# FLOW PAST AN ACCELERATED HORIZONTAL CYLINDER IN A ROTATING FLUID

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Abstract: An analysis is presented to study the laminar flow of a viscous incompressible fluid past an accelerated horizontal cylinder in a rotating fluid. The exact solutions of the dimensionless governing partial differential equations are obtained by Laplace transform technique. Axial and transverse components of velocity profile, skin friction are derived and effects of rotation parameter and time on these components are presented in graphs. As a departure from the analysis concerning to the horizontal accelerated plate in a rotating fluid, the present analysis shows no inertial oscillation, whatever may be the time small or large.

Keywords: horizontal cylinder; rotating fluid; Laplace transformation

2000 Mathematics Subject Classification: 76U05

Nomenclature:

| | |
|---|---|
| $J_0$ | Bessel function of first kind and order zero |
| $J_1$ | Bessel function of first kind and order one |
| $K_0$ | Modified Bessel function of second kind and order zero |
| $K_1$ | Modified Bessel function of second kind and order one |
| $r$ | radial coordinate |
| $R$ | dimensionless radial coordinate |
| $S$ | dummy real variable used in integral |
| $t'$ | time |
| $t$ | dimensionless time |
| $u$ | axial component of velocity |
| v | transverse component of velocity |
| $U$ | dimensionless axial velocity |
| $V$ | dimensionless transverse velocity |
| $Y_0$ | Bessel function of second kind and order zero |
| $Y_1$ | Bessel function of second kind and order one |
| $\omega$ | rotation of the cylinder |
| $\Omega$ | dimensionless rotation parameter |

## 1. Introduction:

The rotating fluid flows over solid surface plays a significant role in many engineering and industrial applications. Greenspan and Howard [1] initiated the study of the dynamics of the spin-up of an incompressible viscous rotating fluid. Many researchers have studied the flow past plate in a rotating fluid with varied physical situations, for example, Deka *et al*. [2], Kang and Xu [3] and Hayat *et al*. [4]. Khan et. al. [5] presented the closed form solutions for accelerated MHD flow of non-Newtonian fluid in a rotating frame. Deka and Paul [6] studied the flow of a viscous fluid past an impulsively started horizontal cylinder in a rotating fluid. They have demonstrated that for smaller time the rotation leads to oscillatory motion where as the flow approaches steady state at larger time. Recently, Deka *et al.* [7] presented the closed form solution to the flow of a viscous fluid past an accelerated vertical circular cylinder in a rotating fluid.

In the present paper the flow of a viscous incompressible fluid past an accelerated horizontal cylinder in a rotating fluid is considered. This is among the simplest examples of the way the Coriolis force due to the rotation manifests itself in changing the pattern of flows around cylinder. Here, a semi-infinite mass of an incompressible viscous fluid bounded by an infinite solid circular cylinder is initially rotating with uniform angular velocity about an axis normal to the cylinder. The attempt has been made to investigate the subsequent flow when the cylinder started impulsively from rest (relative to the rotating fluid) moves with uniform acceleration in its own plane. The analysis reveals that the Coriolis forces induce a flow parallel to the cylinder but transverse to the main flow direction.

## 2. Mathematical analysis:

Consider an unsteady flow of an incompressible viscous fluid past an infinite horizontal cylinder of radius $r_0$. Here the $x$-axis is taken horizontally along the axis of the cylinder and the radial co-ordinate $r$ is taken normal to the cylinder. Initially, the fluid and the cylinder rotate in unison with a uniform angular velocity $\omega$ about $x$-axis. Relative to the rotating fluid, the cylinder is impulsively started from rest and then moves with uniform acceleration along $x$-axis. Due to the horizontal homogeneity of the flow problem the flow quantities depend on $r$ and $t$ only. As a rotating system is not an inertial system, the Coriolis force will exhibit. Taking the Coriolis force into account and assuming that the pressure is uniform in the whole flow field, the equations of motion for the unsteady flow are as follows:

$$\frac{\partial u}{\partial t'} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + 2\omega\,\mathrm{v} \qquad (1)$$

$$\frac{\partial \mathrm{v}}{\partial t'} = \nu\left(\frac{\partial^2 \mathrm{v}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathrm{v}}{\partial r}\right) - 2\omega\, u \qquad (2)$$

where the last terms in equations (1) and (2) are due to the Coriolis force.

