Stiffness analysis of symmetric cross-ply laminated composite plates

Mukul Saxena[1], Sushen Kirtania[2]
[1]Student, Department of Mechanical Engineering, Tezpur University
Napaam - 784028, Assam. India.
mukulsaxena11@rediffmail.com
[2]Assistant Professor, Department of Mechanical Engineering, Tezpur University
Napaam - 784028, Assam. India.
sushen@tezu.ernet.in

Abstract: Stiffness of a cross-ply laminated composite plate has been investigated with aspect ratios (AR) for different orientation sequences of laminate using finite element method (FEM). A simply supported plate with uniform distributed load has been considered for present analysis. A MATLAB code has been developed to find out the deflection of the laminated composite plates. The same has also been analyzed using ANSYS software. The computational results have been compared with the theoretical results (classical lamination theory) and a good agreement has been found. When number of lamina is increased with 90^0 domination or decreased with 0^0 domination under the condition AR less than 1 and increased with 0^0 domination or decreased with 90^0 domination under the condition AR greater than 1, higher stiffness was observed.

Keywords: Laminated composites plates, classical lamination theory, finite element analysis, stiffness.

1. Introduction

Israelites using straw reinforced clay bricks are an early example of composites application. The individual constituents viz. clay and straw, could not serve the function by alone but did when put together. Indicative examples include the use of bamboo shoots reinforced mud walls, glued laminated wood by Egyptians and laminated metals used in forging swords. In early era, the modern composites were used in different fields of innovation. A structural composite can be defined as a system of material consisting of two or more insoluble phases on a macroscopic scale for remarkable mechanical performance which cannot serve by the constituent acting alone. Discontinuous, stiffer and stronger phase is reinforcement, whereas the less stiff and weaker phase is matrix. Composites can be classified by geometry of the reinforcement viz. particulate, flake and fibers. Long fibers in various forms are inherently much stiffer and stronger than the same material in bulk form. Obviously, then, the geometry of fiber and physical makeup are somehow crucial to the evaluation of its strength and must be considered for structural applications. Fiber reinforced composite (FRC) lamina is generally having thickness on the order of 0.125mm can be shown in Figure 1 [1-3].

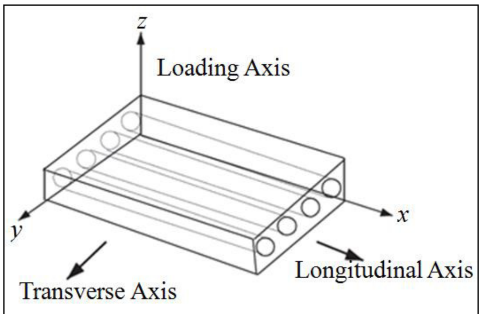

Figure 1: A unidirectional fiber reinforced lamina.

A composite laminate is two or more laminae bonded together in the direction of lamina thickness to act as an integral structural element. Laminae principal material directions are oriented to produce a structural element capable of resisting load in several directions. Each lamina can be spot by its location in the laminate, its material, and its orientation with reference axis.

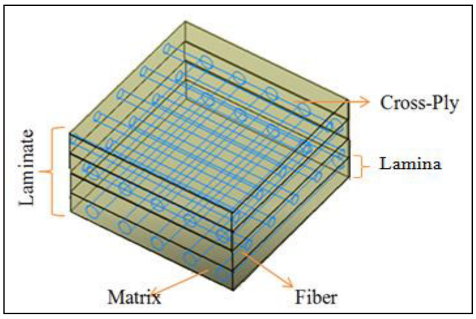

Figure 2: Schematic diagram of a [0/90/0/90] cross-ply laminate.

A laminate is called symmetric cross-ply if the material, angle, and the thickness of plies are the same above and below the midplane and also if only 0^0 and 90^0 plies have been used to make laminates, such as $[0/90_2/0/90_2/0]$. Figure 2 shows the cross-ply composite laminate having four laminae with laminate code [0/90/0/90].

2. Methodology

2.1 Analytical methodology

Classical laminate theory (CLT) has been used to calculate deformation of composite plate. For CLT, due to simplicity over energy and variational principles, Newtonian approach has been used in which summing up forces and moments on the plate is often used to develop the governing differential equations. The governing equations consisting the behaviour of the boundary conditions. In the present analysis, assumptions made by Kirchhoff's hypothesis have been used.

