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BILATERALIST DETOURS: FROM INTUITIONIST TO CLASSICAL LOGIC AND BACK

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Abstract

There is widespread agreement that while on a Dummettian theory of meaning the justified logic is intuitionist, as its constants are governed by harmonious rules of inference, the situation is reversed on Huw Price's bilateralist account, where meanings are specified in terms of primitive speech acts assertion and denial. In bilateral logics, the rules for classical negation are in harmony. However, as it is possible to construct an intuitionist bilateral logic with harmonious rules, there is no formal argument against intuitionism from the bilateralist perspective. Price gives an informal argument for classical negation based on a pragmatic notion of belief, characterised in terms of the differences they make to speakers' actions. The main part of this paper puts Price's argument under close scrutiny by regimenting it and isolating principles Price is committed to. It is shown that Price should draw a distinction between A or $\neg A$ making a difference. According to Price, if A makes a difference to us, we treat it as decidable. This material allows the intuitionist to block Price's argument. Abandoning classical logic also brings advantages, as within intuitionist logic there is a precise meaning to what it might mean to treat A as decidable: it is to assume $A \vee \neg A$.

1. Harmony, Negation and Denial

It is generally agreed that the rules for intuitionist negation in standard systems of natural deduction are *in harmony*, whereas those for classical negation are not. Grounds for and consequences of asserting $\neg A$ in intuitionist logic are in balance:

$$\neg I\text{:}\ \begin{array}{c}\overline{A}^{\,i}\\ \Pi\\ \dfrac{\bot}{\neg A}\,i\end{array} \qquad \neg E\text{:}\ \frac{\neg A \quad A}{\bot} \qquad \bot E\text{:}\ \frac{\bot}{B}$$

The elimination rule for $\bot$ is in harmony with the empty introduction rule: $\bot$ has no grounds for its assertion.

To formalise classical negation, we need to add, e.g., one of the following:

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$$
\begin{array}{c}
\overline{\neg A}^{\,i} \\
\Pi \\
\underline{\quad\bot\quad}_{\,i} \\
A
\end{array}
\qquad
\dfrac{\neg\neg A}{A}
\qquad
\dfrac{\begin{array}{cc} \overline{A}^{\,i} & \overline{\neg A}^{\,i} \\ \Pi & \Xi \\ C & C \end{array}}{C}\,i
\qquad
\dfrac{}{A \vee \neg A}
$$

This creates a misbalance between the consequences of asserting $\neg\neg A$ and the grounds for asserting it: we get more out of $\neg\neg A$ than we put in, so to speak.

There is also widespread agreement that the situation is reversed if we follow Huw Price and change an aspect of the Dummettian account of meaning, which provides the philosophical backbone of the notion of harmony and the context in which it was developed.[1] Dummett argues that the meanings of sentences are determined by the conditions under which they are correctly assertible. Price proposes an alternative which 'takes the fundamental notion for a recursive theory of sense to be not assertion conditions alone, but these in conjunction with rejection, or *denial* conditions.' (Price (1983): 162) Price's account is intended to remain within the Dummettian paradigm and to respect Dummettian constraints on the form of a theory of meaning. Meaning is characterised in terms of speakers' behaviour and use of language. Price argues that on his alternative, negation must be classical, so that he can answer a well-known Dummettian challenge: to provide a semantic theory satisfying Dummettian criteria relative to which classical logic is justified. Price's arguments for classical logic emulate Dummett's own arguments for intuitionist logic, and arguably he meets Dummett on largely shared grounds.

Ian Rumfitt proposes a system of natural deduction that captures Price's idea with regard to the logical constants: they are governed not only by rules specifying the grounds and consequences of the assertions of sentences with them as main operator, but also by rules specifying the grounds and consequences of the denials of such sentences (Rumfitt (2000)). Rumfitt's *bilateral* logic builds on work by Timothy Smiley, who agrees that Price's account has advantages over the usual 'unilateral' one and formalises a classical logic for assertions and denials in a system of natural deduction in sequent calculus style (Smiley (1996)). Rumfitt's rules for classical negation are in harmony:

$$
+\neg I\text{:}\ \frac{-\,A}{+\,\neg A}
\qquad
+\neg E\text{:}\ \frac{+\,\neg A}{-\,A}
\qquad
-\neg I\text{:}\ \frac{+\,A}{-\,\neg A}
\qquad
-\neg E\text{:}\ \frac{-\,\neg A}{+\,A}
$$

[1] For details, see (Dummett (1978): 220ff) and (Dummett (1993): 245ff). For an overview, see (Kürbis (2015a)). For a discussion of negation within that framework, see (Kürbis (2015b)).

An intuitionist must reject $-\neg E$. According to Rumfitt, this creates a misbalance between the grounds for a denial of $\neg A$ and the consequences of denying it: we get less out of denying $\neg A$ than we put in, so to speak. Thus, he concludes, the bilateral framework justifies classical and rules out intuitionist logic.

