The Supply Chain Integrated Inventory Model for Single Product, Multi-Buyer and Multi-Vendor

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***Abstract*—This paper presents an extended integrated inventory model with multi vendors that produce one item product and supply it to multi buyers. It is assumed that vendor production rates are not equal and the demands of buyers assumed to be normally distributed and independent. The model formulation was made for each buyer and vendor. Then, each cost component from the buyer and vendor is combined to form the total inventory costs. This model uses the service level as constraint. The final mathematical model consists of formulations of the total inventory costs with certain service level.**

Keywords— *multi-vendor, multi-buyer, integrated inventory system, supply chain*

1. Introduction

A supply chain consists of all parties involved, both directly and indirectly to fulfill a customer demands. The parties involved in supply chain include manufacturers, suppliers, transporters, warehouses, retailers and customers. The success of supply chain is measured in terms of supply chain profitability instead of profit at an individual party [1]. Therefore, a single party optimal strategy is not suitable for today's global competition and in supply chain profitability. With the supply chain concept, many firms realize that inventories across supply chain could be managed efficiently through a coordination between parties [2]. Integrating vendor and buyer inventory system is an important factor to lower the joint total cost of supply chain and the supply chain would gain competitive advantage [3].

The coordination between vendor and buyer to integrate vendor and buyer inventory system has attracted many researchers and has been studied for years. The focus of this integrated approach is on the production-inventory decisions of supply chain partners while minimizing total inventory cost [2]. The idea of the integrated inventory system first was introduced for a single vendor and single buyer [4]. It showed that a coordinated inventory system policy was more desirable than individual optimal policy. This model was modified by incorporating a finite production rate with a lot-for-lot policy for the vendor [5] and relaxed for the lot-for-lot policy and proposed a more general joint economic lot sizing model [6]. In 1995, a model was proposed where successive shipment sizes increase by a ratio equal to the production rate divided by the demand rate [7]. Other related integrated inventory system between single vendor and single buyer can be referred to, for example [8]-[14]. In 2016, a model was introduced for single vendor and multi buyer integrated production-inventory model for food products that incorporates the quality degradation [15]. In 2017, this model was expanded to a proposed single vendor single manufacturer integrated inventory model with stochastic demand and variable production rate [16]. Based on this study, the variation of the production rate enables the manufacturer to reduce lead times and to accommodate the demand uncertainty. The vendor shipped finished products in multiples full truckloads to the manufacturer. In 2018, a single vendor single buyer model with stochastic demand and transportation cost was introduced with the transportation cost was a function of shipping weight, distance and transportation modes [3].

Research done in 1995 was the first integrated inventory system that considered single vendor and

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multi buyers model [17]. This model was assumed that the vendor had an advantage over the buyer by allowing shipments during the production period. In 2001, this model was expanded by considering that vendor offer a price discount and persuade the buyers to have the product replenishment on a schedule given by the vendor[18]. The optimal replenishment period and price discount quantity were derived using Stackelberg game approach. In 2001, a single vendor multi buyers integrated model was also proposed by extending the idea of synchronization so that the total inventory cost could be reduced [19]. In 2013, a similar topic, a single vendor multi buyers integrated production-inventory model, was also introduced by considering the controllable lead time and service level constraint [2]. In this research, the buyer's demand was assumed to be independent normally distributed and the lead time of each buyer could be reduced with addition cost called the crashing cost. The buyers reviewed their inventories using the continuous review policy and the unfulfilled demand at the buyers was backordered.

The other integrated inventory system of the parties in supply chain were also done by [20]-[22]. In 2014, a model was also introduced by considering three parties in the integrated inventory system, single supplier, single vendor, and single buyer. In this integrated inventory system, a single vendor procured raw materials from a single supplier, processed the raw materials and shipped the finished products to a single buyer who stored the products in a warehouse before shipping them to the end customer. The deterministic demand was assumed and dependent on the level of items displayed. In 2016, this model was extended by proposing a three-party integrated joint economic lot sizing model for single supplier, single vendor and multi buyers which was considered by using common replenishment time period [20]. The demand rate depended on the displayed stock level. In 2017, an integrated production-inventory model for multi vendors and single buyer was developed [22]. This model considered a shortage permitted based on shortage being allowed to occur only for the buyer. In 2018, a proposed single-vendor and single buyer integrated inventory model was also introduced by assuming that the lead time a linear function of batch size, setup time and transportation time [23]. The objective function was to decide an optimal decision that minimize the expected total annual cost. A vendor-buyer integrated model was introduced in 2019 that consider the items with constant deteriorating rate and increasing time-varying demand rate [24]. Joint decision on integrated supplier selection was also considered by considering the purchase discount given by supplier and the product demand [25].