2.1.1. Kirchhoff's hypothesis

A side view of a plate in the Cartesian coordinate system as shown in Figure 3.

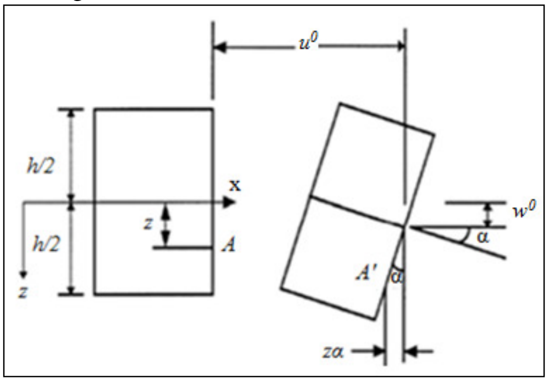

Figure 3: Relationship between displacements through the thickness of plate to midplane displacement, curvatures [1].

Considering the origin of the plate is at the midplane of the plate, that is, $z = 0$. Assume u^0, v^0 and w^0 to be displacements in the *x, y* and *z* directions, respectively, at the midplane and u, v, and w are the displacements at a point in the *x, y,* and *z* directions, respectively. $\frac{\partial w^o}{\partial y}$ and $\frac{\partial w^o}{\partial x}$ are the rotations about the *x* and *y* axes, respectively. Displacements *u, v* and *w* in the *x, y* and *z* directions are:

$$u(x,y,z) = u^o(x,y) - z\frac{\partial w^o(x,y)}{\partial x} \quad (1)$$

$$v(x,y,z) = v^o(x,y) - z\frac{\partial w^o(x,y)}{\partial y} \quad (2)$$

$$w(x,y,z) = w^o(x,y) \quad (3)$$

In addition to Kirchhoff's hypothesis, it has been assumed that the layers are perfectly bonded to each other and each lamina to be elastic.

2.1.2. Governing equations

Using strain relations:

$$\varepsilon_x(x,y) = \frac{\partial u(x,y)}{\partial x} = \frac{\partial u^0}{\partial x} - z\frac{\partial^2 w^0}{\partial x^2} \quad (4)$$

$$\varepsilon_y(x,y) = \frac{\partial v(x,y)}{\partial y} = \frac{\partial v^0}{\partial y} - z\frac{\partial^2 w^0}{\partial y^2} \quad (5)$$

$$\gamma_{xy}(x,y) = \frac{\partial v(x,y)}{\partial x} + \frac{\partial u(x,y)}{\partial y} = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} - 2z\frac{\partial^2 w^0}{\partial x \partial y} \quad (6)$$

By the definition of midplane curvature and strains:

$$k_x(x,y) = -\frac{\partial^2 w^0(x,y)}{\partial x^2}\ ;\ k_y(x,y) = \frac{\partial^2 w^0(x,y)}{\partial y^2} \quad (7, 8)$$

$$k_{xy}(x,y) = -2\frac{\partial^2 w^0(x,y)}{\partial x \partial y}\ ;\ \varepsilon_x^0(x,y) = \frac{\partial u^0(x,y)}{\partial x} \quad (9, 10)$$

$$\varepsilon_y^0(x,y) = \frac{\partial v^0(x,y)}{\partial y} \quad (11)$$

$$\gamma_{xy}^0(x,y) = \frac{\partial v^0(x,y)}{\partial x} + \frac{\partial u^0(x,y)}{\partial y} \quad (12)$$

Where,
ε and γ denotes midplane normal and shear strain, respectively and k denotes midplane curvature.
From equations (4) to (12), laminate strain can be written as:

$$\begin{bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}\varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0\end{bmatrix} + z\begin{bmatrix}k_x \\ k_y \\ k_{xy}\end{bmatrix} \quad (13)$$

Global stress can be find out by stress and strain relation for a laminate:

$$\begin{bmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}}\end{bmatrix}\begin{bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{bmatrix} \quad (14)$$

