# MAKING THE LANGUAGE OF SECONDARY MATH CLASSROOMS MORE COMPREHENSIBLE FOR ENGLISH LEARNERS 

by

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## Summary of Project

Research question: How can secondary mathematics teachers make language features in the mathematical register more comprehensible for English learners within a mainstream mathematics content classroom?

This project is a curriculum supplement for a MN standards-based algebra course. It was designed specifically for a 9th-grade algebra 1 class that covers $8^{\text {th }}$ grade linear algebra concepts as defined by the Minnesota State Standards (Minnesota Department of Education, 2007). The class primarily covers 8th grade standards and many students in the class were unsuccessful in a linear algebra class during the previous school year. Typically, there are between 50\%-80\% EL students at WIDA levels 1-3 enrolled in this course as it covers foundational material students need for higher level classes. This project provides eight activities teachers can implement to help beginning ELs (WIDA levels 1-3) access the language of lines. The activities focus on helping ELs build language skills within the math classroom.

There were two frameworks used in the design of the project. The first framework is Backward Design (Wiggins \& McTighe, 2011) and the second is the Mutually Adaptive Learning Paradigm (MALP) (Marshall \& DeCapua, 2013). The Backwards Design framework was used to create resources focused on the standards, while MALP ensured that these resources were aimed at specifically helping ELs access the language and mathematical content. The basic premise of MALP is starting with a topic students are comfortable with to teach a new language skill. Once students understand the language skill it can then be applied to the course content. The goal is to provide scaffolds so that students are not asked to interact with content material using language skills they are unfamiliar with.

Being a curricular supplement, this project is designed for an algebra course that already has a curriculum in place. While some of the activities can be used in place of existing materials, it is not intended to be a complete curriculum. As such, I did not create presentations or lessons for any topics, nor did I create units that are sufficient to fully teach a topic. It is expected that teachers already have materials for teaching the entire algebra course and I have created eight activities and given ideas for how to implement them into classroom lessons and existing assessments. The activities are unique in content and strategy. Ideally teachers can adapt each activity for more topics and have a toolbox of eight different strategies to help ELs within their classroom. These activities are designed for linear equations, forms of a line, and interpreting linear graphs, but all the activities could be adapted for any topic in a math classroom. Each activity includes a teacher version which includes suggested answers and 'teacher talk' tips for successful implementation. There is also a student version which can be printed and handed directly to students. Introducing each activity is an informational page which explains the rationale behind the activity and gives multiple ideas for how to use it in a classroom. The final section of the informational page explains how the activity can be used as an assessment tool. Sometimes the activity itself can be used as an assessment or included on an existing assessment, other times this section gives ideas for how to modify assessments so ELs are not prevented from accurately demonstrating their content knowledge because of their English skills.

## Activities to Support Language Use in a Mainstream Algebra Classroom

The following activities are designed to support English Learners (ELs) in a mainstream algebra classroom. The focus of the activities is to help students move from informal, daily language to formal academic language. Ideas for how to implement each activity are given, but these are only suggestions, all activities can be used in a variety of ways. Additionally, ideas for using each activity as an assessment are also included. Each activity will first have the teacher copy followed by a student copy. Keep in mind, since the goal is language development included answers are only suggestions.

Activities have been designed using a combination of Understanding by Design (Wiggins \& McTighe, 2011) and the Mutually Adaptive Learning Paradigm (MALP). The basic premise of MALP is to start with something students are familiar with when teaching a new language skill, once the skill is comfortable and familiar then it is applied to the new content language. In this way students are only wrestling with one new element at a time, first a new language skill, second new content (Marshall \& DeCapua, 2004)

Many activities are adapted from or inspired by activities found in The ELL Teachers' Toolbox (Ferlazzo \& Hull, 2018).

Included activities are aligned with the following 2007 Minnesota Academic Standards for Mathematics:
8.2.1.3 Understand that a function is linear if it can be expressed in the form $f(x)=m x+b$ or if its graph is a straight line.
8.2.2.1 Represent linear functions with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another.
8.2.2.2 Identify graphical properties of linear functions including slopes and intercepts. Know that the slope equals the rate of change, and that the $y$-intercept is zero when the function represents a proportional relationship.
8.2.4.1 Use linear equations to represent situations involving a constant rate of change, including proportional and non-proportional relationships.
8.2.4.3 Express linear equations in slope-intercept, point-slope and standard forms, and convert between these forms. Given sufficient information, find an equation of a line.
8.3.2.1 Understand and apply the relationships between the slopes of parallel lines and between the slopes of perpendicular lines. Dynamic graphing software may be used to examine these relationships.

