Concept Stability Based Isolated Maximal Cliques Detection in Dynamic Social Networks

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Abstract. As the network security gradually deviates from the virtual environment to the real environment, the security problems caused by abnormal users in social networks are becoming increasingly prominent. These abnormal users usually form a group which can be regarded as an isolated network. This paper aims to detect the isolated maximal cliques from a dynamic social network for identifying the abnormal users in order to cut off the source of fake information in time. By virtue of concept stability, an isolated maximal clique detection approach is proposed. Experimental results shown that the proposed algorithm has a high F-measure value for detecting the isolated maximal cliques in social network.

Keywords: Isolated Maximal Clique · Concept Stability · Formal Concept Analysis.

1 Introduction

5G technology not only will comprehensively promote the development of the Internet of Things, big data and artificial intelligence, but also will further enhance the degree of integration between the network and the entity. Thus, network security will gradually deviate from the virtual environment and move to the real environment. For instance, the security problems caused by abnormal users in social networks are becoming increasingly prominent. By establishing a large number of fake users and stealing normal users, criminals push fake advertisements or phishing websites on social networks, and organize "water army" to post malicious comments. Incidents of normal users being deceived in social networks often occur, causing that normal users have a crisis of trust in social networks. In fact, abnormal users usually have some hidden common characteristics. For example, the IP addresses of some abnormal users are the same. Therefore, identifying and detecting abnormal users to ensure user safety in social networks is a key issue by data mining and other techniques.

Actually, these abnormal users usually have the following characteristics: 1) abnormal users usually rarely establish contact with normal users; 2) there are usually some hidden connections among abnormal users. Abstractly, these abnormal users usually form a group which can be regarded as an isolated network.

It implies that the abnormal user data represents the outlier data that are inconsistent with other existing data models in social network analysis. Outlier mining is an important research field of data mining. Outliers contain some potentially valuable information: isolated clique helps both in getting faster algorithms than for the enumeration of maximal general cliques and in filtering out cliques with special semantics[1]. This paper aims to extract as much potential information as possible with focus on the dynamic isolated cliques detection.

Social networks usually consist of nodes, such as social members or organizations, and edges indicating the relationships between these social members. Generally, a social network is mathematically formalized as a graph $G = (V, E)$ where V denotes the set of objects and E denotes the set of relationships between objects. There are two types of social networks: static social network and dynamic social network. In the real world, most social networks are dynamically evolving, that is, the nodes in the network often change, and the relationships among nodes also often change. For instance, a newcomer joins an existing community, or someone in a community may be affected by a specific event and migrate to another community. As the ownership of the nodes changes constantly, so does the structure of the network.

Clique is a common existing topological structure in the network. It is composed of nodes and their common attributes and reveals the characteristics of social networks and the commonality of groups. Clique detection not only plays an important role in many applications, such as recommendation system [2] and public opinion monitoring, but also provides an effective coarse knowledge granularity for understanding social network structure [3]. However, finding a maximum clique is not only NP-hard but also hard to approximate within a factor of $n^{(1-\epsilon)}$ [4]. There are numerous computational approaches for maximal clique finding and enumeration. Ito, Iwama and Osumi [5] studyed the linear-time enumeration of isolated cliques. Roughly speaking, isolation means that the connection of the maximal clique to the rest of the graph is limited, that is, there are few edges with one endpoint in the clique and one endpoint outside the clique [6].

However, there is no previous work on isolated maximal clique detection using Formal Concept Analysis (FCA). With the help of FCA's powerful analysis ability, our recent research [7] has proved that a particular concept, called the equiconcept in a concept lattice represents the maximal cliques of social graph. This paper can be considered a continuation of previous work. Specifically, this paper takes into account the dynamic environment and focuses on the detection and changes of isolated maximal cliques under dynamic social networks by using concept stability. Concept stability is an effective measure of FCA for selecting interesting concepts and noise reduction [8]. And it reflects the dependency of the intent on the particular object of the extent [9].

