

УДК 621.396.677

Directional Characteristics of Circular Scanning Aperture

Vasily S. Panko^{a*},**Yury P. Salomatov^a and Mikhail I. Sugak^b**^a*Siberian Federal University*

79 Svobodny, Krasnoyarsk, 660041, Russia

^b*Saint Petersburg Electrotechnical University «LETI»*

5 Professora Popova Str., St. Petersburg, 197376, Russia

Received 17.07.2013, received in revised form 23.08.2014, accepted 10.09.2014

In this paper explicit analytical expressions are derived which describe basic directional characteristics of circular scanning aperture: radiation pattern for different kinds of amplitude distribution, beamwidth, part of radiated power contained in the main lobe, directivity. Consideration is based on the theory of continuous ring radiator within the scalar approximation.

Keywords: scanning, circular aperture, ring antenna

Характеристики направленности круглой сканирующей апертуры

В.С. Панько^a,**Ю.П. Саломатов^a, М.И. Сугак^b**^a*Сибирский федеральный университет*

Россия, 660041, Красноярск, пр., Свободный, 79

^b*Санкт-Петербургский государственный**электротехнический университет «ЛЭТИ»*

Россия, 197376, Санкт-Петербург, ул. Попова, 5

В данной статье приведены явные аналитические выражения, описывающие основные характеристики направленности круглой сканирующей апертуры: диаграмма направленности для различных видов амплитудного распределения, ширина главного лепестка, часть излучаемой мощности, содержащейся в главном лепестке, коэффициент направленного действия. Рассмотрение основано на теории непрерывного кольцевого излучателя в скалярном приближении.

Ключевые слова: сканирование, круговая апертура, кольцевая антенна.

© Siberian Federal University. All rights reserved

* Corresponding author E-mail address: vpanko@sfu-kras.ru

1. Introduction

Directional properties of the radiating circular aperture are studied in the literature in detail [1, 2]. However, all presented results usually describe a uniform-phase aperture, while characteristics of the scanning aperture are determined numerically. In this paper we present explicit analytical expressions describing the main directional characteristics of circular scanning aperture: radiation pattern for different kinds of amplitude distribution, beamwidth, part of radiated power contained in the main lobe, directivity. Also we analyze the directivity changing while scanning. All consideration is based on the theory of continuous ring radiator within the scalar approximation.

2. Directional characteristics

Radiation pattern

Radiation pattern of circular aperture can be written in the form of Huygens-Green integral [3]:

$$f(\theta, \varphi) = \int_0^R \int_0^{2\pi} \dot{I}(r, \varphi') e^{ikr \sin \theta \cos(\varphi' - \varphi)} r d\varphi' dr, \tag{1}$$

where $\dot{I}(r, \varphi')$ – field amplitude and phase distribution over the aperture, $k = 2\pi / \lambda$, (θ, φ) – angles of spherical coordinate system, r, φ' – integration point coordinates (Fig. 1).

Various kinds of amplitude and phase distribution are considered in [4]. For circular scanning aperture examination we suppose axially symmetric amplitude distribution:

$$|\dot{I}(r, \varphi')| = I(r),$$

and phase distribution which establishes maximum radiation in direction (θ_0, φ_0) , i.e.

$$\Psi(r, \theta_0, \varphi_0, \varphi') = kr \sin \theta_0 \cos(\varphi_0 - \varphi').$$

Then

$$f(\theta, \varphi) = \int_0^R I(r) r \int_0^{2\pi} e^{ikr[\sin \theta \cos(\varphi' - \varphi) - \sin \theta_0 \cos(\varphi_0 - \varphi')]} d\varphi' dr.$$

Internal integral here represents radiation pattern of a continuous ring radiator, and it can be written as [5]:

$$F(r, \theta, \varphi, \theta_0, \varphi_0) = J_0\left(kr \sqrt{\sin^2 \theta + \sin^2 \theta_0 - 2 \sin \theta \sin \theta_0 \cos(\varphi - \varphi_0)}\right),$$

