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Adjoint-Based Well Placement Optimisation for Enhanced Oil Recovery (EOR) under Geological Uncertainty: From Seismic to Production

Emmanuel I. Epelle and Dimitrios I. Gerogiorgis*

Institute for Materials and Processes (IMP), School of Engineering, University of Edinburgh, The King's Buildings, Edinburgh, EH9 3FB, United kingdom

*Corresponding author: D.Gerogiorgis@ed.ac.uk (+44 131 6517072)

ABSTRACT

The development of new and existing oil fields is very expensive and technically challenging; this provides an important economic incentive to improve oil recovery via efficient computational optimisation. Decisions related to these activities may be significantly aided by sound and proven mathematical-oriented methods. The use of intuitive engineering judgement alone cannot guarantee sustainable profitability over long periods especially under geological (reservoir model) uncertainty. To capture the uncertainties in the subsurface geological/reservoir model in this work, geostatistical realisations of the model are obtained using available information (permeabilities and porosities). We also apply specialised algorithms within the MATLAB Reservoir Simulation Toolbox – MRST (interfaced with PETREL^{TM}) to optimally vary the well locations and production rates, thus maximising the field's oil recovery. A new modification of an existing pseudowell-based injection well placement algorithm is presented herein with the application of adjoint-computed gradients of an auxiliary objective function (the Lorenz coefficient). The difficulty of this problem is characterised by the presence of discrete variables, nonlinear and nonconvex objective function and constraints. The developed workflow is applied to a realistic case study, for which robust optimality is demonstrated using the worst case realisation for the determination of optimal well locations and controls. A comparative investigation of optimising injection well location vs. simultaneous optimisation of injection and production well placement is also presented. In our presented case study, we further discover that increasing the optimisation search space does not necessarily guarantee improved results.

1.0 Introduction

Huge expenditures, rigorous exploratory efforts and long evaluation periods are required to prove the existence of petroleum resources and the profitability of a new oil and gas prospect. For this reason, improving the performance of existing fields over an extended time horizon has received significant industrial attention for long-term operational sustainability. This has resulted in the concept of closed-loop reservoir management (Jansen et al., 2009; Barros et al., 2019), of which the application of optimisation methodologies is a key attribute. This enhances the field development, operational decision making tasks and workflows compared to traditional heuristic methods (Jesamani et al., 2016). A field's oil recovery depends on the well locations and the reservoir's geological properties. Hence, well placement optimisation at the early planning stages of reservoir development is necessary to achieve the best possible economic benefits. Controlled numerical flow experiments (reservoir simulation) that quantify fluid flow behaviour with respect to well positions can be used to realistically describe the subsurface flow phenomena at time scales usually associated with reservoir management (Epelle and Gerogiorgis, 2019a,b; Møyner et al., 2015). However, full-scale reservoir simulations are computationally expensive and this limits the number of trials and iterations that can be performed in the search for an optimal operation strategy.

Production optimisation of petroleum systems (Fig. 1) involves the evaluation of well controls (production and injection rates and well pressures) to maximise field profitability (usually written in terms of the Net Present Value – NPV). Since the fundamental relationship between reservoir flow dynamics and the well control variables are nonlinear; finding the set of optimal controls can be challenging. The presence of operational constraints on the well bottomhole pressures, production and injection rates, water cut and pressure drop also add to this complexity. Furthermore, geological uncertainty is a key concern because, it is difficult to gain accurate knowledge of the spatial distribution of the reservoir's properties, and how this changes with time. This uncertainty must be quantified by obtaining multiple realisations of the uncertain parameters and probabilistically incorporating these realisations into the optimisation formulation; thus capturing fundamental reservoir engineering knowledge. Neglecting the effect of uncertainty could result in poorly placed wells, premature water breakthrough and low production rates, despite the maintenance of reservoir pressure via water injection.

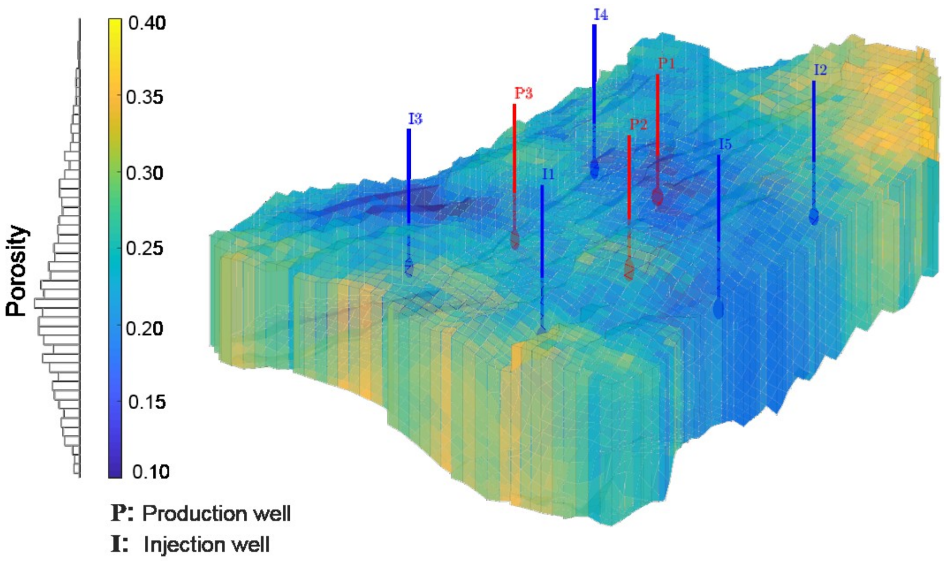

Figure 1: Reservoir model showing the spatial variability of the porosity and the initial well locations.

Numerous calls to a subsurface simulator, the absence of gradient information and the necessity for global search methods have warranted the application of derivative-free optimisation methodologies in well placement optimisation problems (Awotunde, 2014). These methods include Hooke-Jeeves direct search (HJDS), generalised pattern search (GPS), genetic algorithms (GA), Mesh Adaptive Direct Search (MADS), particle swarm optimisation (PSO) techniques etc. (Ciaurri et al., 2010). However, gradient based methods can be effective when gradients are available through adjoint methods. The optimisation surface of well placement problems can be very rough and nonconvex; thus resulting in discontinuous gradients and numerous optima. This results in the difficulty of finding a global solution when gradient based methods are employed. Furthermore, adjoint methods for computing gradients are invasive with respect to the subsurface flow simulator. Their implementation requires a detailed knowledge/full access to the simulator source code, compared to the non-invasive technique of derivative-free methods which treat the simulator as a black box (only values of the cost functions are required). This treatment is also made possible by using special constraint handling techniques such as penalty functions and filter methods (Datta-Gupta et al., 1995; Wamg et al., 2012; Bellout et al., 2012; Gildin et al., 2013; Narasingam et al., 2018; Beykal et al., 2018; Jesmani et al., 2020).

It is worth re-emphasizing that incorporating specific geological knowledge about the reservoir into the optimisation problem is a vital step for the determination of feasible well locations that may be explored by the algorithm; thus enabling practical solutions. Although, these optimisation methodologies may be prone to multiplicity of solutions after several runs for varying problem configurations, candidate solutions may be evaluated by the engineering and geology teams for the most practical and cost-effective implementation. Our focus in this study is thus an integration of vital geological considerations when carrying out well placement optimisation studies with robust algorithms.

2.0 Relevant Literature

The application of mathematical optimisation to well placement problems has generally been carried out using gradient-based approaches (based on adjoint methods and finite differencing), mixed integer programming (MILPs/MINLPs), simulated annealing, genetic algorithms, cat swarm optimisation and particle swarm optimisation (Bangerth et al., 2006; Onwunalu and Durlofsky, 2010; Tavallali et al., 2013; Isebor et al., 2014; Hongwei et al., 2018; Tavallali et al., 2016). In this section we briefly discuss some key contributions in this field. A more thorough review of well placement studies is presented the perspective article of Tavallali and Karimi (2016).

Simultaneous parametric optimisation of well patterns, well types and location has received considerable attention in the past decades (Yeten et al., 2003; Onwunalu and Durlofsky, 2010; Yasari et al., 2013; Fan et al., 2018). In these studies, the well arrangements are constrained by repeated patterns which are common in industrial field operations (5-spot pattern, 7 spot pattern etc.). Ciaurri et al. (2011) applied a PSO algorithm for perturbing well locations within four pre-defined patterns in a repeated manner. This procedure may force some wells that were originally within the reservoir boundary to be shifted out thus leading to suboptimal results. However, this approach was shown to be more computationally tractable than an exhaustive approach. A 22% increment in NPV was attained by performing a well-by-well perturbation compared to well placement optimisation via well pattern description. Ciaurri et al. (2010) performed a computational analysis of non-invasive derivative free optimisation methods to some production optimisation problems of practical relevance. Their findings indicate that the Hooke-Jeeves direct search (HJDS) is the recommended approach when distributed computing resources are unavailable. They also highlighted that a rigorous iterative process is often required when handling constraints using the penalty function method; a filter-based method was proposed as a reliable alternative when hybridized with a generalised pattern search procedure. It was shown in the work of Onwunalu and Durlofsky, (2010), that the PSO algorithm (with fewer function evaluations) represents a viable alternative to the widely adopted GA algorithm for well placement problems. A benchmarking study by Minton (2012) arrived at a similar conclusion for the PSO algorithm (in comparison to GA). Other methods such as the simulated annealing and hill climbing algorithms showed poor performance.

Jesmani et al. (2016) introduced a decoder method for handling constraints in well placement optimisation problems. The decoder procedure maps the feasible search space onto a cube; thus avoiding repeated parameter tuning. When compared with the penalty function method, it was shown in most cases that the decoder method outperforms the penalty function method. A joint approach for well placement and control optimisation was proposed by Bellout et al. (2012). Derivate-free (pattern search) methods were adopted for the well placement problem, whereas, sequential programming using adjoint-computed gradients was implemented for determining optimal well controls. They realised up to 20% increment compared to sequential methods, although with an order of magnitude increase in the number of functional evaluations. A hybrid algorithm that utilises PSO and GPS was applied for the simultaneous optimisation of well placement and controls in the work of Humphries and Haynes (2014). The compared the quality of solutions obtained using a joint approach and a decoupled/sequential approach. They discovered that in 3 out of 5 experiments, the decoupled approach found better solutions than the joint method. Isebor et al. (2014) developed a hybrid PSO-MADS algorithm that outperforms standalone MADS and PSO procedures and a relaxed Branch and Bound method (B&B), when solving a well placement optimisation problem formulated as an MINLP. The B&B method was the computationally least efficient (with the highest number of iterations required). The reduction of computational complexity using proxy models in well placement optimisation problems has also received some attention, with key contributions from Guyaguler (2002) and Pouladi et al. (2017). Kriging and fast marching methods were respectively applied. Furthermore, several studies (Bouzarkouna, 2012; Wang et al., 2012; Li et al., 2013) have accounted for uncertainty using stochastic optimisation techniques, such as GA, PSO, Covariance Matrix Evolution Strategy (CMA-ES) and Simultaneous Perturbation Stochastic Approximation (SPSA). Uncertainty reduction (selection of a subset of geological realisations from a superset) has also been applied for computational time reduction in the work of Rahim and Li (2015). Their well placement optimisation problem was formulated as an MILP and solver using the NOMADS optimiser (Nonlinear Optimisation with MADS algorithm).

Research contributions that tackle well placement optimisation problems using gradient-based optimisation methods are scarce. Zandvliet et al. (2007) introduced an adjoint method for computing gradients using pseudowells. The main advantage of this algorithm over previous approaches is that improving directions for all wells are computed only in one forward and backward simulation. It was discovered that the algorithm converges to similar optimal NPV values with different initial well locations. This gradient based method was extended by Wang et al. (2007); in their algorithm, the optimisation problem is initialised by adding an injection well to every grid block that does not contain a production well. This number of injection wells is successively reduced by assigning a drilling cost to each well and constraining the total maximum injection rate that should be allocated to all injection wells remaining at each iteration. By developing a continuous functional approximation to an original discrete-parameter well placement problem, Sarma et al. (2008) applied gradient-based algorithms to efficiently determine optimal well locations. Kourounis et al. (2014) developed adjoint procedures for general compositional problems (which are more challenging than oil-water systems). They successfully applied these procedures with a new heuristic method of handling constraints in a gradient-based optimisation framework to optimise wells controls using different real field examples. More recently, Volkov and Bellout (2018) introduced a novel gradient-based approach based on finite difference approximation of augmented Lagrangian derivatives. They coupled their adjoint formulation with a Sequential Quadratic Programming (SQP) solver, which enabled fast convergence and efficient constraint handling.

The difference between the methodology presented in this paper and previous studies are the following:

- This study implements a modification of an existing pseudowell-based injection well placement algorithm (Zandvliet et al., 2007) with the application of an auxiliary objective function (the Lorenz coefficient), for the improvement of oil displacement efficiency. We also modify the well placement algorithm in MRST to include both injection and production wells.
- We computationally compare the different methods of optimising the well positions (injectors only, producers first and injectors next and injectors first and producers next) and examine their efficiencies in terms of the final recoveries and the Lorentz coefficient. Flow diagnostics via enhanced visualisation tools are applied to gain insights into fluid distributions in the reservoir under geological uncertainty.
- This study implements an efficient adjoint formulation in MRST for speedy computation of gradients used in the optimisation formulation (for both well placement and control); thus resulting in reduced number of iterations and computational times compared to previous studies that apply evolutionary and gradient-based algorithms (Tavallali et al., 2013; Isebor et al., 2014; Onwunalu and Durlofsky, 2010).
- We conduct an investigation on the influence of the solution search space for each well on the final oil recovery, final oil saturation and computational cost.