Rumfitt's system requires rules coordinating assertions and denials. He thinks of these as *structural rules* that are part of the formal framework of the logic as opposed to the operational rules, which determine the meanings of the logical constants within that framework. Rumfitt has two options. One is to add a rule of *Reductio* that specifies directly how to proceed if the same formula has been derived as asserted and denied: if Γ, $\alpha \vdash \beta$ and Γ, $\alpha \vdash \beta^*$, then $\Gamma \vdash \alpha^*$, where α and β are assertions or denials and $*$ exchanges + and −. The other option is to add rules of *Reductio* and Non-Contradiction for the symbol $\perp$, which registers that an impasse has been reached in a deduction: if Γ, $\alpha \vdash \perp$, then $\Gamma \vdash \alpha^*$, and from α, α^* infer $\perp$.

The foundations of Price's approach have been challenged, most notably by Dummett himself in response to Rumfitt (Dummett (2002)). Dummett is sceptical about dual component theories of meaning, such as bilateralism, as the two components in terms of which meanings are specified need to be in harmony, so one of them should be superfluous.[2] Price's account goes against a widely accepted view, stemming from Frege and popularised by Geach, that a primitive notion of denial is unnecessary. To explain certain inferences, we need to appeal to negation and assertion and then we can define denial in terms of them (see (Frege (1918): 149ff) and (Geach (1972): 260)). Imogen Dickie questions whether the approach is of much use in the realism/anti-realism debate, as an unbiased look at all that may count as denials reveals them to be too untidy an assortment of linguistic acts and too unspecific to be of any use in codifying inferences, whereas assuming them to be sufficiently specific amounts to assuming bivalence (Dickie (2010): 175 & 180). Mark Textor questions whether there is any evidence in linguistic practice that there is a primitive speech act of denial prior to the assertion of negated sentences (Textor (2011)).

Once the foundations are accepted, it looks as if Price's account lends itself better to classical than to intuitionist logic. Humberstone shows how to translate his version of classical bilateral logic into sequent calculi with multiple conclusions (Humberstone (2000): 346 & 352ff). These calculi formalise classical logic so naturally that it has been quipped that like Monsieur Jourdain, classical logicians have been speaking multiple conclusions

[2] Rumfitt evades the issue of how to formulate a suitable notion of harmony applying to the speech acts assertion and denial. Bernhard Weiss takes on Dummett's challenge and argues for a dual component theory of meaning from a thoroughly Dummettian perspective (Weiss (2007)).

all along.[3] Humberstone expresses doubt whether the intuitionist version of his bilateral logic can properly be said to incorporate the fundamental bilateralist idea that rules for assertion and denial count as structural rules: the intuitionist versions of the rules ‘come to look simply like the result of re-notating familiar rules governing intuitionistic negation’ (Humberstone (2000): 366). Gibbard proposes that bilateral logic could also be a constructive logic with strong negation (Gibbard (2002): 302).[4] Rumfitt responds and argues once more why he thinks that only a classical version of bilateral logic satisfies his criteria (Rumfitt (2002): 309ff). Such consensus asks to be challenged.

2. Intuitionist Bilateralism

It is undeniable that dropping $-\neg E$ or weakening it while keeping $-\neg I$ creates a misbalance in the rejective negation rules of bilateral logic. But this does not show that it is impossible to formalise an intuitionist bilateral logic with harmonious rules. It is open to the intuitionist to change *both* rejective negation rules and adopt the following pair:

$$-\neg I_{Int}: \quad \dfrac{\begin{array}{c}\overline{-A}^{\,i}\\ \Pi \\ \alpha\end{array} \qquad \begin{array}{c}\overline{-A}^{\,i}\\ \Xi \\ \alpha^*\end{array}}{-\neg A}\,i \qquad\qquad -\neg E_{Int}: \quad \dfrac{-\neg A \qquad -A}{\beta}$$

These rules are in harmony. The intuitionist could adopt alternative mirroring assertive negation rules:

$$-\neg I': \quad \dfrac{\begin{array}{c}\overline{+A}^{\,i}\\ \Pi \\ \alpha\end{array} \qquad \begin{array}{c}\overline{+A}^{\,i}\\ \Xi \\ \alpha^*\end{array}}{+\neg A}\,i \qquad\qquad +\neg E': \quad \dfrac{+\neg A \qquad +A}{\beta}$$

[3] We could even define the validity of multiple conclusion sequents in terms of assertion and denial: $\Gamma \vdash \Delta$ is valid if and only if it is impossible that all formulas Γ are assertible while all formulas Δ are deniable (see Restall (2005)). This answers Dummett’s worry that we cannot understand multiple conclusions independently of a classical concept of disjunction (Dummett (1993): 187f). Notice that if we accept that if ‘It is assertible that *A*’ is assertible, then ‘*A*’ is not deniable, then it follows from this definition of validity that ‘It is assertible that *A*’ entails ‘*A*’, and, more controversially, if we if accept that if ‘*A*’ is assertible, then ‘It is assertible that *A*’ is not deniable, then ‘*A*’ entails ‘It is assertible that *A*’. This is congenial to Dummett, but may not be so to everyone who favours multiple conclusions.

[4] Heinrich Wansing championed and substantiated this view at ‘How to Say “Yes” or “No”’ in Lecce. See also (Wansing (2016)).