## Developing Definitions

## Rationale:

Mathematical definitions involve precise language and many students are not familiar with this type of precision. Many are not familiar with the idea of a formal definition in general. To teach students what a formal definition is and how to interpret them, create a formal definition of something students are familiar with first (MALP). Once students understand the concept of writing definitions, then they can attempt to write definitions of mathematical terms. The exercise will also help students interpret definitions themselves. Feel free to use this to create definitions of any math terms (integer, whole number, polynomial, square root, etc.) Some examples have been included.

## Ideas for use:

Do the pizza example on the first day of school, use the same format for all definitions throughout the course.

Do the pizza example together, have students create definitions of something they are familiar with (food, animal, person, sports, etc). Then write definitions of math words students are familiar with (add, equation, multiply, etc). Finally, have students write definitions for known math words on their own.

Construct definitions together during the notes/lesson and include them in a math journal.
As a review activity have small groups of students construct definitions for terms they know. Each group shares their definitions. Or, do this on the first day of class, write definitions for words they should know.

## Assessment Options:

Have students create a definition on a summative test.
Instead of a 'matching' section, have students identify terms given a word bank and the completed definition chart.

Have students create definitions in pairs/groups, listen to their conversations or collect their definitions to find misconceptions in the class.

Use the activity on the first day to see what students know (or don't remember) about previous material.

## Developing Definitions (Teacher Version)

What is a formal, mathematical definition? In math, we use language that is very specific. Definitions explain what things are in math. We are picky about definitions. A definition is a description that is ALWAYS true for EVERY item. There are NO EXCEPTIONS to definitions.

Example: Pizza

| What does every pizza have? | How is every pizza organized? |
| :--- | :--- |
| (. Emphasize that these features MUST be | (emphasize that this must be true for ALL pizzas. The |
| present in ALL pizzas, they are what make a |  |
| pizza a pizza) | organization is partly what makes it a pizza. Use 'circle' |
| *crust | as a non-example since pizzas can be other shapes) |
| *sauce | *crust on bottom |
| *toppings | *sauce and toppings on top of crust |
|  | *baked |

Definition: A pizza is a baked food that has a crust on the bottom with sauce and toppings on top.

## Definition of: __fish

| What does every_fish__ have? | More information about ALL__._. |
| :--- | :--- |
| *scales | *live in water |
| $*$ fins | *animal |
| $*$ gills | *breathes water, not air |
|  |  |
|  |  |

Definition: A fish is an animal that lives in the water and has scales, fins, and gills and breathes water.

## Definition of: Linear Equation

What does each variable represent?

Has ordered pairs as a solution ( $\mathrm{x}, \mathrm{y}$ )
What operations are being used? List at least 3 words you could use for each one.

Can use many operations, but must make a straight line when graphed.

Definition: A linear equation is any equation that has 2 variables and makes a straight line when graphed. Explain if students are ready: although $y=2$ only has one variable visible, solutions of the line all have 2 variables ( $x, 2$ ).

## Definition of: Y-Intercept $\quad(0, y)$

| What does each variable represent? What operations are being used? List at least 3 words <br> you could use for each one. <br> Y: is the y-coordinate of an ordered pair No operations. <br> This is where a line touches the y-axis |
| :--- |
| Definition: The y-intercept is the ordered pair where a line touches the y-axis. It always has the ordered <br> pair $(0, y)$. |

## Definition of: X-Intercept $\quad(x, 0)$

What does each variable represent?
What operations are being used? List at least 3 words you could use for each one.
$\mathrm{X}: \mathrm{x}$ coordinate of the ordered pair
No operations
Where the line crosses the x -axis

Definition: The x -intercept is where the line crosses the x -axis. It always has the ordered pair ( $\mathrm{x}, 0$ )

## Definition of: Direct Variation $y=k x$

What does each variable represent?
y: y-coordinate, output
k : constant of variation
x : x -coordinate, input

Definition: Direct variation is a linear equation where the $y$-coordinate is equal to the constant of variation times the x -coordinate.