These 4 points constitute the novel elements of this paper.

3.0 Methodology

The general procedure begins by constructing a static reservoir model, generating several realisations of the reservoir model and applying the optimisation algorithms to our case study. Before providing details of each procedure, the assumptions made in this work are stated.

- Reservoir rock and fluid properties are fully available (3D dimensions, permeability, porosity, fluid density etc.).
- The reservoir is considered infinite acting with many permeable fault boundaries.
- Economic parameters such as the unit volumetric costs for oil and gas sales and water treatment costs are available.
- Reservoir fluids are oil and water.
- Number of production and injection wells are known
- Production time horizon is known ($T = 1 – 5$ years).
- Injection and production wells are vertical.

3.1 Static modelling

The first step in this stage involves the mapping of horizons and faults from the available seismic data in PETREL^{TM} (Fig. 2a and 2b). This is followed by the creation of a surface maps that mark the reservoir's boundary (Fig. 2c). Well long interpretations are carried out next to identify the productive geological zones based on the reservoir's lithology, porosity and fluid resistivity (Fig. 2d). This is the zone that would be perforated for fluid flow into the wellbore. The result of this interpretation is the final static model as shown in Fig. 2e which is upscaled for the dynamic simulation purposes (Fig. 2f). The field contains 5 injection wells and 3 production wells in operation.

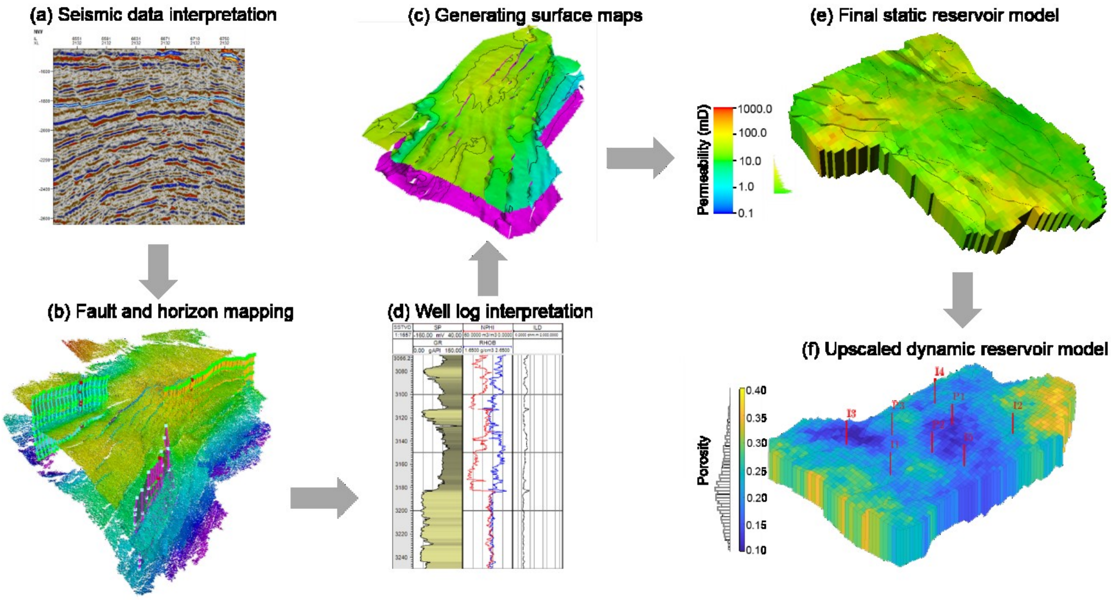

Figure 2: Static & dynamic model development (from data interpretation in PETREL^{TM} to optimisation in MATLAB).

3.2 Incorporating uncertainty

Geological uncertainty exists because it is difficult to know the exact properties of every section of the realistic reservoir (Rahim and Li, 2015). Using the Sequential Gaussian Simulation (SGS) functionality of PETREL^{TM}, 50 realisations of the reservoir's permeability (horizontal and vertical) are generated (Fig. 3). The grid structure and the fluid and rock properties of each realisation are imported into MATLAB (where optimisation tasks are performed using the MRST toolbox). In ranking these realisations, the pressure distribution, flow capacity, storage capacity and the Lorenz coefficient of the reservoir are computed for each scenario to assess the uncertainty of the water flooding operation while assuming all realisations are equiprobable. Further details of the ranking procedure can be found in Shook et al. (2009), Møyner et al. (2015) and Lie et al. (2019).

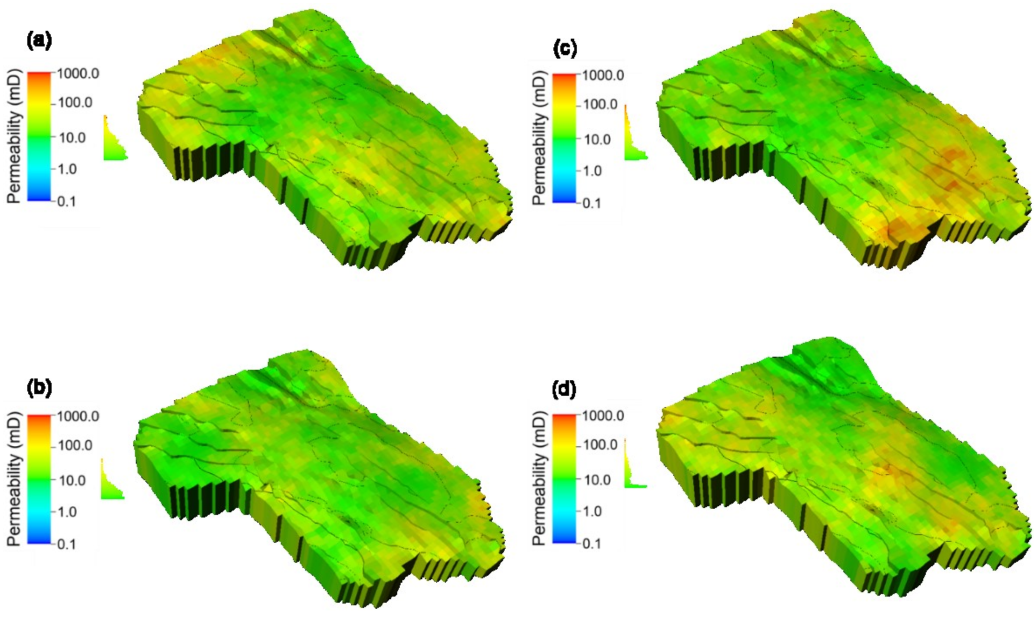

Figure 3: Geological realisations implemented (4 out of 50).

Since a rigorous treatment of geological uncertainty via stochastic optimisation (Wang et al., 2012; Li et al., 2013; Rahim and Li, 2015) is beyond the scope of this paper, it was necessary to ensure obtain operationally feasible realisations without numerical artefacts in the simulated field. SGS, a popular method was chosen to stochastically populate a grid with a Gaussian random field (such as permeability). The method involves visiting each node of the computational grid sequentially and evaluating the probability distribution based on previous values using kriging (Nussbaumer et al., 2017). The order in which the nodes are visited and simulated (simulation path) may influence the accuracy of simulations; however studies that examine and quantify the influence of the simulation path on the existence and magnitude of possible statistical bias are scarce. PETREL's implementation of the SGS algorithm used herein is an industrially accepted and widely applied method in the oil and gas industry. The robustness of this implementation guarantees successful application to even more complex reservoirs with challenging geological features (such as high/low permeability channels and reservoir compartmentalisation). We have also applied SGS to the Norne field, as shown in Fig. 16.

3.3 Dynamic modelling and optimisation formulation

Optimisation tasks are carried out over the worst case scenario after ranking the geological realisations; thus ensuring robust feasibility of the obtained solution (worst-case optimisation) (Krishnamoorthy, 2016). Although this is very conservative approach, it is justifiable when limited production data is available for history matching purposes (Jansen et al., 2012), especially at the early stages of field operation. Optimising the expected value of the objective (a less conservative approach) can be pursued with good knowledge of the field's production history. The mathematical formulation of the reservoir model, objective function, operational constraints and adopted solution strategy are shown in Table 1.

The dynamic reservoir model (Eqs. 1–3) describes the flow field in the reservoir and the time-of-flight (TOF, the time required for a fluid particle to travel along a streamline from its starting point to the current position). The pressure is denoted as p, the TOF, τ, the Darcy velocity, $\vec{v}$, the reservoir's storage capacity, ϕ, the permeability tensor $\mathbf{K}$, and the fluid mobility, λ_f. Flow in the reservoir can be driven by wells, n_{bh}, which are controlled by the bottomhole pressure (BHP) and wells, n_r, which are controlled by the flow rate. Both well types, n_w, have perforations, n_{pf}, through which fluid flows from the reservoir into the wellbore. All wells are modelled using the Peaceman well model (Eq. 4) in which the well perforation fluxes are denoted by q_{pf}. The index of the well to which perforation number j belongs is denoted as $N_w(j)$; k is the well index and W^j_{pf} is the Peaceman well index. Furthermore, a set of controls (in the form of closure relations) for each well type is specified (Eqs. 5 and 6); where $\mathbf{u}$ is the control vector. Asides the well placement optimisation, rate control optimisation is subsequently performed on the optimally located wells. The first objective function (Eq. 7) is applied to the well placement optimisation task; the objective function is based on the Lorenz coefficient (Eq. 7), which is written in terms of the flow capacity, F, and the storage capacity, ϕ. This coefficient measures how the oil displacement efficiency for a given well pattern differs from that of an ideal (piston-like) displacement pattern in the reservoir (Shook et al. 2009). Thus, this coefficient is a measure of the optimality of the water flooding operation and hence the oil recovery in the reservoir. A simplified NPV expression (without installation cost of wells and other factors) is utilised as the objective function of the rate control procedure (Eq. 8). T represents the length of the time horizon, q_c and q_{ci} are the field production and injection rates of components c (oil and water) respectively. The revenues and costs of production and injection of components c are denoted as r_c and r_{ci} respectively and d is the discount rate. $\boldsymbol{\mathcal{C}}$, $\boldsymbol{\mathcal{T}}$, $\boldsymbol{\mathcal{V}}$, $\boldsymbol{\mathcal{Q}}$ and $\boldsymbol{\mathcal{P}}$ represent the discretised system of equations in terms of variables, q_{pf}, p_{bh}, p, v, τ. The production and injection rates are constrained according to Eq. 10 and we ensure voidage replacement by enforcing Eq. 11.

Table 1: Modelling and optimisation framework.

Reservoir Model

$$\mathcal{P}(q, \vec{v}) = \nabla \times \vec{v} - q = 0 \tag{1}$$

$$\mathcal{V}(p, \vec{v}) = \vec{v} + \mathbf{K}\lambda \nabla p = 0 \tag{2}$$

$$\mathcal{T}(\tau, \vec{v}) = \vec{v} \times \nabla \tau - \phi = 0 \tag{3}$$

$$\mathcal{Q}^j(q_{pf}, q_{pf}, p) = q_{pf}^j - W_{pf}^j \lambda \left[p_{bh}^{N_w(j)} - p\right] = 0;\; j = 1, \dots, n_{pf} \tag{4}$$

$$\mathcal{C}_{bh} = u_{bh}^k - p_{bh}^k = 0; k = 1, \dots, n_{bh} \tag{5}$$

$$\mathcal{C}_r = u_r^k - \sum_{j \in N_{pf}(k)} q_{pf}^j = 0; k = 1, \dots, n_r \tag{6}$$

Objective Function & Constraints

$$L_{c,o} = 2\int_0^1 [F(\phi) - \phi]\, S_o d\phi \tag{7}$$

$$NPV(T) = \int_{t=0}^{T} \sum_{c=o,w} (r_c q_c + r_{ci} q_{ci})\, (1+d)^{-1} dt \tag{8}$$

$$u \leftarrow u - P\left(\alpha \frac{dG_\lambda}{d\boldsymbol{u}}\right)^T \tag{9}$$

$$q_{c,min} \leq q_c \leq q_{c,max} \tag{10}$$

$$\sum q_i \equiv \sum q_p \tag{11}$$

Solution Strategy

$$\begin{bmatrix} 0 & \partial_v \mathcal{P} & \partial_q \mathcal{P} & 0 \\ \partial_p \mathcal{V} & \partial_v \mathcal{V} & 0 & 0 \\ \partial_p \mathcal{Q} & 0 & \partial_q \mathcal{Q} & \partial_{p_{bh}} \mathcal{Q} \\ 0 & 0 & \partial_q \mathcal{C} & \partial_{p_{bh}} \mathcal{C} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \\ \mathbf{p}_{bh} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

Adjoint Formulation

$$G_\lambda = G[\boldsymbol{x}(\boldsymbol{u}), \boldsymbol{u}] + \lambda^T g[\boldsymbol{x}(\boldsymbol{u}), \boldsymbol{u}] \tag{13}$$

$$\frac{dG_\lambda}{d\boldsymbol{u}} = \frac{\partial G}{\partial \boldsymbol{u}} + \left(\frac{\partial G}{\partial \boldsymbol{x}} + \lambda^T \frac{\partial g}{\partial \boldsymbol{x}}\right) \frac{\partial x}{\partial \boldsymbol{u}} + \lambda^T \frac{\partial g}{\partial \boldsymbol{u}} + \boldsymbol{g}^T \frac{\partial \lambda}{\partial \boldsymbol{u}} \tag{14}$$

$$\left(\frac{\partial g}{\partial \boldsymbol{x}}\right)^T \lambda = J^T \lambda = -\left(\frac{\partial G}{\partial \boldsymbol{x}}\right)^T \tag{15}$$

$$\frac{dG_\lambda}{d\boldsymbol{u}} = \frac{\partial G}{\partial \boldsymbol{u}} + \lambda^T \frac{\partial g}{\partial \boldsymbol{u}} \tag{16}$$

To perform optimisation computations, the primary variables (pressure, rates and TOF) in Eqs. 1–3 are solved for and the objective function gradients are computed for a set of controls. The solution strategy (two-point flux approximation for spatial discretisation – Eq. 12) minimises computational workload and makes it adaptable to different linear algebraic solvers. The adjoint equations comprises the Lagrange function for the problem (Eq. 13), its derivatives (Eq. 14) and simplifications (Eqs. 15 and16) that yield an objective function which depends on the state variables, **x** and not on the control variables **u**. $\mathbf{x}^T$ is a vector of the solution quantities $\mathbf{p}^T$, $\mathbf{v}^T$, τ^T, $\mathbf{q}^T$ and $\mathbf{p}_{bh}^T$. G[**x**(**u**)] represents the objective function and **g**[**x**(**u**),**u**] = 0 represents a set a constraints; λ is the Lagrange multiplier, **J**, the Jacobian, and superscript *T*, the matrix transpose.