Different from the existing algorithms on maximal clique detection, we adopt the concept stability measure for mining isolated maximal cliques from a dynamic social network. The major contributions of this paper are summarized as follows:

– Our research scenario is a dynamic social network, which is closer to the actual situation. Then, by using the concept stability, the bridge between FCA and network topology, isolated maximal cliques can be easily detected in dynamic situations.
– A concept stability based isolated maximal cliques detection approach is proposed. First, we propose and prove the concept stability based isolated maximal cliques detection theorem. Besides, some interesting properties of concept stability are presented as well. Then, based on the proposed theorem, an algorithm of detecting isolated maximal clique with concept stability is devised.
– We conduct the experiments on two classical networking datasets for validating the effectiveness of the proposed algorithm. Experimental results have shown the proposed algorithm has a high F-measure value for detecting the isolated maximal cliques in social network.

The remainder of this paper is structured as follow. Section 2 sketches the preliminaries about clique, FCA and concept stability. Then, the problem definition of isolated maximal clique detection is presented in Section 3. Section 4 describes concept stability based isolated maximal clique detection approach from a dynamic social network. Experimental results are reported in Section 5. Finally, Section 6 concludes this paper.

2 Preliminaries

In this section, we briefly outline the key notions used in the rest of this paper, including three aspects: clique, FCA and concept stability.

2.1 Clique

Definition 1. *[3] (Clique): Given an undirected graph $G = (V, E)$, a clique $Q \subseteq G$ is a subset of the vertices such that every two distinct vertices are adjacent.*

Definition 2. *[7] (Maximal Clique): Given an undirected graph $G = (V, E)$, a clique $Q \subseteq G$ is maximal if it cant be extended by including one more adjacent vertex.*

Definition 3. *[10] (Isolated Maximal Clique): Let $G = (V, E)$ be an undirected graph. A maximal clique $Q \subseteq G$ is isolated if for any two vertices $v_i \in V$,$v_j \notin V$, there doesn't exist an edge $(v_i, v_j) \in E$. That is, there is no edge that connects an object in the maximal clique to any object outside it.*

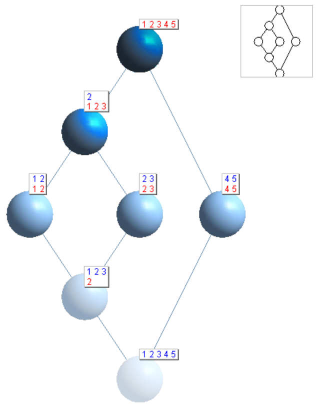
2.2 FCA

Formal Concept Analysis (FCA) is a mathematical theory oriented at data analysis and visualization. It provides tools for understanding the data by representing it as a hierarchy of concepts or, more exactly, a concept lattice.

Definition 4. *[3] (Formal Context): A formal context is formed as a triple $K = (U, A, I)$, where U is a set of objects, A represents a set of attributes, and I is the binary relation between U and A (i.e.., $I \subseteq O \otimes A$). Suppose $o \in U$ and a $\in A$, each (o, a)$\in$ I denotes that object o has the attribute a, otherwise (o, a) $\notin$ I.*

U\A	1	2	3	4	5
1	×	×			
2	×	×	×		
3		×	×		
4				×	×
5				×	×

(a) Formal Context.



(b) Concept Lattice.

Fig. 1. Example of formal context and its corresponding concept lattice.

Example 1. Fig.1(a) shows a formal context. The set of objects U and the set of attributes A are $\{1, 2, 3, 4, 5\}$, and in which " $\times$ " denotes that there exists the binary relation between U and A. For example, the object "1" has the attributes "1" and "2".

Definition 5. *[3] For a formal context $K = (U, A, I)$, the operators $\uparrow$ and $\downarrow$ on $X \subseteq U$ and $Y \subseteq A$ are respectively defined as*

$$X^{\uparrow} = \{a \in A | \forall x \in X, (x, a) \in I\} \tag{1}$$

$$Y^{\downarrow} = \{x \in U | \forall a \in Y, (x, a) \in I\} \tag{2}$$

$\forall x \in U$, let $\{x\}^{\uparrow}$=$x^{\uparrow}$, and $\forall a \in A$, let $\{a\}^{\downarrow} \in a^{\downarrow}$.