The initial and boundary conditions are:

$$\left.\begin{array}{llll} t' \le 0: & u = 0, & \mathrm{v} = 0, & \forall\, r \\ t' > 0: & u = ct', & \mathrm{v} = 0, & \text{at} \quad r = r_0 \\ & u \to 0, & \mathrm{v} \to 0, & \text{as} \quad r \to \infty \end{array}\right\} \qquad (3)$$

where $c > 0$ is a constant. Introducing the non-dimensional quantities,

$$U = \frac{u\nu}{c r_0^2}, \quad V = \frac{\mathrm{v}\nu}{c r_0^2}, \quad R = \frac{r}{r_0}, \quad t = \frac{t'\nu}{r_0^2}, \quad \Omega = \frac{r_0^2}{\nu} w \qquad (4)$$

the equations (1) and (2) reduce to,

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} + 2\Omega V \qquad (5)$$

$$\frac{\partial V}{\partial T} = \frac{\partial^2 V}{\partial R^2} + \frac{1}{R}\frac{\partial V}{\partial R} - 2\Omega U \qquad 6)$$

with the initial and boundary conditions as:

$$\left.\begin{array}{llll} t \le 0: & U = 0, & V = 0 & \forall\, R \\ t > 0: & U = t, & V = 0 & \text{at} \quad R = 1 \\ & U \to 0, & V \to 0 & \text{as} \quad R \to \infty \end{array}\right\} \qquad (7)$$

Equation (5) and (6) can be combined to give,

$$\frac{\partial W}{\partial t} = \frac{\partial^2 W}{\partial R^2} + \frac{1}{R}\frac{\partial W}{\partial R} - 2i\,\Omega W \qquad (8)$$

where

$$W(R,t) = U(R,t) + iV(R,t) \quad (9)$$

3. Solution technique:

Using the Laplace transform technique, the solution to the equation (8) subject to the initial and boundary conditions in (7) is obtained as,

$$W = t\frac{K_0(R\sqrt{M})}{K_0(\sqrt{M})} + \frac{K_1(\sqrt{M})K_0(R\sqrt{M}) - RK_0(\sqrt{M})K_1(R\sqrt{M})}{2\sqrt{M}K_0^2(\sqrt{M})} - \frac{2e^{-Mt}}{\pi}\int_0^\infty \frac{e^{-tS^2}}{(S^2+M)^2}\Gamma(R,S)SdS \quad (10)$$

where $M = 2i\Omega$ and $\Gamma(R,S) = \dfrac{J_0(RS)Y_0(S) - Y_0(RS)J_0(S)}{J_0^2(S) + Y_0^2(S)}$

The modified Bessel functions of second kind ($K_0$ and $K_1$) and the last term of equation (10) are complex. During computation, the real and complex quantities are separated to calculate the axial and transverse velocity components.

In non-dimensional form, the skin friction, $\tau = \tau_x + i\tau_y = -(\partial W/\partial R)_{R=1}$ at the surface of the cylinder is obtained from equation (10) as:

$$\tau = \frac{1}{2} + t\frac{\sqrt{M}K_1(\sqrt{M})}{K_0(\sqrt{M})} - \frac{K_1^2(\sqrt{M})}{2K_0^2(\sqrt{M})} - \frac{2e^{-Mt}}{\pi}\int_0^\infty \frac{e^{-tS^2}}{(S^2+M)^2}\frac{J_1(S)Y_0(S) - Y_1(S)J_0(S)}{J_0^2(S)+Y_0^2(S)}S^2dS \quad (11)$$

where $\tau_x$, $\tau_y$ are axial and transverse components of skin friction.

4. Results and discussions:

In order to get a physical insight into the problem, the numerical values of the axial and transverse components of velocity and skin friction are obtained for different values of physical parameters involved in the flow field. The computed results are presented in Figures 1-8.