3.4 Well placement algorithm:

As depicted in Fig. 4, the algorithm begins by adding pseudowells with a zero-rate in the region around each injector and computes the gradients of the added wells (based on the Lorenz coefficient – the objective function). The original well is then shifted to the position of the pseudowell with the largest gradient. This process is repeated until all wells are stationary. Since vertical wells that penetrate the entire depth of the reservoir are assumed in this work, the optimisation variables also consist of the wells' area locations (x, y) – integer variables. Thus, the number of variables is bound to increase with increasing number of wells in the field. In a field with many wells (in hundreds for example), it becomes essential to impose an inter-well minimum distance constraint to avoid well interference effects during production. The multiplicity of infeasible solutions and the presence of integer variables complicates the computations, especially when the number of wells is an unknown variable.

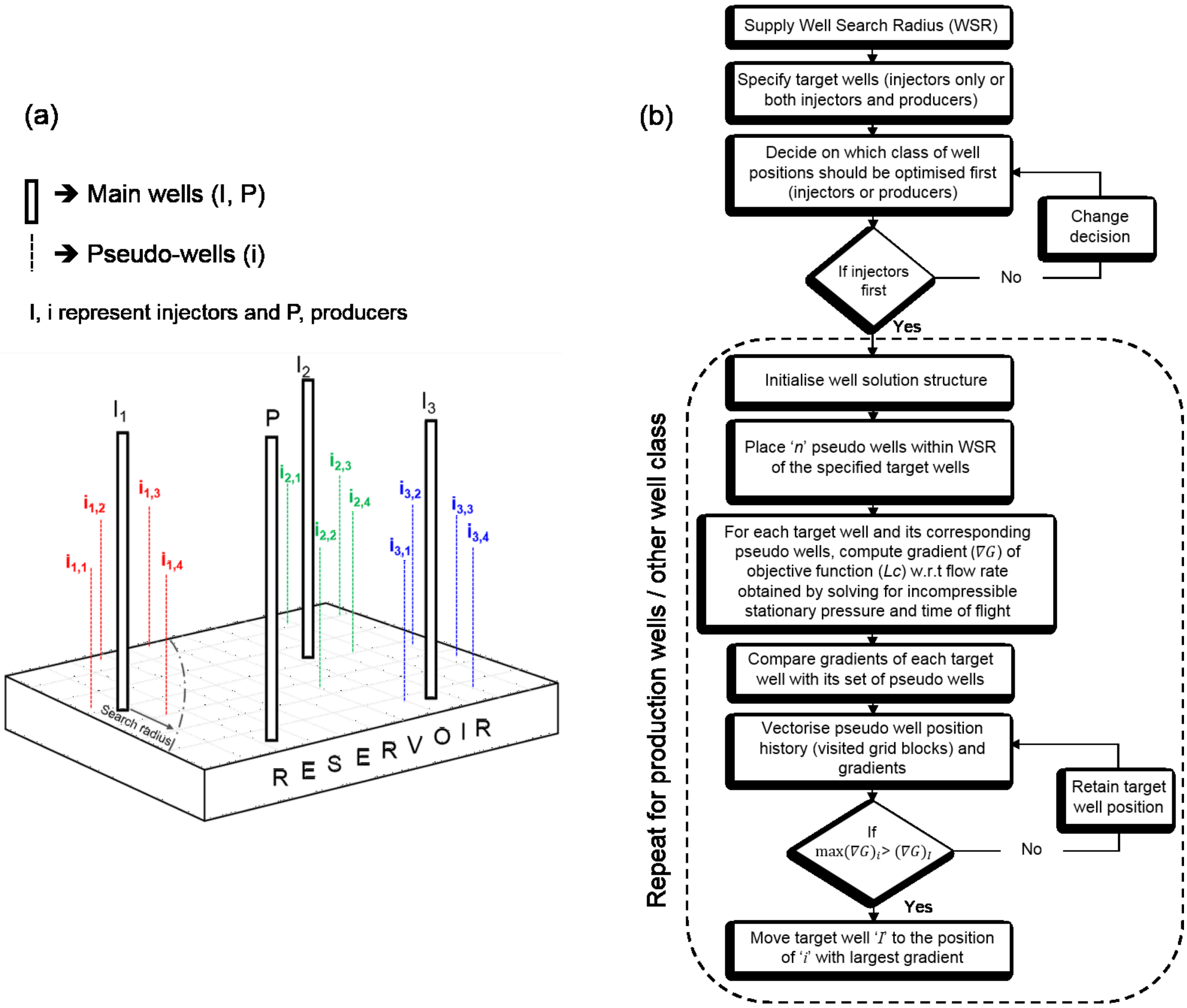

Figure 4: Well placement algorithm showing pseudowells and search radius; a radius of 10 grid cells around each well was implemented in CS1 and CS2 respectively.

All optimisation tasks are performed using the Matlab Reservoir Simulation Toolbox (MRST) (Møyner et al. 2015; Lie, 2019). MRST is an open-source simulator that also allows the inclusion of custom modelling, optimisation and computational methods for specific problems. We apply the two-phase fluid model of MRST which implements the simplified Corey model with relative permeabilities for the oil and water phases calculated as: $k_{rw} = (S_w)^2$ and $k_{ro} = (1 - S_w)^2$.

3.5 Optimal well controls

A steepest-descent algorithm is implemented for finding optimal controls (Eq. 9). This utilises the *NPV* objective function (Eq. 8), the well rates bounds (maximum and minimum) (Eq. 10) and voidage replacement constraints (Eq. 11) which are supplied in the code; α represents the step size and P is a projection to the constraints. While evaluating the objective, the value of α is adjusted and the algorithm stops when the improvement in the objective function between 2 successive iterations is within a specified tolerance (5×10^{-4}).

Table 2: Reservoir simulation parameters used in this study

Parameter	Value
Grid-cell dimensions	19,063 m × 15,503 m × 1,469 m
Grid size	47 × 58 × 1
Fluid densities, ρ_o and ρ_w	859 and 1,014 kg.m^{-3}
Fluid viscosities, μ_o and μ_w	2 and 0.5 cP
Initial reservoir pressure, P_0	500 bar
Well BHP range, P_{bh}	150 – 700 bar
Control interval, t	50 days
Maximum water injection rates, Q_i	500 m^3.day^{-1}
Maximum oil production rates, Q_p	500 m^3.day^{-1}

4.0 Results and Discussion

4.1 Optimising injection well placement (Case study 1 – CS1)

In carrying out the optimisation procedure it is assumed that the production wells have been drilled whereas the injection wells are yet to be drilled. Thus, the aim of the optimisation task to determine the optimal injection well positions that yields the best possible displacement of the residual oil in the reservoir. Details of the fluid properties and reservoir geometry are given in Table 1.

The initial injector placement was done in such a manner as to maintain good hydraulic connectivity between the injection and production wells given the faulted nature of the reservoir – this is based on reservoir engineering judgement (Fig. 5a). However, on applying the well placement algorithm, optimal injector locations that guarantee improved oil sweep are obtained. This can be observed in the oil saturation plots for both placement patterns (unexplored regions of the reservoir - the yellow patches in Fig 3a are absent in Fig. 5b). The saturation plots are obtained by running a simulation (production forecast) for a 5-year production horizon. Furthermore, the well paths taken by the algorithm during the search for optimal injector well position are shown in Fig. 5c. The Lorenz coefficient (a measure of reservoir heterogeneity and the efficiency of oil displacement) is also shown for the two placement scenarios. This coefficient is a function of the flow and storage capacities (F and ϕ). It was highlighted as the most robust measure of reservoir heterogeneity and the best metric for ranking earth models in the work of Shook et al. (2009). A smaller value of this parameter represents a better displacement scenario; this is the case with the optimised well positions as shown in Fig. 5b compared to Fig. 5a.

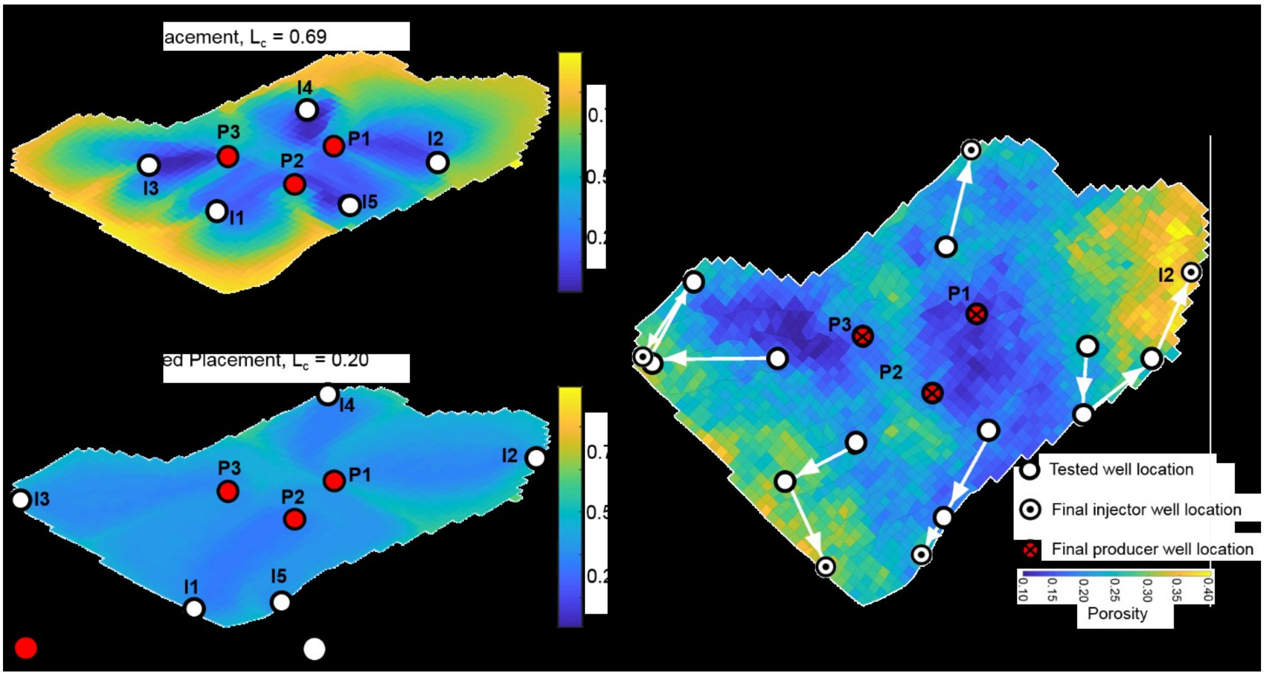

Figure 5: Oil saturation distribution with the initial well placements (a), optimal (b) well placements and well paths taken during optimisation computations (c). *PX* represents the production wells and *IX* the injection wells; where *X* is the well number (CS1).

F/ϕ denotes the ratio of flow capacity of the reservoir to its storage capacity (Fig. 6a). For a perfect/idealised displacement of oil in the reservoir, the *F/ϕ* ratio = 1 (this is never he case in real field operations). It is observed that the optimised well placement yields an *F/ϕ* curve closer to an idealized displacement scenario compared to initial well positions. In order to further validate the optimality of the new well configurations determined by the algorithm, we run multiphase flow simulations for a production timeframe of five years and obtain the oil recovery over this period. It is shown in Fig. 6b that the oil recovery of the optimised well placement far supersedes that of the initial well placement (twice the recovery of the initial placement at the end of the production forecast – Fig. 6b). This is indicative of the fact that intuitive-based well placements will hardly yield similar performance and oil recovery (field profitability) to that obtained by sound mathematical-based techniques. The well placement algorithm has thus capitalised on the underlying permeability distribution for the optimal determination of injection well locations.

In determining the optimal controls, a production horizon of 500 days is considered with a time step of 50 days. This is facilitates better understanding of the rates to apply over a shorter duration, since well controls in practice are typically carried out frequently. Re-optimising after this 500-day time horizon can always be done when significant changes in reservoir dynamics (such as the onset of water or gas coning, significant changes in pressure responses due to a previously undetected sealing fault etc.) occur. These changes could also affect the well dynamics and cause problems such as slugging. The results of the rate controls are shown next.

