INTRINSIC PROBLEMS OF MICROSCOPIC THEORIES OF SUPERFLUIDITY AND SUPERCONDUCTIVITY DEVELOPED BY USING PLANE WAVE REPRESENTATION OF PARTICLES

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Abstract: In this paper we discover intrinsic problems of microscopic theories of superfluidity/ superconductivity developed by using single particle basis with plane wave representation of particles. Such theories are found to be inconsistent with certain physical realities attained by the system in its low energy states and they cannot reveal complete and clear understanding of the said phenomenon with experimentally matching results. Here we also conclude that pair of particle basis is more appropriate for developing the micro-scopic theories of widely different systems such as liquid ^{4}He, liquid ^{3}He, *N*−conduction electrons, etc. which exhibit superfluidity/ superconductivity at low temperatures.

Keywords: BEC, He-II, superfluid, theory of superfluids

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1. Introduction:

'It is not that the theory fails to account for one phenomenon or another but that it embodies intrinsic difficulties', was stated once by London [1] (quoted also in [2]) in relation to several difficulties of Landau's two fluid theory which were discussed and analysed later in [3-5]. This statement applies to several theories in physics which have been in the process of finding their final shape for the last several decades. For example, we have microscopic theories of a system of interacting bosons (SIB) such as liquid ^{4}He. Ever since Bogoliubov [6] and Feynman and coworkers [7-10] laid the foundations of these theories, numerous papers (as reviewed recently in [11-15]) have been published during the last 7 decades with a view to conclude the final theory but with limited success. Interest in the subject grew further after the discovery of the so called Bose Einstein condensation (BEC) in trapped dilute gases (TDG) [16, 17] but it is clear that a microscopic theory that explains the experimental properties of liquid ^{4}He and BEC of TDG at quantitative scale has been awaited until recently when the author achieved the breakthrough in field [18, 19]. This development has been greatly helped by his study of the wave mechanics of two hard core particles trapped in 1-D box [20]. To facilitate the study reported here, we identify the said theories [6-10] (updated and reviewed in [11-15]) as conventional microscopic theories (CMTs), while the recently developed Jain's theory [18] as non-conventional microscopic theory (NCMT). This development motivated us to discover the intrinsic problems for which CMTs could not achieve desired success. To this effect we analyze the basic premises (summed up in Section2) of CMTs for their inconsistencies with certain physical realities of the system and this fact is identified as a main source of the said problems of CMTs.

2. Basic premises of CMTs [6-15]:

(1). Presumed existence of $p = 0$ condensate: Superfluid phase of a SIB is presumed to have the existence of $p = 0$ condensate (a macroscopically large fraction of particles occupying a single particle state of momentum $p = 0$ [3]) as the origin of its superfluidity and related aspects.

(2). Single particle basis (SPB): A many body quantum system (MBQS) such as liquid ^{4}He can be described by using SPB with plane wave representation of particles.

This, explicitly, means that: (i) a single particle is presumed to represent the basic unit of the system, (ii) particles in the system are supposed to occupy different quantum states of a single particle trapped in a box of volume V of the system, (iii) each particle is described by a plane wave,

$$u_{\mathbf{p}}(\mathbf{b}) = V^{-1/2} \exp(i\mathbf{p}.\mathbf{b}), \quad (1)$$

where **p** and **b**, respectively, represent its momentum (with p expressed in wave number) and position vectors (henceforth V would be treated as unity), and (iv) **p** and corresponding energy $\frac{\hbar^2 p^2}{2m}$ of each particle stay as good quantum numbers in every state of the system.

3. Basic premises of CMTs and their intrinsic problems:

As discussed in [21, 22], no experimental study of He-II could directly or indirectly confirm the existence of p = 0 condensate, beyond doubt. In addition, several prominent scientists in the field [21-23] also expressed their doubts about the existence of p = 0 condensate in He-II and similar states of TDG. In fact our recent theoretical study [19] clearly concludes that the laws of nature which demand that the ground state of a physical system has least possible energy do not favour the existence of p = 0 condensate in any SIB. Interestingly, this inference is consistent with our NCMT [18] developed by using first quantization method. In what follows, the possibilities of the presumed existence of p = 0 condensate in the superfluid state of a SIB has to be ruled out without any reservation. Evidently, the first intrinsic problem of CMTs of liquid ^{4}He and similar systems [6-15] has been their presumption of the existence of p = 0 condensate as the origin of their superfluidity and related aspects.