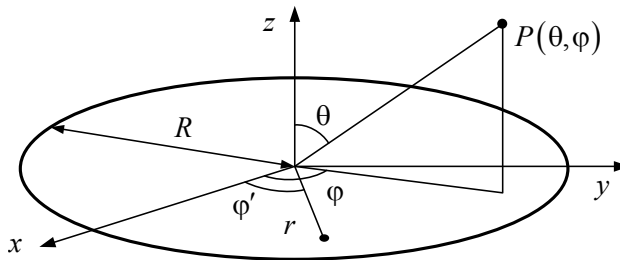


Fig. 1. Circular aperture

where $J_0(z)$ is the Bessel function of the first kind of order zero. Then radiation pattern of circular aperture takes the form:

$$f(\theta, \varphi) = \int_0^R I(r) r J_0[kr \cdot \delta(\theta, \varphi, \theta_0, \varphi_0)] dr$$

where

$$\delta(\theta, \varphi, \theta_0, \varphi_0) = \sqrt{\sin^2 \theta + \sin^2 \theta_0 - 2 \sin \theta \sin \theta_0 \cos(\varphi - \varphi_0)}.$$

Now we can obtain the expression of radiation pattern for scanning circular aperture with uniform amplitude distribution $I(r) = I_0$:

$$F(\theta, \varphi, \theta_0, \varphi_0) = \frac{2J_1[kR\delta(\theta, \varphi, \theta_0, \varphi_0)]}{kR\delta(\theta, \varphi, \theta_0, \varphi_0)}, \quad (2)$$

where $J_1(z)$ is the Bessel function of the first kind of first order.

For axially symmetric amplitude distributions which taper towards aperture edge:

$$I(r) = (1 - \Delta) + \Delta \left[1 - \left(\frac{r}{R} \right)^2 \right]^n, \quad (3)$$

where $1 - \Delta$ – excitation level at the aperture edge, $n = 1, 2, \dots$, we can obtain analogically

$$f(\theta, \varphi) = (1 - \Delta) \Lambda_1(u) + \frac{\Delta}{n+1} \Lambda_{n+1}(u), \quad (4)$$

where $\Lambda_n(u) = J_n(u) n! / \left(\frac{u}{2} \right)^n$, $u = kR\delta(\theta, \varphi)$, J_n – the Bessel function of the first kind of n -th order.

Expressions (2), (4) represent radiation patterns of circular aperture for arbitrary direction of radiation maximum (θ_0, φ_0) . Fig. 2 demonstrates cut section of radiation pattern in $\varphi = 0$ plane for aperture with $R = 4\lambda$ and amplitude distribution of (3), where $\Delta = 1$ and $n = 1$.

Using of presented expressions, we can determine other directional properties of circular aperture: beamwidth for arbitrary direction of main lobe; directivity; fraction of radiated power concentrated within the main lobe.

Beamwidth of scanning circular aperture

Beamwidth of circular aperture radiation pattern in different cut sections can be derived as [5]:

$$2\Delta\theta = \frac{0.365\lambda}{R\sqrt{0.5 \cos \theta_0}}, \quad 0 \leq \theta_0 < \frac{\pi}{2}, \quad \varphi = \varphi_0 = \text{const}. \quad (5)$$

Here R – aperture radius. Fig. 3 represents change of beam width depending on the direction of the radiation maximum θ_0 ($\varphi_0 = 0$).

Fraction of power, concentrated in main lobe

Fraction of radiated power, concentrated within main lobe of circular aperture radiation pattern can be written as

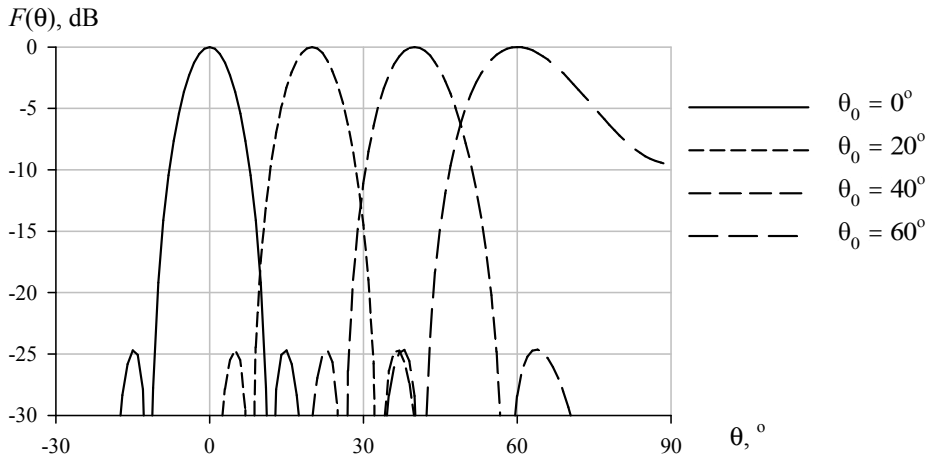


Fig. 2. Radiation pattern of circular aperture $R = 4\lambda$ with taper amplitude distribution (3) for different direction of radiation maximum θ_0