Where, σ_x, σ_y and τ_{xy} are stresses in *x, y* axes and *xy* plane, respectively. ε_x, ε_y and γ_{xy} are strain in *x, y* axes and *xy* plane, respectively. $\left[\overline{Q_{ij}}\right]$ is transformed reduced stiffness matrix. Coefficients of transformed reduced stiffness matrix are:

$$\overline{Q_{11}} = Q_{11}C^4 + Q_{22}S^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \quad (15)$$

$$\overline{Q_{12}} = (Q_{11} + Q_{22} - 4Q_{66})S^2C^2 + Q_{12}(C^4 + S^4) \quad (16)$$

$$\overline{Q_{22}} = Q_{11}S^4 + Q_{22}C^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \quad (17)$$

$$\overline{Q_{16}} = (Q_{11} - Q_{12} - 2Q_{66})C^3S - (Q_{22} - Q_{12} - 2Q_{66})S^3C \quad (18)$$

$$\overline{Q_{26}} = (Q_{11} - Q_{22} - 2Q_{66})CS^3 - (Q_{22} - Q_{12} - 2Q_{66})C^3S \quad (19)$$

$$\overline{Q_{66}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})S^2C^2 + Q_{66}(S^4 + C^4) \quad (20)$$

Where,
$C = \cos\theta$, $S = \sin\theta$ and coefficients of stiffness matrix can be found as:

$$Q_{11} = E_1/(1-\nu_{21}\nu_{12})\ ;\ Q_{12} = \nu_{12}E_2/(1-\nu_{21}\nu_{12})$$

$$Q_{22} = E_2/(1-\nu_{21}\nu_{12})\ ;\ Q_{66} = G_{12}$$

Where,
E_1, E_2 = Longitudinal, Transverse elastic modulus,
ν_{12}, ν_{21} = Major, Minor Poisson's ratio,
G_{12} = Shear modulus.

Resultant forces and moments can be found as:

$$N_x = \int_{-h/2}^{h/2} \sigma_x dz\ ;\ N_y = \int_{-h/2}^{h/2} \sigma_y dz \quad (21, 22)$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} dz\ ;\ M_x = \int_{-h/2}^{h/2} \sigma_x z dz \quad (23, 24)$$

$$M_y = \int_{-h/2}^{h/2} \sigma_y z dz\ ;\ M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} z dz \quad (25, 26)$$

$$\begin{bmatrix}N_x \\ N_y \\ N_{xy}\end{bmatrix} = \sum_{k=1}^{n}\int_{h_{k-1}}^{h_k}\begin{bmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{bmatrix}dz \quad (27)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} zdz \quad (28)$$

Where $h = \sum_{k=1}^{n} t_k$, t_k is the lamina thickness.

N_x, N_y, N_{xy} and M_x, M_y, M_{xy} are resultant forces and moments in *x*, *y* axes and *xy* plane, respectively.

So, stiffness matrices can be found by these generalized equations:

$$\left[A_{ij}\right] = \sum_{k=1}^{n} \left[\overline{Q_{ij}}\right]_k (h_k - h_{k-1}) \quad (29)$$

$$\left[B_{ij}\right] = \frac{1}{2} \sum_{k=1}^{n} \left[\overline{Q_{ij}}\right]_k (h_k^2 - h_{k-1}^2) \quad (30)$$

$$\left[D_{ij}\right] = \frac{1}{3} \sum_{k=1}^{n} \left[\overline{Q_{ij}}\right]_k (h_k^3 - h_{k-1}^3) \quad (31)$$

Where,

[*A*], [*B*], and [*D*] matrixes are called Extensional, Coupling, and Bending stiffness matrix, respectively.

Using equations (13) to (31) gives six simultaneous linear equations which can be written as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (32)$$

Equilibrium equations for laminated composite plate can be derived from the principle of virtual work [2]:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 ; \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \quad (33,34)$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + q(x,y) = 0 \quad (35)$$

Where,

$$N_x = A_{11}\frac{\partial u^o}{\partial x} + A_{12}\frac{\partial v^o}{\partial y} \quad (36)$$

$$N_y = A_{12}\frac{\partial u^o}{\partial x} + A_{22}\frac{\partial v^o}{\partial y} \quad (37)$$

$$N_{xy} = A_{66}\left(\frac{\partial u^o}{\partial y} + \frac{\partial v^o}{\partial x}\right) \quad (38)$$

$$M_x = -D_{11}\frac{\partial^2 w^o}{\partial x^2} - D_{12}\frac{\partial^2 w^o}{\partial y^2} \quad (39)$$

$$M_y = -D_{12}\frac{\partial^2 w^o}{\partial x^2} - D_{22}\frac{\partial^2 w^o}{\partial y^2} \quad (40)$$