Definition 6. *[3] (Concept): For a formal context $K = (U, A, I)$,if (X, Y) satisfies $X^{\uparrow} = Y$ and $Y^{\downarrow} = X$, $(X, Y)$is called a concept, X is the extent of the concept, Y is the intent of the concept.*

Definition 7. *[3] (Equiconcept): For a formal context $K = (U, A, I)$,if (X, Y) satisfies $X^{\uparrow} = Y$, $Y^{\downarrow} = X$ and $X = Y$, $(X, Y)$is called an equiconcept, where X is called extent, and Y is called intent.*

Definition 8. *(Timed Equiconcept): A equiconcept $<(X,Y),t>$ is timed with t means that the concept (X,Y) is an equiconcept at time t. In the remainder, we use abbreviation form $<X,t>$ for timed equiconcept.*

Definition 9. *[3] Let $C(K)$ denote the set of all formal concepts of the formal context $K=(U,A,I)$. If $(X_1,B_1),(X_2,B_2)\in C(K)$, then let*

$$(X_1,B_1)\le(X_2,B_2)\Leftrightarrow X_1\subseteq X_2(\Leftrightarrow B_1\supseteq B_2) \tag{3}$$

then "$\le$" is a partial relation of $C(K)$.

Definition 10. *[3] (Concept Lattice): A concept lattice $L=(C(K),\le)$ can be obtained by all formal concepts $C(K)$ of a context K with the partial order $\le$. Its graphical representation is a Hasse diagram.*

Example 2. Fig.1(b) illustrates the concept lattice for the context of Fig.1(a). Each blue node represents a concept. The upper labels and lower labels of the nodes represent intents and extents of the concepts, respectively. Thus, we can obtain equiconcepts $(\{1,2\},\{1,2\})$, $(\{3,4\},\{3,4\})$ and $(\{4,5\},\{4,5\})$.

2.3 Concept Stability

Definition 11. *(Stability): Given a formal context $K=(U,A,I)$ and a concept $c=(X,Y)$ of K. The intensional stability essentially depicts a proportion of the subsets of X whose closure is equal to Y. It is defined as follows [11, 12]:*

$$\sigma(c)=\frac{\left|\left\{x\in\ \varphi(X)\middle|x^{\uparrow}=Y\right\}\right|}{2^{|X|}} \tag{4}$$

Actually, intensional stability measures the dependency between the intent Y and the object of the extend X. The stability index [13] can be computed by locating the closed set X's associated minimal generator. In this paper, we calculate concept stability by invoking the DFSP algorithm [13] that is the first algorithm handles efficiently and straightforwardly concept stability computation.

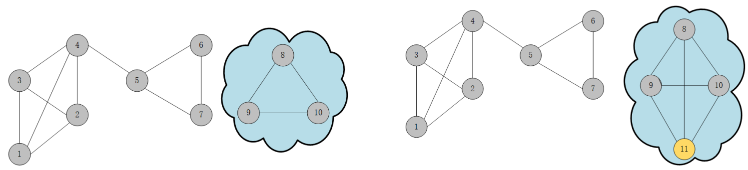
3 Problem Statement

In this paper, we will mainly investigate how to detect the isolated maximal clique in a dynamic social network by using the theory of FCA and concept stability. A formal description of this problem is given below.

Given a dynamic social network $G=\{G_1,G_2,G_3\ldots G_t\}$, where $G_t=(V_t,E_t)$ represents the network structure at time t. The G_t at each moment can be regarded as a static network in which the vertices set V_t includes the individuals in the social network, and the edge set $E_t=\{(v,w)|v,w\in V_t)\}$ represents the relationship between individuals. The isolated maximal clique detection problem is to detect all isolated maximal clique at every moment by using unique properties of concept stability.

In order to illustrate the problem addressed in this paper, a simple example of isolated maximal clique identification is given in Fig.2.