## Definition of: Slope-Intercept Form $y=m x+b$

| What does each variable represent? | What operations are being used? List at least 3 words <br> you could use for each one. |
| :--- | :--- |
| Y: output value | = equal to, equals, is |
| M: slope | x times, multiply, of |
| X: input value | +add, sum, combine |
| B: y-intercept |  |

Definition: Slope-intercept form is a linear equation where the output value is equal to slope times the input plus the y-intercept.

## Definition of: Point-Slope Form $\quad y-y_{1}=m\left(x-x_{1}\right)$

What does each variable represent?
y : output value
$y_{1}$ : the $y$-coordinate of the ordered pair
m: slope
$x$ : input value
$\mathrm{x}_{1}$ : x -coordinate of the ordered pair
What operations are being used? List at least 3 words
you could use for each one.
= equals, equal to, is

- subtract, minus, less, difference
x multiply, times, of
() distribution

Definition: Point slope form a linear equation where the output value minus the y-coordinate of a point is equal to the slope times the difference of the input minus the x -coordinate of the point.

## Developing Definitions

What is a formal, mathematical definition? In math, we use language that is very specific. Not just any words will do. Definitions explain what things are in math. We are picky about definitions. A definition is a description that is ALWAYS true for EVERY item. There are NO EXCEPTIONS to these definitions.

## Example: Pizza

| What does every pizza have? | How is every pizza organized? |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

## Definition of:

| What does every ___ have? | More information about ALL__. |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

## Definition:

## Definition of:

| What does each variable represent? | What operations are being used? List at least 3 words <br> you could use for each one. |
| :--- | :--- |
|  |  |
|  |  |
| Definition: |  |

## Definition of:

What does each variable represent?
What operations are being used? List at least 3 words you could use for each one.

## Definition:

## Rate of Change

## Rationale:

Many students struggle describing the rate of change. While we naturally use many different phrases to mean the same thing (Ex: 15 miles per hour, you go 15 miles in one hour, in one hour you can travel 15 miles, etc.) most beginning ELs will not recognize these different phrases as meaning the same thing. The sentence frame provided may seem stringent and will not always create the most natural sounding phrase, however, it does always connect the table to the language. If students can make that connection they will have a better understanding of the rate of change as a concept. As their English improves, they will become more comfortable rephrasing the sentence frame. For level 1 and 2 students, the sentence frame provides a bridge between the concept and the language.

Note: One easy "help" to make the frame sound more natural is to add in the words the number of anytime the $x$ or $y$ variable is counting a noun. See number 1 for an example of this. Level 1 students may not understand this change, then it is more important to stick to the frame and have an awkward sounding sentence. Level 2 and 3 students will likely be able to make this change as needed.

## Ideas for use:

Incorporate the sentence frame into your lesson. Write it on the front board etc. Give it as an option for all students so they can see the connection with the labels, then let students who feel comfortable rephrase it as they choose.

Practice the sentence frame orally in small groups.
Have students practice the frame orally in pairs.
Have students write/copy the frame and fill in the blanks.
Instead of having students write the sentence, accept an oral answer using the sentence frame.

## Assessment Options:

In general, writing is more difficult than speaking for many students, as a formative assessment have them say the explanation of rate of change without writing it.

On a summative assessment, accept an oral explanation of rate of change instead of a written answer.

Provide the sentence frame on a summative test for ELs to fill in. They can do the calculations on their own, but the sentence frame will help them write the explanation.

## Rate of Change (Teacher version)


change in $x$
no labels
slope

The only difference between rate of change and slope is the labels.
Make sure to simplify when comparing.

| Hours <br> worked <br> $(\mathrm{x})$ | Money <br> earned <br> $(\mathrm{y})$ |
| :---: | :---: |
| 2 | 30 |
| 4 | +30 |
| 6 | 60 |
| 8 | +2 |
| 8 | +30 |
| 2 | +30 |

$$
\frac{\text { change in } y}{\text { change in } x}=\frac{+30}{+2}=\frac{15}{1}
$$

Money earned goes up/down by 15 as hours worked goes up/down by $\underline{1}$. y -label x-label

Use the sentence frame to describe the rate of change for each table.
$\qquad$ goes up/down by $\qquad$ as $\qquad$ goes up/down by $\qquad$ . y-label x -label
1)

| Students <br> $(\mathrm{x})$ | Pencils <br> $(\mathrm{y})$ |
| :---: | :---: |
| 5 | 60 |
| 10 | 45 |
| 15 | 30 |
| 20 | 15 |