$$M_{xy} = -2D_{66}\frac{\partial^2 w^o}{\partial x \partial y} \quad (41)$$

Where, *q(x,y)* is the transverse load as sown in Figure 4. Substituting expressions from equations (36) to (41) into the governing equations of plate (33), (34) and (35), we get the equilibrium equations that can govern the response of a laminated plate [2]:

$$A_{11}\frac{\partial^2 u^o}{\partial x^2} + A_{66}\frac{\partial^2 u^o}{\partial y^2} + (A_{12} + A_{66})\frac{\partial^2 v^o}{\partial x \partial y} = 0 \quad (42)$$

$$(A_{12} + A_{66})\frac{\partial^2 u^o}{\partial x \partial y} + A_{66}\frac{\partial^2 v^o}{\partial x^2} + A_{22}\frac{\partial^2 v^o}{\partial y^2} = 0 \quad (43)$$

$$D_{11}\frac{\partial^4 w^o}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w^o}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w^o}{\partial y^4} = 0 \quad (44)$$

For a cross-ply laminate the coupling stiffness matrix and the terms A_{16}, A_{26}, D_{16} and D_{26} in the extensional and bending stiffness matrix should be zero.

2.1.3. Boundary conditions

The composite plate is simply supported at all four edges with uniformly distributed load. Mathematical function for boundary conditions for simply supported plate can be defined as [2]:

$$\begin{aligned} x = 0, a : w = 0, M_x = -D_{11}w,_{xx} - D_{12}w,_{yy} = 0 \\ y = 0, b : w = 0, M_y = -D_{12}w,_{xx} - D_{22}w,_{yy} = 0 \end{aligned} \quad (45)$$

2.1.4. Deflection function

For present analysis a fiber reinforced laminated composite plate of length *a*, width *b* and thickness *h* is considered with transverse loading of load *q*(*x*,*y*). Figure 4 shows the geometry and loading on composite laminated plate [2].

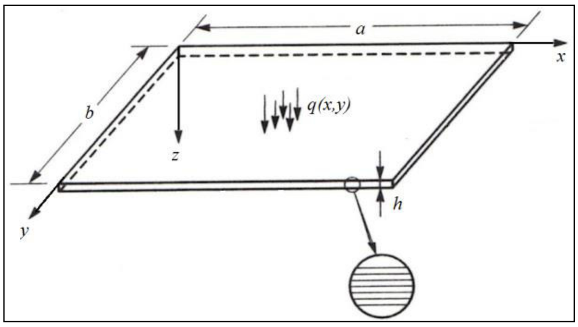

Figure 4: Geometry of plate along with applied load.

For a plate, the transverse deflections function can be described by a differential equation of equilibrium [2]:

$$D_{11}w,_{xxxx} + 2(D_{12} + 2D_{66})w,_{xxyy} + D_{22}w,_{yyyy} = q(x,y) \quad (46)$$

The boundary conditions from equation (45) and differential equation of equilibrium (46) can be satisfied by:

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right) \quad (47)$$

Exact solution has been found when:

$$a_{mn} = \frac{a^4 q_{mn}}{\pi^4\left[D_{11}m^4 + 2(D_{12} + 2D_{66})(mnR)^2 + D_{22}(nR)^4\right]} \quad (48)$$

So, for a uniform load, the solution is easily shown to be:

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{16q_0\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{\pi^6 mn[D_{11}(\frac{m}{a})^4+2(D_{12}+2D_{66})(\frac{m}{a})^2(\frac{n}{b})^2+D_{22}(\frac{n}{b})^4]}$$

Where m, n = 1,3,5…

2.2 Finite element modeling

Finite element analysis of laminated composite plates has been performed using ANSYS software. SHELL99 embodied in ANSYS has been used for analysis. The input parameters of a composite plate can be modified easily due to it being a shell element. For SHELL99 element, the composite theory was adopted for deflection analysis in the thickness direction. SHELL99 allows 250 layers and if more than a user-input constitutive matrix is also available. Element has six degrees of freedom at each node viz. translations in the nodal *x, y* and *z* directions and rotations about the nodal *x, y* and *z* axes. Properties of lamina are used in the present analysis which is listed in Table 1 [5].

TABLE 1: Properties of lamina [5]

E_x	25×10^6 N/m^2
$E_y = E_z$	1×10^6 N/m^2
$\nu_{xy} = \nu_{xz}$	0.25
$G_{xy} = G_{zx}$	0.5×10^6 N/m^2
G_{yz}	0.2×10^6 N/m^2

And $\nu_{yz} = \nu_{yx} = \nu_{xy}\left(\frac{E_y}{E_x}\right)$.

