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Study of some rare decays of B_s meson in the fourth generation model

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We study some rare decays of the B_s meson governed by the quark-level transitions $b \to s$, in the fourth generation model popularly known as SM4. Recently, it has been shown that SM4, which is a simple extension of the SM3, can successfully explain several anomalies observed in the *CP* violation parameters of B and B_s mesons. We find that in this model, due to the additional contributions coming from the heavy t' quark in the loop, the branching ratios and other observables in rare B_s decays deviate significantly from their SM values. Some of these modes are within the reach of LHCb experiments, and a search for such channels is strongly argued.

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I. INTRODUCTION

The spectacular performance of the two asymmetric B factories by Belle and *BABAR* provided us with a unique opportunity to understand the origin of *CP* violation in a very precise way. Although the results from the B factories do not provide us with any clear evidence of new physics, there are a few cases that were observed in the last few years which have 2σ–3σ deviations from their corresponding standard model (SM) expectations [1]. For example, the difference in the direct *CP* asymmetry parameters between $B^- \to \pi^0 K^-$ and $\bar{B}^0 \to \pi^+ K^-$ is expected to be negligibly small in the SM, but was found to be nearly 15%. The measurement of mixing-induced *CP* asymmetry in several $b \to s$ penguin decays is not found to be the same as that of $B_d \to J/\psi K_s$. Recently, a very large *CP* asymmetry has been observed by the CDF and D0 collaborations [2,3] in the tagged analysis of $B_s \to J/\psi \phi$ with a value $S_{\psi\phi} \in [0.24, 1.36]$. Within the SM this asymmetry is expected to be vanishingly small, which basically comes from the $B_s - \bar{B}_s$ mixing phase. It should be noted that all these deviations are associated with the flavor changing neutral current (FCNC) transitions $b \to s$. It is well known that the FCNC decays are forbidden at the tree level in the SM and therefore play a very crucial role to look for the possible existence of new physics (NP).

In this paper we would like to study some rare decays of the B_s meson involving $b \to s$ transitions. The study of the B_s meson has attracted significant attention in recent times because a huge number of B_s mesons are expected to be produced in the currently running LHCb experiment, which opened up the possibility to study B_s mesons with high statistical precision. These studies will not only play a dominant role to corroborate the results of B mesons, but they will also look for possible hints of new physics. Here we will consider the decay channels $B_s \to \phi\pi$, $B_s \to \phi\gamma$, $B_s \to \gamma\gamma$, and $B_s \to \mu^+\mu^-\gamma$, which are highly suppressed in the SM. We intend to analyze these decay channels both in the SM and in the fourth quark generation model [4], usually known as SM4. SM4 is a simple extension of the standard model with three generations (SM3) with the additional up-type (t') and down-type (b') quarks. It has been shown in Ref. [5] that the addition of a fourth family of quarks with $m_{t'}$ in the range (400–600) GeV provides a simple explanation for the several deviations that have been observed involving *CP* asymmetries in the B, B_s decays. The implications of the fourth generation in various B decays are discussed in [6–10]. The experimental search for fourth generation quarks has also received significant attention recently due to the operation of the Large Hadron Collider. The CMS Collaboration put a lower bound on the mass of t' as $m_{t'} \gtrsim 450$ GeV [11] and excluded the b'-quark mass in the region 255 GeV $< m_{b'} <$ 361 GeV at 95% C.L. [12].

The paper is organized as follows. In Sec. II we discuss the nonleptonic decay process $B_s \to \phi\pi$. The radiative decays $B_s \to \phi\gamma$ and $B_s \to \gamma\gamma$ are discussed in Secs. III and IV. The process $B_s \to \mu^+\mu^-\gamma$ is presented in Sec. V, and Sec. VI contains the conclusions.

II. $B_s \to \phi\pi$ PROCESS

In this section we will discuss the nonleptonic decay mode $B_s \to \phi\pi$ which receives a dominant contribution from electroweak penguins $b \to sq\bar{q}$ ($q = u$, d), as the QCD penguins are Okubo-Zweig-Iizuka suppressed and the color-suppressed tree contribution $b \to u\bar{u}s$ is doubly Cabibbo suppressed. Therefore, this process is expected to be highly suppressed in the SM and hence serves as a suitable place to search for new physics. This decay mode has been studied in the SM using the QCD factorization approach [13] and in the model with nonuniversal Z' bosons [14].

The relevant effective Hamiltonian describing this process is given by [15]

$$\mathcal{H}_{\rm eff}^{\rm SM} = \frac{G_F}{\sqrt{2}}\Big[V_{ub}V_{us}^* \sum_{i=1,2} C_i(\mu)O_i - V_{tb}V_{ts}^* \sum_{i=3}^{10} C_i(\mu)O_i\Big], \tag{1}$$

where $C_i(\mu)$'s are the Wilson coefficients evaluated at the b-quark mass scale, $O_{1,2}$ are the tree-level current-current operators, O_{3-6} are the QCD penguin operators, and O_{7-10} are the electroweak penguin operators.

Here we will use the QCD factorization approach to evaluate the hadronic matrix elements as discussed in [16]. The matrix elements describing the $B_s \to \phi$ transition can be parametrized in terms of various form factors [17] as

$$\begin{aligned}\langle \phi(p', \epsilon)|\bar{s}\gamma_\mu(1-\gamma_5)b|B_s(p)\rangle &= -i\epsilon_\mu^*(m_{B_s} + m_\phi)A_1(q^2) + i(p+p')_\mu(\epsilon^*\cdot q) \\ &\times \frac{A_2(q^2)}{m_{B_s}+m_\phi} + iq_\mu(\epsilon^*\cdot q)\frac{2m_\phi}{q^2}(A_3(q^2) \\ &- A_0(q^2)) + \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho p'^\sigma \frac{2V(q^2)}{m_{B_s}+m_\phi},\end{aligned} \tag{2}$$

where p and p' are the momenta of B_s and ϕ mesons, $q = p - p'$ is the momentum transfer, and $A_{1-3}(q^2)$ and $V(q^2)$ are various form factors describing the transition process. Using the decay constant of the π^0 meson as

$$\left\langle \pi^0(q)\left|\frac{\bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d}{\sqrt{2}}\right|0\right\rangle = i\frac{f_\pi}{\sqrt{2}}q^\mu, \tag{3}$$

one can obtain the transition amplitude for the process

$$\begin{aligned}\mathcal{A}(B_s\to\phi\pi) &= \frac{G_F}{2}f_\pi(\epsilon^*\cdot q)2m_\phi A_0(q^2) \\ &\quad\times\left(V_{ub}V_{us}^* a_2 - \frac{3}{2}V_{tb}V_{ts}^*(-a_7+a_9)\right),\end{aligned} \tag{4}$$

where $\lambda_q = V_{qb}V_{qs}^*$. The parameters a_i are related to the Wilson coefficients C_i, and the corresponding expressions can be found in Ref. [16].

The corresponding decay width is given as

$$\Gamma(B_s\to\phi\pi) = \frac{|\mathbf{p}_{\rm cm}|^3}{8\pi m_\phi^2}\left|\frac{\mathcal{A}(B_s\to\phi\pi)}{\epsilon^*\cdot q}\right|^2, \tag{5}$$

where $\mathbf{p}_{\rm cm}$ is the center of mass momentum of the outgoing particles.