So far, there is no integrated production-inventory model for multi vendors and multi buyers. So in order to enrich the research in the integrated production-inventory model, this paper proposed a development of model with normally distributed demand. This paper developed an integrated production-inventory model for single vendor multi buyers based on the model proposed in [2]. As assumes in [2], that the lead time was not identic, this paper also assumes non-identical lead time of buyers. In the proposed model, we relaxes the controllable lead time and considers the service level constraint (SLC) that corresponds to each buyer caused by the difficulties in estimating the stock-out cost in inventory system.

2. Material and Method

This model is made by using some following assumptions:

1. There are multiple vendors that supplied single item product for multiple buyers.
2. Demand of each buyer is independent and normally distributed.
3. Rate of demand for buyer j is D_J.
4. Vendor i will produce $\mathrm{m_i}\sum_{j=1}^{n} Q_{ij}$ units with production rate P_i (note that $\mathrm{P_i} > \sum_{j=1}^{n} D_{ij}$) in one set up, but shipping $\mathrm{Q_i} = \sum_{j=1}^{n} Q_{ij}$ units for m_i times to fulfill all buyers' demand.
5. Vendor i will ship $Q_{ij} = \frac{\mathrm{D_{ij}}}{\mathrm{D_j}} Q_i$ units to buyer j, with $D_{ij} = \frac{\mathrm{P_i}}{\sum_{i=1}^{\mathrm{M}} \mathrm{P_i}}$ and $\mathrm{D_j} = \sum_{j=1}^{M} D_{ij}$.
6. All buyers using continuous review policy and order when the inventory level reaches the reorder point.

7. Every stock out from buyer will be backordered.

Notations that used in the model are:

Q_{ij}	=	units of item sent to buyer *j* by vendor *i* (decision variable)
D_{ij}	=	demand from buyer *j* to vendor *i*
D_j	=	total demand of buyer *j*
r_j	=	reorder point buyer *j*
s_j	=	safety stock buyer *j*
m_{ij}	=	number of lots delivered from the vendor *i* to buyer *j* (decision variable)
P_i	=	production rate of vendor *i*
A_{ij}	=	ordering cost from buyer *j* to vendor *i*
L_{ij}	=	lead time from vendor *i* to buyer *j*
k_j	=	safety factor of buyer *j* (decision variable)
N	=	number of buyers
M	=	number of vendors
ϕ	=	probability density function of standard normal distribution
Φ	=	cumulative density function of standard normal distribution

3 Results and Discussion

This model is developed based on a model by [2]. The model developed in this research involves multi-vendor and multi-buyer. Buyer *j* has demand rate of D_j units per unit time. Each buyer applies a continuous review policy, so that if the inventory level reaches the reorder point (r_j), the order is placed into the system. Vendor *i* ships as many as Q_i units in one shipment with the total number of shipments of m_i times per unit time. Q_i units are distributed to all buyers, so buyer *j* will receive Q_{ii} units from vendor *i*. Each buyer receives product from all vendors and the amount is proportional to the production rate of each vendor. An illustration can be seen in Figure 1.

Buyer *j* cost components are ordering cost and holding cost. That are:

$$\text{ordering cost } = \sum_{i=1}^{M}\left(A_{ij}\times\frac{D_{ij}}{Q_{ij}}\right) \tag{1}$$

$$\text{holding cost } = \sum_{i=1}^{M}\left(\frac{Q_{ij}}{2}+k_j\sigma_j\sqrt{L_{ij}}\right) \tag{2}$$

Total inventory cost for buyer *j* is:

$$TEC_{bj}=\sum_{i=1}^{M}\left(A_{ij}\times\frac{D_{ij}}{Q_{ij}}\right)+\sum_{i=1}^{M}\left(\frac{Q_{ij}}{2}+k_j\sigma_j\sqrt{L_{ij}}\right) \tag{3}$$

While for vendor, expected inventory level is the difference between vendor's inventory level and buyer's inventory level.

$$\bar{I}_{vi}=\left[\left(m_iQ_i\left(\frac{Q_i}{P_i}+\frac{(m_i-1)Q_i}{D_i}\right)-\frac{m_i^2Q_i^2}{2P_i}\right)-\sum_{j=1}^{N}\frac{Q_{ij}^2}{D_{ij}}\left(1+2+\ldots+(m-1)\right)\right]\frac{\sum_{j=1}^{N}D_{ij}}{m_iQ_i} \tag{4}$$