A full system of intuitionist bilateral logic requires changing Rumfitt's rejective rules for conjunction and implication and the structural rules for the coordination of assertions and denials. We need *Reductio*$_{Int}$: if $\Gamma, +A \vdash \beta$ and $\Gamma, +A \vdash \beta^*$, then $\Gamma \vdash -A$, and *ex contradictione quodlibet*: $\alpha, \alpha^* \vdash \beta$ (or more economically $+A, -A \vdash +B$). The resulting intuitionist system meets all the requirements Rumfitt imposes for a bilateral logic to be satisfactory.[5]

Reductio may display a symmetry that the combination of *Reductio*$_{Int}$ and *ex contradictione quodlibet* does not display. This is no objection. In the present discussion, the only relevant notion of symmetry applying to logics is the notion of harmony, but it does not apply to structural rules. Rumfitt writes that *Reductio* holds by stipulation: 'as a matter of simple definition, then, quite independently of the soundness of double negation elimination, the double conjugate α^{**} is strictly identical with α itself.' (Rumfitt (2000): 804) The intuitionist is at liberty to adopt an analogous attitude.[6]

Bilateralism fails where unilateralism succeeds. On Dummett's account, intuitionist logic is justified, and classical logic ruled out, by proof-theoretic considerations. On Rumfitt's account, classical logic is justified, but, as a closer look reveals, so is intuitionist logic. What is worse, Rumfitt argues that the cost of the complications of bilateral logic is outweighed by the benefit of justifying classical while ruling out intuitionist logic. As there are no such benefits, we should give up on the bilateral framework and opt for the simpler unilateral one for methodological reasons.

From the formal perspective, bilateralists have no argument against intuitionist logic. Price builds his case for classical logic on an informal argument in which his semantic theory is underpinned by a pragmatic notion of belief. This is vital to his view, as 'there is a fundamental tension between a plausible pragmatic account of belief and the rejection of [double negation elimination]' (Price (2016): 7). This argument, despite the interest Price's account has attracted, is virtually ignored in the literature. So much so that the objective of the paper just quoted is 'to call attention to [this] pragmatist element', as he considers it 'to be significant, and to deserve more attention than it has received in the recent discussions' (Price (2016): 3). If Price's informal argument is mentioned at all, it is quickly dismissed as begging the question against the intuitionist (see, e.g., (Rumfitt (2000): 814) and (Gibbard (2002): 300ff)). In the rest of this paper, I will put this argument under close scrutiny. The conclusion will be that, far from begging any questions against the intuitionist, Price's account of belief may well go better with intuitionist than with classical logic.

[5] For details, see (Kürbis (2016)).

[6] Adding the other half of *Reductio* to intuitionist bilateral logic gives classical bilateral logic. Whether a logic is classical or intuitionist depends on which rule of *Reductio* is adopted, but we have not been given a means of deciding between them.

3. Price's Argument for Classical Negation

Price in fact gives two arguments that a semantic theory incorporating a pragmatic account of belief allows the derivation of the properties of classical negation. In an earlier argument, beliefs are characterised in terms of the differences they make to an agent's dispositions to act (Price (1983)). In a later argument, beliefs are characterised in terms of the possibilities discounted by an agent: 'to judge that *P* is to turn one's back to [...] all the ways in which it would be the case that ~*P*. [...] Therefore, believing *P* and believing ~~*P* come to the same thing: both amount to a disposition to discount the possibility that ~*P*.' (Price (1990): 232) This begs the question against the intuitionist, so I will focus on the earlier argument, which constitutes an attempt to meet the intuitionist on neutral grounds.[7]

For ease of exposition, I'll quote passages containing elements of Price's account of belief and his argument for double negation elimination in full:

> This association of denial with disagreement offers an account of the nature of the inconsistency to which we are led if we find that, on certain hypotheses, we are led to assert both that T and that not-T. For to assert (or agree) that not-T is to deny (or disagree) that T; and we are thus led both to agree and to disagree that T. And the conventions governing these activities rule this out.
>
> […] We take 'Not-S' to be deniable when some consequence of 'Not-S' is deniable. How can we be sure this is precisely when 'S' is assertible?
>
> Suppose 'T' is taken to be a consequence of 'Not-S'. Then we are disposed to agree that T when we agree that not-S; that is, when we disagree that S. So if we disagree that T — that is, agree that not-T — then we cannot hold open the possibility that we might disagree that S. We want to show that this means that we must agree that S.
>
> It seems to me that the demonstration must depend on the familiar claim that the state of agreeing, or believing, that S is to be understood — *inter alia*, but centrally — as a disposition to behave as if S, whenever, in the light of one's desires, the expected utility of one's various possible future actions depends on whether S. Roughly, to believe that S is to be disposed to act as if S, whenever one thinks that it makes a difference whether S. Similarly, to disbelieve that S is roughly to be disposed to act on the assumption that not-S — that is to ignore the possibility that S — whenever one thinks it makes a difference to the outcome of one's action whether S.
>
> Let us therefore suppose that we cannot hold open the possibility of disagreeing, or coming to disbelieve, that S (because if we were to do so we would

[7] Characterising the content of propositions in terms of what they exclude is relevant to a falsificationist account of meaning. Interestingly, although the two approaches look orthogonal, Rumfitt hints at a connection between his falsificationist semantics for classical logic and bilateralism (Rumfitt (2007): footnote 5).