Now we discuss the CP violating observables for this process. To obtain these observables, we can symbolically represent the amplitude (4) as

$$\begin{aligned}\mathcal{A}(\bar{B}_s\to\phi\pi) &= (\epsilon^*\cdot q)[\lambda_u A_u - \lambda_t A_t] \\ &= -\lambda_t A_t(\epsilon^*\cdot q)[1 - rae^{-i(\pi+\beta_s+\gamma+\delta)}],\end{aligned} \tag{6}$$

where $a = |\lambda_u/\lambda_t|$, $-\gamma$ is the weak phase of V_{ub}, $(\pi+\beta_s)$ is the weak phase of λ_t, $r = |A_u/A_t|$, and δ is the relative strong phase between A_t and A_u. From the above amplitude, the direct and mixing-induced CP asymmetry parameters can be obtained as

$$\begin{aligned}A_{\phi\pi} &= \frac{2ra\sin\delta\sin(\beta_s+\gamma)}{1+(ra)^2+2ra\cos\delta\cos(\beta_s+\gamma)}, \\ S_{\phi\pi} &= -\frac{2ra\cos\delta\sin(\beta_s+\gamma)+(ra)^2\sin(2\beta_s+2\gamma)}{1+(ra)^2+2ra\cos\delta\cos(\beta_s+\gamma)}.\end{aligned} \tag{7}$$

For numerical evaluation, we use the particle masses, lifetime of B_s meson from [18]. For the Cabibbo-Kobayashi-Maskawa (CKM) elements, we use the Wolfenstein parametrization with the values of the parameters as $\lambda = 0.2253 \pm 0.0007$, $A = 0.808^{+0.022}_{-0.015}$, $\bar{\rho} = 0.132^{+0.022}_{-0.014}$, $\bar{\eta} = 0.341 \pm 0.013$. The parameters of the QCD factorization approach and the value of the form factor used, $A_0^{B_s\to\phi} = 0.32 \pm 0.01$, are taken from [13].

With these inputs we obtain the branching ratio for this process as

$$\mathrm{Br}(B_s\to\phi\pi) = (1.26\pm 0.32)\times 10^{-7}, \tag{8}$$

which is consistent with the prediction of [13,14].

The CP violating observables are found to be

$$S_{\phi\pi} = -0.23, \qquad A_{\phi\pi} = 0.1. \tag{9}$$

Our predicted direct CP asymmetry is lower than the prediction of [13]. This difference arises mainly because the subleading power corrections to the color-suppressed tree amplitude a_2 have been included in Ref. [13], which introduces a large strong phase.

Now we will analyze this process in the fourth generation model. In the presence of a sequential fourth generation, there will be additional contributions due to the t' quark in the loop diagrams. Furthermore, due to the additional fourth generation there will be mixing between the b' quark and the three down-type quarks of the standard model; the resulting mixing matrix will become a 4×4 matrix ($V_{\rm CKM4}$), and the unitarity condition becomes $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$, where $\lambda_q = V_{qb}V_{qs}^*$. The parametrization of this unitary matrix requires six mixing angles and three phases. The existence of the two extra phases provides the possibility of an extra source of CP violation [19]. In the presence of a fourth generation there will be an additional contribution both to the $B_s\to\phi\pi$ decay amplitude and to the $B_s - \bar{B}_s$ mixing phenomenon. However, since the new physics contribution to the $B_s - \bar{B}_s$ mixing amplitude due to a fourth generation model has been discussed in Ref. [9], we will simply quote the results from there.

Now we will consider the additional contribution to the decay amplitude due to the fourth quark generation model. In this model the new contributions are due to the t' quark in the penguin loops. Thus, the modified Hamiltonian becomes

TABLE I. Numerical values of the Wilson coefficients C_i' for $m_{t'} = 400$ and 500 GeV.

t' mass	C_7'	C_8'	C_9'	C_{10}'
$m_{t'} = 400$ GeV	4.453×10^{-3}	2.115×10^{-3}	-0.029	0.006
$m_{t'} = 500$ GeV	7.311×10^{-3}	3.199×10^{-3}	-0.041	0.009

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\lambda_u(C_1O_1 + C_2O_2) - \lambda_t\sum_{i=3}^{10} C_iO_i - \lambda_{t'}\sum_{i=3}^{10} C_i'O_i\right], \quad (10)$$

where C_i''s are the effective Wilson coefficients due to the t' quark in the loop. To find the new contribution due to the fourth generation effect, first we have to evaluate the new Wilson coefficients C_i'. The values of these coefficients at the M_W scale can be obtained from the corresponding contribution from the t quark by replacing the mass of the t quark by the t' mass in the Inami-Lim functions [20]. These values can then be evolved to the m_b scale using the renormalization group equation [15]. Thus, the obtained values of $C_{i=7-10}'(m_b)$ for two representative sets of values, i.e., $m_{t'} = 400$ and 500 GeV, are as presented in Table I.

Thus, in the presence of the fourth generation model, one can obtain the transition amplitude for the $B_s \to \phi\pi$ process from Eq. (10), which can be symbolically represented as

$$\mathcal{A}(\bar{B}_s \to \phi\pi) = (\epsilon^* \cdot q)(\lambda_u A_u - \lambda_t A_t - \lambda_{t'}A_{t'}) - \lambda_t A_t(\epsilon^* \cdot q)[1 + rae^{i(\beta_s+\gamma-\delta)} + r'be^{i(\phi_s-\beta_s+\delta_1)}], \quad (11)$$

where $b = |\lambda_{t'}/\lambda_t|$, $r' = |A_{t'}/A_t|$, and δ_1 is the relative strong phase between $A_{t'}$ and A_t. From the above amplitude, the CP averaged branching ratio, and direct and mixing-induced CP asymmetry parameters can be obtained as

$$\mathrm{Br} = \mathrm{Br}^{\rm SM}X, \qquad A_{\phi\pi} = \frac{Y}{X}, \qquad S_{\phi\pi} = -\frac{Z}{X}, \quad (12)$$

with

$$\begin{aligned} X &= 1 + (ra)^2 + (r'b)^2 + 2ra\cos\delta\cos(\beta_s+\gamma) \\ &\quad + 2r'b\cos\delta_1\cos(\phi_s-\beta_s) + 2rr'ab\cos(\phi_s+\gamma) \\ &\quad \times\cos(\delta+\delta_1), \\ Y &= 2ra\sin\delta\sin(\beta_s+\gamma) + 2r'b\sin\delta_1\sin(\phi_s-\beta_s) \\ &\quad + 2rr'ab\sin(\phi_s+\gamma)\sin(\delta+\delta_1), \\ Z &= \sin2\theta + 2ra\cos\delta\sin(\beta_s+\gamma+2\theta) \\ &\quad - 2r'b\cos\delta_1\sin(\phi_s-\beta_s-2\theta) \\ &\quad + r^2a^2\sin(2\beta_s+2\gamma+2\theta) \\ &\quad - r'^2b^2\sin(2\phi_s-2\beta_s-2\theta) \\ &\quad - 2rr'ab\cos(\delta+\delta_1)\sin(\phi_s-2\beta_s-\gamma-2\theta). \end{aligned} \quad (13)$$

In Eq. (13), 2θ is the additional contribution to the $B_s - \bar{B}_s$ mixing phase in the fourth generation, and the expression for it can be found in Ref. [9].

