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Computing Truth of Logical Statements in Multi-Agents' Environment

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This paper describes logical models and computational algorithms for logical statements (specs) including various versions of Chance Discovery (CD). The approach is based at temporal multi-agent logic. Prime question is how to express most essential properties of CD in terms of temporal logic (branching time multi-agents' logic or a linear one), how to define CD by formulas in logical language. We, as an example, introduce several formulas in the language of temporal multi-agent logic which may express essential properties of CD. Then we study computational questions (in particular, using some light modification of the standard filtration technique we show that the constructed logic has the finite-model property with effectively computable upper bound; this proves that the logic is decidable and provides a decision algorithm). At the final part of the paper we consider interpretation of CD via uncertainty and plausibility in an extension of the linear temporal logic LTL and computation for truth values (satisfiability) of its formulas.

Keywords: temporal logics, multi-agent logics, chance discovery, CD, Kripke-Hintikka models.

Introduction

This paper is aimed to suggest a computational framework for truth of statements (specs) including various versions of the Chance Discovery (CD in the sequel) operation. The approach essentially depends on how to define operation of CD in logical terms - ion terms of suggested symbolic mathematical models. A "chance" is usually meant as a new event/situation that can be conceived either as an opportunity or as a risk in the future. In own turn, "discovery" of chances is of crucial importance since it may have a significant impact on human decision making. Generally, chance discovery aims to provide means for inventing or surviving the future, rather than simply predicting the future, though merely prediction may also have a vital importance. This may lead to recommendations, precautions, attempts to secure situation and future consequences.

Chance Discovery, as a discipline (CD in the sequel, cf. Ohsawa and McBurney [23], Abe and Ohsawa [1]), initially appeared in Japanese school as a direction in Artificial Intelligence (AI) and Computer Science (CS). Investigations in CD analyzes important events with uncertain information, incomplete past data, so to say, *chance* events, where *a chance* is defined as some event which is significant for decision-making in a specified domain.

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CD primarily works with construction and study various methods for discovering chance events. Contemporary research in AI for CD mostly is directed to practical applications, this field forms now a solid branch in AI, knowledge representation (KR) and information technologies (IT) (cf. e.g. in papers [1–4, 13, 15, 24–26, 26]). Since a time ago (about 2007) some steps to develop a technique for study CD in terms of mathematical logic based on Kripke/Hintikka like models also were undertaken (cf. e.g. Rybakov [31]). This approach looks enough promising for theoretical analysis of the essence of CD as a phenomenon.

1. Background, aims

Being interested to describe and formalize properties of CD in logical systems originating in AI (as well as some interpretations of CD-like operations via existence, possibility to achieve an information (sometimes without direct reference to CD)) a logical research of CD was undertaken in, e.g., Rybakov [28] or [27, 29–32]). In these papers, interpretation of CD was primarily modeled via introductions of new logical operations based on possibility to happen (possibility to occur etc).

In current paper we would like to investigate CD via fixed and known already logical language — multi-agent temporal logic (with basic idea — to study how to model CD in this language).

Nowadays multi-agent systems (with autonomous or interacting, say competitive) agents is an active research area. Technique and research outputs are various, diverse and work well in many areas. Areas of applications are, indeed, utterly diverse, but, anyway they are primarily focused to IT (cf. Nguyen et al [19–21], Arisha et al [5], Avouris [6], Hendler [14]). For analysis of implementations these systems, one need a logical language and technique to reason about agents' knowledge and properties (e.g. various technique of mathematical (symbolic) logic is widely used, cf. [10–12]), in particular, multi-agents' modal logics were successfully implemented. Multi-agent epistemic logics have found various applications in fields ranging from AI domains such as robotics, planning, and motivation analysis in natural language, to negotiation and game theory in economics, to distributed systems analysis and protocol authentication in computer security.

These applications are based at the fact that intelligent agents must be able to reason about knowledge. Multi-agents' logics, in particular, appeared in the research about knowledge representation and reasoning about knowledge and beliefs (cf. for example, [10–12]).

Contemporary research in computer science oriented to knowledge representation actively uses various languages and logical systems to capture elements of human reasoning and computational aspects. These logical systems provide us with various inference capabilities to deduce implicit knowledge from the explicitly represented knowledge, as well as with explicit, mathematically precise, description of properties of the objects. To mention some initial typical logical tools cf. ones from Brachman and Schmolze (1985, [8]), Moses and Shoham (1993, [16]), Nebel (1990, [18]), Quantz and Schmits (1994, [22]). Research of agent-decision oriented systems paid many attention to formal descriptions of the meaning of agents knowledge.

For multi-agents' logical systems, an essential question was what is common part of the agents' knowledge, what is a shared knowledge and what is a common knowledge. Some initial concepts concerning this subject can be found in Barwise (1988, [7]), Niegerand and Tuttle (1993, [17]), Dvorek and Moses (1990, [9]). Well developed approach to common knowledge logics was summarized in the book: Fagin R., Halpern J., Moses Y., Vardi M. (1995, [10]). Multi-agent logic based on the linear temporal logic LTL was studied in Rybakov (2009, [30]), where some decision algorithms were found. Recently a look to chance of possibility via modal logic was investigated in Steinberg ([34]). This issue study a theory based upon a proposition that the (non-epistemic) modal realm is tripartite: truths about possible worlds supervene on modal truths, which in turn supervene on truths about objective chances.

This our paper is devoted to the question how to express most essential properties of CD in terms of temporal logic. Basic attention is directed to the prime question: what could be definition of CD in terms of temporal multi-agent logic, which ones adequately (or close to it) expresses CD. We suggest several formulas in the language of branching temporal multi-agent logic which may express essential properties of CD. Then we study the question of decidability for suggested logic. Using some light modification of the standard filtration technique we show that the logic has the finite-model property with effectively computable upper bound. This proves that the logic is decidable and provides a decision algorithm. At the final part of the paper we consider interpretation of CD via uncertainty and plausibility in an extension of the linear temporal logic LTL and computation for truth values (satisfiability) of its formulas, results about decidability such logic and computational algorithms for satisfiability are obtained.