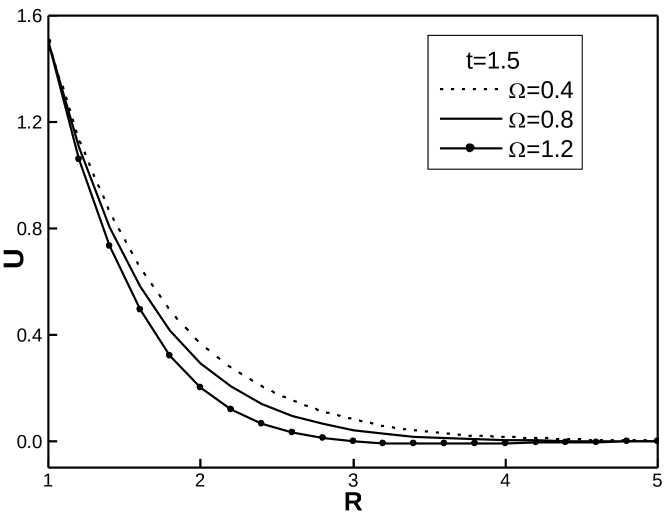


Figure 1: Axial velocity profiles for different $\Omega$ at t=1.5

Figure 1 and Figure 2 respectively show the axial velocity $U$ and transverse velocity $V$ along the radial direction of the cylinder for different values of the rotation parameter $\Omega$ at time $t = 1.5$. It is observed that with increase of rotation parameter $\Omega$, the axial velocity $U$ decreases whereas the transverse velocity $V$ increases. The negative sign of $V$ in the Figure 2 indicates that this component is transverse to the main flow direction in the clockwise direction. Figure 3 and Figure 4 depict the axial velocity and transverse velocity against $R$ for $\Omega = 0.8$. It is observed from the figures that at a fixed point for a fixed value of the rotation parameter the axial as well as transverse velocity increases with increasing time.