> agree that T, and we disagree that T). Then in assessing the expected utilities of possible future actions, we can discount those outcomes which depend on it not being the case that S (for such an outcome would give us grounds for disagreeing that S, and this is just the possibility we rule out). In other words, we can proceed, whenever it makes a difference whether S, on the assumption that S. This is precisely what agreeing (or believing) that S amounts to.
>
> We have thus shown that a denial condition for 'Not-S' is an assertion condition for 'S'. Conversely, if we agree that S, then we cannot hold open the possibility of disagreeing that S — that is, of agreeing that not-S; so we are bound to disagree that not-S. Thus an assertion condition of 'S' is a denial condition of 'Not-S'. (Price (1983): 169ff)

No doubt this is all very sketchy. My aim, however, is not to criticise Price's account of belief. I see no *prima facie* reason why Price shouldn't be able to spell out the details so as to answer some obvious objections that even the most benign reader will have. For my purpose, it suffices that the essential strands of an argument for double negation elimination are reasonably clear.

The text suggests a number of economies. The argument is intended to establish something about assertion and denial, but much of it is expressed in terms of agreement and disagreement. There is also a shift between these notions and belief and disbelief. I will treat them all as stylistic variants and paraphrase the argument in terms of agreement and disagreement. Price also draws no distinction between 'to have a disposition to behave' and 'to have a disposition to act', nor between 'we cannot hold open the possibility of φ-ing' and 'we cannot hold open the possibility that we might φ'. I also treat 'to ignore the possibility that' and 'to discount' as variants of the latter two, so that they all mean the same thing.

Price's account of belief is non-standard as, contrary to received wisdom, he stipulates that our beliefs are consistent. When put into context, this is not outrageous: for Price, agents' beliefs are inextricably tied to the actions they can perform, and inconsistent actions cannot be performed together. 'Beliefs ain't just in the head', as he might put it. Price's account is not as non-standard as to entail that beliefs are *complete* in the sense that for any A, an agent either believes that A or believes that $\neg A$. Price bases his distinction between truth and assertibility on a distinction between *disagreeing* that A, i.e. agreeing that $\neg A$, and *declining to agree* that $\neg A$, i.e. not agreeing that A (see Price (1983): 164, 169).

Price shifts between what is a consequence and what is taken to be a consequence. I presume there is no difference between the two. 'Consequence' should mean 'logical consequence', as Price aims to establish that A is a logical consequence of $\neg\neg A$. All our actions are in accordance with the laws of logic. We cannot do anything that goes against them. Thus we always agree with them, as agreement is characterised in terms of actions. Hence, if B is a consequence of A, we agree that this is so. Conversely, as our agreements are consistent, if we agree that B is a consequence of A, then this must be the case.[8]

Finally, although Price draws a distinction between 'being disposed to agree (disagree)' and 'to agree (disagree)', we can ignore it. The third paragraph of the quoted passage states that (i) if T is a consequence of $\neg S$, then, if we agree that $\neg S$, we are disposed to agree that T. From this Price derives that (ii) if we disagree that T (agree that $\neg T$), then we cannot hold open the possibility that we disagree that S (agree that $\neg S$). (i) suggests the general claim that (iii) if B is a consequence of A, then, if we agree that A, we are disposed to agree that B. A complementary claim which is equally plausible is that (iv) if B is a consequence of A, then, if we disagree that B, we are disposed to disagree that A. So, by (iii), as A is a consequence of A, (v.a) if we agree that A, then we are disposed to agree that A, and either by replacing A by $\neg A$ or by (iv), (v.b) if we disagree that A, then we are disposed to disagree that A. It follows that, if T is a consequence of $\neg S$, then, by (iv), if we disagree that T, then we are disposed to disagree that $\neg S$. Price needs a claim connecting dispositions to agree with not holding possibilities open to be able to derive from this the conclusion of (ii) 'we cannot hold open the possibility that we disagree that S (agree that $\neg S$)'. The following suggests itself: if we are disposed to disagree that $\neg S$, we cannot hold open the possibility that we agree that $\neg S$. Generalising, (vi) if B is a consequence of A, then if we are disposed to disagree that B, then we cannot hold open the possibility that we agree that A, and, complementarily, (vii) if B is a consequence of A, if we are disposed to agree that A, then we cannot hold open the possibility that we disagree that B. Price's argument is supposed to establish that (viii) if we cannot hold open the possibility that we disagree that S, then we agree that S. As A is a consequence of A, by (vii), if we are disposed to agree that A, then we cannot hold open the possibility that we disagree that A, hence, by (viii), we agree that A. So, by (v.a), agreeing that A is equivalent to being disposed to agree that A. Similarly, as A is a consequence of A, by (vi), if we are disposed to disagree that A, then we cannot hold open the possibility that we agree that A. The complement to (viii), if we cannot hold open the possibility that we agree that A, then we disagree that A, should also hold, so that, by (v.b), disagreeing that A and having a disposition to disagree that A are also equivalent.