2. Definitions, notation, known facts

At the beginning we recall technique, notation and known facts necessary for reading this paper. Common approach to construct logical language for multi-agents' logic starts from fixing a countable set P of propositional letters and the standard set of Boolean logical operations $\wedge, \vee, \rightarrow$ and $\neg$. Then, usually, a finite tuple of modal-like unary logical operations $K_1, ..., K_n$ is added.

The definition of wff (well formed formulas) is as usual (for any given formula A, K_iA has the informal meaning: the i-th agent knows A.

Derived operations CK_i (i-th agent may know) are defined as follows $CK_iA := \neg K_i \neg A$. Yet, we use standard unary modal (temporal) operation $\Box$, with meaning $\Box A$ says that the statement A always will be true.

Derived operation $\Diamond$ is as follows: $\Diamond A := \neg \Box \neg A$. In the sequel we will refer to operations $\Diamond$ and $\Box$ as temporal ones (in fact they are modal, but we see them as temporal directed to only future, they speak only about future (without considering past)).

Semantic models for this language are possible-worlds models for knowledge (offered e.g. in [10]). We extend this approach to models describing agents' logic for evolving systems (like dilative internet network). We consider Kripke-like models $\mathcal{M} := \langle S, R, R_1, ..., R_n, V \rangle$.

Here, S is the base (nonempty) set, R is a binary reflexive and transitive relations (transition/connection relation), all R_i are binary equivalence relations, where for all i, $aR_ib \Rightarrow aRb$; V is a valuation for propositional letters, i.e. V maps propositional letters in subsets of S. For any $a \in S$, $C(a)$ is the cluster containing a, i.e. $C(a) := \{b \mid b \in S,\ aRb \text{ and } bRa\ \}$. Just a cluster is the cluster $C(a)$ for some a. In accordance with definitions for relations R_i, any R_i is an equivalence relation on each cluster $C(a)$.

Conceived interpretation for such models is as follows: S is a set of all current and all future (planned after updating/creation) web sites. Any $C(a)$ is the set of web sites built to current time point (today, which is identified by the state a). R is the *administrator* accessibility relation, for today — the set $C(a)$, and all future (planned) web sites. For any R_i, R_i is the set of all web sites at the current time point available for the agent (user) i (by login, password, etc.).

To fix notation, for any propositional letter p, expression $\forall a \in S, a \Vdash_V p$ means $a \in V(p)$. The valuation V can be extended from propositional letters to formulas constructed by Boolean operations in the standard way:

$$a \Vdash_V A \wedge B \Leftrightarrow_{def} a \Vdash_V A \text{ and } a \Vdash_V B;$$

$$a \Vdash_V A \vee B \Leftrightarrow_{def} a \Vdash_V A \text{ or } a \Vdash_V B;$$

$$a \Vdash_V \neg A \Leftrightarrow_{def} not(a \Vdash_V A);$$

$a \Vdash_V A \to B \Leftrightarrow_{def} a \Vdash_V B$ or $a \Vdash_V \neg A$.

For agents' knowledge operation the rules are:

$a \Vdash_V K_i A \Leftrightarrow_{def} \forall b \in S(aR_i b \Rightarrow b \Vdash_V A)$;

$a \Vdash_V CK_i A \Leftrightarrow_{def} \exists b \in S(aR_i b \wedge b \Vdash_V A)$.

For temporal operations,

$a \Vdash_V \Box A \Leftrightarrow_{def} \forall b \in S(aRb \Rightarrow b \Vdash_V A)$;

$a \Vdash_V \Diamond A \Leftrightarrow_{def} \exists b \in S(aRb \wedge b \Vdash_V A)$.

The proposed language is rather flexible and expressible, E.g., as it is well known, the compound *know* operation E_n, may be introduced as *all agents know* A: $E_n A := \bigwedge_{\forall i \in [1,n]} K_i A$.

The formulas $CK_i A \to \Diamond A$ says that if the agent i may know A then the supervisor also may know it.

If the formula $CK_i A \to CK_j A$ is true in a model for all formulas A, then the agent j has priority against the agent j: j may know all what may know i.

We say that a formula A is true in a model $\mathcal{M}$ if for all a from the base set of $\mathcal{M}$, $a \Vdash_V A$.

We do not specify that our models $\mathcal{M}$ in consideration are finite (though practical applications evidently assume this). This is because if models have large amount of states and the size of the model is not effectively bound (upper bound in consideration is not known), the behavior of the models and tools applied to check truth of statements have no difference with the case of infinite models.

Definition 1. *Transition multi-agents' logic* L^T_{MA} *is the set of all formulas which are true in all models* $\mathcal{M}$.

The aim of this paper is to construct a framework suitable for reasoning about chance discovery in multi-agents' logic. For example, to find am algorithm which would allow to determine equivalence of the statements (so to say, to recognize equivalent specifications on internet network properties). In next section we suggest some interpretations for CD in chosen logical language.

3. Possible interpretations for chance discovery

As we know, chance discovery (CD) is a possibility to discover (observe) a rare event which is difficult to recognize and to identify. In offered language we can suggest several representations for CD.

(1) For all natural numbers k, where $1 \leqslant k \leqslant n/10$ and any formula A,

$$CD_1 A := \Box\Diamond A \wedge \Box(\neg A \to \neg \bigvee_{X \subseteq [1,n], ||X|| > k} \bigwedge_{i \in X} CK_i A).$$

This formula says that always there will be a chance to discover A in future (that A will be true), but in any state where A is not true there are no chance that more than tenths part of agents will know that A might be true.

(2) For all natural numbers k, where $1 \leqslant k \leqslant n/10$ and any formula A,

$$CD_2 A := \Box[\neg A \to \bigvee_{i \in [1,n]} CK_i A] \wedge \Box(\neg A \to \neg \bigvee_{X \subseteq [1,n], ||X|| > k} \bigwedge_{i \in X} CK_i A).$$

This formula says more definite information about chances to observe A: for any state, at least one agent knows that A might be true, but again, in any state where A is not true, there are no chance that more than 10ths part of agents will know that A might be true.