For a numerical evaluation using the values of the new Wilson coefficients as presented in Table I, we obtain $r \approx 7.79$, $\delta \approx 25.9°$, $r' = 3.48(5.03)$, and $\delta_1 \approx -0.1°(-0.1°)$ for $m_{t'} = 400(500)$ GeV. For the new CKM elements $\lambda_{t'}$, we use the allowed range of $|\lambda_{t'}| = (0.08\text{–}1.4) \times 10^{-2}$ $[(0.06\text{–}0.9)\times 10^{-2}]$ and $\phi_s = (0 \to 80)°$ $[\phi_s = (0 \to 80)°]$

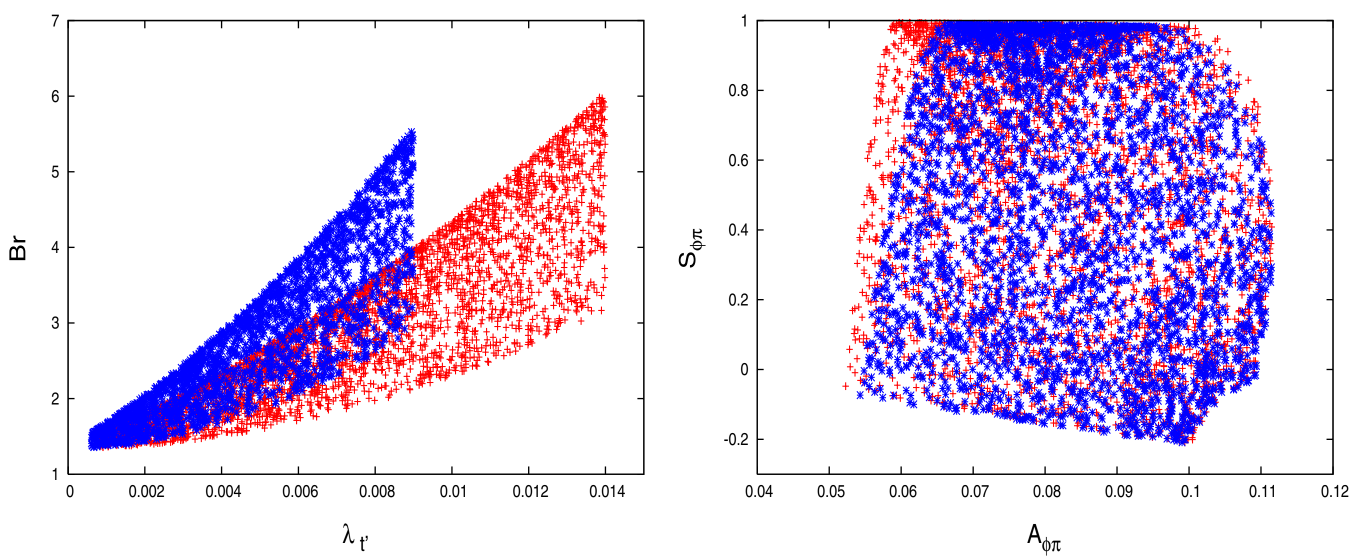

FIG. 1 (color online). Variation of the CP averaged branching ratio in units of 10^{-7} (left panel) and the correlation plot between the mixing-induced CP asymmetry $S_{\phi\pi}$ and the direct CP asymmetry parameter $A_{\phi\pi}$ (right panel) for the $B_s \to \phi\pi$ process. The red/grey (blue/black) regions correspond to $m_{t'} = 400$ GeV (500 GeV).

for $m_{t'} = 400$ GeV [500 GeV], extracted using the available observables mediated through $b \to s$ transitions [5]. Now varying λ_t' and ϕ_s in their allowed ranges, we show the variation of the branching ratio in the left panel of Fig. 1 and the correlation plot between the CP violating parameters in the right panel. From the figure it can be seen that the branching ratio is significantly enhanced from its SM value, and large mixing-induced CP violation ($S_{\phi\pi}$) could be possible for this decay mode in the fourth generation model. However, the direct CP asymmetry does not deviate much from the corresponding SM value. It should also be noted that the branching ratio decreases slowly with the increase of the t' mass. However, there is no significant $m_{t'}$ dependence of the CP violating observables.

III. $B_s \to \phi\gamma$

Here we will consider the decay channel $B_s \to \phi\gamma$, which is induced by the quark-level transition $\bar{b} \to \bar{s}\gamma$. This mode is the strange counterpart of the $B \to K^*\gamma$, which is very clean to analyze. Compared to the B_d system, the elements of the B_s system are the small mixing phase ϕ_s and the large width difference $\Delta\Gamma_s$ of the $B_s - \bar{B}_s$ system. The branching ratio of this mode was recently reported by the Belle Collaboration [21],

$$\mathrm{Br}\,(B_s \to \phi\gamma) = (57^{+18}_{-15}(\mathrm{stat})^{+12}_{-11}(\mathrm{syst})) \times 10^{-6}. \quad (14)$$

In the standard model the CP averaged branching ratio of this mode is predicted to be [22]

$$\mathrm{Br}\,(B_s \to \phi\gamma) = (39.4 \pm 10.7 \pm 5.3) \times 10^{-6}. \quad (15)$$

Although the SM prediction seems to be consistent with the observed value, the presence of large experimental uncertainties makes it difficult to infer/rule out the presence of new physics from this mode.

The transition process $b \to s\gamma$ can be described by the dipole-type effective Hamiltonian, which is given as [23]

$$\mathcal{H}_{\mathrm{eff}} = -\frac{4G_F}{\sqrt{2}}\lambda_t C_7(m_b) O_7, \quad (16)$$

where C_7 is the Wilson coefficient and O_7 is the electromagnetic dipole operator given as

$$O_7 = \frac{e}{32\pi^2} F_{\mu\nu}[m_b \bar{s}\sigma^{\mu\nu}(1+\gamma_5)b + m_s \bar{s}\sigma^{\mu\nu}(1-\gamma_5)b]. \quad (17)$$

The expression for calculating the Wilson coefficient $C_7(\mu)$ is given in [23].

The matrix elements of the various hadronic currents between the initial B_s and the final ϕ mesons are parametrized in terms of various form factors as [17]

$$\begin{aligned}&\langle \phi(p', \epsilon|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B_s\rangle \\ &= i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho p'^\sigma 2T_1(q^2) + T_2(q^2)[\epsilon^*_\mu(m^2_{B_s} - m^2_\phi) \\ &\quad - (\epsilon^* \cdot q)(p+p')_\mu],\end{aligned} \quad (18)$$

with $T_1(0) = T_2(0)$ and $q = p - p'$. With these definitions of the form factors, one can obtain the corresponding decay width as

$$\Gamma(B_s \to \phi\gamma) = \frac{\alpha G_F^2}{32\pi^4}|V_{tb}V^*_{ts}|^2|C_7^{eff}|^2 m_b^2 m^3_{B_s}|T_1(0)|^2 \times \left(1 - \frac{m^2_\phi}{m^2_{B_s}}\right)^3. \quad (19)$$

Using the value of the form factor $T_1(0) = 0.349 \pm 0.033$ [17], $C_7(m_b) = -0.31$, and the values of the other parameters as discussed in Sec. II, we obtain the branching ratio as

$$\mathrm{Br}\,(B_s \to \phi\gamma) = (39.9 \pm 12.3) \times 10^{-6}. \quad (20)$$