2) 

| Hours <br> $(\mathrm{x})$ | Temperature <br> $(\mathrm{y})$ |
| :---: | :---: |
| 1 | 75 |
| 2 | 71 |
| 3 | 67 |
| 4 | 63 |

3) 

| Temperature <br> degrees $F$ <br> $(x)$ | Inches of <br> snow (y) |
| :---: | :---: |
| 33 | 0 |
| 31 | 2 |
| 29 | 4 |
| 27 | 6 |

1) The number of pencils goes down by 3 as the number of students goes up by 1 .
2) The temperature goes down by 4 as the hours goes up by 1 .
3) The inches of snow goes up by 1 as the temperature goes down by 1 .


The only difference between rate of change and slope is the labels.
Make sure to simplify when comparing.

| Hours <br> worked <br> $(\mathrm{x})$ | Money <br> earned <br> $(\mathrm{y})$ |
| :---: | :---: |
| 2 | 30 |
| 4 | 60 |
| 6 | +30 |
| 8 | 120 |
| 6 | +30 |
| 2 | +30 |

$$
\frac{\text { change in } y}{\text { change in } x}=\frac{+30}{+2}=\frac{15}{1}
$$

Money earned goes up/down by 15 as hours worked goes up/down by 1 . y-label x-label

Use the sentence frame to describe the rate of change for each table.
$\qquad$ goes up/down by $\qquad$ as $\qquad$ goes up/down by $\qquad$ . y-label
"
$\qquad$ x -label
1)

| Students <br> $(\mathrm{x})$ | Pencils <br> $(\mathrm{y})$ |
| :---: | :---: |
| 5 | 60 |
| 10 | 45 |
| 15 | 30 |
| 20 | 15 |

2) 

| Hours <br> $(\mathrm{x})$ | Temperature <br> $(\mathrm{y})$ |
| :---: | :---: |
| 1 | 75 |
| 2 | 71 |
| 3 | 67 |
| 4 | 63 |

3) 

| Temperature <br> degrees $F$ <br> (x) | Inches of <br> snow (y) |
| :---: | :---: |
| 33 | 0 |
| 31 | 2 |
| 29 | 4 |
| 27 | 6 |

1) 
2) 
3) 

## Identifying Slope and Intercepts - Sentence Navigators

Rationale: These sentence navigators can be used at the same time or at various times, reorganize them to fit your specific need. The abstract concept of identifying parts of a line can be confusing to all students, these navigators are one way to help students link the vocabulary with the concept. These will help students become more comfortable with how to talk about parts of a line and how to answer questions about lines. No answer key has been provided, instead a blank template follows the activity.
*Note: adapted from The ELL Teacher's Toolbox by Larry Ferlazzo and Katie Hull Sypnieski No answer key has been provided, instead a blank template follows the activity.

## Ideas for Use:

Do one or two navigators together so students understand how they work. Have students complete them individually or in pairs/groups.

Do Think-Pair-Share and have pairs share out their answers once completed.
Once students complete their sentences, have them practice reading them out loud to a partner.

## Assessment:

Accept these navigators in place of a regular worksheet or specific problems focused on identifying slope and/or intercepts.

Modify tests to include a sentence navigator in place of open-ended "identify" questions. It will help ELs understand what is being asked and give some direction on what they need to identify.

## Identifying Slope and Intercepts - Sentence Navigators

Example: Make sure to use correct capitalization

|  | What is the cat doing? |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | the | cat | are | reading |
|  | Cat | am | eating |  |
|  | A | dog | is | sleeping |

Directions: Answer the question about each graph. Circle words to make a correct sentence.

|  | 1) What is the slope of the graph? |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | The | y-intercept | is |
|  | the | slope | was | 2 |  |  |
|  |  |  |  |  |  |  |


|  |  | 2) What is the slope of the graph? |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |



|  |  | 4) What is the slope of the graph? |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


|  |  |  | 5) What is the slope of the graph? |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | slope | is |
|  |  |  | A | -2 |  |  |




|  | 7) What is the y-intercept of the graph? |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | the | y-intercept | am | 1 |  |
|  | The | x-intercept | are | -1 |  |
|  |  |  |  |  |  |


|  | 8) What is the $x$-intercept of the graph? |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |



|  |  | 10) What is the y-intercept of the graph? |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | The | y-intercept |

Blank Sentence Navigators


## Direct Variation

## Rationale:

Direct variation word problems use very specific language in a set pattern. If students know this pattern they can accurately interpret these word problems. This is not uncommon with other topics in mathematics, some of the same ideas can be used with those topics as well.