Example 3. Fig.2 is a topology graph of a dynamic network $G = \{G_a, G_b\}$ at different time slots. The shadow part ($\{8, 9, 10\}$) represents the isolated maximal clique at time a. When the time goes from a to b, the node 11 is added. Obviously, maximal isolated cliques ($\{8, 9, 10\}$) are changing to ($\{8, 9, 10, 11\}$) over time.



(a) t_a (b) t_b

Fig. 2. Simple example of isolated maximal clique detection.

4 Concept Stability Based Isolated Maximal Clique Detection

This section provides a new isolated maximal clique detection approach based on concept stability. To clarify our approach, we will detail our study through the following issues: 1) construct formal contexts for a dynamic social network; 2) explore the relation between concept stability and network topology; 3) propose an algorithm for detecting isolated maximal clique.

4.1 Formal Context Construction from Dynamic Social Network

We divide the dynamic social network $G = \{G_1, G_2, G_3 \ldots G_t\}$ into t (t is the number of time nodes) static network G_t according to the time nodes. A static social network G_t can be formalized as a classical mathematical relationship visualized as an undirected graph. We utilize the modified adjacency matrix as a formal context of G_t. Then, by invoking incremental concept lattice generation algorithm, we get all concepts and construct the concept lattice of static network G_t.

The modified adjacency matrix is defined as follows:

Definition 12. *[3] (Modified Adjacency Matrix): Let G be a graph with n vertices that are assumed to be ordered from v_1 to v_n. The $n \times n$ matrix A' is called a modified adjacency matrix, in which*

$$A' = \begin{cases} a_{ij} = 1 & \textit{if there exists an edge from } v_i \textit{ to } v_j \textit{ and } i \neq j \\ a_{ij} = 1 & \textit{if } i = j \\ a_{ij} = 0 & \textit{otherwise} \end{cases} \tag{5}$$

Example 4. Continue Example 3, the constructed formal context K_a is presented in Table 1. And then we build the corresponding concept lattices L_a as shown in Fig.3.

Table 1. Formal context K_a of G_a.

G_a	1	2	3	4	5	6	7	8	9	10
1	×	×	×	×						
2	×	×	×	×						
3	×	×	×	×						
4	×	×	×	×	×					
5				×	×	×	×			
6					×	×	×			
7					×	×	×			
8								×	×	×
9								×	×	×
10								×	×	×

4.2 Isolated Maximal Clique Detection

This section introduces several interesting properties about concept stability and provides the isolated maximal clique detection algorithm as shown in $Algorithm 1$. In addition, the complexity of algorithm is analyzed and an illustrative example is presented to show that how does the algorithm run.

Proposition 1. *[10] Given a social graph $G = (V, E)$ and its corresponding concept lattice $L(K)$, if a maximal clique $\mathcal{C}$ is isolated, there is an equiconcept (A, B) corresponding to $\mathcal{C}$ and its stability equals to $\frac{2^{|A|}-1}{2^{|A|}}$.*

Proposition 2. *Given a social graph $G = (V, E)$ and its corresponding concept lattice $L(K)$, the equiconcept (A, B) corresponding to an isolated maximal clique locates between the top and bottom of the concept lattice.*

Proof. Suppose a concept (X, Y) is a son concept of the concept (A, B), then by Definition 9 , $X \subseteq A$ and $B \subseteq Y$. Thus $\exists x \in X \Rightarrow x \in A$. Due to $|x^{\uparrow}| \geq |Y|$, and B is a subset of Y, thus $|x^{\uparrow}| > |B|$. That means x is associated with items not included in B. Moreover, corresponding node x is associated with nodes outside the isolated clique. This goes against Definition 3 of isolated maximal clique. Therefore, (A, B) cannot have son concept. Similarly, it can be proved that it is impossible (A, B) have father concept. Thus, the equiconcept (A, B) corresponding to an isolated maximal clique locates between the top and bottom of the concept lattice.