As is well known the rare radiative decays of B mesons are particularly sensitive to the contributions from new physics. The $V - A$ structure of the weak interactions can be tested in FCNC decays of the type $b \to (s, d)\gamma$, since the emitted photon is predominantly left-handed. The crucial point is that the leading operator $O_7 \sim \bar{s}\sigma_{\mu\nu}F^{\mu\nu}b_{L(R)}$ necessitates a helicity flip on the external quark legs, which introduces a natural hierarchy between the left- and right-handed components of the order $m_{d,s}/m_b$. However, it is difficult to measure the helicity of the photon directly. It was pointed out long ago that the time dependent CP asymmetry is an indirect measure of the photon helicity [24], since it is caused by the interference of left- and right-handed helicity amplitudes. The final state in $B_s \to \phi\gamma$ is not a pure CP eigenstate. Rather, in the SM it consists of an equal mixture of positive and negative eigenvalues. Thus, due to an almost complete cancellation between positive and negative CP eigenstates, the asymmetries in $b \to s\gamma$ are very small. They are given by m_s/m_b, where the quark masses are current quark masses.

The normalized CP asymmetry for the $B_s \to \phi\gamma$ is defined as follows [25]:

$$A_{CP}(B_s \to \phi\gamma) = \frac{\Gamma(\bar{B}_s \to \phi_s\gamma) - \Gamma(B_s \to \phi\gamma)}{\Gamma(\bar{B}_s \to \phi_s\gamma) + \Gamma(B_s \to \phi\gamma)}, \quad (21)$$

where the left- and right-handed photon contributions are added incoherently, i.e., $\Gamma(B_s \to \phi\gamma) = \Gamma(B_s \to \phi\gamma_L) + \Gamma(B_s \to \phi\gamma_R)$. It is well known that the neutral mesons exhibit the time dependent CP asymmetry through mixing, i.e., if the particle and the antiparticle decay into a common final state f. In $B_s \to \phi\gamma$ this amounts to

$$B_s \to \phi\gamma_{L(R)} \leftarrow \bar{B}_s. \quad (22)$$

With $|q/p| = 1$, the CP asymmetry assumes the following generic time dependent form:

$$A_{CP}(t) = \frac{S\sin(\Delta m_s t) - C\cos(\Delta m_s t)}{\cosh\frac{\Delta\Gamma_s t}{2} - H\sinh\Delta\Gamma_s t2}. \tag{23}$$

In terms of the left- and right-handed amplitudes

$$\begin{aligned} \mathcal{A}_{L(R)} &= \mathcal{A}(B_s \to \phi\gamma_{L(R)}), \\ \bar{\mathcal{A}}_{L(R)} &= \mathcal{A}(\bar{B}_s \to \phi\gamma_{L(R)}), \end{aligned} \tag{24}$$

the form of the observables C, S, and H can be found as

$$\begin{aligned} C &= \frac{(|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2) - (|\bar{\mathcal{A}}_L|^2 + |\bar{\mathcal{A}}_R|^2)}{|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2 + |\bar{\mathcal{A}}_L|^2 + |\bar{\mathcal{A}}_R|^2}, \\ S &= \frac{2\,\mathrm{Im}[\frac{q}{p}(\bar{\mathcal{A}}_L\mathcal{A}_L^* + \bar{\mathcal{A}}_R\mathcal{A}_R^*)]}{|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2 + |\bar{\mathcal{A}}_L|^2 + |\bar{\mathcal{A}}_R|^2}, \\ H &= \frac{2\,\mathrm{Re}[\frac{q}{p}(\bar{\mathcal{A}}_L\mathcal{A}_L^* + \bar{\mathcal{A}}_R\mathcal{A}_R^*)]}{|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2 + |\bar{\mathcal{A}}_L|^2 + |\bar{\mathcal{A}}_R|^2}. \end{aligned} \tag{25}$$

In the standard model the leading operator O_7 allows the $\bar{B}_s(B_s)$ meson to decay predominantly into a left (right)-handed photon, whereas the $B_s(\bar{B}_s)$ meson decays into the left (right)-handed photon suppressed by an m_s/m_b chirality factor. Because of the interference between mixing and decay in $B_s \to \phi\gamma$, a single weak decay amplitude proportional to λ_t is exactly canceled by the mixing phase, and hence one can obtain $S_{\phi\gamma} = 0$ and $H_{\phi\gamma} = 2m_s/m_b$ [25].

The situation can be significantly modified in certain models beyond the standard model by new terms in the decay amplitudes and also by the new contribution to the $B_s - \bar{B}_s$ mixing. In this section we will study the effect of the fourth quark generation on the various decay observables. In the presence of the fourth generation, the Wilson coefficients C_7 will be modified due to the new contributions arising from the virtual t' quark in the loop. Thus, these modified coefficients can be represented as

$$C_7^{\mathrm{tot}}(\mu) = C_7(\mu) + \frac{\lambda_{t'}}{\lambda_t}C_7'(\mu). \tag{26}$$

The new coefficients C_7' can be calculated at the M_W scale by replacing the t-quark mass by $m_{t'}$ in the loop functions. These coefficients are then evolved to the b scale using the renormalization group equation as discussed in [15]. The values of the new Wilson coefficients at the m_b scale for $m_{t'} = 400$ GeV are given by $C_7'(m_b) = -0.375$.

Thus, including the new physics contribution due to the fourth generation effect, the branching ratio can be obtained from Eq. (19) by replacing C_7 by C_7^{tot}, and the CP violating parameters are given as

$$S_{\phi\gamma} = \frac{m_s}{m_b}\bigg(\frac{-C_7^2\sin2\theta + 2aC_7C_7'\sin(\phi_s - \beta_s - 2\theta) + a^2C_7'^2\sin2(\phi_s - \beta_s - \theta)}{C_7^2 + a^2C_7'^2 + 2aC_7C_7'\cos(\phi_s - \beta_s)}\bigg), \tag{27}$$

$$H_{\phi\gamma} = \frac{m_s}{m_b}\bigg(\frac{C_7^2\cos2\theta + a^2C_7'^2\cos2(\phi_s - \beta_s - \theta) + 2aC_7C_7'\cos(\phi_s - \beta_s - 2\theta)}{C_7^2 + a^2C_7'^2 + 2aC_7C_7'\cos(\phi_s - \beta_s)}\bigg), \tag{28}$$

where 2θ is the new contribution to the $B_s - \bar{B}_s$ mixing phase due to the fourth generation. Now varying $\lambda_{t'}$ in the range $(0.08–1.4) \times 10^{-2}$ and ϕ_s in the range $(0°–80°)$, we show in Fig. 2 the CP averaged branching ratio (left panel) and the correlation plot between the CP violating observables (right panel). From the figure it can be seen that small but nonzero CP violating observables could be possible in the fourth generation model, while the branching ratio is still consistent with the observed value. Furthermore, in this case the branching ratio also decreases with the increase of t' mass.