Note: Students may struggle to identify what the variable is, remind students that in a word problem the variable is always something that can be counted. This means that the variable is always a countable noun (person, place or thing). This can help eliminate students guessing random words.

## Ideas for Use:

Use a highlighter to highlight the words varies directly, then use different colors and highlight the dependent variable, independent variable, and constant of variation.

Do a couple of examples with highlighting for the entire class during the lesson.
Have students work in pairs and take turns identifying/highlighting a different piece of the word problem. They can write the equation together.

Have students use highlighters on worksheets/homework

## Assessment Options:

Have students highlight the word problem to define the dependent and independent variables instead of writing them.

Instead of writing their answer, let students point and/or give an oral answer to identify the variables. Accept an oral equation as well.

## Direct Variation (Teacher Version)



Remember that the $y$ variable is the output or the dependent variable. Also, $x$ is the input or the independent variable. The $k$ is the constant of variation or the rate of change, it will always be a number with a label.

All direct variation problems follow the same pattern:
Dependent variable varies directly with the independent variable.
When you read a direct variation problem, look for the words varies directly. Once you find these words then look in front for the dependent variable and after for the independent variable.

Example: The total cost of buying grapes at the store varies directly with the number of pounds at a rate of $\$ 0.50$ per pound.

Identify: dependent variable: total cost
Equation: $y=0.50 x$
Independent variable: number of pounds
Constant of Variation: \$0.50 per pound
*Teacher talk: 1) remind students that variables are counting things, this means that when identifying variables they are always looking for nouns (person/place/thing). There are 2 nouns before the words varies directly, both "total cost" and "grapes" help students see that total cost is the variable because this is the number we want to know, also it is before the verb 'buying,' generally variables come before the verb. 2) the word rate is a good clue for identifying $k$ as it links back to rate of change.

## Practice:

1) The distance you drive in the car varies directly with the speed you are traveling. If you are traveling at a rate of 65 miles per hour, write an equation for this relationship.

Identify: dependent variable: distance
Equation: $y=65 x$
Independent variable: speed
Constant of variation: 65 miles per hour
2) The amount of money you earn varies directly with how many hours you work at a rate of $\$ 13.50$ per hour.

Identify: dependent variable: money you earn
Equation: $y=13.50 x$
Independent variable: hours you work
Constant of variation: $\$ 13.50$ per hour
3) The height of bamboo varies directly with how old it is at a rate of 36 inches per day.

Identify: dependent variable: height
Equation: $y=36 x$
Independent variable: how old (days)
Constant of variation: 36 inches per day
4) The total cost of buying gas for your car varies directly with the gallons of gas at rate of $\$ 3.00$ per gallon.

Identify: dependent variable: total cost
Equation: $y=3 x$
Independent variable: gallons of gas
Constant of variation: $\$ 3.00$ per gallon

## Direct Variation



Example: The total cost of buying grapes at the store varies directly with the number of pounds at a rate of $\$ 0.50$ per pound.

Identify: dependent variable: total cost
Equation: $y=0.50 x$
Independent variable: number of pounds
Constant of Variation: $\$ 0.50$ per pound

## Practice:

1) The distance you drive in the car varies directly with the speed you are traveling at a rate of 65 miles per hour.

Identify: dependent variable:

## Equation:

Independent variable:
Constant of variation:
2) The amount of money you earn varies directly with how many hours you work at a rate of $\$ 13.50$ per hour.

Identify: dependent variable:
Equation:
Independent variable:
Constant of variation:
3) The height of bamboo varies directly with how old it is at a rate of 36 inches per day.

Identify: dependent variable:
Equation:
Independent variable:
Constant of variation:
4) The total cost of buying gas for your car varies directly with the gallons of gas at rate of $\$ 3.00$ per gallon.