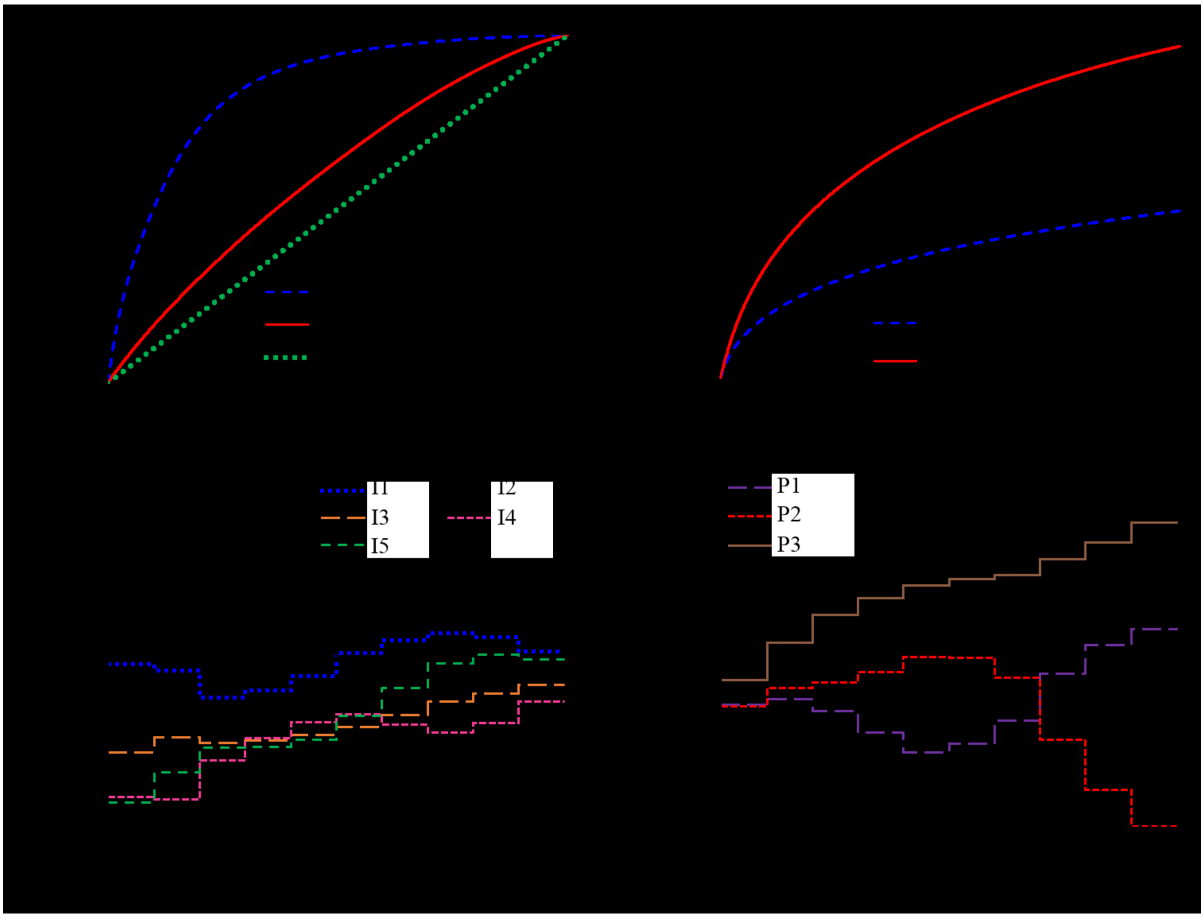

Figure 6: Oil displacement efficiency – F-ϕ diagram (a), percentage oil recovery and optimal controls for the injection (c) and production wells in the field (CS1).

The optimal control configurations of the injection and production wells based on the new well placements are shown Figs. 6c and 6d respectively. The application of the rate optimisation algorithm, which is based on the NPV indicates that injection well I2 with a steady decreasing injection rate at each timestep should be allocated the highest injection rate at the start of production. Next in magnitude is I1 with a relatively lower injection rate. I4 has the lowest injection rate compared to other injection wells and may be considered the least effective. However, to fully ascertain the efficiencies of these wells, further evaluation using well allocation factors is necessary (Fig. 8). Since operators have control over the injection rates at the surface, it can be said that the rate optimisation algorithm also inherently solves a rate allocation problem over a practical time step of 50 days (this is reasonable since reservoir dynamics is slow-paced). The production rate responses from the different wells indicates that P3 is the most productive well and significantly contributes to the overall field NPV.

To further evaluate the performance of the water flooding operations, we examine drainage volumes of the respective wells before and after the well placement optimisation (Fig. 7). The intersecting drainage volumes implies that injection and production wells have good connectivity. Although the drainage volume of P1 is increased on applying our optimisation algorithm, it is observed that P2 and P3 have lower drainage volumes in the optimal scenario. This can be likened to a compensating action (by the algorithm) to ensure drainage of the remaining reservoir regions. The algorithm has incorporated the local permeability distribution around the wells and their new positions to determine which well should drain most regions of the reservoir to ensure increased profitability (in this case, P1).

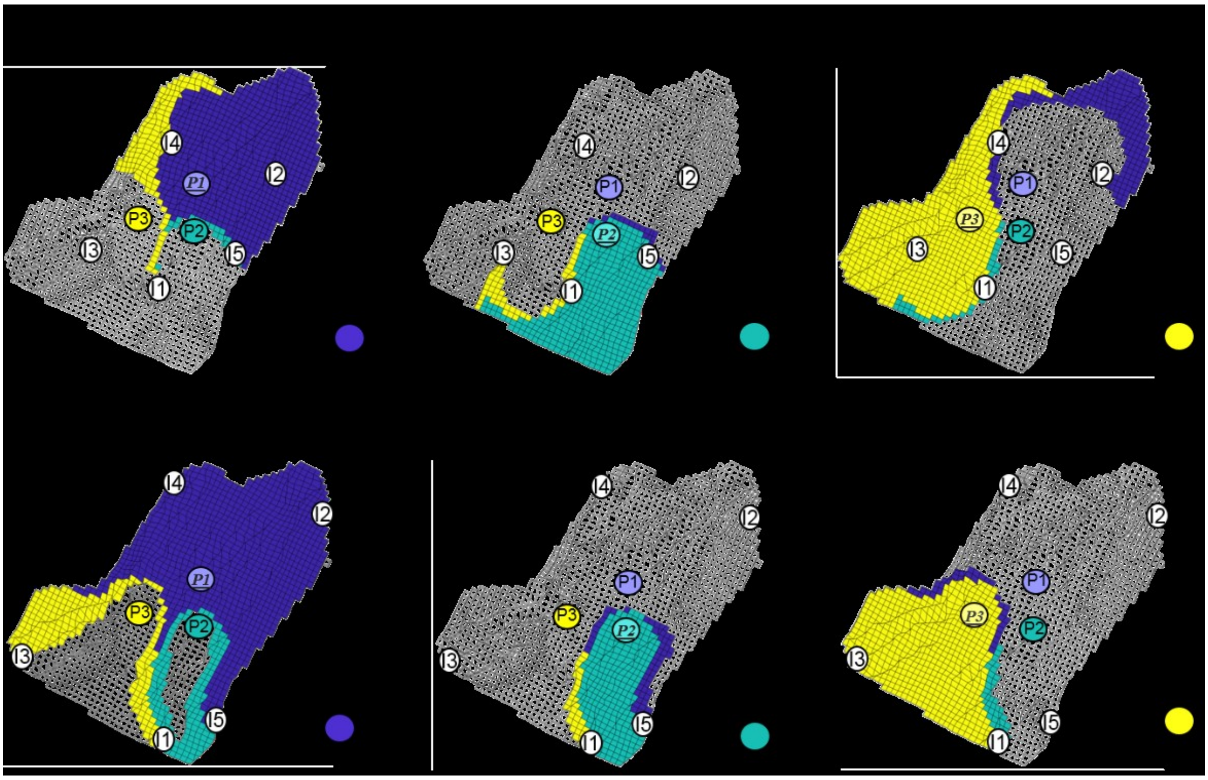

Figure 7: Drainage volumes for the initial and optimised well placements (CS1).

Fig. 8 shows the well allocation factors (the fraction of a producer's inflow that can be attributed to a particular injection well); it quantifies the influences that well pairs have on each other. As shown in Fig. 8, the production rate of P1 was initially only influenced by 3 production wells; however, the optimised scenario shows that better performance can be obtained when all wells contribute to its production. However, this is not always the case. For well P2, we observe an increased contribution of I1 compared to the approximately equal contributions from I1 and I5 respectively in the initial scenario. Despite the relatively low injection rate of I5 (Fig. 6), its contribution to the oil production is still somewhat significant. It is important to bear in mind that the oil production reflected in Fig. 8 relates to that obtainable via water injection, whereas, in Fig 6d, the rate response observed is due to both the effects of the reservoir's primary energy and the injection support. The applied methodology has quantified the performance of these wells; thus enabling decision support.

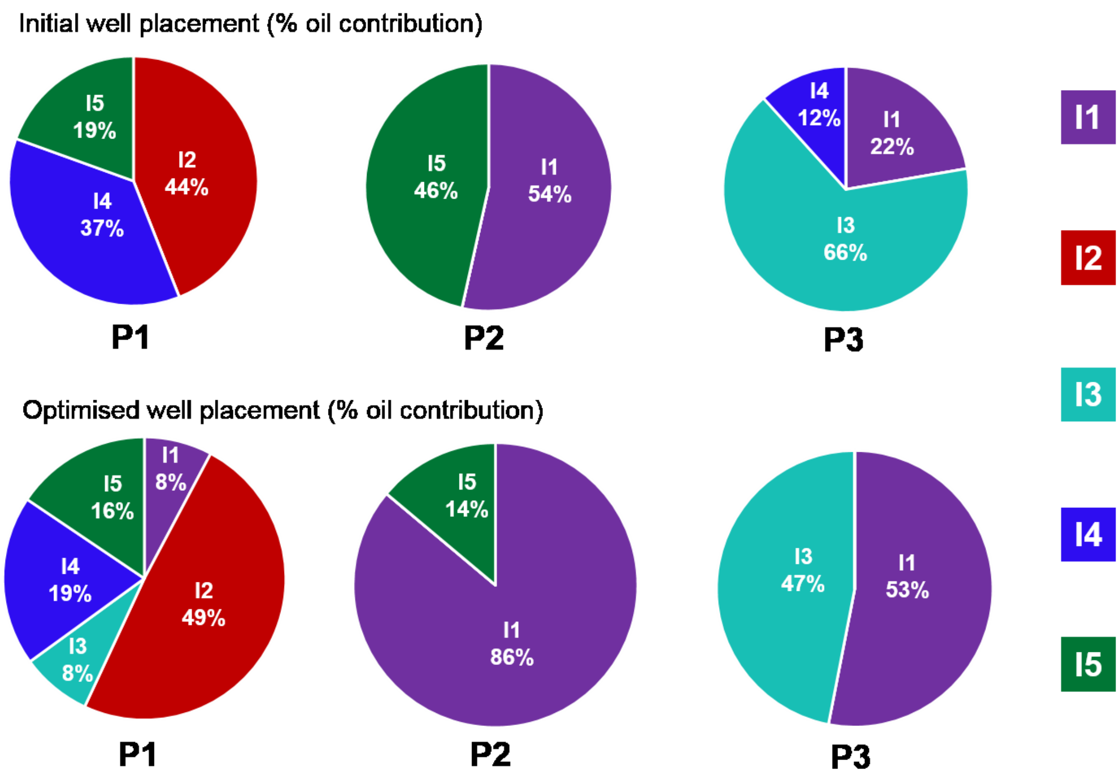

Figure 8: Well allocation factors for the initial and optimised well placements (CS1).

4.2 Optimising injection and production wells simultaneously (Case study 2 – CS2)

The results presented thus far have only considered injector well placement, while retaining the positions of the production wells. In this section we simultaneously account for both well types by performing algorithmic adjustments of their positions in a similar way to the previous case study. However, we implement a looped optimisation algorithm in 2 ways. In the first scenario, the production well positions are optimised first in the loop, after which the injection wells positions are optimised. This scenario is denoted as CS2-PI. The second scenario is the reverse of the first and is denoted as CS2-IP. In addition to the well paths taken during the optimisation procedure and the Lorentz coefficient, the recovery at the half way point (T = 2.5 years) is plotted and compared with CS1.

Table 3: Performance evaluation of CS1 and CS2

Case Study	L_c of optimised placement (-)	Oil recovery at T = 2.5 years (%)	Oil recovery at T = 5 years (%)	Computational Time (mins)
CS1	0.20	60	77	2.38
CS2-PI	0.20	58	74	3.14
CS2-IP	0.14	62	80	2.42

It is observed that the Lorentz coefficients for both CS1 and CS2-PI are the same; however there is a difference of 2% in the final oil recovery as shown in Table 1. While it is expected that a combined optimisation of injector and producer well placement would yield better oil recovery compared to optimisation of the injection well placements alone, this study has shown that this is not always the case.