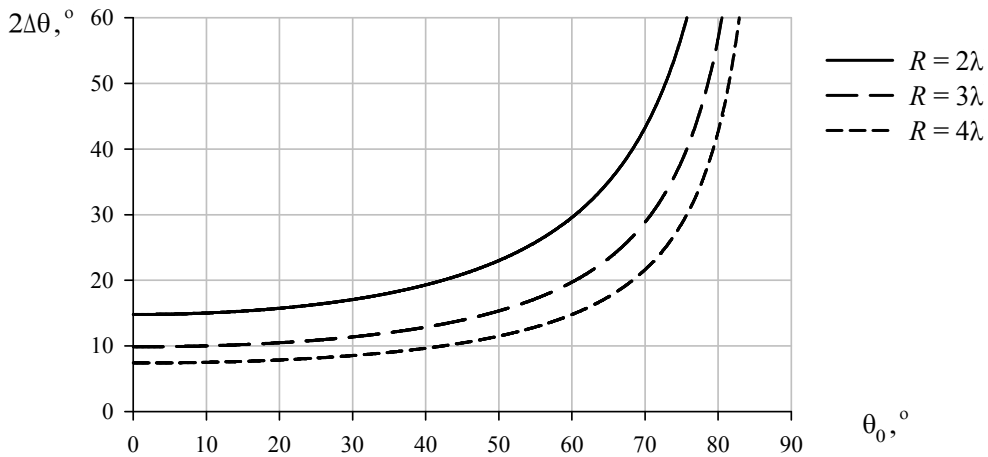


Fig. 3. Beamwidth of circular aperture radiation pattern for different R and radiation maximum direction

$$P_0 \cong \int_0^{2\pi} \int_0^{\Delta\theta} F^2(\theta, \varphi) \sin\theta d\theta d\varphi.$$

In [3] it is stated that factor $(1 + \cos \theta) / 2$ is excluded from (1). This factor for small θ can be approximated as $\sqrt{\cos\theta}$. It allows us to make a suggestion that circular aperture with uniform amplitude distribution consists of “elementary” radiators with radiation patterns in the form of $\sqrt{\cos\theta}$. In this way we can obtain:

$$P_0 \cong \frac{8\pi}{k^2 R^2} \int_0^{\Delta\theta} \frac{J_1^2(kR \sin\theta)}{kR \sin\theta} d(kR \sin\theta).$$

By changing of variable $t = kR \sin \theta$:

$$P_0 \cong \frac{8\pi}{k^2 R^2} \int_0^{kR \sin \Delta\theta} \frac{J_1^2(t)}{t} dt. \quad (6)$$

For power allocated in the whole radiation pattern, supposing radiation in upper semi-sphere only $\Delta\theta = \pi / 2$,

$$P_\Sigma \cong \frac{8\pi}{k^2 R^2} \int_0^{kR} \frac{J_1^2(t)}{t} dt.$$

These expressions can be transformed in the following manner. Since

$$\int_0^x x J_0^2(x) dx = \frac{x^2}{2} [J_0^2(x) + J_1^2(x)] \text{ and}$$

$$\int_0^x \frac{1}{x} J_n^2(x) dx = \frac{1}{2n} \left[1 + J_0^2(x) + J_n^2(x) - 2 \sum_{k=1}^n J_k^2(x) \right],$$

([4], p. 38), we can get

$$\int_0^x \frac{1}{x} J_1^2(x) dx = \frac{1}{2} [1 + J_0^2(x) - J_1^2(x)].$$

By integrating of (6) and using the results from [5], we can write:

$$P_0 \cong \frac{4\pi}{(kR)^2} [1 + J_0^2(kR \sin \Delta\theta) - J_1^2(kR \sin \Delta\theta)], \quad (7)$$

$$P_\Sigma \cong \frac{4\pi}{(kR)^2} [1 + J_0^2(kR) - J_1^2(kR)]. \quad (8)$$

Fraction of power, concentrated within main lobe, will be equal to:

$$\frac{P_0}{P_\Sigma} = \frac{1 + J_0^2(kR \sin \Delta\theta) - J_1^2(kR \sin \Delta\theta)}{1 + J_0^2(kR) - J_1^2(kR)}. \quad (9)$$

