# SYNTHESIS PROGRAM OF HARMONIC RACK-AND-PINION DRIVE CAM GEAR 

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Harmonic rack-and-pinion drive (HRPD) is a new type of rotary motion converter to the translational motion. The principle of the orthogonal interaction pushers and trapezoidal (tapered) teeth of rack is used in HRPD.

Pushers bear either rollers on themselves or teeth with a profile similar to the tooth profile rack (Figure 1). Pushers movement carried out by the drive shaft equipped with cams or eccentric. The number of pushers in the drive may be in the range of 3-8 and more. The number of pushers, number of teeth on the pusher and the module of teeth can be assigned such as to provide the multiband and multipair in engagement. This contributes to a high smoothness and accuracy of movements rack and ensures good tractive ability.


Figure 1 - Diagram of the HRPD unit with pushers carrying multiblade comb
Intermediate links in the form wedge-pushers and multicam driving shaft are used in HRGD. When mounted on pusher rollers, the outer surface of which interacts with the cam roller rotational speed reaching $4000 \mathrm{~min}^{-1}$ and more. Such rotational speeds require the use in the designs of cams mechanism of precision bearings, taking measures to reduce vibration activity and increased longevity, which leads to higher prices for products. HRGD scheme with bearings, built-in pushers, eliminates this drawback, since In this case, the cam interacts with inner ring bearings.

When HRGD designing first of all it is necessary to perform the chart synthesis of pusher motion relatively to the pin rack. Pusher with a thickness of tooth on the top $s_{a t}$ at interaction with pins touches them with wedge two sides $\left(\Omega, \Pi_{0}\right)$. In the extreme upper position pusher change of interacting wedge parties is provided automatically. And in the bottom position on the chart it is necessary to foresee the pusher down in order to change the interacting wedge surface. Diagram of the pusher relative to the rack takes the form (Figure $2)$.

In the diagram let's note 7 sections: 0-1 - lower pusher down, 1-2 - pusher acceleration; 2-3 - linear motion at pusher lifting; 3-4 - pusher run-out at the rise (the change
of the working surfaces of the wedge pusher takes place here); the other three sections are symmetrical to the previous.


Figure 2 - Diagram of relative pusher movement
Initial data: $P_{p}$ - step of pin rack; $Z$ - the number of pushers; $n$ - the number of simultaneously interacting pushers; $\alpha_{p}$ - profile pusher angle; $s_{a t}$ - thickness top pusher; $n_{b}$ shaft rotational speed; $P_{p}^{\prime}$ - calculated step, $D_{\mathrm{p}}$ - shaft diameter.

To review profiling procedures let's consider the first four sections. On condition that the initial rotation angle $\varphi_{10}=0$ will form the equation for the rotation angle of the shaft section 1-2 (acceleration pusher):

$$
\begin{equation*}
\varphi_{20}=\frac{2 \cdot s_{a t} \cdot \pi}{P_{p}} . \tag{1}
\end{equation*}
$$

Value of the expression will also correspond to the rotation angle of the cam on the given section. The equation of the pusher movement on this section is

$$
\begin{equation*}
y_{12 m}=\alpha_{c} \cdot h_{t}, \tag{2}
\end{equation*}
$$

where $h_{t}$ is the height of the rack tooth; $\alpha_{c}$ - the coefficient of pusher lift at the acceleration. Let's form the equation of the shaft rotation angle for the following sector - 2-3 (linear motion at the pusher lift):

$$
\begin{equation*}
\varphi_{30}=\frac{2 \pi \cdot\left(s_{a t}+\beta \cdot P_{p}^{\prime}\right)}{P_{p}} \tag{3}
\end{equation*}
$$

where $\beta$ is the coefficient of the angle cam at the acceleration. The angle numerical value of the cam will be the difference between the values obtained in (1) and (2) expressions. The pusher movement equation will be:

$$
\begin{equation*}
y_{23 m}=y_{12 m}+h_{a}, \tag{4}
\end{equation*}
$$

where $h_{a}$ is the height of the active tooth part. Then section 3-4 (pusher run-out at the lift), the equation of rotational angle and the equation of pusher movement, respectively, equal to

$$
\begin{gather*}
\varphi_{40}=\frac{2 \pi \cdot\left(s_{a t}+\beta \cdot P_{p}^{\prime}+h_{a} \cdot \operatorname{tg} \alpha_{p}\right)}{P_{p}}  \tag{5}\\
y_{34 m}=y_{23 m}+\gamma_{c} h_{t} \tag{6}
\end{gather*}
$$

where $\gamma_{\mathrm{c}}$ is the coefficient of pusher lift at the run-out. For this part of motion is necessary to know the rotation angle in the fifth point, which equals to:

$$
\begin{equation*}
\varphi_{50}=\frac{2 \pi \cdot\left(s_{a t}+\left(\beta+\delta_{c}\right) \cdot P_{p}^{\prime}+h_{a} \cdot \operatorname{tg} \alpha_{p}\right)}{P_{p}} \tag{7}
\end{equation*}
$$

where $\delta_{c}$ is the coefficient of the cam rotation angle at the run-out.
According to the considered dependence the program of the pusher movement diagram synthesis at the pusher lift has been developed in MathCad. Trajectory of the pusher
movement in depending on the rotation angle is shown in Figure 3. Also graphs of velocity and acceleration pusher obtained by dependences relationships are shown here.


Figure 3 - Graphs of the pusher lift as a function of $x$ from angle camshaft: movement -a , an analog of speed $-b$ and the analogue of acceleration $-c$

For the design of the cam of drive it is necessary to consider some features of the cam geometry contact with the inner race, built-in to the pusher.

The cam and rings contact points coincide with the axis $Y$ (Figure 4) only in 0, 3 and 6 points of the diagram (Figure 2). When you rotate a cam 1 around an $O_{1}$ axis in the direction clockwise to the position $\rho_{i}$ radius-vector, growing at the lift, comes into contact with the bearing ring 2 in the K point.

Total normal $n-n$, through the K and $O_{2}$ points, forms the angle $\gamma_{i}$ together with the radius-vector $\rho_{i}$, equal to the angle of pressure $\theta_{i}$. Radius-vector $\rho_{i}$ with the $Y$ axis coordinate cam system forms an $\alpha_{i}$ angle. The distance between the axis of the cam rotation $O_{1}$ and the center bearing $O_{2}$ is the difference between the initial spacing on centres $a_{0}$ and the current pusher movement $h_{i}$.

Let's make synthesis of the cam profile by means of the expressions (2), (4) and (6) conversions into the polar coordinates. For this purpose it's necessary to calculate radiusvector's values $\rho_{i}$ in the K contact point, $\alpha_{i}$ angle for the cam $\varphi_{i}$ angle position and $S_{i}$ the pusher movement.

As a synthesis basis of cam profile, in our case we can take the well-known method of synthesis for cam mechanisms with the dish pusher. Preliminary studies have shown that the pressure angle in the mechanism does not exceed $\theta_{i}=14^{0}$. Taken into account this, the minimal radius of the cam, made in concert with the shaft, can be taken from the design considerations depending on

$$
\begin{equation*}
r_{0}=0,5 d+0,5 \text { мм } \tag{9}
\end{equation*}
$$

where $d$ is the diameter of the driving shaft selected by the condition of its bending stiffness.
Let's perform definition of cam polar coordinates by the following algorithm:

1. Determine the side $\mathrm{O}_{1} \mathrm{~L}$ of the triangle $\mathrm{O}_{1} \mathrm{BL}$

$$
\begin{equation*}
O_{1} L=O_{1} B \cdot \cos \varphi_{i} \tag{13}
\end{equation*}
$$

1. LK segment of the construction is equal to the segment $\mathrm{O}_{1} \mathrm{~m}$. Hence

$$
\begin{equation*}
L K=r_{0}+S_{i} \tag{14}
\end{equation*}
$$

2. We calculate the polar radius vector

$$
\begin{equation*}
\rho_{i}=\sqrt{\left(O_{1} L\right)^{2}+(L K)^{2}} \tag{15}
\end{equation*}
$$

4. We determin the angle between the radius vector $\rho_{i}$ and the normal $n-n$

$$
\begin{equation*}
\gamma_{i}=\theta_{i}=\operatorname{arctg}\left(\frac{O_{1} L}{L K}\right) \tag{16}
\end{equation*}
$$

5. We define the polar angle

$$
\begin{equation*}
\alpha_{i}=\theta_{i}+\varphi_{i} \tag{17}
\end{equation*}
$$



Figure 4 - Scheme of cam 1interaction with the bearing 2
In accordance with the developed algorithm calculated the flat cam parameters, including the radius of profile curvature, built-in to the MathCad of the pusher lift have been calculated (Figure 5).


Figure 5 - Estimated cam profile and its radii of curvature at the pusher lift section
The cam profile is evenly convex with a maximum radius $\mathrm{R} \approx 23,9 \mathrm{~mm}$. The cam can interact with the inner bearing ring with a diameter of which is not less than 35 mm .

At the synthesis of electronic cam model the work of this software module has been relied by using ActiveX technology for communication with the CAD-system by means of API SolidWorks.

The software module is used in the performed final qualification work at the pin rack design with a rollers step on the rack of 20 mm .