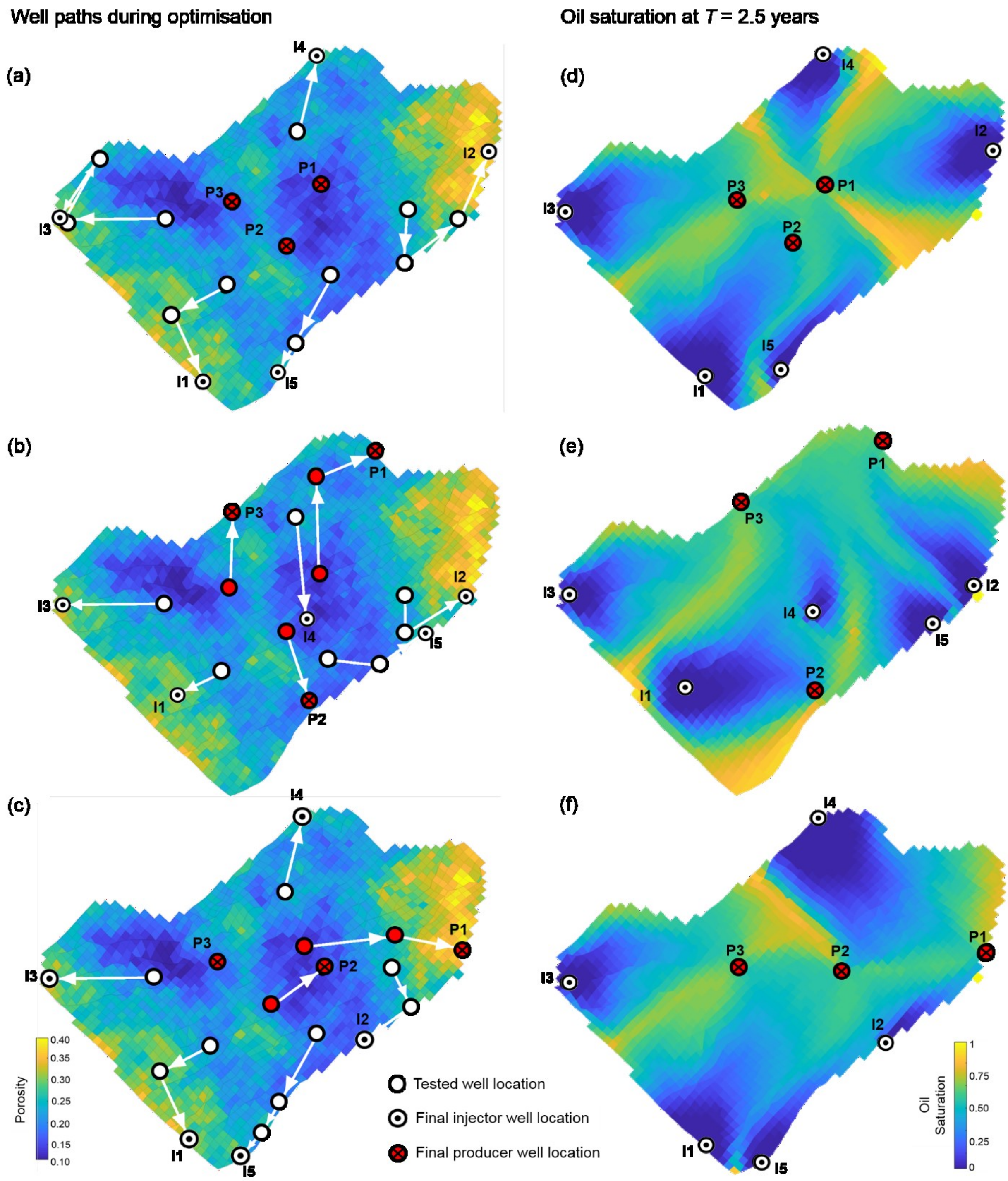

Figure 9: Paths taken by the well placement algorithm and the oil saturation for CS1 (a, d) CS2-PI (b, e) and CS2-IP (c, f) respectively.

Based on this observation, it can be concluded that the performance of the algorithm shows some sensitivity to the well type used in the outer loop. CS2-IP showed the best performance with a 30% decrease in the Lorentz coefficient and a corresponding 3% increase in recovery at the end of the production horizon compared to CS1. With an increased time horizon, this difference in the recovery is expected to increase further. For the production field considered in this work, it can be stated that the position of the injectors is more influential on the oil recovery obtained than the producers' placements.

As observed in Fig. 9, the well paths taken during the optimisation and the resulting reservoir fluid distributions differ significantly in all case studies. For each well position, it is observed that the best position is found in less than 4 iterative steps; thus demonstrating the computationally efficiency of the pseudo-well algorithm implemented herein. An exhaustive search approach (implemented in most evolutionary algorithms) may not achieve similar performance. In CS2-IP, the algorithm has retained the position of P3 as the optimal; the production wells are also somewhat aligned (Fig. 9f) along a straight line, thus creating a staggered injector-producer arrangement. We can also visualise the potential for water breakthrough at the respective production wells from the oil saturation plots (Figs, 10d, e, and f). In CS1, the production wells maintain a fixed centralised pattern and may experience water breakthrough at the same time. Although the location of P1 in CS2-PI is farthest from the injection wells, the fluid flow pattern/direction in the reservoir is generally towards this well. This may induce faster water breakthrough despite a high oil production rate. Furthermore, compared to CS2-PI in which the injector I2 is situated in the high porosity region (Fig. 9e), well P1 has been moved to this position in CS2-IP (Fig. 9f). It is thus shown that the production well can take better advantage of this high porosity region for enhancing oil recovery as shown in Table 2. Although these computations are based on the initial well positions in Fig. 1, preliminary tests were performed to by changing the initial well locations and convergence to similar well configurations in Fig 10 were obtained; this is an inherent advantage of the well placement algorithm as also demonstrated by Zandvliet et al. (2008).

Based on the superior performance of CS2-IP over CS2-PI, CS2-IP is the preferred case study for carrying out further rate control optimisation computations and flow diagnostics (in terms of drainage volumes and well allocation factors), which we present next.

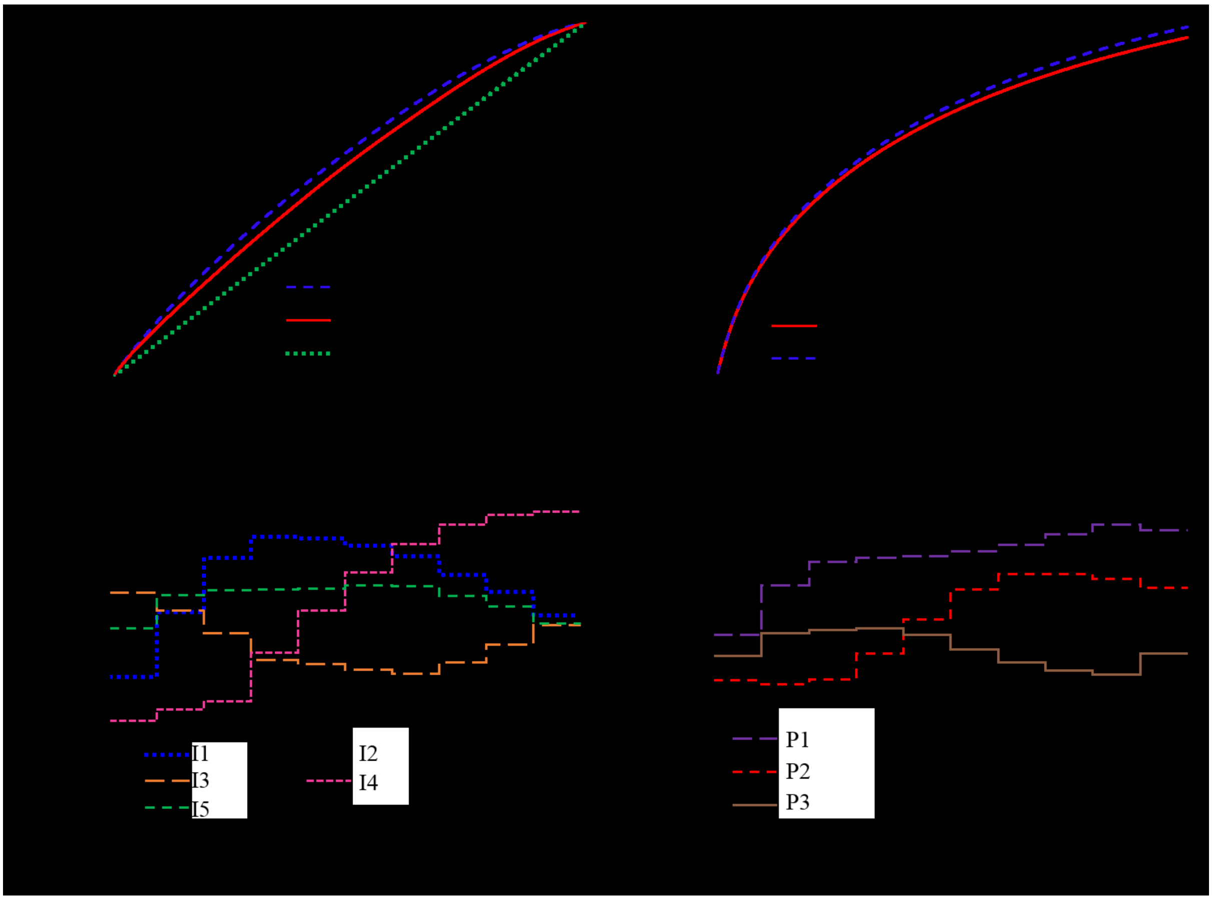

Figure 10: Oil displacement efficiency – F-ϕ diagram (a), percentage oil recovery and optimal controls for the injection (c) and production wells in the field (CS2-IP).

It is observed in Fig. 10a that the F- ϕ curve obtained for CS2-IP is closer to the idealised scenario than CS1. This improvement by increasing the degrees of freedom of well positions is also shown in the oil recovery plots (Fig. 10b) – final recoveries are also shown in Table 3. On analysing the water injection controls for the respective wells (Fig. 10c) it is observed that I1 and I4 are rate dominant wells. However, I2 can be shut until the 450th day, compared to CS1 in which the water injection rate at the start of the production horizon was the highest. The new position of P1 in CS2-IP has now made it the best performing production well as shown in Fig. 10d, whereas P2 dominates P3 from the 200th day of production. Despite the retainment of P3 at its initial location (Fig. 9c), the optimal rate controls have changed (Fig. 10d) in comparison to CS1 (Fig. 6d). The difference in the reservoir fluid distribution and resulting mobility changes around each well are the most likely reasons for this observation.

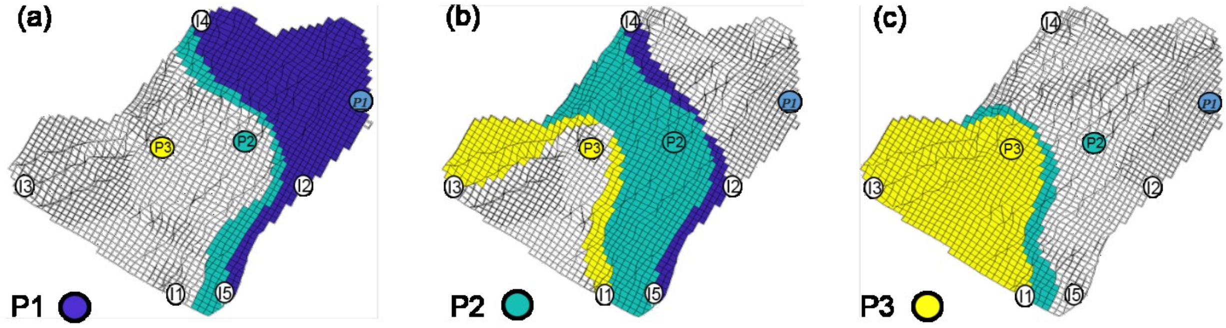

Figure 11: Drainage volumes for the optimised well placements (CS2-IP).

The drained volumes for the production wells in CS2-IP are shown in Fig. 11. Compared to CS1, in which P2 drained a considerably smaller portion of the reservoir (Fig. 7e), the drained volume here (Fig. 11b) is larger. In addition, the drained volume of P1 has been reduced in this case study compared to CS1. It is also observed that a rather equal split in the drained volumes of each production well of CS2-IP exists compared to a scenario in which the production well locations are not optimised (CS1). Thus, the result of our positional optimisation of both well types is to equilibrate the drainage volumes between all production wells. The results also show that a production well's performance may be significantly hindered by the local reservoir permeability distribution, which is in turn a function of its location in the reservoir.

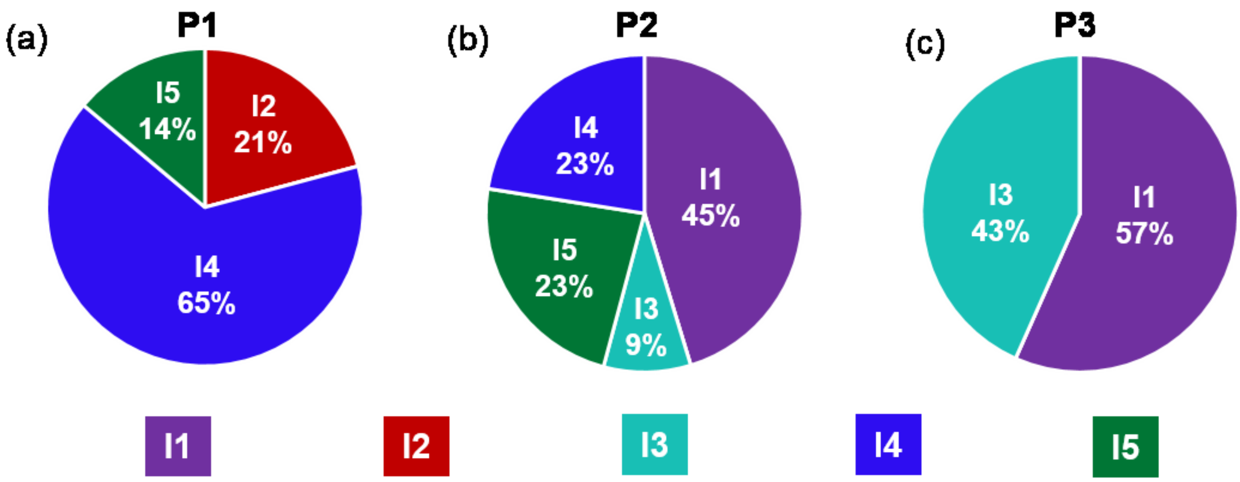

Figure 12: Well allocation factors for the optimised well placements (CS2-IP).