(3) For all natural numbers k, where $1 \leqslant k \leqslant n/10$ and any formula A,

$$CD_3A := \Box[\neg A \rightarrow \bigwedge_{i\in[1,n]} \neg CK_iA] \wedge \ \Box\Diamond A \wedge \Box(\neg A \rightarrow \neg \bigvee_{X\subseteq[1,n],||X||>k} \bigwedge_{i\in X} CK_iA).$$

Now the formula says that in any state in future where A is not true, no one agent knows that A might be true, but always, in some state in future, A definitely will be true, though again, as before, in any state where A is not true, there are no chance that more than 10ths part of agents will know that A might be true (so to say to emphasize that truth of A is rare).

(4) $$CD_4A := \Diamond\Box\neg A \wedge \Diamond A \wedge \Box(\Diamond A \rightarrow \neg \bigvee_{X\subseteq[1,n],||X||>k} \bigwedge_{i\in X} CK_iA).$$

This says to us that at current state there is a chance to discover A, but in some future state this chance will be lost. Besides in any future were this chance will exists, not more than k agents may know A.

Based at these examples we conclude that the chosen language is adequate to express various aspects of CD. In described manner we could vary portions of agents able to observe CD, we can express that, since at a state where no one agent might know statement A, that always will happen in future, or yet A never will be true in future always, etc. Now, in next section, we would like to describe mathematical part of our research with the aim to construct deciding algorithms.

4. Deciding algorithm for L^T_{MA}

The awaking problem here is how to determine whether two given statements (say A and B) (specifications — in programming terms) are equivalent (are the same from logical viewpoint).

By equivalence we mean that A and B have the same truth value in any state of any model. In logical framework the equivalence may be expressed in the following way. Let

$$A \equiv B := (A \rightarrow B) \wedge (B \rightarrow A).$$

Then, if $A \equiv B \in L^T_{MA}$ then A and B have always the same truth value, i.e. we can tell that these A and B are equivalent.

Therefore if the logic L^T_{MA} is decidable (which means there is (and known) an algorithms which checks, for any formula A, if $A \in L^T_{MA}$ holds) we will have an algorithm recognizing equivalent statements. Hence, we only need to show decidability of L^T_{MA}.

This may be done by simple variation of standard filtration technique for multi-modal logics extended to temporal logic which we draft (briefly describe) below.

Given a model $\mathcal{M} := \langle S, R, R_1, ..., R_n, V\rangle$ and a formula φ with letters from the domain of V. Assume $\mathcal{M}$ disproved φ, i.e. there is $a \in S$ such that $a \Vdash_V \neg\varphi$. We evidently may assume that φ contains only operations CK_i and $\Diamond$ among its non-boolean operations.

First stage of our filtration will work with clusters $C(a)$ of this model. Let $Sub(\varphi)$ be the set of all subformulas of φ and

$$Sb := Sub(\varphi) \cup \{CK_i\psi \mid \psi \in Sub(\varphi)\}.$$

Let $\equiv_{Sb}$ be the equivalence relation on $C(a)$, where

$$a \equiv_{Sb} b \Leftrightarrow \forall\psi \in Sb\ [a \Vdash_V \psi \ \Leftrightarrow\ b \Vdash_V \psi].$$

Then the quotient-set $C(a)_{\equiv_{Sb}} := \{[b]_{\equiv_{Sb}} \mid b \in C(a)\}$ has at most $2^{||Sb||}$ elements. We define relations R_i on $C(a)_{\equiv_{Sb}}$ by

$$[b_1]_{\equiv_{Sb}} R_i [b_2]_{\equiv_{Sb}} \Leftrightarrow \forall CK_i\psi \in Sb \ \ [b_1 \Vdash_V CK_i\psi \ \Leftrightarrow \ b_2 \Vdash_V CK_i\psi].$$

If we only transform one chosen cluster $C(a)$ in described above way, we, by standard induction on length formulas, may easily obtain that

$$\forall \psi \in Sb \ \ [\ [b]_{\equiv_{Sb}} \Vdash_V \psi \Leftrightarrow b \Vdash_V \psi.]$$

Using similar filtration in any separate cluster of $\mathcal{M}$, we may assume that $\mathcal{M}$ contains only clusters with at most $2^{||Sb||}$ elements. So, there is only a finite, effectively evaluated set of such non-isomorphic clusters. Denote it by Cl.

Now we perform the second stage filtration. Let $Sb_1 := Sb \cup \{\Diamond\psi \mid \psi \in Sb\}$. For all $C(a), C(b) \in Cl$ we set

$$C(a)RC(b) \Leftrightarrow \forall \Diamond\psi \in Sb_1[b \Vdash_V \Diamond\psi \Rightarrow a \Vdash_V \Diamond\psi].$$

It is not difficult to verify by induction on the length of formulas, that after this transformation the resulting model will have at the elements the same truth values of formulas as the original elements had in the original model. So, this model will also disprove the formula φ. Hence, this way we may prove the following

Theorem 1. *The logic L^T_{MA} is decidable. There is an algorithm checking for any given formula φ, if $\varphi \in L^T_{MA}$ holds.*

As we noted earlier, having at our disposal an algorithm for decidability L^T_{MA}, we can determine equivalence of the statements about CD in suggested framework. Though, it is relevant to confess that the computational efficiency of the suggested algorithm is too high, since it is based on standard filtration technique (and then unavoidably brings margins over exponential bound).