IV. $B_s \to \gamma\gamma$

Now we will discuss the decay process $B_s \to \gamma\gamma$. At the quark level this process is similar to $b \to s\gamma$. Up to the correction of order $1/M_W^2$, the effective Hamiltonian for $b \to s\gamma\gamma$ at scales $\mu_b = O(m_b)$ is identical to the one for $b \to s\gamma$. The $B_s \to \gamma\gamma$ process has been studied extensively in the SM and in various new physics scenarios [26–31]. The present experimental limit on the decay $B_s \to \gamma\gamma$ is [21]

$$\mathrm{Br}\,(B_s \to \gamma\gamma) \le 8.7 \times 10^{-6}(90\%\ \mathrm{C.L.}). \tag{29}$$

We expect, with the continuous accumulation of the experimental data, that the situation will improve and the branching ratio will be more precise.

The effective Hamiltonian for this process is given by Eq. (16). To calculate the decay amplitude for this process one may follow the procedure discussed in Ref. [29]. In order to calculate the matrix element of Eq. (16) for $B_s(p_B) \to \gamma(k_1)\gamma(k_2)$, one can work in the weak binding approximation and assume that both the b and s quarks are at rest in the B_s meson and that the b quark carries most of the meson energy and its four velocity can be treated as equal to that of B_s. Hence one may write the b-quark momentum as $p_b = m_b v$, where v is the common four velocity of b and B_s. Thus we have

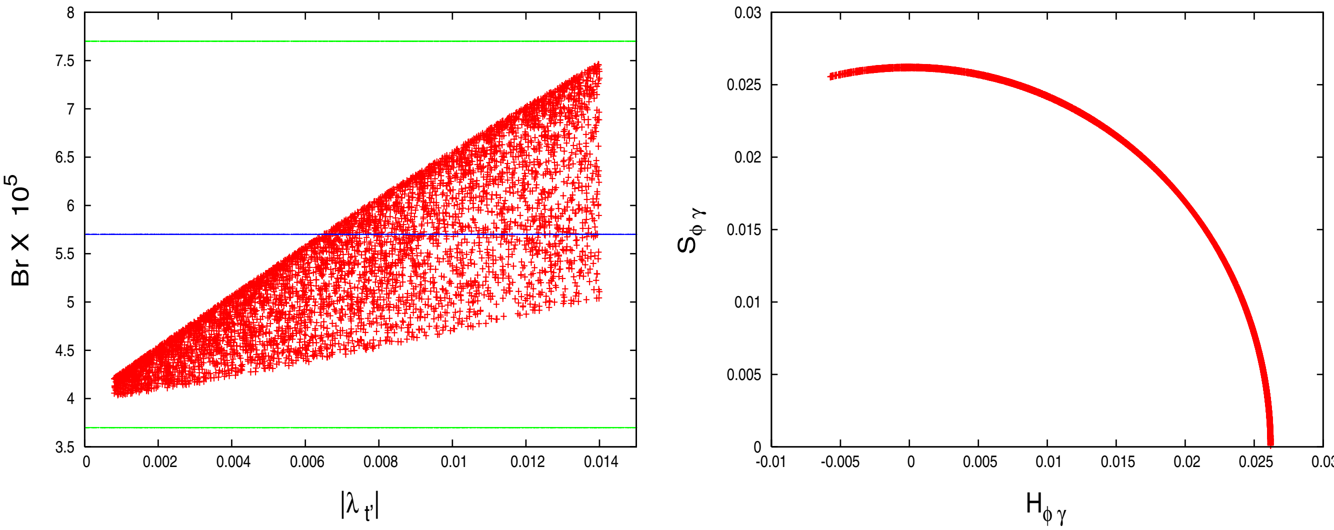

FIG. 2 (color online). Variation of the CP averaged branching ratio (left panel) and the correlation plot between the CP violating observables $S_{\phi\gamma}$ and $H_{\phi\gamma}$ (right panel) for the $B_s \to \phi\gamma$ process. The horizontal blue/black line on the left panel is the central value of the measured branching ratio, whereas the green/light grey lines represent the corresponding 1-sigma range.

$$\begin{aligned} p_b \cdot k_1 = m_b v \cdot k_1 &= \frac{1}{2} m_b m_{B_s} = p_b \cdot k_2 p_s \cdot k_1 \\ &= (p - k_1 - k_2) \cdot k_1 = -\frac{m_{B_s}}{2}(m_{B_s} - m_b) \\ &= p_s \cdot k_2. \end{aligned} \tag{30}$$

The amplitude for $B_s \to \gamma\gamma$ can be computed using the following matrix elements:

$$\begin{aligned} \langle 0|\bar{s}\gamma^\mu\gamma_5 b|B_s(p_B)\rangle &= i f_{B_s} p_B^\mu, \\ \langle 0|\bar{s}\gamma_5 b|B_s\rangle &= i f_{B_s} M_{B_s}. \end{aligned} \tag{31}$$

Thus, one can obtain the total amplitude for this process containing CP even and CP odd parts as

$$\mathcal{A}(B_s \to \gamma\gamma) = M^+ F_{\mu\nu}F^{\mu\nu} + iM^- F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{32}$$

with

$$M^+ = -\frac{4\sqrt{2}\alpha G_F}{9\pi} f_{B_s} m_{B_s} V_{tb} V_{ts}^* \left(B m_b K(m_b^2) + \frac{3C_7}{8\bar{\Lambda}} \right) \tag{33}$$

and

$$\begin{aligned} M^- = \frac{4\sqrt{2}\alpha G_F}{9\pi} f_{B_s} m_{B_s} V_{tb} V_{ts}^* \bigg(& \sum_q m_{B_s} A_q J(m_q^2) \\ & + m_b B L(m_b^2) + \frac{3C_7}{8\bar{\Lambda}} \bigg), \end{aligned} \tag{34}$$

where $\bar{\Lambda} = m_{B_s} - m_b$. The parameters A_q are related to the Wilson coefficients C_i, which are evaluated at the m_b scale as

$$\begin{aligned} A_u &= (C_3 - C_5)N_c + (C_4 - C_6), \\ A_d &= \tfrac{1}{4}[(C_3 - C_5)N_c + (C_4 - C_6)], \\ A_c &= (C_1 + C_3 - C_5)N_c + C_2 + C_4 - C_6, \\ A_s &= A_b = \tfrac{1}{4}[(C_3 + C_4 - C_5)N_c + (C_3 + C_4 - C_6),] \\ B &= C = -\tfrac{1}{4}(C_6 N_c + C_5). \end{aligned} \tag{35}$$

The functions $J(m^2)$, $K(m^2)$, and $L(m^2)$ are defined as

$$J(m^2) = I_{11}(m^2), \qquad K(m^2) = 4I_{11}(m^2) - I_{00}(m^2), \qquad L(m^2) = I_{00}(m^2), \tag{36}$$

with

$$I_{pq}(m^2) = \int_0^1 dx \int_0^{1-x} dy \frac{x^p y^q}{m^2 - 2k_1 \cdot k_2 xy - i\epsilon}. \tag{37}$$

Thus, one can obtain the decay width of $B_s \to \gamma\gamma$ as

$$\Gamma(B_s \to \gamma\gamma) = \frac{m_{B_s}^3}{16\pi}(|M^+|^2 + |M^-|^2). \tag{38}$$

To obtain the numerical results we use the parameters as presented in Sec. II. Thus, we obtain the branching ratio as

$$\mathrm{Br}(B_s \to \gamma\gamma) = (1.8 \pm 0.4) \times 10^{-7}, \tag{39}$$

which is lower than the present experimental upper bound [21].