The well allocation factors of CS2-IP show that injection well I2, does not contribute to the oil production of any other well except P1 (Fig. 12a). This may also be observed in Fig. 9f in which the magnitude of the water front from I2 is relatively low and only in the direction of P1. Injectors I1 and I4 have significant contributions to the respective production wells. The high injection rates as displayed in Fig. 10c is the reason for this observation, hence, they may be termed the high efficiency injectors. Furthermore injection wells I1 and I3 maintain good hydraulic connectivity with P3 and are the main contributors to its performance (Fig. 12c). Since P2 and P3 are aligned (Fig. 9f) along the flow path from I3 (with P3 being the first encountered well by the pressure wave), P2 thus receives a relatively lower contribution from injector I3 (Fig. 12b). P1 which is further up the reservoir receives no support from this injection well I3 (Fig. 12a).

4.3 Pressure distribution in the reservoir

The reservoir pressure distributions before and after the well placement optimisation are shown in Fig. 13. It is observed that the optimal placement of the wells has resulted in slower decline in the reservoir pressure over the production time horizon of 5 years. An improved pressure support by changing the well locations has thus been demonstrated (similar results are obtained for CS1 and CS2-IP respectively). Nonetheless, compared to a primary production scenario, the pressure decline in the initial case will be slower, due to the action of injectors; thus favouring improved oil recovery. The regions of lowest pressure are mostly around the production well positions (due to their draining action); however, the lower region of the field (orange coloured) represent areas for which new wells might be drilled after an in-depth analysis of the water breakthrough times for the respective production wells.

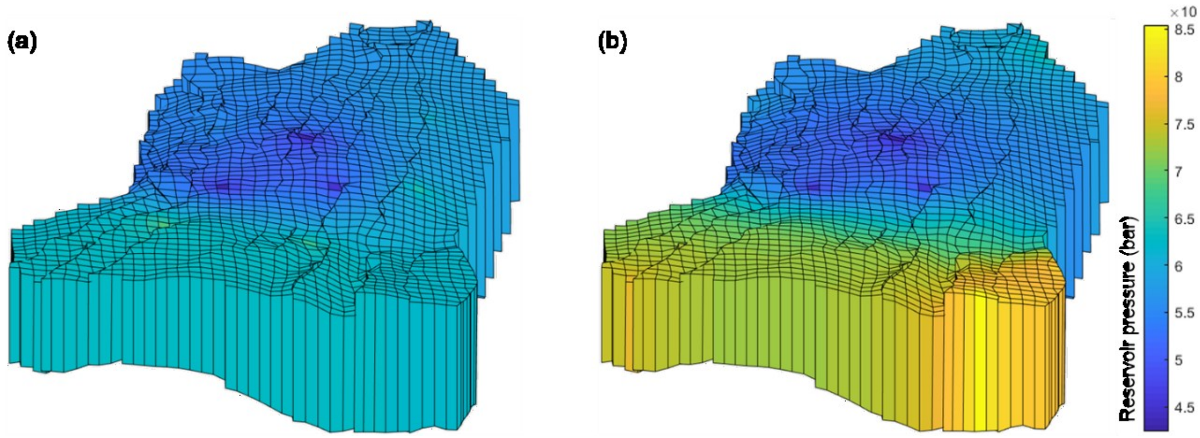

Figure 13: Reservoir pressure distribution for the initial (a) and optimised (b) well placements.

4.4 Computational Performance of CS1 & CS2

Well placement optimisation and rate control optimisation are the three separate computations performed in this work. All three steps were completed in less than 15 mins of computational time (Table 3 and Fig. 14). The application of the Lorentz coefficient as the ranking metric and the objective function of the well placement problem ensures that only a few timesteps of a given streamline model computation are required to achieve an optimal solution (Shook et al., 2009). Thus, the reservoir heterogeneity (irrespective of the number of cells) is easily and quickly assessible. In addition to the computational time required for the rate control optimisation step, we present the time required for the entire workflow (Fig. 14). It is observed that the bulk of the time is spent on the static model development and preliminary dynamic simulations to ascertain its performance.

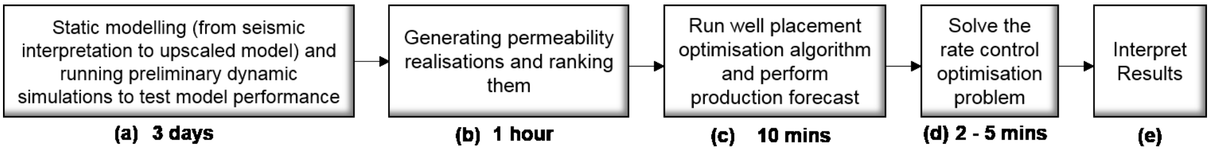

Figure 14: Time requirement for each step of the workflow for CS1 and CS2

The evolution of the NPV objective function for the rate control optimisation is shown in Fig. 15. As expected the NPV of the optimised scenario (CS1) is significantly higher (47%) than that obtained for the initial scenario. As expected the NPV of CS2-IP is higher (by 5%) than CS1. It is observed that within the first 5 iterations, the algorithm is able to find a near optimal solution; the solution of CS2-IP is obtained with fewer number of iterations. Compared to a methodology that requires numerous direct calls to a high-fidelity simulator or an approximation of the simulator's output (as shown in Epelle and Gerogiorgis, 2019c), the herein implemented algorithm attains optimality in fewer iterations. All computations were performed on an Intel Core i7-6700 CPU @ 3.40GHz machine with 4 processing cores.

Although the presented case study is somewhat small (in terms of the number of wells), such rapid computational performance is also expected when the problem size increases (Møyner et al., 2015). It is also worth mentioning that the computational performance observed herein is a consequence of the sequential optimisation of placement and controls performed. Our rationale for choosing the sequential approach (as opposed to a joint approach) in this work is based on the findings of Humphries and Haynes (2014). They demonstrated via some cases studies, superior performance of a decoupled/sequential approach compared to a joint optimisation method. A possible reason for this is that the initially fixed control scheme is one that is favourable for finding good solutions of well placements. As such the solution/search space of the problem is significantly reduced, thus yielding a more thorough exploration by the algorithm. Despite the fact that the solution pace of the joint approach contains all solution the decoupled approach could possibly find, it becomes harder to find these solutions when the search space is larger (Humphries and Haynes, 2014). This concept of limiting the search space for optimal well placements has also been demonstrated by Onwunalu and Durlosfsky (2010).

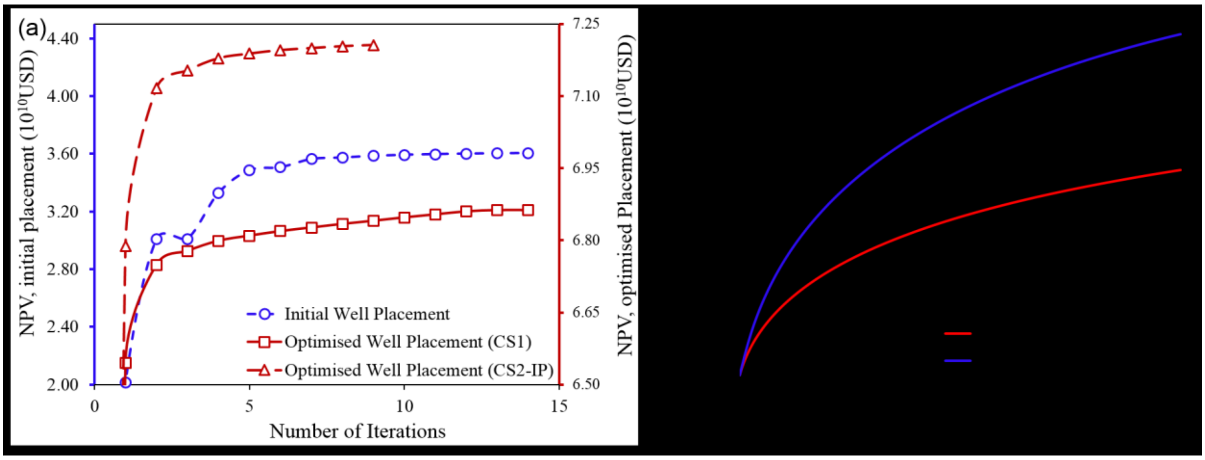

Figure 15: Objective function evolution for rate control optimisation for CS1 and CS2 (a); oil recovery obtained when NPV and Lc are used as the objective function for well placement optimisation.

It may be also useful to optimise the well controls at specific time intervals after optimising the well placements. This utilises the current pressure distribution, fluid saturation profile and other fluid and rock properties in the reservoir which may have changed significantly. Streamline simulation could also be performed to examine the dynamic injection efficiency, so that if there is a drastic change in this efficiency, the re-optimisation of the well controls is justifiable. Although this will inevitable increase the number of functional evaluations, it is expected that the computational cost will be reasonable, given the robust gradient formulation implemented. Fig. 15b shows that well placement optimisation based on *Lc* yields a higher recovery than when the *NPV* is the objective function; thus justifying our multi-objective optimisation strategy.

4.5 Case Study 3 (Analysis of optimisation search space for each well)

In this case study, we demonstrate the applicability of the presented approach to a more challenging and bigger scenario. We mainly investigate the sensitivity of the optimal oil recovery to the supplied search radius in the well placement algorithm (when optimising both injector and producer placements). In doing so, the strategy presented in CS2-IP is implemented. A more complex geological model of the Norne Field is applied, consisting of 20 production wells and 10 injection wells. The Norne Field (NTNU, 2012) is compartmentalised (2 parts which are in communication through the drilled wells – Fig. 16). All drilled wells since 2006 have been included with their original locations retained but with horizontal well configurations changed to vertical. Furthermore, the reservoir fluid considered is the same as that used in CS1 and CS2 respectively. The robustness of the SGS algorithm is again demonstrated with successful generation of permeability distributions as seen in Fig. 16 (a-d).

The resultant effect of 30 wells present in this case study is that the well search radius (WSR) is limited to avoid severe well interference problems and its negative impact on production. Compared to CS1 and CS2 in which the search radius was defied as 10 grid blocks, detailed analysis on this case study utilises a maximum of 5 grid blocks. The well paths taken during the optimisation procedure are shown in Figs. 17-19. It is observed that the number of well positions explored by the optimisation algorithm increases as the search radius increases. However, the result of this increased search area is not necessarily and increase in the corresponding oil recovery (Table 4).

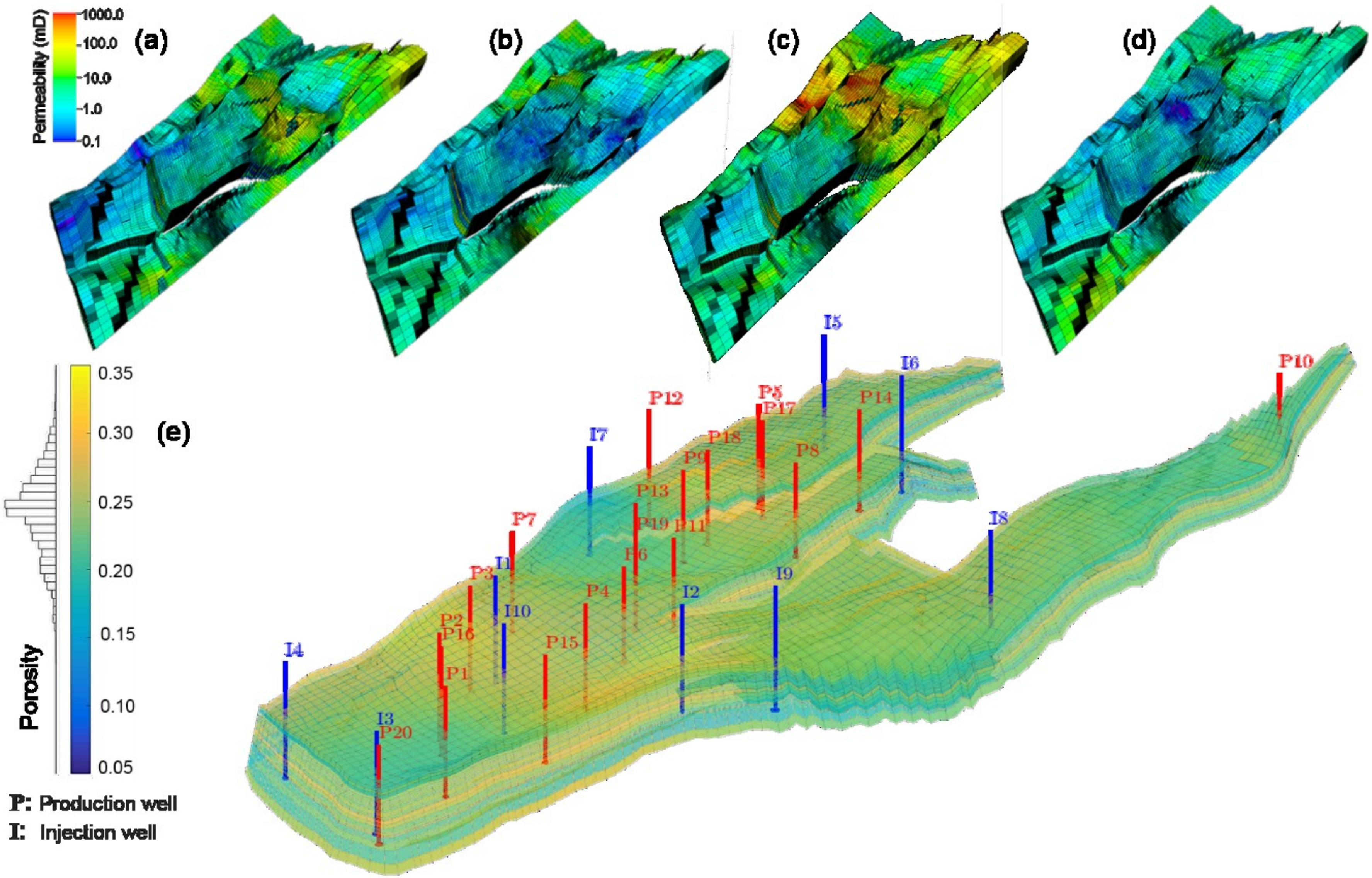

Figure 16: The Norne Field showing different realisations of the permeability distribution (with active and inactive cells), the field porosity distribution and the initial locations of the wells.

