)$. Fault occur at 4-6

Table 4.7: Critical Clearing Time Using Controlling u.e.p Method for Multi-machine System Using COI Reference Model

S.No.	FB	FL	Fault clearing type	T_{cr}	V_{cr}
1	4	7	Line trip	0.31s	4.3721
2	6	6	Line trip	0.30s	2.4477
3	5	8	Line trip	0.31s	5.2536
4	6	5	Line trip	0.34s	3.5133
5	7	9	Line trip	0.34s	2.0789
6	9	4	Line trip	0.31s	5.1221

FB:Fault bus; FL: Fault line; T_{cr}: Critical clearing time; V_{cr}: Critical energy

Table 4.8: Critical Clearing Time Using Closest u.e.p Method for Multi-machine System Using COI Reference Model

S.No.	FB	FL	Fault clearing type	T_{cr}	V_{cr}
1	4	7	Line trip	0.26s	3.1673
2	6	6	Line trip	0.24s	6.1137
3	5	8	Line trip	0.27s	3.1065
4	6	5	Line trip	0.27s	4.5721
5	7	9	Line trip	0.28s	5.2248
6	9	4	Line trip	0.24s	4.3129

FB:Fault bus; FL: Fault line; T_{cr}: Critical clearing time; V_{cr}: Critical energy

the critical transient energy $V_{cr} = 4.3129$ and Critical clearing time is $T_{cr} = 0.24s$. This is shown that the system is stable up to clearing time $T_c = 0.24s$.

Table 4.9 shows that comparison of all the three methods with time-domain approach.

Table 4.9: Comparison of All the Three Methods for Multi-machine System With Time-domain Approach

S.No.	FB	FL	Time-domain	LUEP method	CUEP method	PEBS Method
1	4	7	0.32s	0.28s	0.32s	0.30s
2	6	6	0.32s	0.24s	0.30s	0.30s
3	5	8	0.33s	0.27s	0.32s	0.28s
4	6	5	0.34s	0.26s	0.33s	0.28s
5	7	9	0.36s	0.29s	0.32s	0.31s
6	9	4	0.27s	0.28s	0.31s	0.29s

FB:Fault bus; FL: Fault line;

By observing the results, we can say that controlling unstable equilibrium point (u.e.p) method is close to the time-domain approach.

Chapter 5

Conclusion and Future Scope of Research

5.1 Conclusion

The present thesis attempts to investigate the transient stability of power system using energy functions. Normally, step-by-step numerical integrations are used to evaluate the transient stability of power system. This step-by-step (time-domain) simulation approach is, however, an extremely time consuming process. Hence, an alternative approach to transient stability assessment can use the energy functions. These energy functions eliminates the post-fault integration and, thus, reduces the simulation time.

The thesis presents a procedure for finding stable equilibrium points, critical energy (V_{cr}) and critical clearing time (CCT). The methods like closest unstable equilibrium point (u.e.p) method, controlling unstable equilibrium point (u.e.p) method and potential energy boundary surface method (PEBS) are discussed to determine the transient stability of power system. These methods are studied using center of inertia technique and relative rotor angle technique.

Simulation results are discussed for both single machine infinite bus system and multi-machine system using center of inertia technique and relative rotor angle technique. Comparison is done for all the three methods with time-domain approach. The method of controlling unstable equilibrium point is used to estimate the critical clearing time. It is observed that the CUEP method does produce a more accurate result compared to other energy function-based methods.

5.2 Future Scope of Research

The thesis presents the study of different energy function methods to estimate the critical clearing time (CCT). The present work is based on the classical multi-machine power system model. The loads are assumed to be constant impedances. The critical clearing time (CCT) resulting from the present work is an approximate one. Hence, there is a need to model the elements of power system in a greater detail to get more precise CCT. A robust energy function algorithm for analysing the detailed model may lead to a more stable and robust power system operation.

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