Figure 5 shows isometric view of laminated composite plate with FE mesh.

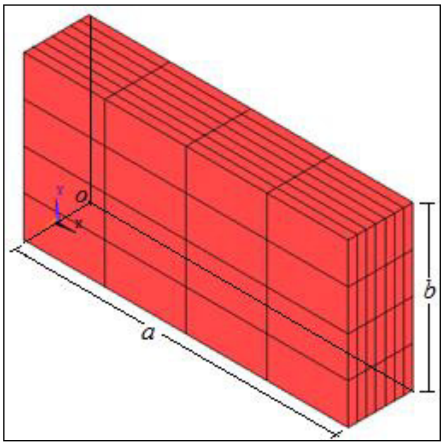

Figure 5: Isometric view of laminated composite plate with FE mesh.

Length of composite plate *a*, width *b* and thickness *h* has been used for the finite element analysis. A pressure of 1.0 N/m^2 is applied on all the nodes along the surface area of the plate which will exactly work as uniformly distributed load.

2.2.1. Boundary conditions

Considering origin at *O*(0,0) and using SSSS [9]type boundary condition. At x = 0 and *a*, the plate is constrained in the *y* and *z* directions and at *y* = 0 and *b*, it is constrained in the *x* and *z* directions. Finite element model of laminated composite plate along with boundary conditions is shown in Figure 6.

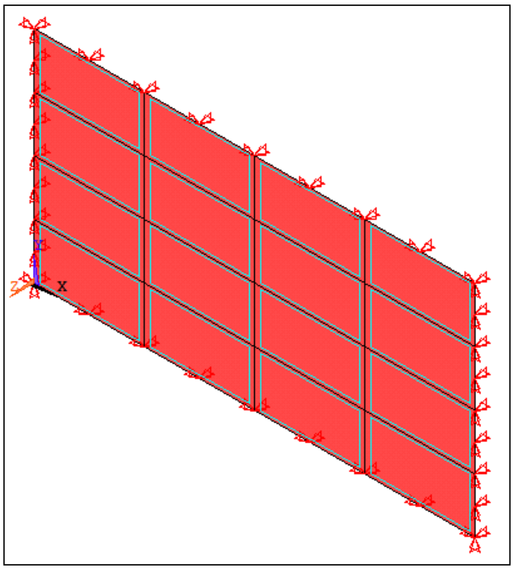

Figure 6: Finite element model of laminated composite plate along with boundary conditions.

3. Results and discussion

Deflection of laminated composite plate has been investigated for different aspect ratios (AR). Different orientation sequences have been considered for analysis, when AR ≥ 1 and when AR ≤ 1. Deflections have been found for different orientation sequences of laminate for a constant AR. For all the cases thickness and width of plate has been kept constant with the increase in number of laminas.

3.1. Deflection of plate with aspect ratio

3.1.1. For aspect ratios≤ 1

Stiffness of composite plate has been found with different orientation sequences of lamina for AR≤ 1. Figure 7(a) shows the deflection of composite plate with ARs.

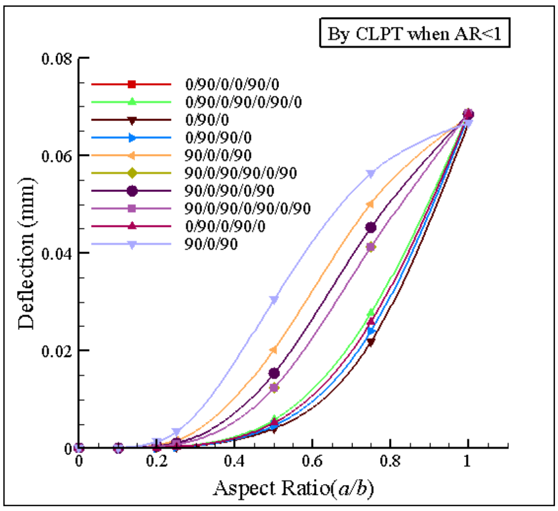

Figure 7(a): Deflection of a composite plate with ARs≤ 1 using CLT.

It can be seen from Figure 7(a) that deflection of the plate increase with increase in ARs. It can also be seen from Figure 7(a) that maximum and minimum deflection is for [90/0/90] and [0/90/0] orientation sequence and all the laminates have approximately same deflection value when AR is equal to one.
The same model and boundary conditions has been considered for FE analysis.