[8] Price's account of belief has another non-standard feature: our beliefs are closed under entailment. This is not outrageous either, if we keep in mind the robust pragmatic nature of the account. It is worth comparing Price's views with Davidson's on meaning and interpretation.

4. Price's Principles

With the stratifications of the previous section in place, we can formulate principles Price is committed to. Most philosophers follow Frege and Geach and agree that speech acts cannot be embedded, so that an expression such as $+A\& -A \supset \perp$ is illformed.[9] What can certainly be embedded, however, are ascriptions of beliefs. We can extract the following four symmetric claims from Price's argument:

(1) If we disagree that A, then we cannot hold open the possibility that we agree that A.

(2) If we cannot hold open the possibility that we agree that A, then we disagree that A.

(3) If we agree that A, then we cannot hold open the possibility that we disagree that A.

(4) If we cannot hold open the possibility that we disagree that A, then we agree that A.

(1)-(3) are unproblematic, (4) is supposed to be established by Price's argument.

The principles of Price's account match rules of bilateral logic. Prices uses 'to agree that $\neg A$' and 'to disagree that A' interchangebly, which corresponds to $+\neg I$ and $+\neg E$. The consistency of belief means that if we agree that A, we do not disagree that A (agree that $\neg A$), which corresponds to Non-Contradiction. We can read 'We cannot hold open the possibility that we agree that A' as 'There is a deduction of α and $\alpha*$ from $+A$' or alternatively 'There is a deduction of $\perp$ from $+A$', and analogously for 'agree' replaced by 'disagree'. Then (1) and (3) correspond Non-Contradiction, so one of them is redundant. Replacing 'we disagree that A' with 'we agree that $\neg A$' in (1) and (2) gives principles corresponding to the assertive negation rules $+\neg I'$ and $+\neg E'$. Replacing A for $\neg A$ in (1) and (2) and replacing the resulting 'we agree that $\neg A$' with the equivalent 'we disagree that A' gives $-\neg I_{Int}$ and $-\neg E_{Int}$.

We can replace (3), which is redundant, and (4), which is to be established by Price's argument, by

(5) If we disagree that $\neg A$, then we cannot hold open the possibility that we disagree that A.

(6) If we cannot hold open the possibility that we disagree that A, then we disagree that $\neg A$.

These correspond directly to $-\neg I_{Int}$ and $-\neg E_{Int}$.

[9] In the present context, the most interesting philosopher to voice doubt about the strict truth of this doctrine is Dummett himself (Dummett (1981): 335-348)

A principle corresponding to $Reductio_{Int}$ follows from the principle corresponding to $+I'$: if agreeing that A entails that we agree and disagree that B, then we agree that $\neg A$, so we disagree that A. Given the parallelism between principles for agreement and disagreement and applying vacuous discharge, we also have a principle corresponding to *ex contradictione quodlibet*.

Next we'll look into the details of Price's argument for (4), which corresponds to the classical half of *Reductio*: if $\Gamma, -A \vdash \beta$ and $\Gamma, -A \vdash +\beta^*$, then $\Gamma \vdash +A$. Most people agree, I think, that there is a flaw somewhere in the argument, although there is disagreement over where it is. With the material at hand, we can pinpoint exactly where the argument fails.

5. Agreement, Disagreement, Dispositions and Differences

Price's argument turns around the notions of making a difference and dispositions to act in terms of which agreement and disagreement are characterised. I will occasionally abbreviate 'We agree that' by 'agree', 'We disagree that' by 'disagree', 'We are disposed to act as if' by 'act' and 'It makes a difference to us whether' by 'difference'. From the fourth paragraph of the quoted passage we get:[10]

(7) agree $A =$ (difference $A \rightsquigarrow$ act A)

(8) disagree $A =$ (difference $A \rightsquigarrow$ act $\neg A$)

The consistency of beliefs imposes restrictions on the interpretation of the conditional $\rightsquigarrow$. It cannot be material, $\Box\!\!\rightarrow$, or strict, lest classical logic entails that everything makes a difference, everything possibly makes a difference in the sense that $\neg$(difference $A \;\Box\!\!\rightarrow \perp$), or everything possibly makes a difference. All are unreasonable, even ignoring the fact that it would also hold for contradictions.[11]

[10] It is an oddity that Price appeals to negation in this characterisation of disagreement, although negation is supposed to be defined in terms of it. We should expect a primitive notion equivalent to having a disposition to act as if $\neg A$ that does not contain negation.