In the sequential fourth generation model there exist additional contributions to $b \to s\gamma$ induced by the fourth generation up-type quarks t'. The new Wilson coefficients can be obtained from those of their t counterparts by replacing the mass of the t quark by t' at the M_W scale, which is then evolved to the m_b scale using the renormalization group approach. As discussed in the previous

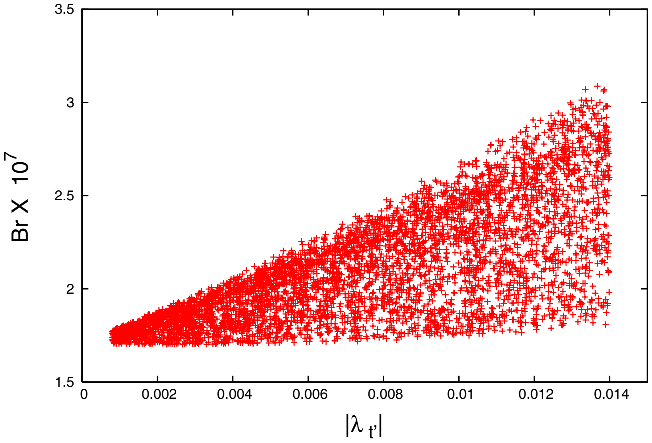

FIG. 3 (color online). Variation of the branching ratio for the $B_s \to \gamma\gamma$ process.

section the values of the new Wilson coefficients at the m_b scale for $m_{t'} = 400$ GeV are given by $C_7'(m_b) = -0.375$. At the scale m_b, the modified Wilson coefficient of the dipole operator becomes

$$C_7^{\rm tot}(m_b) = C_7(m_b) + \frac{V_{t'b}V_{t's}^*}{V_{tb}V_{ts}^*}C_7'(m_b). \tag{40}$$

Now varying $\lambda_{t'}$ in the range $(0.08\text{–}1.4) \times 10^{-2}$ and ϕ_s in the range (0°–80°), we show in Fig. 3 the branching ratio for the $B_s \to \gamma\gamma$ process. From the figure we see that the branching ratio can be enhanced from its SM value, but the enhancement is not so significant.

V. $B_s^0 \to \mu^+\mu^-\gamma$ PROCESS

Now let us consider the radiative dileptonic decay modes $B_s \to \mu^+\mu^-\gamma$, which are also very sensitive to the existence of new physics beyond the SM. Because of the presence of the photon in the final state, this decay mode is free from helicity suppression, but it is further suppressed by a factor of α with respect to the pure-leptonic $B_s \to \mu^+\mu^-$ process. However, in spite of this α suppression, the radiative leptonic decay $B_s \to \mu^+\mu^-\gamma$ has a comparable decay rate to that of purely leptonic ones.

The effective Hamiltonian describing this process $B_s \to \mu^+\mu^-$ is [15]

$$\mathcal{H}_{\rm eff} = \frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{ts}^*\bigg[C_9^{\rm eff}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l) + C_{10}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu\gamma_5 l) - \frac{2C_7 m_b}{q^2} \times (\bar{s}i\sigma_{\mu\nu}q^\nu P_R b)(\bar{l}\gamma^\mu l)\bigg], \tag{41}$$

where l is the shorthand notation for μ, $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, and q is the momentum transfer. C_i's are the Wilson coefficients evaluated at the b-quark mass scale in next-leading-logarithmic order with the values [32]

$$C_7^{\rm eff} = -0.31, \qquad C_9 = 4.154, \qquad C_{10} = -4.261. \tag{42}$$

The coefficient $C_9^{\rm eff}$ has a perturbative part and a resonance part which come from the long-distance effects due to the conversion of the real $c\bar{c}$ into the lepton pair l^+l^-. Hence, $C_9^{\rm eff}$ can be written as

$$C_9^{\rm eff} = C_9 + Y(s) + C_9^{\rm res}, \tag{43}$$

where the function $Y(s)$ denotes the perturbative part coming from one-loop matrix elements of the four quark operators and is given in Ref. [23]. The long-distance resonance effect is given as [33]

$$C_9^{\rm res} = \frac{3\pi}{\alpha^2}(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \times \sum_{J/\psi,\psi'}\kappa\frac{m_{V_i}\Gamma(V_i \to l^+l^-)}{m_{V_i}^2 - s - im_{V_i}\Gamma_{V_i}}, \tag{44}$$

where the phenomenological parameter κ is taken to be 2.3, so as to reproduce the correct branching ratio $\mathcal{B}(B \to J/\psi K^* \to K^* l^+l^-) = \mathcal{B}(B \to J/\psi K^*)\mathcal{B}(J/\psi \to l^+l^-)$. In this analysis, we will consider only the contributions arising from two dominant resonances, i.e., J/ψ and ψ'.

The matrix element for the decay $B_s \to \mu^+\mu^-\gamma$ can be obtained from that of the $B_s \to \mu^+\mu^-$ one by attaching the photon line to any of the charged external fermion lines. In order to calculate the amplitude, when the photon is radiated from the initial fermions [structure dependent part], we need to evaluate the matrix elements of the quark currents present in (41) between the emitted photon and the initial B_s meson. These matrix elements can be obtained by considering the transition of a B_s meson to a virtual photon with momentum k. In this case the form factors depend on two variables, i.e., k^2 (the photon virtuality) and the square of momentum transfer $q^2 = (p_B - k)^2$. By imposing gauge invariance, one can obtain several relations among the form factors at $k^2 = 0$. These relations can be used to reduce the number of independent form factors for the transition of the B_s meson to a real photon. Thus, the matrix elements for the $B_s \to \gamma$ transition, induced by vector, axial-vector, tensor, and pseudotensor currents, can be parametrized as [34]

$$\begin{aligned}
\langle\gamma(k,\varepsilon)|\bar{s}\gamma_\mu\gamma_5 b|B_s(p_B)\rangle &= ie[\varepsilon_\mu^*(p_B\cdot k) - (\varepsilon^*\cdot p_B)k_\mu] \times \frac{F_A}{m_{B_s}},\\
\langle\gamma(k,\varepsilon)|\bar{s}\gamma_\mu b|B_s(p_B)\rangle &= e\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha k^\beta\frac{F_V}{m_{B_s}},\\
\langle\gamma(k,\varepsilon)|\bar{s}\sigma_{\mu\nu}q^\nu\gamma_5 b|B_s(p_B)\rangle &= e[\varepsilon_\mu^*(p_B\cdot k) - (\varepsilon^*\cdot p_B)k_\mu]F_{TA},\\
\langle\gamma(k,\varepsilon)|\bar{s}\sigma_{\mu\nu}q^\nu b|B_s(p_B)\rangle &= e\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha k^\beta F_{TV},
\end{aligned} \tag{45}$$

TABLE II. The parameters for $B_s \to \gamma$ form factors.

Parameter	F_V	F_{TV}	F_A	F_{TA}
β (GeV^{-1})	0.28	0.30	0.26	0.33
Δ (GeV)	0.04	0.04	0.30	0.30

where ε and k are the polarization vector and the four-momentum of the photon, p_B is the momentum of the initial B_s meson, and F_i's are the various form factors.