5. CD via uncertainty and plausibility

Here we would like to discuss another possible way to (conventionally) define and to model CD, - via plausibility and uncertainty, being based at linear temporal logic LTL — the one, which is widely accepted at computer science. We start from definition of semantic bases at which our logic will be based. In this part we have to essentially verbatim repeat some part from [33], where the basic results towards computational algorithm were obtained. Actually we here just use these results to give new attractive and useful definition for CD via this framework. The models we will use hear are the following Kripe/Hintikka-like frames: $\mathcal{N}_C := \langle \cup_{i\in N} C(i), R, R_1, \dots R_m, Next\rangle$ which are tuples, where N is the set of natural numbers, C(i) are some disjoint nonempty sets ($C(i) \cap C(j) = \varnothing$ if $i \neq j$); $R, R_1, \dots R_m$ are binary accessibility relations on $\cup_{i\in N} C(i)$. The relation R represents linear flow of time: for all elements a and b from the set $\cup_{i\in N} C(i)$,

$$\forall a, b \in \cup_{i\in N} C(i) \ [aRb \Leftrightarrow [a \in C(i) \ and \ b \in C(j) \ and \ i \leqslant j]].$$

Relations R_j represent agents' accessibility relations in time clusters $C(i)$: any R_j is a reflexive, transitive and symmetric relation, and

$$\forall a, b \in \cup_{i\in N} C(i) \ [\ aR_jb \Rightarrow [a, b \in C(i) \text{ for some } \ i)]].$$

The binary relation $Next$ distinguishes worlds of neighboring time clusters:

$$\forall a, b \in \cup_{i\in N} C(i) \ [a \ Next \ b \Leftrightarrow [\exists i((a \in C(i))\&(b \in C(i+1))]],$$

i.e a $Next$ b says that b is situated in time cluster next to one containing a.

An well accepted general idea in CS is that the such frames $\mathcal{N}_C$ represent possible unbounded (in time) computation with multi-possessors, that is any $i \in N$ (any natural number i) simulated time tick i, any $C(i)$ consists of processors (computational units) evolved in computation at tick time i. Any R_j is an accessibility relation for the agent j between these computational units. R is the accessibility relation in $\mathcal{N}_C$ in time from now to future, as it is specified above, it is a linear relation. Within any $C(i)$ (which means in the same time moment) all computational units ($u \in C(i)$) are mutually accessible by time (but not by agents accessibility relations). Agents are - users, administrators of networks, agents looking through brigs between computational threads, etc.

Results (states) of computation are modeled (rather just encoded) by sets of of propositions (propositional letters) P, any $p \in P$ simply names a property (like p = computational thread t_p is faulty terminated, etc.). For any $\mathcal{N}_C$ and a set of propositions P, a model based on $\mathcal{N}_C$ and V is the tuple $\mathcal{M} := \langle \cup_{i \in N} C(i), R, R_1, \ldots R_m, Next, V \rangle$, where V is a mapping of P is the set of all subsets of $\cup_{i \in N} C(i)$, i.e. $\forall p \in P, V(p) \subseteq \cup_{i \in N} C(i)$.

For fixing a logical language describing events happening while computation we use a (potentially infinite) set of propositional letters P and following symbols for logical operations: Boolean operations $\wedge, \vee, \rightarrow, \neg$, temporal binary operations: $\mathbf{N}$ (next), $\mathbf{U}$ (until), $\mathbf{U}_w$ (weak until) and $\mathbf{U}_s$ (strong until), and the unary operation $Next$. Also it includes (unary) operations $\mathbf{K}_j$, $1 \leq j \leq m$ for agents' knowledge, and additional unary operations $\mathbf{CK_L}$ (local knowledge), $\mathbf{CK_G}$ (global knowledge), **IntK** (interactive knowledge), and **Pl** (plausible). To handle uncertainty we add the (unary) operation **Unc**. Formation rules for formulas are as usual: any propositional letter p is a formula, if φ and ψ are formulas then

$\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \rightarrow \psi$, $\neg\varphi$;

$\varphi \mathbf{U} \psi$, $\varphi \mathbf{U}_s \psi$, $\varphi \mathbf{U}_w \psi$, $\mathbf{N}\varphi$;

$\mathbf{CK_L}\varphi$, $\mathbf{CK_G}\varphi$, $\mathbf{K}_j\varphi$, $\mathbf{IntK}\varphi$,

$\mathbf{Unc}\varphi$ and $\mathbf{Pl}\varphi$

are formulas. Informal (intended) meaning of the operations is as follows.

$\mathbf{Unc}\varphi$ means that the statement φ is *uncertain* in the current state of the current time cluster.

$\mathbf{Pl}\varphi$ says that the statement φ is *plausible* in the current state of the current time cluster.

$\mathbf{K}_j\varphi$ means the agent j *knows* φ in the current state of a time cluster.

$\mathbf{CK_L}\varphi$ means that φ is a *local common knowledge* in the in the current state of a time cluster, that all agents knows φ.

$\mathbf{CK_G}\varphi$ means φ is a *global common knowledge* in the in the current state of a time cluster (i.e. that since now all agents will know it always).

$\mathbf{IntK}\varphi$ means that in the current state φ *may be known by interaction between agents.*

$\mathbf{N}\varphi$ has meaning φ holds in the *next time cluster* of states (state);

$\varphi \mathbf{U} \psi$ can be read: φ holds until ψ will hold;

$\varphi \mathbf{U}_w \psi$ has meaning φ weakly holds until ψ will hold;

$\varphi \mathbf{U}_s \psi$ has meaning φ strongly holds until ψ will hold;

For a given frame $\mathcal{N}_C = \langle \cup_{i\in N} C(i), R, R_1, \dots R_m, Next\rangle$, a model

$$\mathcal{M} = \langle \cup_{i\in N} C(i), R, R_1, \dots R_m, Next, V\rangle,$$

is the frame extended with a valuation V.

Truth values of formulas at elements (worlds) of $\mathcal{M}$ may be computed by the following rules (below we denote $(\mathcal{M}, a) \Vdash_V \varphi$ to say that the formula φ is true at a in the frame $\mathcal{N}_C$ with respect to the valuation V).