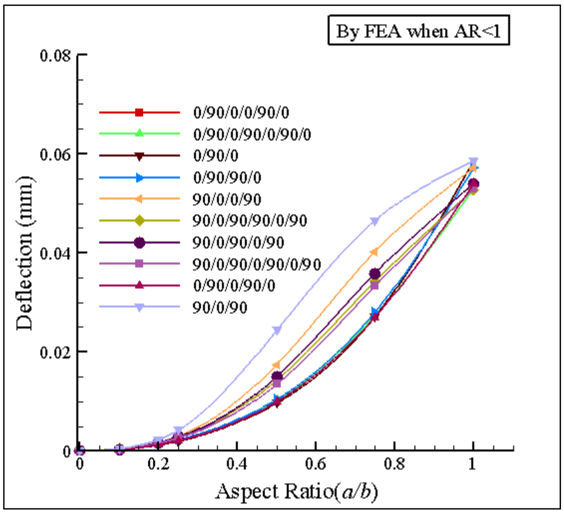

Figure 7(b): Deflection of a composite plate with AR≤ 1 using FEM.

Figure 7(b) shows the deflection of composite plate with ARs using FEM. The trend of curves obtained using FEM are similar that of obtained by CLT.

3.1.2. For aspect ratio ≥ 1

Stiffness of composite plate have been found with different orientation sequences of lamina for AR ≥ 1. Figure 8(a) shows the deflection of composite plate with ARs.

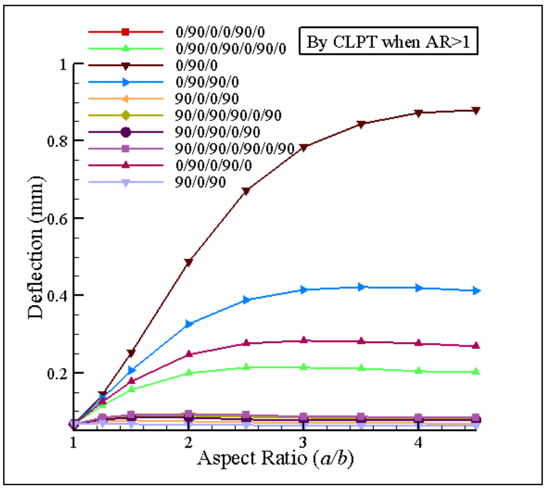

Figure 8(a): Deflection of a composite plate using CLT for AR≥1.

It can be seen from Figure 8(a) that maximum and minimum deflection is for laminate with [0/90/0] and [90/0/90] orientation sequences.

The same model and boundary conditions has been considered for FE analysis. Figure 8(b) shows the deflection of composite plate with ARs using FEM.

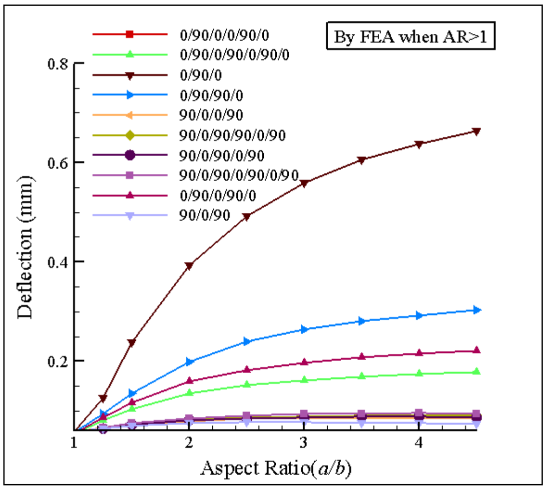

Figure 8(b): Deflection of a composite plate using FEA for AR≥1.

Figure 8(b) shows the deflection of composite plate with ARs using FEM. The trend of curves obtained using FEM are similar that of obtained by CLT.

3.2. Domination of lamina orientation

Domination of lamina means that more number of a particular oriented lamina than others.

3.2.1. Domination of 0^0 lamina

Figure 9 shows the deflection of plate with AR for domination of 0^0 lamina i.e. number of 0^0 lamina is more than that of 90^0 lamina in laminate.