[11] Details. By consistency, if we agree that A, then we do not disagree that A. Suppose $\rightsquigarrow$ is the material conditional. Then if it makes no difference to us whether A, we agree and disagree that A, so by consistency, it is not the case that it makes no difference to us whether A. By classical logic, it makes a difference to us whether A. For an intuitionist, the result may not be quite so unpalatable. Maybe we can never come across a proposition that makes no difference to us, just as we can never come across a proposition that is a counterexample to $A \vee \neg A$. Next, suppose $\rightsquigarrow$ a strict conditional. Then if it is impossible for A to make a difference to us, we agree and disagree with A, so by consistency, it is not the case that it is impossible for A to make a difference to us. By classical logic, it is possible that A makes a difference to us. Again, the result may be less dramatic for an intuitionist. Finally, suppose $\rightsquigarrow$ is a conditional $\Box\!\!\rightarrow$. Then if difference $A \;\Box\!\!\rightarrow \perp$, then difference $A \;\Box\!\!\rightarrow$ act A and difference $A \;\Box\!\!\rightarrow$ act $\neg A$, so by consistency, $\neg$(difference $A \;\Box\!\!\rightarrow \perp$). This already holds in fairly weak conditional logics.

A sensible option is to interpret $A \rightsquigarrow B$ as $\Diamond A \,\&\, (A \,\Box\!\!\rightarrow B)$, often abbreviated by $A \,\Box\!\!\Rightarrow B$. But there is still an obstacle. On Price's account of belief, we agree with the laws of logic, hence disagree with the contradictions. If it is impossible that $\perp$ makes a difference to us, then we neither agree nor disagree with it. So it is logically true that it possibly makes a difference to us whether $\perp$. This is unreasonable.

A minor modification blocks this conclusion. We can draw a distinction between 'It makes a difference to us whether A' and 'It makes a difference to us whether $\neg A$' and modify (8) accordingly:

(9) disagree $A =$ (difference $\neg A \rightsquigarrow$ act $\neg A$)

The modification is motivated purely by the formal properties of making a difference. As will become clear, Price is committed to drawing this distinction also on the basis of the content he gives to the notion.[12]

On to paragraph 5 of the quoted passage. Price considers 'outcomes which depend on it not being the case that S'. I take it that if C depends on it not being the case that S, then C only if $\neg S$, so we can use concepts already introduced: if S is a consequence of C, then if we cannot hold open the possibility that we disagree that S, then we cannot hold open the possibility that we agree that C. This follows from (6) and (vi) of the last but one section, using the equivalence of disagreeing and being disposed to disagree established there. It is of no help in deriving Price's desired conclusion.

Price continues that in the case under consideration, if it makes a difference to us whether A, we can proceed on the assumption that A. It is not farfetched to characterise assuming that A as agreeing that A and seeing what follows. As we will see, Price characterises the notion of making a difference by something very much in that vicinity. The point, however, cannot merely be to make an assumption and seeing what follows. That will only allow us to draw conditional conclusions.

Price's point must be that, in the case under consideration, if it makes a difference to us whether A, then we are allowed to discharge the assumption that we agree that A and proceed to whatever follows. The following suggests itself:

(10) If it makes a difference to us whether A and we cannot hold open the possibility that we disagree that A, then, if we agree that A, then C, then C.

Alternatively, if difference A, we cannot hold open the possibility that we disagree that A and agree A together entail C, then difference A and we cannot hold open the possibility that we disagree that A alone entail C.

[12] Even with this modification, $\rightsquigarrow$ is still best read as $\Box\!\!\Rightarrow$. The other interpretations entail that for every A either it makes a difference to us whether $\neg A$ or it makes a difference to us whether A, or that this disjunction is possible.

We are tantalisingly close to Price's desired result. If we agree that $\neg\neg A$, then we disagree that $\neg A$, so by (1) we cannot hold open the possibility that we agree that $\neg A$, so we cannot hold open the possibility by that we disagree that A. So by (10), if it makes a difference to us whether A, we agree that A.

Yet we are so far. Only on condition that it makes a difference to us whether A is it the case that if we agree that $\neg\neg A$, then we agree that A. But $A \rightarrow (A \mathrel{\Box\!\!\Rightarrow} B)$ does not entail $A \mathrel{\Box\!\!\Rightarrow} B$. Furthermore, if we agree that $\neg\neg A$, this should not entail by itself that it makes a difference to us whether A. Otherwise, as if we agree that $\neg\neg A$, then we agree that A, if we agree that A, it would make a difference to us whether A, so that, assuming $\rightsquigarrow$ satisfies *modus ponens*, if we agree that A we have a disposition to act as if A independently of whether A makes a difference to us. This would spoil the definition of agreement (7).

That it actually makes a difference to us whether A is quite a strong requirement. A weakening of (10) suggests itself according to which it suffices that it is possible that it makes a difference to us whether A, in the sense of possibility used in the definition of agreement. As on the proposed reading of $\rightsquigarrow$ as $\mathrel{\Box\!\!\Rightarrow}$, if we agree that A, then it possibly makes a difference to us whether A, this weakening of the conditions under which agreeing that $\neg\neg A$ entails agreeing that A creates a nice balance: we put into agreeing that A what we can take out. This motivates the modified principle:

(11) If it is possible that it makes a difference to us whether A and we cannot hold open the possibility that we disagree that A, then, if we agree that A, then C, then C.

Alternatively, if $\Diamond$ difference A, we cannot hold open the possibility that we disagree that A and agree A together entail C, then $\Diamond$ difference A and we cannot hold open the possibility that we disagree that A alone entail C. Price can hardly complain if we weaken his conditions for when agreeing to $\neg\neg A$ entails agreeing that A. However, $\Diamond A \rightarrow (A \mathrel{\Box\!\!\Rightarrow} B)$ does not entail $A \mathrel{\Box\!\!\Rightarrow} B$ either, so we are still not where we want to be.