$\forall p \in P, \quad (\mathcal{M}, a) \Vdash_V p \;\Leftrightarrow\; a \in V(p);$

$(\mathcal{M}, a) \Vdash_V \varphi \vee \psi \Leftrightarrow (\mathcal{M}, a) \Vdash_V \varphi \vee (\mathcal{M}, a) \Vdash_V \psi;$

$(\mathcal{M}, a) \Vdash_V \varphi \wedge \psi \Leftrightarrow (\mathcal{M}, a) \Vdash_V \varphi \wedge (\mathcal{M}, a) \Vdash_V \psi;$

$(\mathcal{M}, a) \Vdash_V \rightarrow \wedge\psi \Leftrightarrow not[(\mathcal{M}, a) \Vdash_V \varphi] \vee \; (\mathcal{M}, a) \Vdash_V \psi;$

$(\mathcal{M}, a) \Vdash_V \neg\varphi \Leftrightarrow not[(\mathcal{M}, a) \Vdash_V \varphi];$

$(\mathcal{M}, a) \Vdash_V \mathbf{Unc}\varphi \Leftrightarrow \exists i(a \in C(i) \wedge (\exists b \in C(i)(\mathcal{M}, b) \Vdash_V \varphi \wedge (\exists c \in C(i)(\mathcal{M}, c) \Vdash_V \neg\varphi)).$

That is, we say φ is *uncertain* if there are two states in time cluster $C(i)$, i.e. in time i, where φ is true at one of these states and is false at the another one. This looks as quite plausible way to express uncertainty of the statement φ (though, clearly, only one of possible ones, it could be many various ideas on how to express uncertainty in logical framework).

The next step is for explanation how to compute that the truth of a formula φ is *plausible* at current state. We suggest:

$$(\mathcal{M}, a) \Vdash_V \mathbf{Pl}\varphi \Leftrightarrow \exists i(a \in C(i) \wedge \\ ||\{b \mid b \in C(i)(\mathcal{M}, a) \Vdash_V \varphi\}|| > ||\{b \mid b \in C(i)(\mathcal{M}, a) \text{ not } \Vdash_V \varphi\}||,$$

where, for any set X, $||X||$ is the cardinality of X. For finite X, recall, $||X||$ is the number of elements in X. Therefore the rule above says that the truth of φ in a state of time moment i is *plausible* if there more states in the current time i where φ true then the ones where φ is false (so to say more witnesses for true than witnesses for false, so we use voting principle here). Next rule, $(\mathcal{M}, a) \Vdash_V \mathbf{K}_j\varphi \Leftrightarrow \forall b[(a\ R_j\ b) \Rightarrow (\mathcal{M}, b) \Vdash_V \varphi]$ is at it was in previous sections. So, again, $\mathbf{K}_j\varphi$ says that the agent j knows φ if the formula φ holds in all states available for the agent j from the current state. Next rule,

$(\mathcal{M}, a) \Vdash_V \mathbf{IntK}\varphi \Leftrightarrow \exists a_{i1}, a_{i2}, \dots, a_{ik} \in \mathcal{M}$
$[aR_{i1}a_1R_{i2}a_{i2} \dots R_{ik}a_k \;\&\; (\mathcal{M}, a_k) \Vdash_V \varphi]$ is for agents' interaction.

$\mathbf{IntK}\varphi$ says that φ is known via interaction of agents. As we see, the rule above says us that φ may be known by interaction between the agents if there is a path of transitions by agents accessibility relations which leads to a state where φ holds. So, it looks as passing by agents information that φ holds via their information channels (so it seems to make the term *interaction* relevant).

$(\mathcal{M}, a) \Vdash_V \mathbf{CK_L}\varphi \Leftrightarrow \forall j \forall b[(a\ R_j\ b) \Rightarrow (\mathcal{M}, b) \Vdash_V \varphi.$

So, φ is *local knowledge* if it holds in all states which are accessible in the *current time point* for any agent.

$(\mathcal{M}, a) \Vdash_V \mathbf{CK_G}\varphi \Leftrightarrow \forall b[(a\ R\ b) \Rightarrow (\mathcal{M}, b) \Vdash_V \varphi.$

Thus, φ is of *global common knowledge* if it holds in all states in all future (and current) time clusters. This definition differs from standard meaning of just common knowledge (it does not refer to agents) but is seemed to be meaningful and plausibly justified.

$(\mathcal{M}, a) \Vdash_V \mathbf{N}\varphi \Leftrightarrow \forall b[(a \text{ Next } b) \Rightarrow (\mathcal{M}, b) \Vdash_V \varphi]$;

$(\mathcal{M}, a) \Vdash_V \varphi\mathbf{U}\psi \Leftrightarrow \exists b[(aRb) \wedge ((\mathcal{M}, b) \Vdash_V \psi) \wedge \forall c[(aRcRb) \& \neg(bRc) \Rightarrow (\mathcal{M}, c) \Vdash_V \varphi]]$;

$$(\mathcal{M}, a) \Vdash_V \varphi\mathbf{U}_w\psi \Leftrightarrow \exists b[(aRb) \wedge ((\mathcal{M}, b) \Vdash_V \psi) \wedge \forall c[(aRcRb) \& \neg(bRc) \& \\ \& (c \in C(i)) \Rightarrow \exists d \in C(i)(\mathcal{M}, d) \Vdash_V \varphi]];$$

$$(\mathcal{M}, a) \Vdash_V \varphi\mathbf{U}_s\psi \Leftrightarrow \exists b[(aRb) \wedge b \in C(i) \wedge \forall c \in C(i)((\mathcal{M}, c) \Vdash_V \psi) \wedge \\ \forall c[(aRcRb) \& \neg(bRc) \Rightarrow (\mathcal{M}, c) \Vdash_V \varphi]];$$

Here we have to comment that the rules for computation truth values of temporal operations above unavoidably differ from the ones for LTL itself. The matter is, in our semantic frames, we have time clusters of states for each tick i of time but not merely one state as for LTL itself. It is easy to observe that the computation of the time binary operation $\mathbf{U}$ is only slightly different from the standard one – it is sufficient for ψ to be true as minimum at one state of the achieved in future time cluster. This convention comes from the structure, as we have time clusters $C(i)$ but not time points i, as for LTL. The time operation $\mathbf{U}_w$ more significantly differs from the standard operation $\mathbf{U}$, – it is sufficient for the formula φ to be true only in one state of all possible time clusters before the statement ψ will true inside some state.