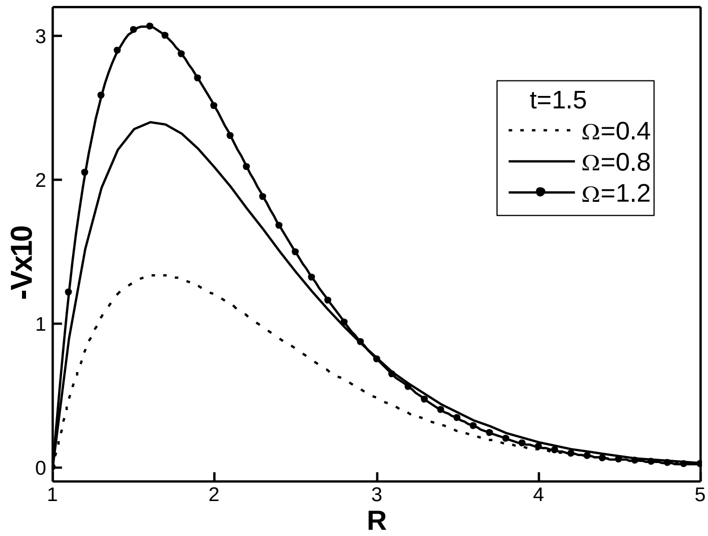


Figure 2: Transverse velocity profiles for different $\Omega$ at t=1.5

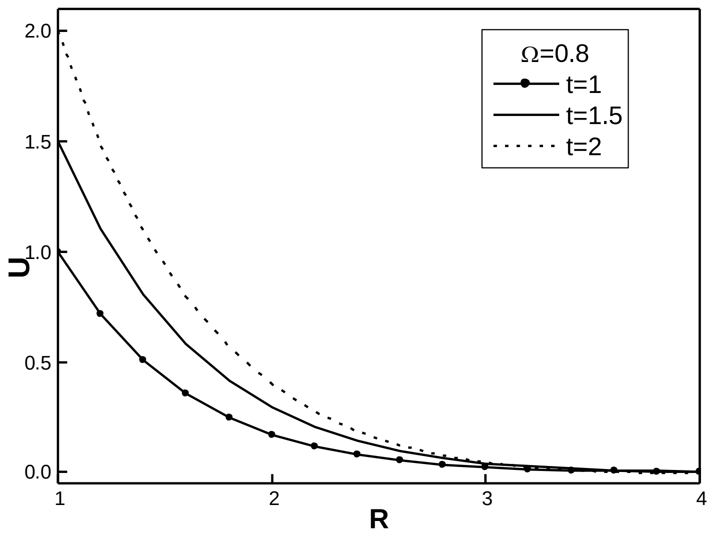


Figure 3: Axial velocity profiles for different $t$ at $\Omega = 0.8$

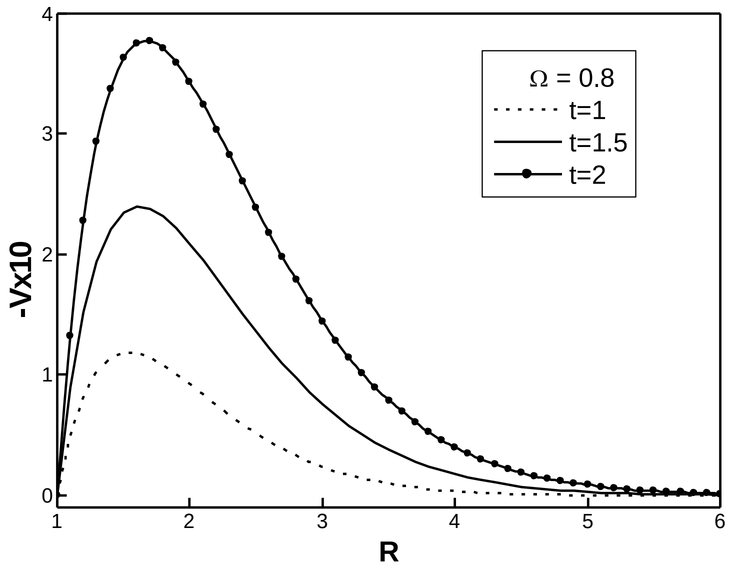


Figure 4: Transverse velocity profiles for different *t* at $\Omega$ = 0.8

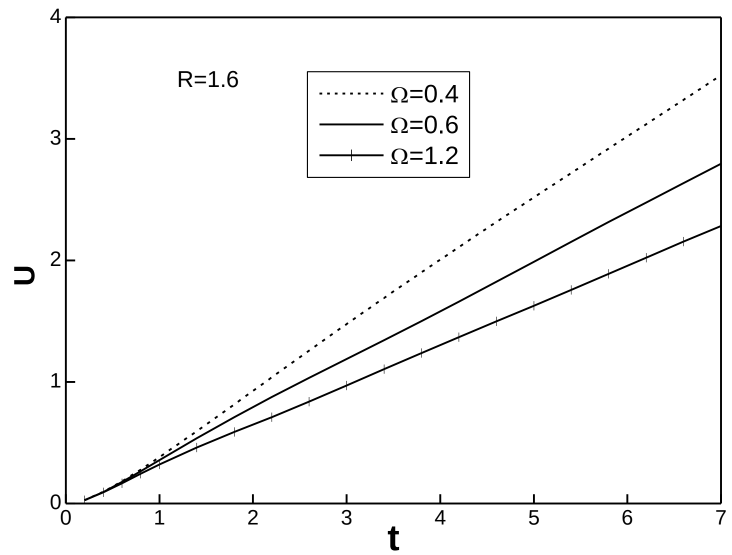


Figure 5: Axial velocity profiles against time for different $\Omega$ and at $R$ = 1.6

At a distance R=1.6 from the surface of the cylinder, the effect of rotation parameter on the velocity components is shown in Figure 5 and Figure 6. These figures show that both the axial and transverse velocity decreases with increase of rotation parameter $\Omega$. Also, magnitude of velocity components increases with increase in time.

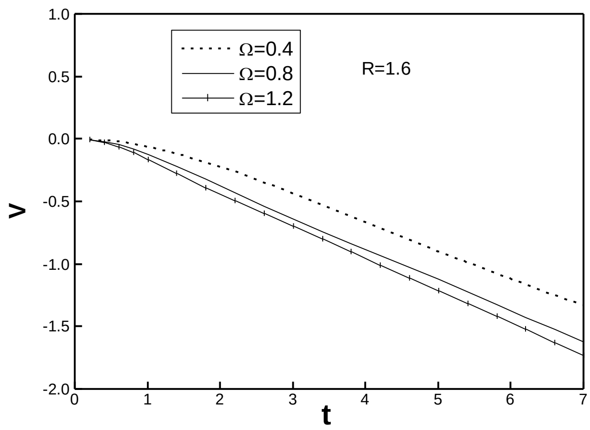


Figure 6: Transverse velocity profiles against time for different $\Omega$ and at $R$ = 1.6