Identify: dependent variable:
Equation:
Independent variable:
Constant of variation:

## Forms of a Line using Inductive Learning

## Rationale:

Inductive Learning is when students are in charge of drawing a conclusion from given information. It may seem obvious to us, but many students do not understand that forms of a line are named after the information used to create the formula. It is important for students to be able to identify what information they are given and then choose which form of a line to use. This activity assumes students are familiar with the terms and concepts of points, slope, intercepts.
*Note: adapted from The ELL Teacher's Toolbox by Larry Ferlazzo and Katie Hull Sypnieski

## Ideas for use:

Before introducing any forms of a line, have the students complete the activity. They should create their groups individually or in pairs.

Use a Think-Pair-Share format.
Use as a review activity once students know the forms of a line. Have them explain how they decided which form each equation was.

Identify the names of each group (slope-intercept, point-slope, standard) and have students take turns identifying and explaining to each other where one equation goes.

## Assessment:

Listen to student explanations about their reasoning for creating each group, this will give an idea of where the class is as a whole and students as individuals.
collect student work to see the groups they created, look for common patterns or misconceptions that might need to be addressed.

## Forms of a Line using Inductive Learning (Teacher Version)

Teacher Talk: Students may create different categories than you are expecting, this is fine! Make sure to acknowledge and affirm the patterns students recognize then point out the three categories we will use in Algebra.

1) Put the equations into 3 different groups.
2) Explain to your partner why you chose to organize them in this way.
3) Compare your groups with your partner's groups, are they the same or different?
$y=4 x-3$
$y-7=3(x-5)$
$4 x+3 y=9$
$y+4=-2(x-5)$
$9 x-5 y=15$
$y=-6 x=7$
$y=\frac{1}{2} x+4$
$y-4=\frac{-2}{3}(x+2)$

| Group 1 | Group 2 | Group 3 |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

In Algebra, we have 3 formulas we use to write linear equations. Each formula has a little different pattern. Can you put the following formulas into your groups?

$$
y=m x+b \quad y-y_{1}=m\left(x-x_{1}\right) \quad A x+B y=C
$$

Some formulas are named based on what information is used to create them.


Some formulas are named because they are applied to more topics in later classes.
Standard Form

$$
A x+B y=C
$$

Teacher Talk: At this time, label each group with its formula name. Explain to students that we will be studying each formula more in depth in upcoming days.

## Forms of a Line using Inductive Learning

## Directions:

1) Put the equations into 3 different groups.
2) Explain to your partner why you chose to organize them in this way.
3) Compare your groups with your partner's groups, are they the same or different?
$y=4 x-3$
$y-7=3(x-5)$
$4 x+3 y=9$
$y+4=-2(x-5)$
$9 x-5 y=15$
$y=-6 x=7$
$y=\frac{1}{2} x+4$
$y-4=\frac{-2}{3}(x+2)$

| Group 1 | Group 2 | Group 3 |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |

In Algebra, we have 3 formulas we use to write linear equations. Each formula has a little different pattern. Can you put the following formulas into your groups?
$y=m x+b$

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

$$
A x+B y=C
$$

Some formulas are named based on what information is used to create them.


Some formulas are named because they are applied to more topics in later classes.
Standard Form
$A x+B y=C$

## Describe the Graph

## Rationale:

This activity is a 'cloze.' It is a way for beginning ELs to interact with mathematical content without getting lost in the language. Beginning students are not yet able to create extensive language on their own, the cloze activity takes away the burden of creating the language while still letting them apply their content knowledge.

## Ideas for Use:

Do the first cloze together as a group talking about why you are choosing each word. Point out connections between the words provided (y-intercept) and how those key words make you look on the graph to fill in the blanks. Then have students complete the second cloze individually or in pairs.

Have students take turns reading the completed sentences out loud to a partner. This is great oral practice.

Have students complete individually and then read aloud to a partner. The partner must listen and see if their answers match. If answers don't match students can explain their reasoning and come to a consensus on a single answer.

## Assessment:

Instead of requiring open ended answers on an assessment, create a cloze that allows students to still show the same content knowledge without requiring language skills that they are not yet capable of. For example:

Original directions: Identify the slope and y-intercept of the graph.
Cloze question: The slope of the graph is $\qquad$ and the $y$-intercept is $\qquad$ .

The original directions may be too abstract for students to understand but the cloze ensures they know what information they are expected to identify.