Thus, the matrix element describing the structure dependent part takes the form

$$\mathcal{M}_{\rm SD} = \frac{\alpha^{3/2} G_F}{\sqrt{2}\pi} V_{tb} V_{ts}^* \{ \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p_B^\alpha k^\beta (A_1 \bar{l} \gamma^\mu l + A_2 \bar{l} \gamma^\mu \gamma_5 l) + i(\varepsilon_\mu^*(k\cdot p_B) - (\varepsilon^* \cdot p_B) k_\mu) \times (B_1 \bar{l}\gamma^\mu l + B_2 \bar{l} \gamma^\mu \gamma_5 l)\}, \tag{46}$$

where

$$A_1 = 2C_7 \frac{m_b}{q^2} F_{TV} + C_9 \frac{F_V}{m_{B_s}}, \qquad A_2 = C_{10} \frac{F_V}{m_{B_s}},$$
$$B_1 = -2C_7 \frac{m_b}{q^2} F_{TA} - C_9 \frac{F_A}{m_{B_s}}, \qquad B_2 = -C_{10} \frac{F_A}{m_{B_s}}. \tag{47}$$

The form factors F_V and F_A have been calculated within the dispersion approach [35]. The q^2 dependence of the form factors is given as [34]

$$F(E_\gamma) = \beta \frac{f_{B_s} m_{B_s}}{\Delta + E_\gamma}, \tag{48}$$

where E_γ is the photon energy, which is related to the momentum transfer q^2 as

$$E_\gamma = \frac{m_{B_s}}{2}\left(1 - \frac{q^2}{m_{B_s}^2}\right). \tag{49}$$

The values of the parameters β and Δ are given in Table II. The same ansatz (48) has also been assumed for the form factors F_{TA} and F_{TV}. We use the decay constant of the B_s meson, which is evaluated in lattice QCD calculation as $f_{B_s} = 232 \pm 10$ MeV [36].

When the photon is radiated from the outgoing lepton pairs, which is known as the internal bremsstrahlung (IB) part, the matrix element is given as

$$\mathcal{M}_{\rm IB} = \frac{\alpha^{3/2} G_F}{\sqrt{2}\pi} V_{tb} V_{ts}^* f_{B_s} m_\mu C_{10} \left[\bar{l}\left(\frac{\varepsilon^* \not{p}_B}{p_+ \cdot k} - \frac{\not{p}_B \varepsilon^*}{p_- \cdot k}\right)\gamma_5 l \right], \tag{50}$$

where p_+ and p_- are the momenta of emitted μ^+ and μ^-, respectively. Thus, the total matrix element for the $B_s \to l^+ l^- \gamma$ process is given as

$$\mathcal{M} = \mathcal{M}_{\rm SD} + \mathcal{M}_{\rm IB}. \tag{51}$$

The differential decay width of the $B_s \to \mu^+\mu^-\gamma$ process, in the rest frame of the B_s meson, is given as

$$\frac{d\Gamma}{ds} = \frac{G_F^2 \alpha^3}{2^{10}\pi^4} |V_{tb} V_{ts}^*|^2 m_{B_s}^3 \Delta_1, \tag{52}$$

where

$$\Delta_1 = \frac{4}{3} m_{B_s}^2 (1-\hat{s})^2 v_l ((\hat{s} + 2r_l)(|A_1|^2 + |B_1|^2) + (\hat{s} - 4r_l)(|A_2|^2 + |B_2|^2) - 64 \frac{f_{B_s}^2}{m_{B_s}^2} \times \frac{r_l}{1-\hat{s}} C_{10}^2 \left((4r_l - \hat{s}^2 - 1) \ln \frac{1+v_l}{1-v_l} + 2\hat{s} v_l \right) - 32 r_l (1-\hat{s})^2 f_{B_s} {\rm Re}(C_{10} A_1^*), \tag{53}$$

with $s = q^2$, $\hat{s} = s/m_{B_s}^2$, $r_l = m_\mu^2/m_{B_s}^2$, $v_l = \sqrt{1 - 4m_\mu^2/q^2}$. The physical region of s is $4m_\mu^2 \le s \le m_{B_s}^2$.

The forward-backward asymmetry is given as

$$A_{FB} = \frac{1}{\Delta_1}\left[2m_{B_s}^2 \hat{s}(1-\hat{s})^3 v_l^2 \,{\rm Re}(A_1^* B_2 + B_1^* A_2) + 32 f_{B_s} r_l (1-\hat{s})^2 \ln\left(\frac{4r_l}{\hat{s}}\right) {\rm Re}(C_{10} B_2^*) \right]. \tag{54}$$

We have shown the variation of the differential decay distribution (52) (in units of 10^{-7}), and the forward-backward asymmetry (54) for $B_s \to \mu^+\mu^-\gamma$ in Fig. 4.

As discussed earlier in the presence of the fourth generation, the Wilson coefficients $C_{7,9,10}$ will be modified due to the new contributions arising from the virtual t' quark in the loop. Thus, these coefficients will be modified as

$$C_7^{\rm tot}(\mu) = C_7(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_7'(\mu),$$
$$C_9^{\rm tot}(\mu) = C_9(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_9'(\mu), \tag{55}$$
$$C_{10}^{\rm tot}(\mu) = C_{10}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_{10}'(\mu).$$

The new coefficients $C_{7,9,10}'$ can be calculated at the M_W scale by replacing the t-quark mass by m_t' in the loop functions as discussed in [23]. These coefficients are then evolved to the b scale using the renormalization group equation. The values of the new Wilson coefficients at the m_b scale for $m_{t'} = 400$ GeV are given by $C_7'(m_b) = -0.375$, $C_9'(m_b) = 5.831$, and $C_{10}' = -17.358$.

Thus, one can obtain the differential branching ratio and the forward-backward asymmetry in SM4 by replacing $C_{7,9,10}$ in Eqs. (52) and (54) by $C_{7,9,10}^{\rm tot}$. Using the values of the $\lambda_{t'}$ and ϕ_s for $m_{t'} = 400$ GeV as discussed earlier, the differential branching ratio and the forward-backward asymmetry for $B_s \to \mu^+\mu^-\gamma$ are presented in Fig. 5, where we have not considered the contributions from

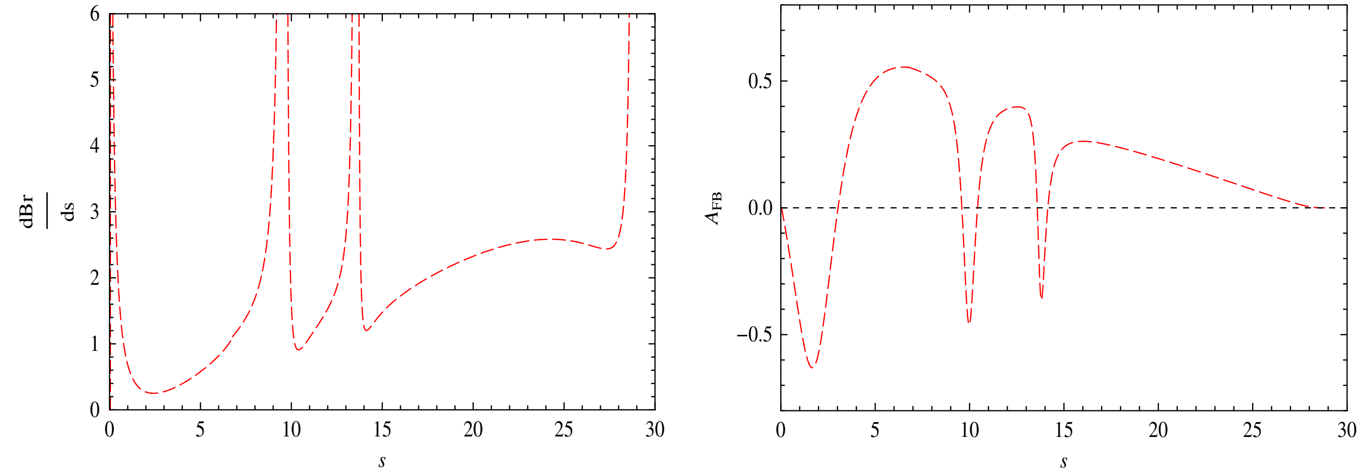

FIG. 4 (color online). Variation of the differential branching ratio (in units of 10^{-10}) (left panel) and the forward-backward asymmetry with respect to the momentum transfer s (right panel) for the $B_s \to \mu^+\mu^-\gamma$ process.