If we agree that $\neg\neg A$, then it possibly makes a difference to us whether $\neg\neg A$, reading $\rightsquigarrow$ in (7) as $\mathrel{\Box\!\!\Rightarrow}$. It also entails that it is possible that it makes a difference to us whether $\neg A$, by (8). Nothing that has been said so far ensures that this entails that it is possible that it makes a difference whether A. We have reached a point where we must look at what Price has to say about the notion of making a difference.

6. Price on Making A Difference

In the article that has been the focus of the discussion so far, Price says next to nothing about what it means that something makes a difference to us. He fills this gap in a later paper:

> 'To imagine a case in which it makes a difference to the outcome of some action whether S is to imagine a case in which there is a procedure available for deciding whether S. What procedure? Simply that of performing the action in question, and seeing what happens — seeing whether the outcome is the one expected if S, or the one expected if ¬S. But the intuitionist agrees that when S is decidable, S follows from ¬¬S. So the crucial pragmatic thought is something like this. To deliberate on the basis of a commitment to a proposition S is to think about hypothetical cases in which it is decidable whether S. So a pragmatic view of belief — a view tied to the role of beliefs in such deliberations — leaves no room for a gap between believing ¬¬S and believing S. [Price accepts] the inference from "it makes a difference whether S" to "S is decidable" [according to his sense of 'decidable'].' (Price (2016): 3ff)

As in similar cases before, I assume that Price's talk of commitments instead of dispositions makes no difference to what is at issue.

Tying the notion of making a difference to decidability puts a strain on how to understand it. It is not an intuitive notion of making a difference, but it is not exactly the technical notion of decidability from formal logic either. Price's idea, however, is clear enough: if something makes a difference to us, then there are only two significant courses of action. This may sound as if it concedes too much to the classicist. Ironically, however, it gives the intuitionist the material to block the inference from 'we agree that $\neg\neg A$' to 'it is possible that it makes a difference to us whether A'.

Intuitionistically, there is a difference between A's decidability and $\neg A$'s decidability. Consider a case where $\neg\neg A$ is provable, but A is not: then $\neg A$ is decidable, as $\neg A \vee \neg\neg A$ is provable. But $A \vee \neg A$ is not provable, as otherwise A would be provable. Collapsing the distinction between the decidability of A and the decidability of $\neg A$ entails that $\vdash \neg\neg A \rightarrow A$. If there is no distinction between 'It makes a difference to us whether A' and 'It makes a difference to us whether $\neg A$', then it is not possible to treat $\neg A$ as decidable, while refraining from treating A as decidable. For then, if it makes a difference to us whether $\neg A$, it makes a difference to us whether A, hence we always must treat both as decidable, if we so treat one of them. If Price wants to collapse the distinction between 'It makes a difference to us whether A' and 'It make a difference to us whether $\neg A$', this needs to be established at a later stage, for instance as a consequences of having justified classical logic. Assuming it at this stage would defeat the purpose of the argument.

Two points emerge from the discussion. First, if an additional argument for the modification (9) of (8) were needed, this is it: the distinction between 'It makes a difference to us whether $\neg A$' and 'It makes a difference to us whether A' is mandatory due to the content Price gives to the notion of making a difference, and thus so is the modification (9) of (8) argued for earlier on purely formal grounds. Secondly, it confirms the connection between making a difference and making assumptions, proposed in (10) and (11).

To establish his desired conclusion, Price would need a principle like this:

(12) $\Diamond$ difference $\neg\neg A / \Diamond$ difference $\neg A \rightarrow \Diamond$ difference A

For an intuitive notion of making a difference, it may seem absurdly stubborn to reject (12): any outcome, whether A, $\neg A$ or $\neg\neg A$ should be of interest, if it makes a difference to us whether one of them is the case, so (12) should be equally blatant. Some such thought may well underlie Price's failure to distinguish between 'It makes a difference to us whether A' and 'It makes a difference to us whether $\neg A$'. Initially, Price gives no explicit content to the notion of making a difference his pragmatic account of belief rests on. He trusts that his readers have an intuitive understanding of what it means that something makes a difference to us and to act in the light of this. Price's notion of making a difference, however, is not an intuitive, but a theoretical one. As shown previously, it is desirable to draw a distinction between 'It makes a difference to us whether A' and 'It makes a difference to us whether $\neg A$' for purely formal reasons connected to how the notion features in principles Price is committed to. That Price's notion of making a difference is not an intuitive one is reinforced by the way he binds it to decidability, itself a highly theoretical notion, no matter what exactly Price's chosen sense of decidability might amount to when applied outside logic and mathematics. That tie gives cogent reasons to reject (12). If a proposition of logic or mathematics is possibly decidable, then it is decidable. So (12) collapses the distinction between treating $\neg\neg A$ and $\neg A$ as decidable while not treating A as decidable, which Price must grant the intuitionist at this stage. The situation is ironic. The only explicit and non-intuitive content Price gives to the notion of making a difference in fact blocks the most obvious route towards his desired conclusion.