## Describe the Graph (Teacher version)

Directions: Use the word bank to complete the sentences about the given graph.
Word Bank
Hours
Money
Rate of change
Slope
Linear
x-intercept
y-intercept
x -axis
$y$-axis
4
3

This graph is linear . The x -axis is counting how many $\qquad$ hours

Saul worked. The $\qquad$ $y$-axis is counting how much money Saul earned.

When Saul worked $\qquad$ hours he earned $\$ 45$. The $\qquad$ rate of change of the graph is $\$ 15$ per hour worked. The $y$-intercept OR x-intercept is 0 .

Word Bank

5
15
-3
cookies
x -axis
$y$-axis
x-intercept
$y$-intercept
person
people


There are 15 cookies at the beginning, this is the $y$-intercept. The $y$-axis is counting how many cookies are left. The $\underline{x}$-axis is counting how many people there are. The rate of change is -3 cookies per __person . The x-intercept is 5 .

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This graph is $\qquad$ . The x-axis is counting how many $\qquad$ Saul worked. The $\qquad$ is counting how much money Saul earned. When

Saul worked $\qquad$ hours he earned \$45. The $\qquad$ of the graph is $\$ 15$ per hour worked. The $\qquad$ is 0 .

Word Bank

5
15
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cookies
x -axis
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 cookies at the beginning, this is the y-intercept. The $\qquad$ is
There are $\qquad$ counting how many cookies are left. The $\qquad$ is counting how many people
there are. The rate of change is -3 $\qquad$ per $\qquad$ . The
$\qquad$ is 5 .

## Identifying Forms of a Line Listening Activity

## Rationale:

Listening activities can help ELs connect the new language they are learning with the math concept they are working with. If students are struggling, a listening activity can help the teacher determine if they are struggling with the mathematical concept or the English language. The teacher can then provide help aimed at the specific problem.

## Ideas for Use:

Model the 'dog' example first so students understand how to record their answers.
Do as a large group 'bell ringer' exercise with the whole class.
Use in small groups led by a peer tutor/para/teacher.
Have students work in partners, then they will practice both speaking and listening as they trade roles. Make sure to only give each student a script for partner A OR partner B.

## Assessment:

On assessments, read directions out loud to students. If they are struggling to understand the written directions then reading them out loud might help them comprehend the directions.

Collect student answer sheets, quickly glance through to see where the misconceptions/mistakes are.

For questions that involve a lot of reading, read them out loud to students. This can be done in a small group but helps eliminate students making mistakes because of not understanding the written word. Also, accept verbal answers from students instead of written sentences.

## Identifying Forms of a Line Listening Activity (Teacher version)

Read aloud:

1) This equation is in slope-intercept form.
2) This equation is in point-slope form.
3) This equation is in standard form.
4) This is the equation of a vertical line.
5) This is the equation of a horizontal line.
6) This is the formula to calculate slope.

Answer Sheet

| Equation | Number |
| :---: | :---: |
| $y-4=m(x-9)$ | 2 |
| $x=4$ | 4 |
| $y=2 x+4$ | 1 |
| $2 x+3 y=8$ | 5 |
| $y=7$ | 6 |
| $\frac{5-7}{3--2}$ |  |

For partners:

## Partner A

Read aloud:

1) This equation is in slope-intercept form.
2) This equation is in standard form.
3) This is the equation of a horizontal line.

## Answers:

| Equation | Number |
| :---: | :---: |
| $y-4=m(x-9)$ | 2 |
| $x=4$ | 4 |
| $y=2 x+4$ | 1 |
| $2 x+3 y=8$ | 5 |
| $y=7$ | 6 |
| $\frac{5-7}{3--2}$ |  |

## Partner B

## Read aloud:

2) This equation is in point-slope form.
3) This is the equation of a vertical line.
4) This is the formula to calculate slope

| Equation | Number |
| :---: | :---: |
| $y-4=m(x-9)$ | 2 |
| $x=4$ | 4 |
| $y=2 x+4$ | 1 |
| $2 x+3 y=8$ | 5 |
| $y=7$ | 6 |
| $\frac{5-7}{3--2}$ |  |

## Identifying Forms of a Line Listening Activity (Teacher version)

Directions: Write down the number by the equation being described.
Example: If you hear "1) The dog is in his house" You should write a number ' 1 ' by the picture of the dog in a house.