A critical examination of a MBQS further reveals that the use of SPB with plane wave representation of a particle is inconsistent with two physical realities (as discussed below) related to the low temperature (LT) states of its particles.

(**Reality-1**): As evident from the experimental observations, it is amply clear that the LT behavior of a MBQS below certain temperature is dominated by the wave nature of its constituent particles and this arises when their de Broglie wave length becomes longer than their inter-particle distance. Since particles in such a situation are bound to have their wave superposition as a natural consequence of wave particle duality, their quantum states, to a good approximation, are described by

$$\Psi(1,2)^{\pm} = \frac{1}{\sqrt{2}}\left[u_{\mathbf{p_1}}(\mathbf{b_1})\, u_{\mathbf{p_2}}(\mathbf{b_2}) \pm u_{\mathbf{p_2}}(\mathbf{b_1})\, u_{\mathbf{p_1}}(\mathbf{b_2})\right] \quad (2)$$

not by plane waves (Eqn.1). We note that Eqn.2 can be reformulated as

$$\psi(r,R) = \sqrt{2}\sin(\mathbf{k}.\mathbf{r}/2)\exp(\mathbf{K}.\mathbf{R}) \quad (3)$$

which, obviously, represents a state of a pair of particles (say P1 and P2); suffix 1 refers to P1 and 2 to P2. Here we have

$$\mathbf{k} = \mathbf{p_2} - \mathbf{p_1} \text{ and } \mathbf{r} = \mathbf{b_2} - \mathbf{b_1} \quad (4)$$

where **k** = relative momentum and **r** = relative position of P1 and P2 and

$$\mathbf{K} = \mathbf{p_2} + \mathbf{p_1} \text{ and } \mathbf{R} = (\mathbf{b_2} + \mathbf{b_1})/2 \quad (5)$$

where **K** = center of mass (CM) momentum and **R** = CM position of the pair.

Analyzing Eqn.(2), we note that $\psi(1, 2)$ is not an eigen function of the momentum operator $(-i\hbar\partial_{r_i})$ or energy operator $(-(\hbar^2/2m)\,\partial r_i^2)$ of any individual particle (i = 1 or 2) and this proves that in the state of wave superposition of P1 and P2, momentum and energy of individual particle do not remain good quantum numbers. In fact $\psi(1, 2)$ or its equivalent ψ(r, R) (Eqns.2 and 3) is an eigen function of only energy operator of the pair. Individuality of each particle loses meaning and a pair of particles clearly represents the basic unit of the system. This physical reality of LT states is, evidently, ignored by different theories of a MBQS like liquid ^{4}He developed by using SPB with plane wave representation of particles.

(Reality-2): When particles of a MBQS lose their kinetic energy (KE) with falling T , their behavior at LTs is, obviously, dominated by inter-particle interactions, $V(r_{ij})$;even the weakest component of $V(r_{ij})$ can demonstrate its presence when they tend to have T = 0. It is not merely a matter of argument or speculation, it is established by experimental observations. For example, it is widely accepted that: (i) liquids ^{4}He and ^{3}He which exhibit superfluidity, respectively, at $T < T_\lambda$ = 2.17 K and $T < T_c$(≈1 mK) do not become solid due to zero-point repulsion $f_o = h^2/4md^3$ between two nearest neighbor particles arising from their zero-point energy, εo = $h^2/8md^2$, and (ii) both these liquids exhibit volume expansion on their cooling through T_λ^+ (slightly above T_λ) and ≈ 0.6K [24] and this behavior is undoubtedly forced by none other than f_o. Evidently, f_o dominates the physical behavior of these systems over the entire range of T in which they exhibit superfluidity. In addition, the physical reality of