intermediate charmonium resonances. From the figure it can be seen that the differential branching ratio of this mode is significantly enhanced from its corresponding SM value, whereas the forward-backward asymmetry is slightly reduced with respect to its SM value. However, the zero position of the forward-backward asymmetry remains unchanged in the fourth quark generation model.

To obtain the branching ratios it is necessary to eliminate the background due to the resonances $J/\psi(\psi')$ with $J/\psi(\psi') \to \mu^+\mu^-$. We use the following veto windows to eliminate these backgrounds,

$$m_{J/\psi} - 0.02 < m_{\mu^+\mu^-} < m_{J/\psi} + 0.02;$$
$$m_{\psi'} - 0.02 < m_{\mu^+\mu^-} < m_{\psi'} + 0.02.$$

Furthermore, it should be noted that the $|\mathcal{M}_{\rm IB}|^2$ has infrared singularity due to the emission of a soft photon. Therefore, to obtain the branching ratio, we impose a cut on the photon energy, which will correspond to the experimental cut imposed on the minimum energy for the detectable photon. We require the photon energy to be larger than 25 MeV, i.e., $E_\gamma \geq \delta m_{B_s}/2$, which corresponds to $s \leq m_{B_s}^2(1-\delta)$, and therefore, we set the cut $\delta \geq 0.01$. Thus, with the above defined veto windows and the infrared cutoff parameter, the total branching ratio for the $B_s \to \mu^+\mu^-\gamma$ process is found to be

$$\begin{aligned} \text{Br}(B_s \to \mu^+\mu^-\gamma) &= 5.6 \times 10^{-9}. \quad \text{(SM)} \\ &< 2.2 \times 10^{-8}, \quad (\text{for } m_{t'} = 400\,\text{GeV}). \end{aligned} \tag{56}$$

The above branching ratio is comparable to that of the corresponding pure-leptonic process $B_s \to \mu^+\mu^-$ whose predicted branching ratio [5] for $m_{t'} = 400$ GeV is

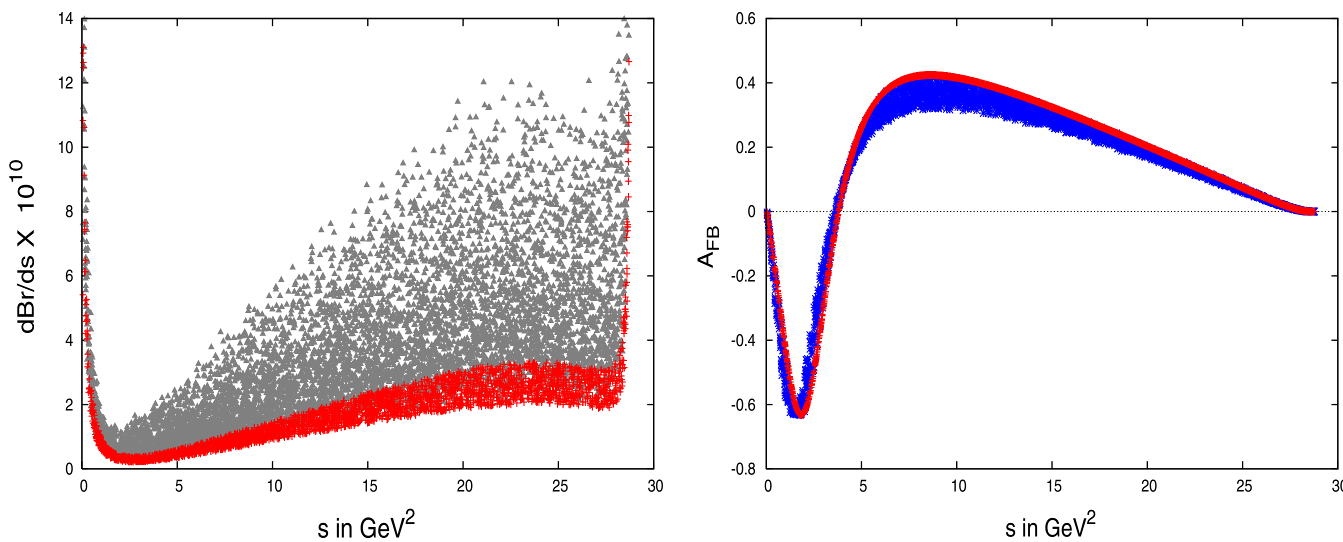

FIG. 5 (color online). Variation of the differential branching ratio (left panel) and the forward-backward asymmetry with respect to the momentum transfer s (right panel) for the $B_s \to \mu^+\mu^-\gamma$ process, in the fourth quark generation model (red regions); the corresponding SM values are shown in blue regions.

$$\mathrm{Br}\,(B_s \to \mu^+\mu^-) < 0.8 \times 10^{-8}. \qquad (57)$$

The LHCb [37] has searched for this process and set the upper limit as $\mathrm{Br}(B_s \to \mu^+\mu^-) < 1.2(1.5) \times 10^{-8}$ at 90% (95%) C.L. Therefore, the $B_s \to \mu^+\mu^-\gamma$ decay channel could also be accessible there, and hopefully it will be observed soon.

VI. CONCLUSION

In this paper we have studied some rare decays of the B_s meson in the fourth quark generation model. The large production of B_s mesons at the LHC opens up the possibility to study B_s mesons with high statistical precision. The decay modes considered here are $B_s \to \phi\pi$, $B_s \to \phi\gamma$, $B_s \to \gamma\gamma$, and $B_s \to \mu^+\mu^-\gamma$, which are highly suppressed in the SM as they occurred only through one-loop diagrams. Therefore, they provide an ideal testing ground to look for new physics. The fourth generation model is a very simple extension of the SM with three generations, and it can easily accommodate the observed anomalies in the B and B_s CP violation parameters for $m_{t'}$ in the range of (400–600) GeV. We found that in the fourth generation model the branching ratios for these processes were enhanced from their corresponding SM values. However, the mixing-induced CP asymmetry of the $B_s \to \phi\pi$ process was enhanced significantly from its SM value. The CP violating observables in $B_s \to \phi\gamma$ were found to be small but nonzero. Some of these branching ratios are within the reach of LHCb experiments; hence the observation of these modes will provide us with indirect evidence for the existence of the fourth quark generation.

ACKNOWLEDGMENTS

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