7. Conclusion

Rumfitt's bilateralism does not exclude intuitionist logic from being justified in his framework, as it is possible to formalise a system of intuitionist bilateral logic that satisfies all the requirements he imposes. Thus there is no formal argument against an intuitionist bilateralism. Price's informal argument for the classical nature of a theory of meaning that takes assertion as well as denial conditions are basic has received a thorough treatment. He stays vindicated that he does not beg the question against the intuitionist, if only because a close look at his account shows that it fails to establish that a denial condition for $\neg A$ is an assertion condition for A.

As indicated in my analysis, there may be more straightforward ways of reading $\rightsquigarrow$ in (7) and (8) open to the intuitionist that are closed to the classicist. There is a further point where the intuitionist has an advantage.

With intuitionist logic, there is an obvious way of making precise what it might mean to treat A as decidable, if it makes a difference to us whether A: to treat A as decidable is to assume $A \vee \neg A$. The intuitionist could put forward a principle such as this one:

(13) difference $A \rightarrow$ agree $(A \vee \neg A)$

or one that connects making a difference to dispositions to act:

(14) difference $A \rightarrow$ act $(A \vee \neg A)$

For a classicist, either principle is vacuous, if we always agree or act in accordance with the laws of logic. Thus the best, if not the only, option of making sense of what it might mean to treat A as decidable is not open to the classicist of Price's persuasion, to whom (13) and (14) have no significant content.

Intuitionist logic may well provide a more congenial background for a development of Price's ideas than classical logic. Combine this with the fact that a satisfactory intuitionist bilateral logic can be formulated and it looks as if a theory that specifies meanings in terms of assertion and denial conditions is no different from the received approach when it come to questions of the justification of logical laws.[13]

References

[1] Dickie, I. (2010). Negation, anti-realism, and the denial defence. *Philosophical Studies 150*(2), 161–185.
[2] Dummett, M. (1978). The philosophical basis of intuitionistic logic. In *Truth and Other Enigmas*, pp. 215–247. London: Duckworth.
[3] Dummett, M. (1981). *Frege. Philosophy of Language* (2nd ed.). London: Duckworth.
[4] Dummett, M. (1993). *The Logical Basis of Metaphysics*. Cambridge, Mass.: Harvard University Press.
[5] Dummett, M. (2002). "Yes", "No" and "Can't Say". *Mind 111*, 289–295.
[6] Frege, G. (1918). Die Verneinung. Eine logische Untersuchung. *Beiträge zur Philosophie des deutschen Idealismus 1*, 148–157.
[7] Geach, P. T. (1972). Assertion. In *Logic Matters*, pp. 254–269. Oxford: Blackwell.

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[8] Gibbard, P. (2002). Price and Rumfitt on rejective negation and classical logic. *Mind 111*, 297–303.

[9] Humberstone, L. (2000). The revival of rejective negation. *Journal of Philosophical Logic 29*, 331–381.

[10] Kürbis, N. (2015a). Proof-theoretic semantics, a problem with negation and prospects for modality. *The Journal of Philosophical Logic 44*, 713–727.

[11] Kürbis, N. (2015b). What is wrong with classical negation? *Grazer Philosophische Studien 92*, 51–85.

[12] Kürbis, N. (2016). Some comments on Ian Rumfitt's bilateralism. *Journal of Philosophical Logic 45* (2016), pp. 623-644.

[13] Price, H. (1983). Sense, assertion, Dummett and denial. *Mind 92*, 161–173.

[14] Price, H. (1990). Why 'Not'? *Mind 99*, 221–238.

[15] Price, H. (2016, acc. 23/06/2016). 'Not' Again. http://prce.hu/w/preprints/NotAgain.pdf.

[16] Restall, G. (2005). Multiple conclusions. In P. Hájek, L. Valdés-Villanueva, and D. Westerståhl (Eds.), *Logic, Methodology and Philosophy of Science: Proceedings of the Twelfth International Congress*, pp. 189–205. London: King's College Publications.

[17] Rumfitt, I. (2000). "Yes" and "No". *Mind 109*, 781–823.

[18] Rumfitt, I. (2002). Unilateralism disarmed: A reply to Dummett and Gibbard. *Mind 111*, 305–312.

[19] Rumfitt, I. (2007). Asserting and excluding: Steps towards an anti-realist account of classical consequence. In R. E. Auxier and L. E. Hahn (Eds.), *The Philosophy of Michael Dummett*, The Library of Living Philosophers, pp. 639–693. Chicago: Open Court.

[20] Smiley, T. (1996). Rejection. *Analysis 56*, 1–9.

[21] Textor, M. (2011). Is 'no' a force indicator? No! *Analysis 71*(3), 448–456.

[22] Wansing, H. (2016). Falsification, natural deduction, and bi-intuitionistic logic. *Journal of Logic and Computation 26*, 425–450.

[23] Weiss, B. (2007). Anti-realist truth and anti-realist meaning. *American Philosophical Quarterly 44*(3), 213–228.

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