With simplify and substitution $Q_{ij}=\frac{D_{ij}}{D_j}Q_i$ and $D_i=\sum_{j=1}^{N}D_{ij}$

$$\bar{I}_{vi}=\frac{(2-m_i)}{2P_i}D_iQ_i+(m_i-1)Q_i\left(1-\sum_{j=1}^{N}\frac{D_{ij}}{D_j^2}\right) \tag{5}$$

Total inventory cost for vendor *i* consist of setup cost and holding cost. So the total inventory cost is:

$$TEC_{vi}=\frac{A_{vi}D_i}{m_iQ_i}+h_{vi}C_{vi}\bar{I}_{vi} \tag{6}$$

$$TEC_{vi}=\frac{A_{vi}D_i}{m_iQ_i}+h_{vi}C_{vi}\bar{I}_{vi}\left(\frac{(2-m_i)}{2P_i}D_iQ_i+(m_i-1)Q_i\left(1-\sum_{j=1}^{N}\frac{D_{ij}}{D_j^2}\right)\right) \tag{7}$$

Total inventory cost for integrated model is:

$$TEC=\sum_{j=1}^{N}TEC_{bj}+\sum_{i=1}^{N}TEC_{vi} \tag{8}$$

$$TEC = \sum_{j=1}^{N}\left(\sum_{i=1}^{M}\left(A_{ij} \times \frac{D_{ij}}{Q_{ij}}\right) + \sum_{i=1}^{M}\left(\frac{Q_{ij}}{2} + k_j \sigma_j \sqrt{L_{ij}}\right)\right) + \sum_{i=1}^{M} \frac{A_{vi} D_i}{m_i Q_i} + h_{vi} C_{vi}\left(\frac{(2-m_i)}{2P_i} D_i Q_i + (m_i - 1)Q_i\left(1 - \sum_{j=1}^{N} \frac{D_{ij}}{D_j^2}\right)\right) \quad (9)$$

In this model, stock out was not calculated. As substitute, this model used service level concept. The service level for buyer j is expressed as the ratio between expected demand shortages at the end of the cycle of buyer j and the quantity available at buyer j for satisfying the demand per cycle. This ratio must not exceed a value of α_j.

Stock out on buyer j occurs if the request during the lead time exceeds the reorder point. Based on [26], if the request from buyer j follows the normal distribution.
The demand shortage expectation is

$$B_j = \sigma_j \sqrt{L_j \psi(k_j)} \quad (10)$$

with $\psi(k_j) = \phi(k_j) - k_j(1 - \Phi(k_j)) > 0$. To meet the service level of $1-\alpha_j$, then:

$$\frac{\sigma_j \sqrt{L_j \psi(k_j)}}{Q_j} \le \alpha_j \quad (11)$$

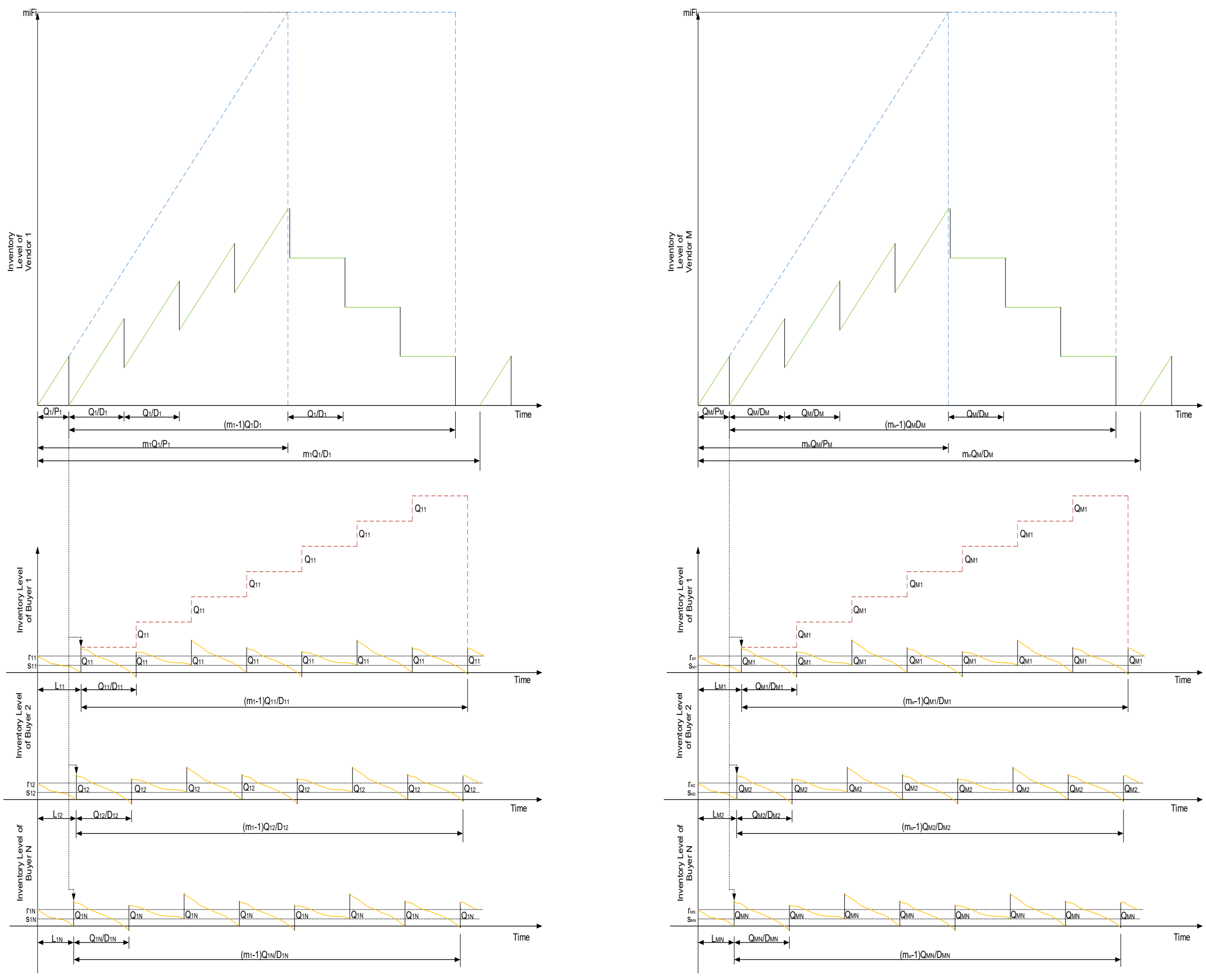

Figure 1. Inventory pattern for buyers and vendors

Substitute $Q_j = \sum_{i=1}^{M}\left(\frac{D_{ij}}{D_j}Q_i\right)$, so:

$$\frac{D_j\sigma_j\sqrt{L_j\psi(k_j)}}{\sum_{i=1}^{M}\left(D_{ij}Qi\right)} \le \alpha_j \tag{12}$$

The mathematical formulation of an integrated inventory system for multi-vendor and multi-buyer with one item is:

$$\min TEC = \sum_{j=1}^{N}\left(\sum_{i=1}^{M}\left(\frac{A_{ij}D_{ij}}{Q_{ij}}\right) + \sum_{i=1}^{M}\left(\frac{Q_{ij}}{2} + k_j\sigma_j\sqrt{L_{ij}}\right)\right)$$

$$+\sum_{i=1}^{M}\frac{A_{vi}D_i}{m_iQ_i} + h_{vi}C_{vi}\left(\frac{(2-m_i)}{2P_i}D_iQ_i + (m_i-1)Q_i\left(1-\sum_{j=1}^{N}\frac{D_{ij}}{D_j^2}\right)\right) \tag{13}$$

$$\text{st. } \frac{D_j\sigma_j\sqrt{L_j\psi(k_j)}}{\sum_{i=1}^{M}\left(D_{ij}Qi\right)} \le \alpha_j \qquad \forall j \tag{14}$$

The integrated production-inventory model for single product multi vendors and multi buyers has developed based on the extension of the model developed in [2]. The model objective is to minimize total inventory cost of integrated/collaborated vendors and buyers. The total cost is calculated for the vendors first and then for the buyers. After getting the total cost for vendors and buyers than the total inventory cost of integrated/collaborated vendors and buyers is obtained by adding them. Since the demand is probabilistic, there is a probability for stockout. Since the estimation of the stock out cost in the inventory system is difficult, then a service level corresponding to each buyer is included in the model and become a constraint in the model. Safety stock calculation is based on the calculation in ref [26].

For further research, the model is tested to solve the integrated production-inventory model for single product multi vendors and multi buyers using the hypothetical data to get the performance of the model and the alternative in solving the problem is proposed. For others, the model can be added by another constraint like warehouse constraint for buyers, budget constraint and so on. The model can also be developed by adding multi product constrain, since many companies deliver or produce multi product in their business.

4. Conclusions

This study has derived an integrated optimization of the total cost model that can be used in the case of single product, multi vendors and multi buyers. The model will generate the quantity of optimal shipping units and the frequency of shipment for each buyer. This model considers the service level as the constraint for each buyer, which of course will be related to the amount of stock out.

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