The temporal operation strong until – $\varphi\mathbf{U}_s\psi$ – expresses the property that there is a time point i, where the formula ψ is true inside $C(i)$ at totally all states and the formula φ holds in any state of all time clusters $C(j)$ with time points j proceeding i.

Now we will define our logic $\mathcal{UIA}_{\mathcal{LTL}}$ for description of interdependencies of operations uncertainty, plausibility, temporal and agents' operations. Recall that, for any given Kripke structure $\mathcal{M} := \langle \mathcal{N}_C, V \rangle$ and any given a formula φ, we use the following definitions. The formula φ is said to be satisfiable in $\mathcal{M}$ (denotation – $\mathcal{M} \Vdash_{Sat} \varphi$) if there exists some state b in the model $\mathcal{M}$ ($b \in \mathcal{N}_C$), at which φ is true: $(\mathcal{M}, b) \Vdash_V \varphi$. The formula φ is called valid at $\mathcal{M}$ (denotation – $\mathcal{M} \Vdash \varphi$) if, for all b from $\mathcal{M}$ ($b \in \mathcal{N}_C$), φ is true at b ($(\mathcal{M}, b) \Vdash_V \varphi$).

Given a frame $\mathcal{N}_C$ and a formula φ, we, as earlier, say that the formula φ is satisfiable in $\mathcal{N}_C$ (denotation $\mathcal{N}_C \Vdash_{Sat} \varphi$) if there exists some valuation V in $\mathcal{N}_C$ such that the following $\langle \mathcal{N}_C, V \rangle \Vdash_{Sat} \varphi$ holds; the formula φ is valid in $\mathcal{N}_C$ (i.e. $\mathcal{N}_C \Vdash \varphi$) if *not*$(\mathcal{N}_C \Vdash_{Sat} \neg\varphi)$ holds. We introduce our logic $\mathcal{UIA}_{\mathcal{LTL}}$ semantically, as the set of all logical laws which are true in our models, more formally,

Definition 2. *The logic $\mathcal{UIA}_{\mathcal{LTL}}$ is the set of all formulas which are valid in all frames $\mathcal{N}_C$.*

A formula φ in the language of $\mathcal{UIA}_{\mathcal{LTL}}$ is called *satisfiable* if and only if there exists some valuation V in a certain Kripke frame $\mathcal{N}_C$, where V which makes φ satisfiable: $\langle \mathcal{N}_C, V \rangle \Vdash_{Sat} \varphi$. It is immediate to observe, that a formula φ is satisfiable if and only if $\neg\varphi$ is not a theorem of $\mathcal{UIA}_{\mathcal{LTL}}$, that is $\neg\varphi$ *not in* $\mathcal{UIA}_{\mathcal{LTL}}$. Conversely, a formula φ is a theorem of logic $\mathcal{UIA}_{\mathcal{LTL}}$ ($\varphi \in \mathcal{UIA}_{\mathcal{LTL}}$) if the formula obtained be negation to φ, – $\neg\varphi$ – is not satisfiable.

Now, before to comment definition of CD in such framework we would like to recall basic results obtained earlier for this logic in [33]. First, recall that using $\mathbf{U}$ and $\mathbf{N}$ we are able to express temporal and modal operations. For example,

Proposition 1. *For any formula φ, (1) $\Diamond\varphi \equiv true\mathbf{U}\varphi \in \mathcal{UIA}_{\mathcal{LTL}}$; (2) $\Box\varphi \equiv \neg(true\mathbf{U}\neg\varphi) \in \mathcal{UIA}_{\mathcal{LTL}}$;*

For example, the temporal operation $\mathbf{F}\varphi$ (*φ holds eventually, in terms of modal logic, φ is possible (denotation $\Diamond\varphi$))* to be determined in our logic as $true\mathbf{U}\varphi$; in own turn, the operation $\mathbf{G}$ ($\mathbf{G}\varphi$ means *φ holds henceforth*) is expressed in $\mathcal{UIA}_{\mathcal{LTL}}$ as $\neg\mathbf{F}\neg\varphi$.

In accepted formalization of uncertainty operation, the following interconnections with possibility and necessity can be easily derived:

Proposition 2. *The following holds:*

(ii) $\mathbf{Unc}\varphi \ \rightarrow \ \Diamond\varphi \wedge \Diamond\neg\varphi \in \mathcal{UIA}_{\mathcal{LTL}}$;

(iii) $\Diamond\varphi \wedge \Diamond\neg\varphi \rightarrow \mathbf{Unc}\varphi \notin \mathcal{UIA}_{\mathcal{LTL}}$;

(iv) $\Box\varphi \rightarrow \neg\mathbf{Unc}\varphi \in \mathcal{UIA}_{\mathcal{LTL}}$;

(v) $\neg\mathbf{Unc}\varphi \rightarrow \Box\varphi$ *not* $\in \mathcal{UIA}_{\mathcal{LTL}}$.

Also, the following simple laws hold for uncertainty:

Proposition 3. *The following holds:*

(i) $\mathbf{Unc}\varphi \ \rightarrow \ \ \mathbf{Unc}\neg\varphi \in \mathcal{UIA}_{\mathcal{LTL}}$;

(vi) $\mathbf{Unc}(\varphi \wedge \psi) \rightarrow \mathbf{Unc}\varphi \vee \mathbf{Unc}\psi \in \mathcal{UIA}_{\mathcal{LTL}}$;

(vii) $\mathbf{Unc}(\varphi \vee \psi) \rightarrow \mathbf{Unc}\varphi \vee \mathbf{Unc}\psi \in \mathcal{UIA}_{\mathcal{LTL}}$.