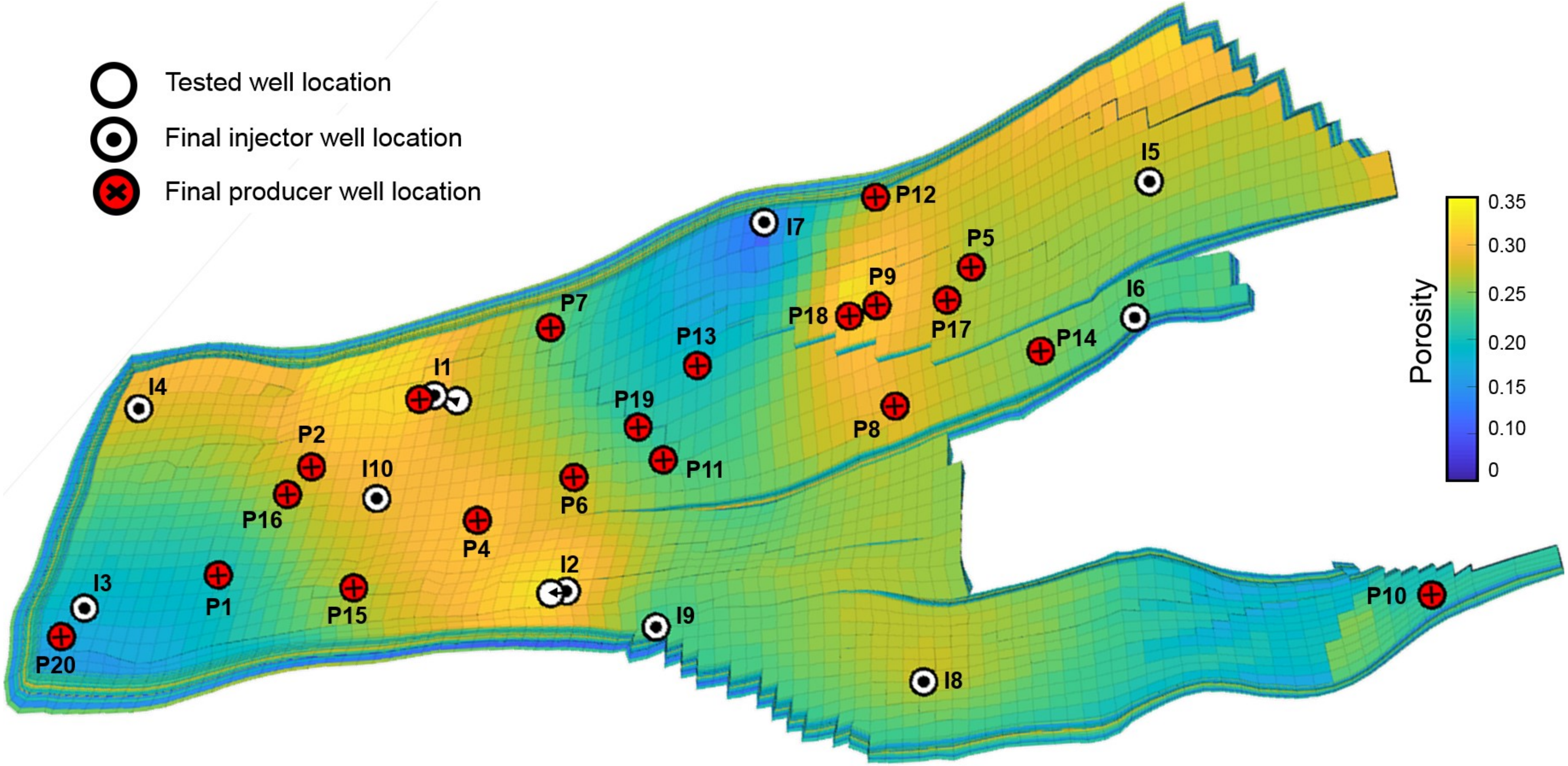

Figure 17: Well path taken during the optimisation procedure for WSR = 3

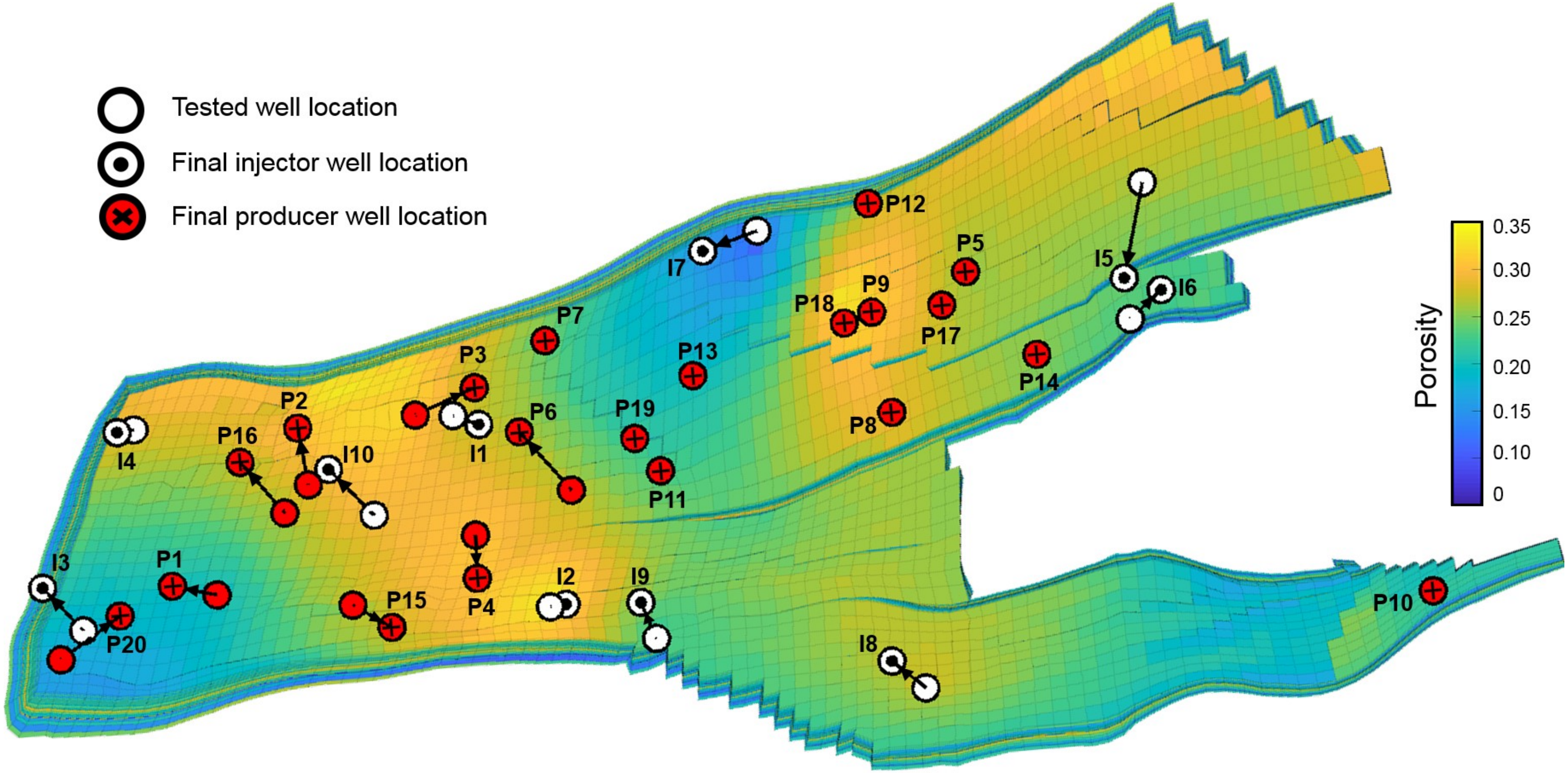

Figure 18: Well path taken during the optimisation procedure for WSR = 4

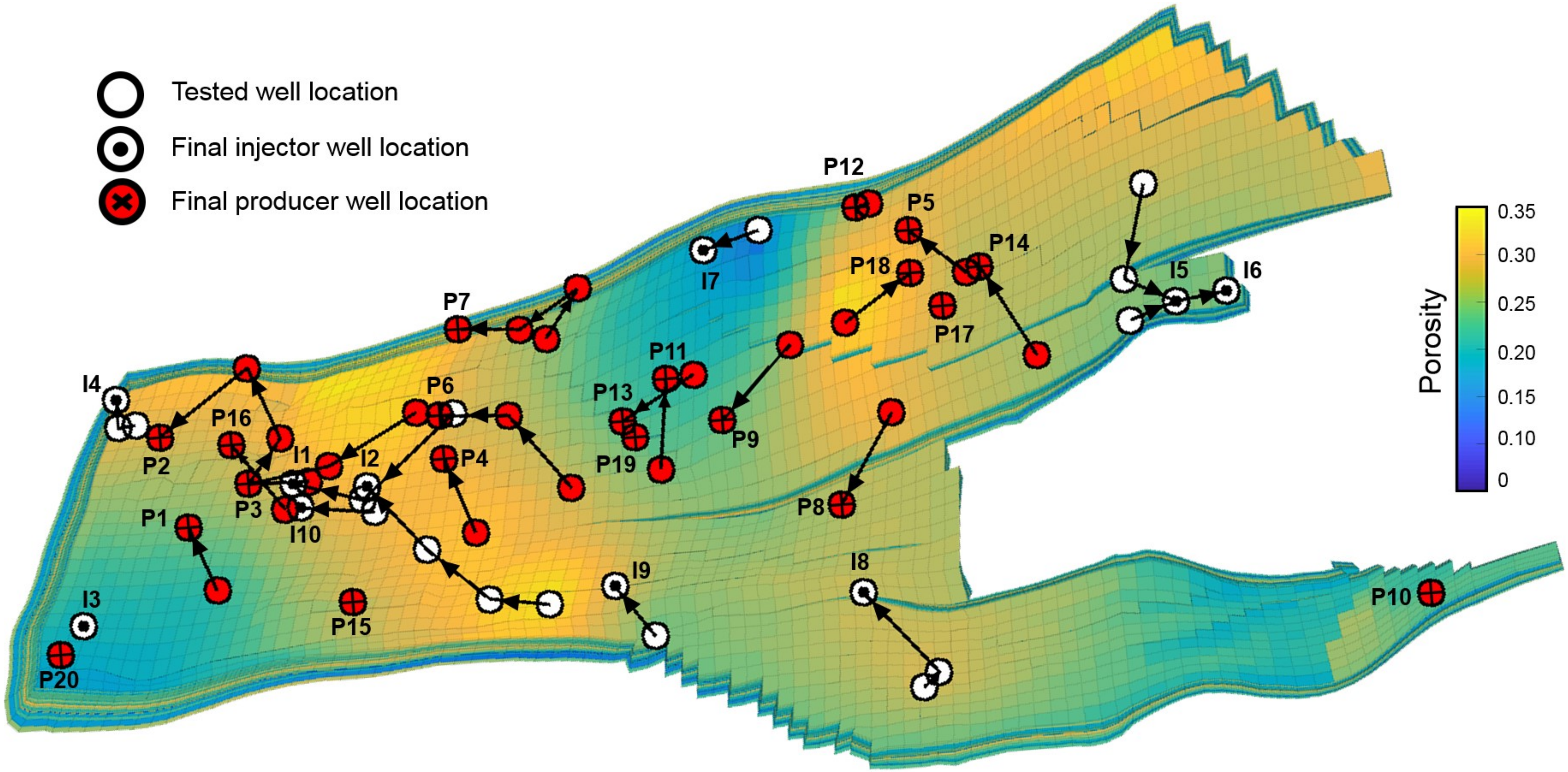

Figure 19: Well path taken during the optimisation procedure for WSR = 5

On performing a two-phase flow simulation for 5 years on the optimised well positions, it is observed in Table 4 that the oil recovery (at 2.5 years) increases from 66% to 70% when WSR increases from 3 to 4, but reduces to 67% when WSR increases to 5. A similar trend is observed for the recovery at 5 years and the *Lc* respectively. On applying a WSR of 10, the resultant effect was operationally infeasible well clusters with a significantly reduced recovery of 30%. Thus, careful selection of the well search radius when using the well placement algorithm is essential, especially in fields with several candidate wells. The inclusion of minimum inter-well distance constraints is a possible remedy for improving the algorithms' performance; however, this is subject to further investigation and is beyond the scope of this study. It is also observed that the computational time required, as seen from Table 4 significantly increases as WSR increases. The time requirements for CS3 are also higher than those of CS1 and CS2 which contain only 8 wells. However, compared to studies such as Tavallali et al. (2013) with a smaller problem size than CS3 (with computational times reaching 27 hours), the run times achieved in this work are significantly shorter.