Listen to each description and write the number in the matching box.

| Equation | Number |
| :---: | :--- |
| $y-4=m(x-9)$ |  |
| $x=4$ |  |
| $y=2 x+4$ |  |
| $2 x+3 y=8$ |  |
| $y=7$ |  |
| $\frac{5-7}{3--2}$ |  |

Example: If you hear "1) The dog is in his house" You should write a number ' 1 ' by the picture of the dog in a house.

| $\cdots$ | 1 |
| :---: | :---: |
| $\because$ |  |
|  |  |
|  |  |

Directions: You each have 3 sentences to read and an answer sheet. Take 2 minutes right now to write in the answers of the 3 sentences you have. When you are finished, take turns reading your sentences out loud to your partner. Make sure to mark down an answer based on what your partner reads to you. At the end compare your answers and see if you agree.

## Partner A

## Read aloud:

1) This equation is in slope-intercept form.
2) This equation is in standard form.
3) This is the equation of a horizontal line.

## Answers:

| Equation | Number |
| :---: | :--- |
| $y-4=m(x-9)$ |  |
| $x=4$ |  |
| $y=2 x+4$ |  |
| $2 x+3 y=8$ |  |
| $y=7$ |  |
| $\frac{5-7}{3--2}$ |  |


| Equation | Number |
| :---: | :--- |
| $y-4=m(x-9)$ |  |
| $x=4$ |  |
| $y=2 x+4$ |  |
| $2 x+3 y=8$ |  |
| $y=7$ |  |
| $\frac{5-7}{3--2}$ |  |

## Parallel and Perpendicular Describe Activity

## Rationale:

Coming up with information about a line without any direction is extremely difficult for students. The partner descriptions not only force students to create a description, but they get feedback from their partner on whether or not their description was understandable. If their partner figured out which graph they were describing then their descriptions made sense! For beginning ELs this task will need to be scaffolded. No teacher answers have been provided, instead the page is designed to be printed double sided and cut in half horizontally with partner A on the top half and partner B on the bottom. Differing levels of scaffolds are on the following page. Beginning ELs may need very specific sentence frames that they can fill in. Advanced ELs can use guiding questions to help them come up with a description. Guiding questions may be helpful for all students until they have done enough description activities that they become confident in the process.

## Ideas for Use:

Go over the provided example in a large group. Give a description and have students identify which animal you have described. Then talk students through coming up with descriptions for the example graph.

Use student-student partners and have them take turns describing and identifying.
Have students audio record their conversation and submit it as part of their assignment.

## Assessment:

Use teacher-student partners and observe students' accuracy
On a test, have students come up individually and give you a verbal description of a graph. Use the same graph for all students, grade on if they can identify at least 2 differentiating features. Use sentence frames for beginning ELs.

## Parallel and Perpendicular Describe Activity

## Partner A:

1) Describe the graph -

2) Choose which graph your partner is describing





## Partner B:

1) Choose which graph your partner is describing.




2) Describe the graph -


## Partner A (continued):

3) Choose which graph your partner is describing.




4) Describe the graph


## Partner B (continued):

3) Describe the graph to your partner -

4) Choose which graph your partner is describing.





## Parallel and Perpendicular Describe Activity - Examples

Example: Listen to the description to figure out which picture is being described.


Practice together: How could you describe the graph?

## Guiding Questions:

Do the lines intersect?
What are the y-intercepts?
Are the slopes positive or negative?
Are the lines parallel, perpendicular, or neither?


## Sentence starters:

Both lines have $\qquad$
One line has $\qquad$
The lines are .....

## Sentence frames:

The lines are $\qquad$ .

One line has a $\qquad$ slope, the other has a $\qquad$ slope.

One line has a y-intercept of $\qquad$ , the other has a y-intercept of $\qquad$ .

## Resources:

Marshall, H. W., \& DeCapua, A. (2013). Making the transition to classroom success: Culturally responsive teaching for struggling second language learners. Ann Arbor, MI: University of Michigan Press.

Ferlazzo, L., \& Hull-Sypnieski, K. (2018). The ELL teacher's toolbox: Hundreds of practical ideas to support your students. San Francisco, CA: Jossey-Bass.

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Wiggins, G., \& McTighe, J. (2011). The understanding by design guide to creating high -quality units (2nd ed.). Alexandria, Virgina: ASCD.