Proposition 3. *Given a concept (A, B), if the $|A| = 1$, then the stability value of the concept (A, B) is equal to 0.5.*

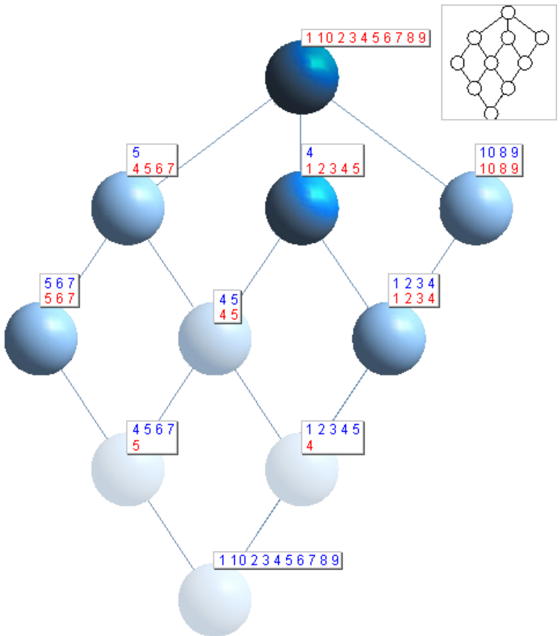


Fig. 3. Concept lattice L_a of K_a.

Proof. According to Definition 11, the intensional stability depicts a proportion of the subsets of A whose closure is equal to B. Since $|A| = 1$, thus $\varphi(A) = \{A, \emptyset\}$. That is, all subsets of A have only the empty set and itself. Due to $A^{\uparrow} = B$ and $\emptyset^{\uparrow} = \emptyset$, only one subset A satisfies the stability condition in Definition 11. So, the stability of (A, B) is equal to $1/2$.

Proposition 4. *Given a social graph $G = (V, E)$ and its corresponding concept lattice $L(K)$, the intensional stability of the son concept of the equiconcept is equal to the extensional stability of the grandfather concept.*

Proof. Suppose the concept (X, Y) is the son concept of a equiconcept, due to the symmetry of the concept lattice of social networks, the concept (Y, X) must be a grandfather concept of the equiconcept. That means, the extent of son concept is the intent of grandfather concept. Thus, the intensional stability of (X, Y) is equal to the extensional stability of (Y, X).

Based on the above property of concept stability, the pseudo code for isolated maximal cliques detection algorithm is given in Algorithm 1. The algorithm goes through two parts. The first part aims to obtain all timed equiconcepts as a basis for mining interesting clique. At the second part, the main purpose of the algorithm is to select interesting concepts that captures isolated cliques using concept stability theory.

Algorithm 1 Concept Stability based Isolated Maximal Cliques Detection Algorithm

Require:
$G = \{G_1, G_2, G_3 \ldots G_t\}$ //t represents *time*
Ensure:
Set of isolated maximal clique Î of G
1: Initialize Î= ∅, Ê= ∅
2: **begin**
3: Construct formal contexts for G_t by Definition 3
4: $L_t \leftarrow$ Build concept lattices of G_t
5: **for** each concept $(A, B) \in L_t$ **do**
6: **if** (A, B) is an equiconcept **then**
7: $\hat{E} \leftarrow \hat{E} \cup < (A, B), t >$
8: $\sigma(A, B) \leftarrow$ calculate the stability index σ of (A,B)
9: **If** $\sigma(A, B) == \frac{2^{|A|}-1}{2^{|A|}}$ **then**
10: $\hat{I} \leftarrow \hat{I} \cup < (A, B), t >$
11: **end if**
12: **end if**
13: **end for**
14: **return** Î
15: **end**

A dynamic network G is the inputs of the algorithm. Our algorithm starts by initializing Î (set of isolated maximal cliques and time nodes) and Ê (set of timed equiconcepts) to ∅(Line 1). Then, we build formal context for G_t. Next, the concept lattice is constructed by our ongoing research which is an algorithm for quickly generating concept lattice based on dynamic network (Line 4). After that, we extract the equiconcepts that captures all maximal cliques from lattices L_t and time dimension into Ê (Lines 5-7). By invoking DFSP to calculate the stability value σ of timed equiconcept (Line 8), it is determined that if its stable value satisfies the Proposition 1(Line 9), its corresponding clique is isolated maximal clique at that moment. DFSP [13] is the first algorithm to deal with the concepts stability calculation efficiently and directly. Lines 10-14 insert the detected isolated maximal clique into Î and return Î.