the existence of electron bubbles in helium liquids [25, 26] establishes how a quantum particle (electron) behaves when it occupies its ground state in a system whose particles have short range repulsion with it. The electron occupies maximum possible space by exerting its f_o on its nearest neighbors and this action calls for an opposing force f_a originating from $V(r_{ij})$ between the said neighbors. Since each atom in liquid ^{4}He or liquid ^{3}He too interacts with its neighbors through a short range repulsion, by analogy with electron in electron bubble, it is also expected to occupy a cavity formed by its neighbors exclusively when it has least possible energy; naturally, the state of such a particle is not represented by a plane wave.

As concluded in [18, 20], the state of such a particle in a MBQS is represented rather by a macro-orbital (a kind of pair waveform like Eqn.3),

$$\xi_i = \sin(\mathbf{q_i}.\mathbf{r_i}) \exp(\mathbf{K_i}.\mathbf{R_i}). \quad (6)$$

It is further evident that the plane wave representation of a particle renders no clue to the reality that particles in their LT states exert f_o on their neighbors, obviously, because the energy of a free particle does not depend on d. All these observations not only establish the inconsistency of SPB with the physical reality that f_o (a kind of two body repulsion) dominates the natural behavior of a MBQS in its LT states but also underlines the use of pair of particle basis (PPB) for the correct understanding of such systems or to convert SPB results into PPB by using appropriate relations and conditions as demonstrated in [19].

Arguably, though the use of SPB with plane wave representation of particles in different theories (such as [6, 10]) is mathematically valid and, in principle, there should be no problem with its use, however, it is frequently observed that something which sounds mathematically correct is not always acceptable in physics. For example, it is well known that mathematically sound solutions of the Schrodinger equation of several systems are accepted only when they are subjected to appropriate boundary conditions [27]. Further, as observed rightly in [28], one finds that the plane wave representation of particles is not always a useful starting point. While in atomic structure, where electrons move around a positively charged point size nucleus, hydrogenic eigenstates are more useful basis functions, for electrons moving in a constant magnetic field, Landau orbitals are more suitable [28]. Guided by these examples it is clear that the use of SPB with plane representation of particles, which appears to be reasonably suitable to describe the HT states of MBQS, does not remain equally appropriate to describe LT states where particles have their wave superposition.

We can have another example to illustrate the suitability or unsuitability of a given set of basis vectors by considering the well understood case of the vibrational dynamics of a polyatomic molecule which can be described, in principle, in terms of the oscillations of: (i) Cartesian coordinates (r_i) of atoms or (ii) internal coordinates (q_i) of a molecule [representing inter-atomic bonds, bond angles, etc.], or (iii) normal coordinates (Q_i) of molecule [29]; these are depicted in Fig.1 for the clarity of their meanings and differences by using water (H_2O) molecule as an example. It is well known that a complete and clear description (having good agreement with experiments) of the said dynamics is obtained only in terms of oscillations of Q_i [not simply of r_i, or q_i]. The reason lies with the fact that only Q_i- oscillations represent the eigen states of the Hamiltonian H of the molecule or in other words the $H-$matrix assumes its diagonal form only when Q_i-oscillations are used as its basis vectors. This clearly demonstrates the need of a suitable set of basis vectors for obtaining complete and clear understanding of the system and experimentally matching theoretical results.

By analogy, since single particle states described by plane waves do not represent the eigen states of the H of a MBQS like liquid ^{4}He and liquid ^{3}He, the $H-$matrix does not assume its diagonal form when plane waves are used as basis vectors. Consequently, different microscopic theories developed by using SPB with plane representation of particles are not expected to render complete and clear understanding of the said systems with results having good agreement with experiments. Most significantly, this is corroborated by the fact that such theories of superconductivity (say BCS theory [30]) or superfluidity [6-15] really achieved limited success in accounting for the experimentally observed LT properties of widely different MBQS such as $N-$ conduction electrons in solids, liquid ^{4}He, liquid ^{3}He, etc. [31].