Table 4: Performance evaluation of CS3

Case Study	L_c	Oil recovery at T = 2.5 years (%)	Oil recovery at T = 5 years (%)	Computational Time (mins)
Initial well positions	0.56	64	76	-
CS3 (WSR =3)	0.53	66	79	47.4
CS3 (WSR =4)	0.50	70	84	71.4
CS3 (WSR =5)	0.52	67	80	205.2

Fig. 20 shows the oil saturation in the field after a production timeframe of 2.5 years for the base case and the different optimal configurations for WSR of 3, 4 and 5 respectively. As can be observed, a WSR of 4 yields the best recovery with a significant portion of the field drained using the same number of wells compared to the other scenarios. Based on the optimal well positions obtained from WSR of 4, the rate control optimisation procedure can be performed to study individual well performance. It is observed that P15 is the best performing well with the highest flowrate (Fig. 21b); whereas, injection well I4 can be allocated the highest water injection rate (Fig. 21c). The optimal well configurations are obtained in 13 iterations and within 15 mins of computational time.

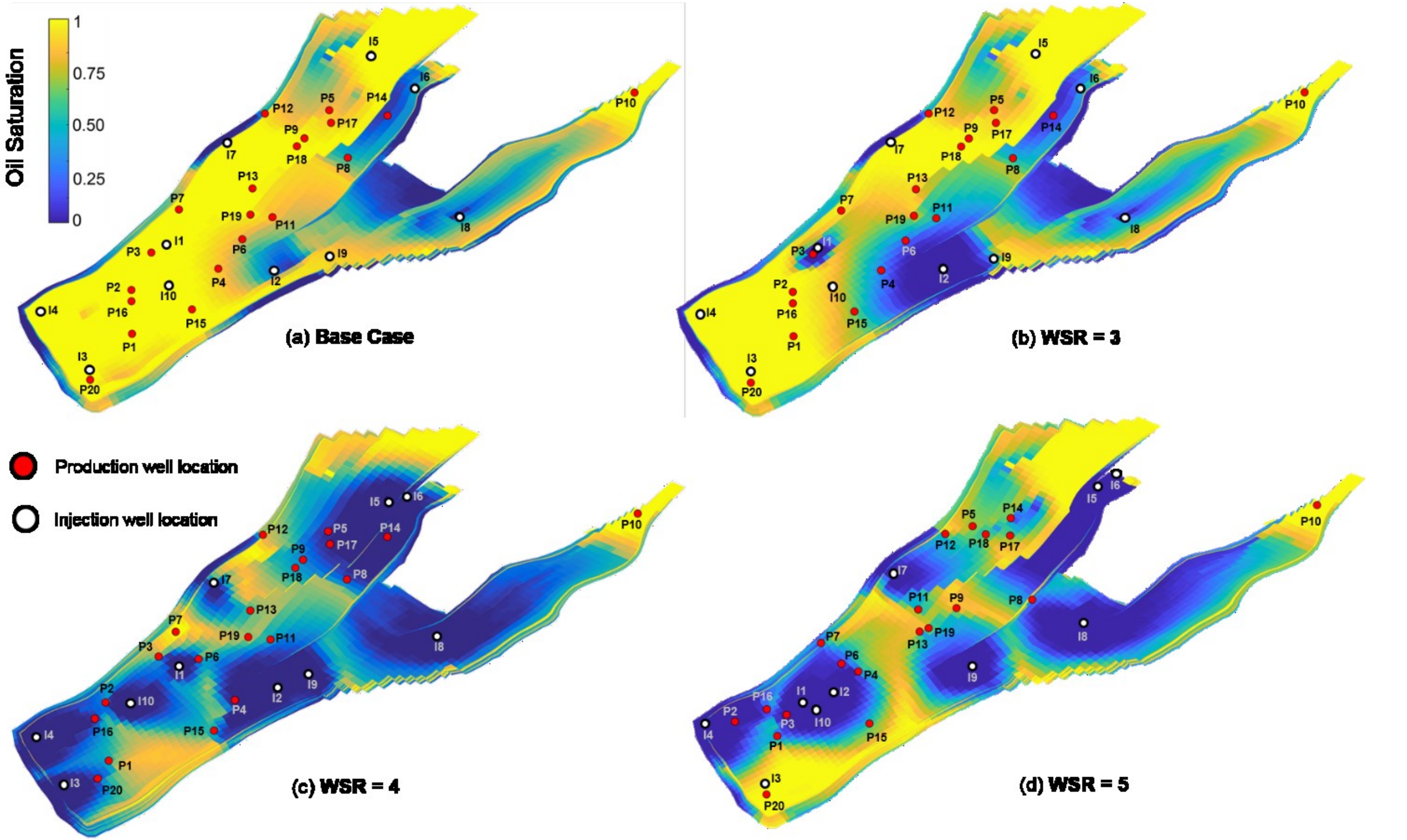

Figure 20: Oil saturation at T = 2.5 years for CS3.

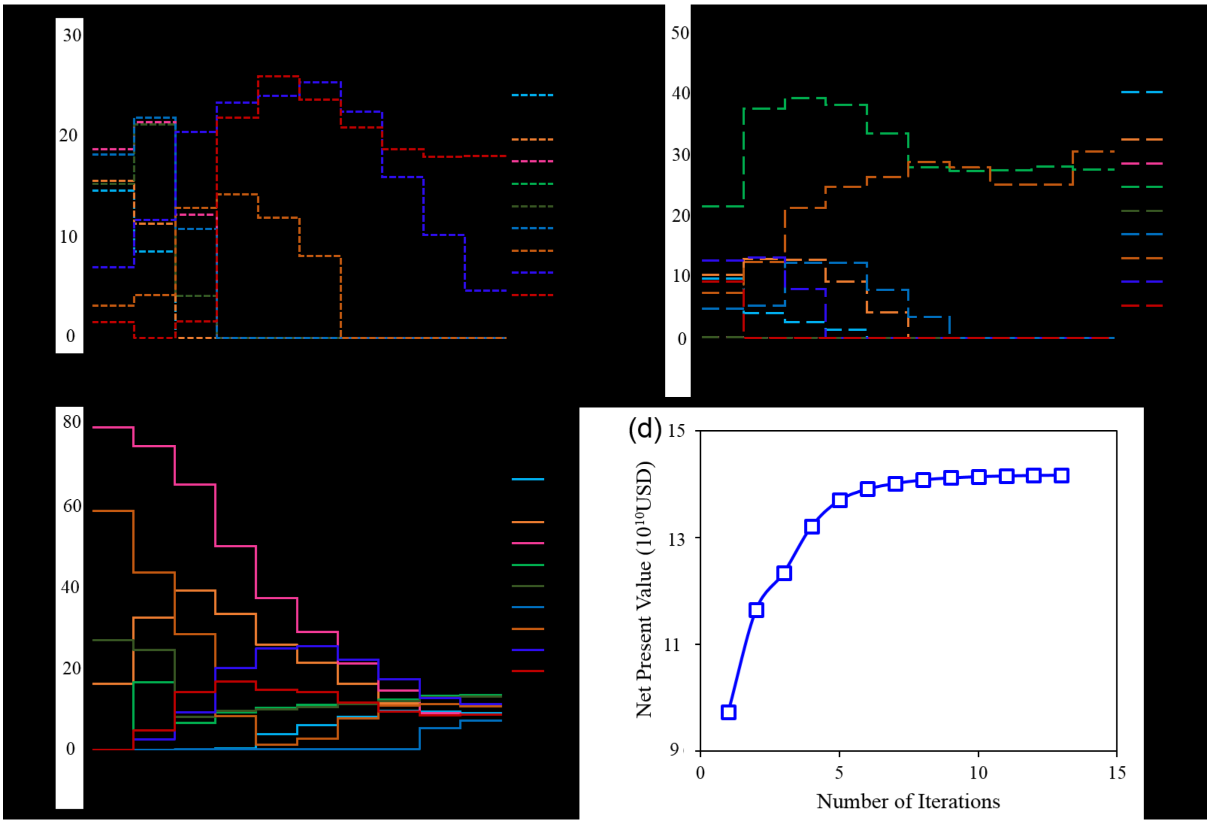

Figure 21: Optimal controls for the injection and production wells of CS3.

5.0 Conclusions and future work

In this work, we have addressed an injection and production well placement and rate control optimisation problem of a realistic field under geological uncertainty. We have incorporated geological uncertainty by using the worst case scenario of the 50 ranked permeability realisations and derived the following conclusions.

- The application of a well placement optimisation algorithm yields a boost in field oil recovery – twice the value obtained using intuition based methods alone.
- Optimal controls for each injection well were determined; thus allowing for a step wise adjustment of surface injection rates and water allocation in real operations.
- Robust computational approach that utilises an adjoint formulation enables rapid optimisation performed in a matter of minutes.
- Optimising both producer and injection well placement resulted in a 30% reduction in the Lorentz coefficient; this translates to 3% increment in recovery. In our case study, the injector well placement significantly affected final oil recovery compared to producer placements.
- The well placement algorithm is sensitive to the type of wells implemented in the outer optimisation loop (whose location is optimised first). The algorithm performs better when injection wells are first optimised in the optimisation loop compared to production wells.
- There is an equilibration of the respective drainage volumes of the production wells by simultaneously optimising the placements of production and injection wells in the field considered in this work.

- It was shown in this study that an increased search space (in terms of the WSR) for the optimisation algorithm does not necessarily guarantee improved oil recovery. The WSR is also a parameter that may be optimally determined for truly optimal well positions to be attained.

Proper calibration of the reservoir model using industrial field data would constitute the next steps of this work. This is expected to increase the reliability of forecasts and the optimal injection strategy obtained herein. Stochastic optimisation (with the probabilities of the respective geological realisations incorporated) of this history-matched model can then be performed to reduce the conservativeness of the optimal solution (obtained from the worst case scenario) and hence increase the field's NPV. The drilling sequence/scheduling problem is also worth considering, since all wells are not drilled simultaneously in practical operations. By introducing horizontal/deviated wells, the complexity of the well placement optimisation procedure increases; this requires further investigation. Simultaneous optimisation of well placement and controls using gradient-based methods is still an open research question that requires further investigations. Finally, the adopted objective functions are nonconvex; hence an efficient subdivision of the parameter space to avoid trapping at local optima is a modification that could enhance the performance of the rate optimisation algorithm.

6.0 Acknowledgements

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7.0 Nomenclature

d : discount rate (%)

F: Flow capacity

g : Constraints

G : Objective function

i_{best} : Pseudo well with highest gradient

IX : Injection well X

J : Jacobian

K : Permeability tensor (mD)

$L_{c,o}$: Lorenz coefficient

n_w : Total number of wells

n_{bh} : Number of BHP controlled wells

n_r : Number of rate controlled wells

$N_{pf}(k)$: Perforation belonging to well k

NPV : Net present value (USD)

$N_w(j)$: Index of well with perforation j

p : Pressure (Pa)

P_{bh} : Bottomhole pressure – BHP (bar)

P_0 : Initial pressure (bar)

PX : Production well X

$\mathcal{P}, \mathcal{V}, \mathcal{T}, \mathcal{Q}, \mathcal{C}$: Discretised system of equations in terms of p, v, τ, q_{pf}, p_{bh}

q : Flowrate (m^3/day)

q_i : Well injection rate (m^3.day^{-1})

q_p : Well production rate (m^3.day^{-1})

Q_i : Maximum injection rate (m^3.day^{-1})

Q_p : Maximum production rate (m^3.day^{-1})

r_c, r_{ci} : Revenue and cost factors (USD/m^3)

S_o : Oil saturation

t : Control time interval

$\boldsymbol{u}$: Control vectors

u_r^k : Rate controlled wells

u_{bh}^k : BHP controlled wells

$\vec{v}$: Darcy velocity (m/s)

W_{pf}^j : Peaceman well index

WSR: Well Search Radius

$\boldsymbol{x}$: State variables

Greek symbols

μ_o : Oil viscosity (cP)

μ_w : Water viscosity (cP)

ϕ: Storage capacity

λ_f : Fluid mobility

λ: Lagrange multiplier

ρ_o : Oil density (kg.m^{-3})

ρ_w : Water density (kg.m^{-3})

τ : Time-of-flight – TOF (day)

Subscripts and superscripts

c : Phase index

j : Perforation index

k : Well index

o : Oil phase

T : Matrix transpose

w : Water phase

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