Complexity Analysis: The first part of algorithm has $O(t_1)$ time complexity which is the time needed to generate concept lattice. Let |Ê| denote the number of timed equiconcepts. Thus. the second part of algorithm has $O(\ |\hat{E}|\ *t_2)$ time complexity, where t_2 is the time needed to calculate stability index by using DFSP.

Illustrative Example: Let us consider the dynamic network $G = \{G_a, G_b\}$ given by Fig.2. First, we build formal context for G_a (Table.1) and G_b. Next, the concept lattices L_a (Fig.3) and L_b are constructed. After that, we extract timed equiconcepts which represent maximal clique in social graph from concept lattice.

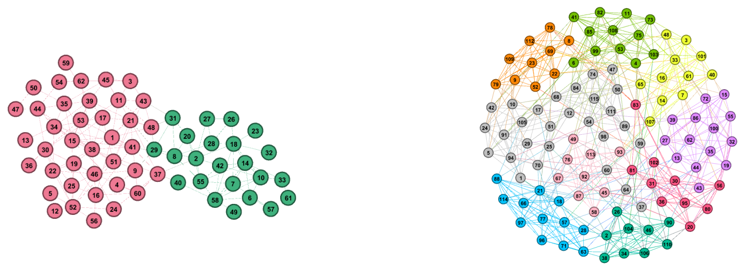
So, we get $<\{1,2,3,4\},ab>, <\{4,5\},ab>, <\{5,6,7\},ab>, <\{8,9,10\},a>$ $, <\{8,9,10,11\},b>$ and then store into $|\hat{E}|$. Finally, by calling the DFSP to calculate the stability value, there are two equiconcepts conform to the Eq.(4). The stability index of $<\{8,9,10\},a>, <\{8,9,10,11\},b>$ respectively equal to $7/8, 15/16$. Hence, clique $\{8,9,10\}$ is an isolated maximal clique at time a; clique $\{8,9,10,11\}$ is an isolated maximal clique at time b.

5 Experiments

In this section, we conduct the experiments on two networking datasets to evaluate the proposed approach. The goal of the experiments is to examine whether using concept stability is feasible and efficient for detecting isolated maximal clique. All algorithms are implemented in JAVA language and are run on an Inter(R) Core (TM) i7-8565U @ 1.80GHz 1.99GHz, 20GB RAM computer.

5.1 Data Set and Configurations

In this paper, two datasets of social network are adopted. Data I [14] is a classical dataset on the social network of frequent associations between 62 dolphins in a community living off Doubtful sound, New Zealand. Data II [15] is a network that represents the schedule of games between college football teams in a single season. Fig.4 shows the visualization of both dolphin living network and football network.



(a) Dolphin living network (b) Football team network

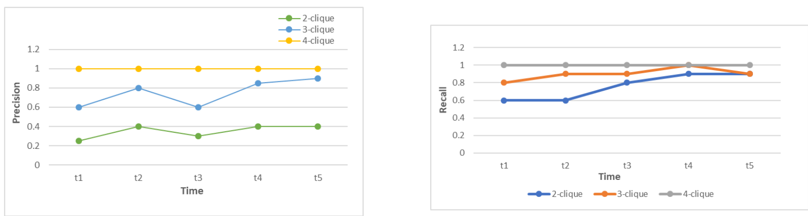
Fig. 4. The visualization of Dolphin living network and Football team network.

Due to the structural specificity of isolated maximal clique, this structure does not exist in most real social networks, but this structure is also the basis for mining isolated communities, so we randomly add data to the existing dataset for synthetic dynamic networks.