4. Conclusions:

This study renders a general principle that any theory, such as BCS theory, developed by using SPB with plane wave representation of particles would not provide a complete, clear and correct microscopic understanding (having close agreement with experiments) of the LT properties, such as, superconductivity or superfluidity and related aspects of a MBQS. To certain extent, the results of such a theory can be made physically meaningful

only when they are transformed to basis vectors (such as macro-orbitals) for which *H*−matrix of such systems

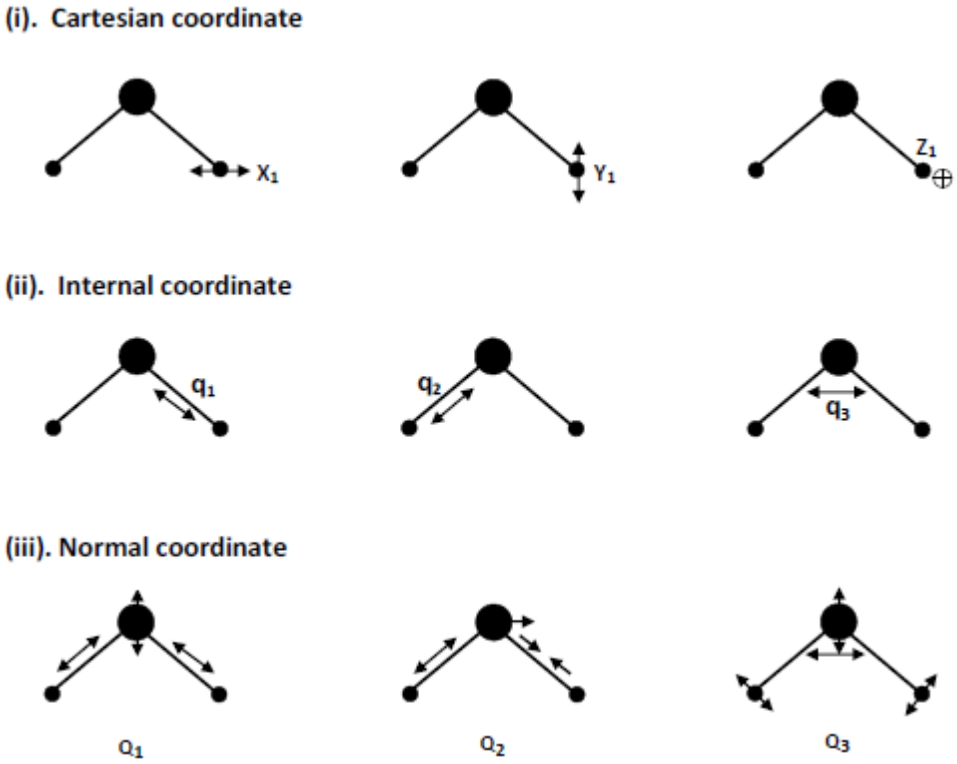

Figure.2 : Different possible basis vectors that can be used to describe the complex vibrational dynamics of a polyatomic molecule (e.g., H_2O molecule): (i) Cartesian coordinates, $\mathbf{r_i}$ (x_i, y_i, and z_i) of individual atoms, (ii) internal coordinates (bonds, bond angles, represented by $\mathbf{q_i}$, and (iii) normal coordinates $\mathbf{Q_i}$.

assumes its diagonal form [18, 32]. This has been demonstrated for liquid ^{4}He and similar systems in [19]. This study also concludes that PPB is more appropriate for developing the microscopic theories of widely different systems such as liquid ^{4}He, liquid ^{3}He, *N*− conduction electrons in solids, etc. and this has been established by our success in formulating these theories [18] and [32]. Evidently, it is not that the BCS theory and other similar theories fail to account for the superconductivity of widely different superconductors but the fact is that they embody intrinsic problems. The same is true with similar microscopic theories of liquid ^{4}He [6-15] and other systems. In view of this fact, we hope that the community of scientists would no longer be skeptic about the theories developed by using first quantization [18, 32].

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