Figures 7 and Figure 8 respectively, show the effect of rotation parameter $\Omega$ on the non- dimensional skin friction $\tau$ drawn against time $t$. It is observed from the figures that both axial and transverse components of skin friction increase with increase in rotation. Further, for a fixed value of $\Omega$, both $\tau_x$ and $\tau_y$ increase with an increase in time. At a fixed instant, an increase in $\Omega$ causes a gradual thinning of the boundary layer on the surface of the cylinder. This results in an increase of the shear stress at the surface with the increased values of $\Omega$. On the other hand, for a fixed value of $\Omega$, an increase in time results in an increase in the velocity of the cylinder, which in turn implies a gradual thinning of the boundary layer on the cylinder with increasing time $t$. It is clear from equation (10) that the flow does not reach steady state for large time and this is to be expected from the physical considerations, since the cylinder started from rest moves with uniform acceleration in its own plane. Accordingly, from equation (11) it is to be expected that the skin friction components too increase with time unboundedly and is shown graphically in figures 7 and 8.

There are few distinctions of the present problem as a departure from the study made by Deka et al. [2] for horizontal accelerated plate. In their analysis they confirmed of the existence of inertial oscillations which grow with time that were the manifestation of the oscillation of the frequency $2\Omega$ due to the presence of rotation, and this oscillation were absent in absence of rotation. The oscillatory behaviours were also shown present in the components of skin friction too. But in the present study of the flow past an accelerated horizontal cylinder in a rotating fluid, the resulting velocity and skin friction components show no oscillatory behaviour, whatever may be the time small or large.

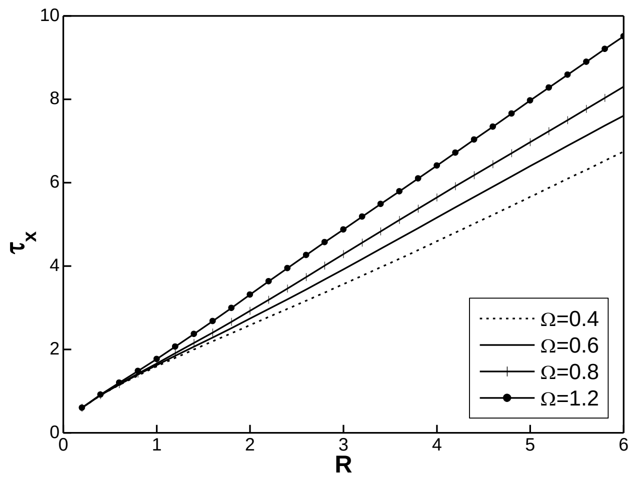


Figure 7: Axial skin friction profiles against time for different $\Omega$

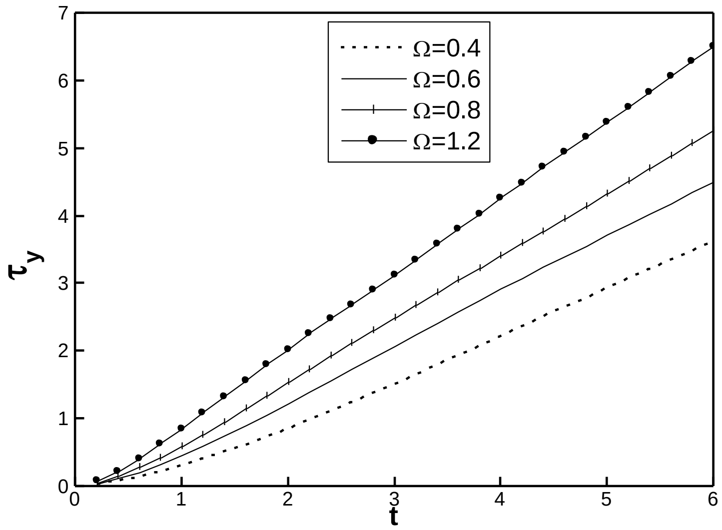


Figure 8: Transverse skin friction profiles against time for different *Ω*

5. Conclusions:

The analytical study to the laminar flow of a viscous incompressible fluid past an accelerated horizontal cylinder in a rotating fluid is presented. As a distinction from the study concerning the flow past an accelerated horizontal plate in a rotating fluid, the present study of rotating fluids in the presence of an accelerated horizontal cylinder shows no inertial oscillation. The axial velocity decreases whereas the transverse velocity increases with increase in rotation parameter. The velocity and skin friction grows with time when the cylinder is started from rest into motion with a uniform acceleration in its own plane.

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