Regarding logical operation *plausibility*, the following properties may be easily derived:

Proposition 4. *The following holds:*
$(i)\neg(\mathbf{Pl}\varphi \wedge \mathbf{Pl}\neg\varphi) \in \mathcal{UIA}_{\mathcal{LTL}}$;

(ii) $\mathbf{Pl}(\varphi \wedge \psi) \rightarrow \mathbf{Pl}\varphi \wedge \mathbf{Pl}\psi \in \mathcal{UIA}_{\mathcal{LTL}}$;

(iii) $\mathbf{Pl}(\varphi \vee \psi) \rightarrow \mathbf{Pl}\varphi \vee \mathbf{Pl}\psi$ *not* $\in \mathcal{UIA}_{\mathcal{LTL}}$;

(iv) $\mathbf{Pl}\varphi \wedge \neg\varphi \rightarrow \mathbf{Unc}\varphi \in \mathcal{UIA}_{\mathcal{LTL}}$;

(v) $\mathbf{Pl}(\varphi \rightarrow \psi) \rightarrow (\mathbf{Pl}\varphi \rightarrow \mathbf{Pl}\psi) \in \mathcal{UIA}_{\mathcal{LTL}}$;

(vi) $\mathbf{Pl}\varphi \rightarrow \mathbf{Pl}\mathbf{Pl}\varphi \in \mathcal{UIA}_{\mathcal{LTL}}$;

(vi) $\mathbf{Pl}\varphi \rightarrow \varphi$ *not* $\in \mathcal{UIA}_{\mathcal{LTL}}$.

Thus, **Pl**, in particular, behaves similar to $K4$-modality but yet has very essential differences. Notice also that based at logical operations derived above we can very simply express some knowledge operations postulated for our language.

Proposition 5. *For any formula* φ,
(i) $\mathbf{CK}_G\varphi \ \equiv \ \Box\varphi \in \mathcal{UIA}_{\mathcal{LTL}}$; (ii) $\mathbf{CK}_E\varphi \ \equiv \wedge_{1\leqslant i\leqslant n}(\mathbf{K}_i\varphi) \in \mathcal{UIA}_{\mathcal{LTL}}$.

Since such possibility to express such knowledge operations, the initially specified language for $\mathcal{UIA}_{\mathcal{LTL}}$ was a bit *superfluous* (but we use it for clarity reasons). Now we can omit operations for environmental and global common knowledge (since they are expressible via others). Usage of our combined language with standard and modified temporal operations together with our various knowledge operations, and logical uncertainty operation gives an impressive power to the formulas. For example, the formula $\mathbf{Unc}\varphi \rightarrow \mathbf{Unc}\psi$, says that uncertainty φ implies uncertainty ψ, which may encode implicit dependence ψ from φ.

For example, the formula $\Box\neg K_1\neg\varphi$ tells us that, for any possible future time cluster $C(i)$ and for any state a situated inside this cluster $C(i)$ the knowledge φ is *discoverable* for the agent 1, it has access to a state b where the statement φ holds. By usage the new (strong and weak) temporal operations $\mathbf{U}_s$ and $\mathbf{U}_s$, we may model (represent) unique features of distribution truth values of formulas inside models.

For example, the formula

$$\Box_w\varphi := \neg(\top\mathbf{U}_s\neg\varphi)$$

codes the *weak necessity*, it says that in any time cluster $C(i)$ there is a state where φ is true. The formula $\neg(\varphi \mathbf{U}_w \Box\varphi) \wedge \Diamond\Box\varphi$ says that, there is an earliest future time cluster $C(i)$, since which φ will be true in all states of all time clusters, but, before $C(i)$ φ fails in a state of any time cluster. These and similar properties just impossible to describe in standard language of LTL, and in standard multi-agents' logic, simply no logical operations which could handle such subtle properties. The logic $\mathcal{UIA}_{\mathcal{LTL}}$ in spite of its high expressive properties, would be not much useful, if we would be not a position to evaluate, which logical laws work for this logic.

This could be done either via construction of an axiomatic system for $\mathcal{UIA}_{\mathcal{LTL}}$ with a collection of axioms and inference rules or by an attempt to find an algorithm, which would recognize logical laws for $\mathcal{UIA}_{\mathcal{LTL}}$, would compute its true and satisfiable formulas. From computational viewpoint the second option is more preferable because this case we are immediately dealing with algorithm determining true theorems of $\mathcal{UIA}_{\mathcal{LTL}}$. Therefore we are working with the second option for finding algorithm solving decidability problem for $\mathcal{UIA}_{\mathcal{LTL}}$.

Before the computational question, we turn to our main aim: how to model CD. In accepted formalization we can now easily define and fix meaning of CD corresponding to intuition for its usage as a term: $CD\varphi := \mathbf{Unc}\varphi \wedge \mathbf{Pl}\neg\varphi$.

This says us that φ is CD–event (it is rare but possible event) if, in current environment, there are more evidences that φ is false, but yet — there are some which determine that φ may be true. This, as it seems, precisely defines CD in accordance with its informal meaning.

Using similar to previous sections technique, constructions and proofs we may derive

Theorem 2. *The logic $\mathcal{UIA}_{\mathcal{LTL}}$ is decidable. The algorithm determining whether a formula φ is a theorem of $\mathcal{UIA}_{\mathcal{LTL}}$ (whether $\varphi \in \mathcal{UIA}_{\mathcal{LTL}}$) is based on verification of validity of inference rules w.r.t. special valuations on specific finite frames of effectively bounded size.*

Similar to [33] we would like to conclude this section with comments how the used approach can be extended to obtain technique which may include some new visions on the studied logical operations. For example, uncertainty operations **Unc** may be defined via the current time cluster $C(i)$ and the next time cluster $C(i+1)$, motivating uncertainty as possible different truth values in the current and the next time cluster $C(i+1)$, i.e.

$$\forall a\ (\mathcal{N}_C, a) \Vdash_V \mathbf{Unc}\varphi \Leftrightarrow \exists b, c \in C(i) \cup C(i+1)[\ ((\mathcal{N}_C, b) \Vdash_V \mathbf{Unc}\varphi)\&((\mathcal{N}_C, c) \Vdash_V \mathbf{Unc}\neg\varphi)].$$

Besides, to go further in this direction, we can consider possible different truth values in a bounded future time. Regarding plausibility operation **Pl**, some other definitions for its truth value may be considered, e.g. we may treat a statement φ as (weakly) plausible if φ holds at least one state of the current time cluster. Also, various operations allowing to model $\mathbf{U}_s$ and $\mathbf{U}_w$ may be suggested. For example, we could consider the following new relation R_s on frames $\mathcal{N}_C$: $\forall i \in N, \forall a, b \in C(i)(aR_sb)$.