Half beamwidth can be found from (5):

$$\Delta\theta = \frac{0.365\lambda\sqrt{2}}{2R}.$$

For large apertures $R \gg \lambda$ half beamwidth has small values $\Delta \ll 1$. Therefore in (9) we can substitute sinus by its argument. Considering that $\lim_{R \rightarrow \infty} J_{0,1}(kR) \rightarrow 0$:

$$\frac{P_0}{P_\Sigma} = 1 + J_0^2(0.365\pi\sqrt{2}) - J_1^2(0.365\pi\sqrt{2}),$$

or

$$\frac{P_0}{P_\Sigma} = 0.869.$$

Directivity

Taking into account definition of P_Σ in (8) we can derive directivity of circular aperture with uniform phase distribution:

$$D = \frac{4\pi}{P_\Sigma} = \frac{(kR)^2}{1 + J_0^2(kR) - J_1^2(kR)}, \text{ while } \frac{R}{\lambda} \gg 1.$$

For apertures with large electrical radii $\lim_{R \rightarrow \infty} J_{0,1}(kR) \rightarrow 0$, then obtain well known formula [1]

$$D = 4\pi \frac{S_A}{\lambda^2},$$

where S_A – aperture physical area.

Let’s examine changing of circular aperture directivity while scanning. Without loss of generality in (2) we always can assign $\varphi_0 = 0$, then radiation pattern takes a form:

$$F(\theta, \varphi, \theta_0) = \frac{2J_1[kR\delta(\theta, \varphi, \theta_0)]}{kR\delta(\theta, \varphi, \theta_0)}.$$

Directivity can be found numerically for every certain θ_0 . Results are presented in Fig. 4.

3. Conclusion

In this paper we derived explicit analytical expressions, which can be used for defining of directional properties of circular scanning aperture without numerical computing.

This work was supported in part by Ministry of education and science of Russian Federation (research in the scope of basic part of government job), Siberian Federal University.

The authors would like to thank the support by the Ministry of Education of Russia together with the Siberian Federal University and JSC “Academician M.F. Reshetnev Information Satellite Systems”, Contract № 02.G25.31.0041.

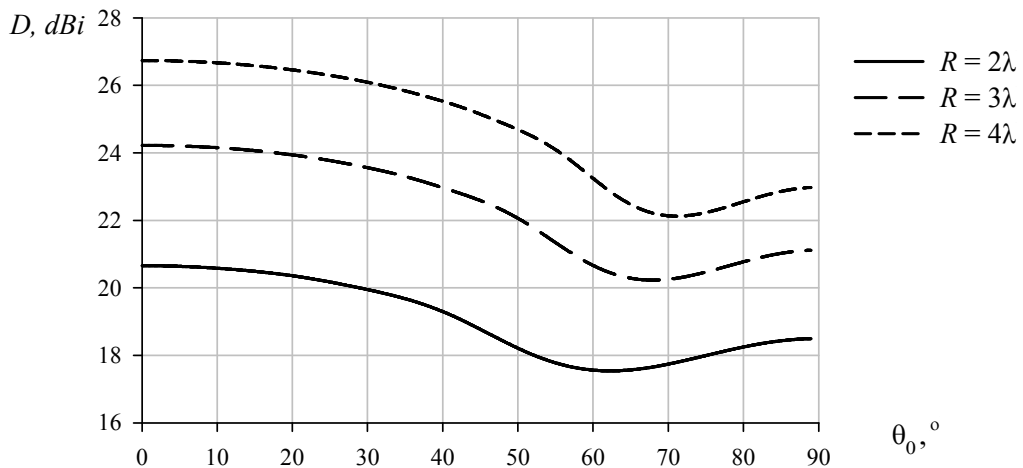


Fig. 4. Directivity of scanning circular aperture

References

- [1] *Марков Г.Т., Сазонов Д.М.* Антенны: учебник для студентов радиотехнических специальностей вузов. М.: Энергия, 1975.
- [2] *Balanis C.A.* Antenna theory: analysis and design. Second ed. John Wiley & Sons, 1997.
- [3] *Корнблит С.* СВЧ-антенны. Оптические принципы в приложении к конструированию СВЧ антенн. М.: Связь, 1980.
- [4] *Прудников А.П., Брычков Ю.А., Маричев О.И.* Интегралы и ряды: в 3 т. М.: ФИЗМАТЛИТ, 2003. Т. 2. 664 с.
- [5] *Саломатов Ю.П., Сугак М.И.* // Антенны. 2010. № 10. С. 17.