5.2 Experimental Results

This section mainly evaluates the proposed approach with two important metrics: *precision/recall* ratio of isolated maximal clique detection. For a given parameter k and time t, a set of isolated maximal cliques are obtained, the corresponding *precision* and *recall* ratio are defined as follows:

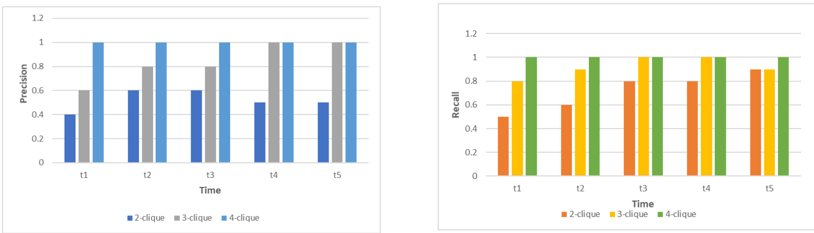
- *Precision* is the ratio of the number of isolated maximal k-cliques in the detection results to the total number of isolated maximal k-cliques obtained by that detection.
- *Recall* is the ratio of the number of isolated maximal k-cliques in the detection results to the total number of existing isolated maximal k-cliques.
- $F1$-*score* is used to evaluate how well an algorithm can find the isolated maximal k-cliques from a social graph by fitting the Precision and Recall, denoted as $F1 = \frac{2*Precision*Recall}{Precision+Recall}$.



(a) (b)

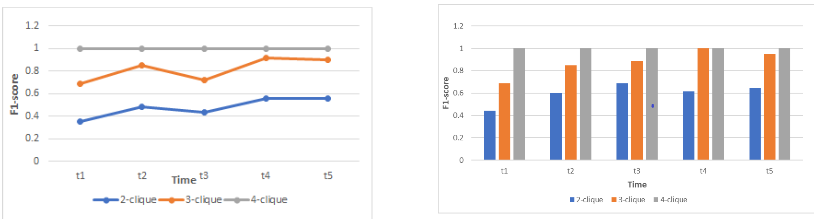
Fig. 5. Isolated maximal cliques detection precision and recall of the proposed algorithm in Dolphin living network. (a)The precision of the proposed algorithm with various k at different time. (b) The recall of the proposed algorithm with various k at different time.

As shown in Fig.5 and Fig.6, the precision and recall of the proposed detection algorithm are measured at different times with different k. Obviously, as the k increases, the detection precision and recall ratio of the isolated maximum k-cliques also increase. In particular, when we try to find the isolated maximum 4-cliques, the precision and recall are reach 100%. In addition, we also evaluated the proposed algorithm with various k values based on the $F1$ score, which is used to evaluate the efficient of each algorithm to find the isolated maximal clique from the social network. Fig.7 reports that the proposed algorithm can effectively find the largest isolated group, especially the group with larger k.



(a) (b)

Fig. 6. Isolated maximal cliques detection precision and recall of the proposed algorithm in Football team network. (a)The precision of the proposed algorithm with various k at different time. (b) The recall of the proposed algorithm with various k at different time.



(a) (b)

Fig. 7. Isolated maximal cliques detection $F1$ score of the proposed algorithm. (a)The $F1$ score of the proposed algorithm with Dolphin living network. (b) The $F1$ score of the proposed algorithm with Football team network.

6 Conclusions

This paper aims to detect the isolated maximal cliques from dynamic social network for identifying the abnormal users in social network to cut off the source of fake information in time, and reduce the occurrence of security incidents. We have proposed the concept stability based isolated maximal clique detection algorithm. Firstly, we used a modified adjacency matrix to construct the formal context and generated the concept lattice of a dynamic social network. Then, some unique properties of concept stability have been presented and proved. Based on the properties, a detection algorithm of isolated maximal clique has been further proposed. The proposed algorithm has been evaluated by using two classical datasets. Experimental results have shown the proposed algorithm has a high $F1$ score for detecting the isolated maximal cliques from dynamic social networks.

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