Here the relation R_s plays especial role: it is an operation to represent local universal modality, say we may interpret it as accessibility relation of a supervise agent (or an omniscient agent) who knows the state of information at any state of the current time cluster $C(i)$. To use this special role of R_s, denote $\Box_s := K_s$, $\Diamond_s := \neg K_s\neg$. Below the notation $\equiv_{sem}$ is meant to say that the truth values of formulas in frames $\mathcal{N}_C$ coincide. It is immediate to see that (i) $\varphi\mathbf{U}_w\psi \equiv_{sem} \Diamond_s\varphi\mathbf{U}\Diamond_s\psi$; (ii) $\varphi\mathbf{U}_s\psi \equiv_{sem} \Box_s\mathbf{U}\Box_s\psi$. Hence, if we possess the operation $\Diamond_s$, it is possible to define via it weak and strong *Until*. If we proceed this way, the resulting logic $\mathcal{UIA}^S_{\mathcal{LTL}}$ in the language with K_s and without $\mathbf{U}_s$ and $\mathbf{U}_w$ obeys the technique presented in this paper for $\mathcal{UIA}_{\mathcal{LTL}}$. It looks very plausible that modifying technique of this paper it is possible to obtain the decidability for such new logics with a similar estimation of complexity.

There are also ways to extend or to change the language in our logic by adding some possible variants of the operation **N**. E.g., take the relation $\mathbf{N}_w$ — weak next with the following interpretation $\forall a\ (\mathcal{M}, a) \Vdash_V \mathbf{N}_w\varphi \Leftrightarrow \exists b[(a \text{ Next } b) \wedge (\mathcal{M}, b) \Vdash_V \varphi]$. The logic with obtained by adjoining this new operation will also be decidable. Also, looking for possible variations, we may take a new special operation $Next_w$ on frames $\mathcal{N}_C$. This operation is assumed to be a restriction of $Next$, e.g., $\forall a, b \in \cup_{i \in N} C(i), a\ Next_w\ b \Rightarrow [a \in C(i)\ for\ some\ i$ and $b \in C(i+1)]$; $\forall a \in \cup_{i \in N} C(i)[a \in C(i) \Rightarrow \exists b \in C(i+1)(a\ Next_w b)\ \wedge \forall c \in C(i) \forall d \in C(i+1)((cNext_w d) \Leftrightarrow (a\ Next_w\ d))]$. Evolving technique suggested in our paper, we may handle this case also, and to obtain decidability results and algorithms for checking satisfiability with similar complexity.

One other way, yet, is to offer some restrictions for agents' accessibility relations R_i, bearing in mind the real world cases, and that the agents are not equitable. So, we may introduce a kind of hierarchy for agents, their accessibility relations, e.g. it may be arbitrary desirable one (of kind $R_i \subseteq R_j$ for supervision). The technique from our paper may be extended to this case as well.

Conclusion

This paper studied Chance Discovery (CD) in terms of temporal multi-agent logic. The basic aim was to identify possible ways for expression CD in terms of temporal multi-agents' logic (how to express CD by formulas in this logical language). In the initial part of our paper we used branching time multi-agent logic and, as an example, introduced several formulas in the language of temporal multi-agent logic which may express essential properties of CD. Then, using some light modification of the standard filtration technique, we showed that the constructed logic has the finite-model property with effectively computable upper bound; this proves that the logic is decidable and provides a decision algorithm. At the final part of the paper we built interpretation of CD via uncertainty and plausibility in an extension of the linear temporal logic LTL (by an multi-agent environment) and illustrated computation for truth values (satisfiability) of its formulas by an constructed algorithm.

There are many ways to deeper and to extend research framework from this paper. First one is to specify more the language or considered formulas to express more precisely desirable properties of CD (e.g. using elements of fuzzy logic, etc). Second one is to built more detail semantic basis for the logic, where more precise description of CD would be possible (e.g. numeric values for CD, possibility theory, etc). Third one is to built more effective deciding algorithms, because the suggested ones are based on computable upper bound of the counter-models with too big size. Next possible direction is to consider some restricted language of the first order logic for modeling CD in predicate logic. Besides, more deep technique to distinguish CD via plausibility and uncertainty (e.g. with measuring bounds for distinction plausibility) would be very relevant.

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Исчисление истинных утверждений с помощью операций теории Chance Discovery в многоагентном окружении

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Представленная статья посвящена построению логических моделей различных версий теории случайных открытий (CO) и описанию вычислительных алгоритмов для логических высказываний. Предлагаемый нами подход основывается на многоагентной временной логике. Главный вопрос состоит в том, как можно было бы выразить самые существенные свойства CO в терминах временной логики, многоагентной логики с ветвящимся временем или линейной логики и вообще как определить CO с помощью формул языка логики. Нами в статье введено несколько формул на языке многоагентной временной логики, которые способны выразить существенные свойства CO. Используя некоторую модифицированную стандартную технику фильтрации, мы показали, что сконструированная таким образом логика имеет свойство финитной аппроксимируемости с эффективно вычислимой верхней границей. Это доказывает, что такая логика разрешима и нами предъявлен алгоритм разрешения. В заключительной части статьи мы рассматриваем интерпретацию CO посредством неопределённости и вероятности в расширении временной линейной логики и вычисление истинностных значений её формул.

Ключевые слова: временные логики, многоагентные логики, случайные открытия, BO, модели Крипке-Хинтикка.