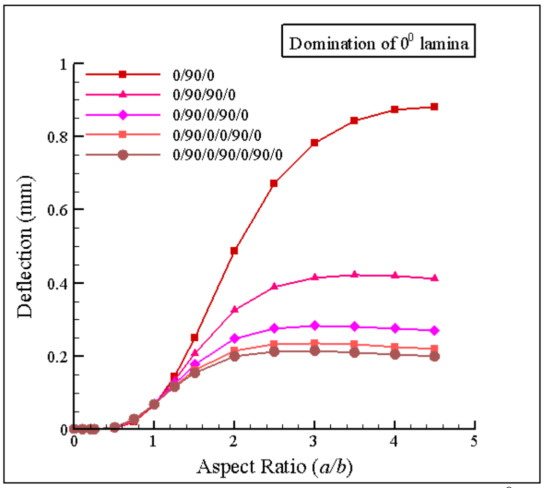

Figure 9: Deflection of plate with AR for 0^0 lamina dominated laminate.

It can be seen from Figure 9 that with increase in number of 0^0 lamina in the laminate sequence it results in higher stiffness for AR≥1 and it is reversed when AR≤1.

3.2.2. Domination of 90^0 lamina

Dominance of 90^0 lamina on stiffness of plate has been shown in Figure 10.

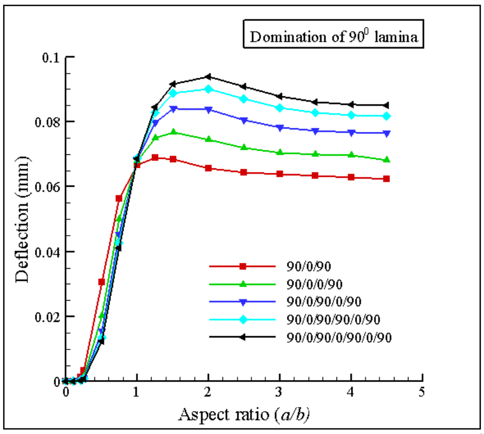

Figure 10: Deflection of plate with AR for 90^0 lamina dominated laminate.

It can be seen from Figure 10 that with decrease in number of 90^0 lamina in the sequence of laminate it results in higher stiffness if AR≥1 and it is reversed when AR≤1. Some of the important values of deflections are compared with theoretical results for different orientation sequence of lamina in laminate. Deflections of laminated composite plates for different orientation sequence are listed in Table 2 for a constant AR of 0.75 and 2.5. It could be observed from Table 2 that the theoretical results are closed if the number of lamina increased in laminate for aspect ratio of 0.75 and 2.5.

TABLE 2: Maximum deflection of composite plate for a constant AR of 0.75 and 2.5

Sequence of laminas in laminate	Deflection (mm)		AR
	CLT	FEA	
0/90/90/0	0.024053	0.028021	0.75
0/90/0/90/0	0.025903	0.026995	
0/90/0/0/90/0	0.027054	0.026662	
0/90/0/90/0/90/0	0.027731	0.027382	
90/0/90	6.43E-02	7.67E-02	2.5
90/0/90/0/90	0.080639	0.085067	
90/0/90/90/0/90	0.086972	0.086342	
90/0/90/0/90/0/90	0.090956	0.09001	

4. Conclusions

Stiffness analysis of symmetric cross-ply laminated composite plates has been investigated for different orientation sequences of laminate with aspect ratios. The same has also been analyzed considering domination of any particular lamina orientation. Some of the important conclusions are given below:

- For AR≤1, the maximum and minimum deflection have been found for ply having sequence [90/0/90] and [0/90/0], respectively, but it has been found reverse when AR≥1.
- With increase in number of lamina for 90^0 domination or decrease in number of lamina for 0^0 domination results in high stiffness of composite plate for AR≤ 1.
- With increase in number of lamina for 0^0 domination or decrease in number of lamina for 90^0 domination results in high stiffness of composite plate for AR≥1.

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Author Profile

Mukul Saxena, B.Tech., is currently pursuing M.Tech. (Applied Mechanics) in the Department of Mechanical Engineering from Tezpur University, Tezpur, Assam, India. He received his bachelor degree in Mechanical Engineering from Uttar Pradesh Technical University, Lucknow (India) in 2014. He His research area includes Finite element analysis, Laminated composite materials.

Sushen Kirtania, M.Tech., is working as an assistant professor in the Department of Mechanical Engineering at Tezpur University, Tezpur, Assam, India. He received hisbachelor degree in Mechanical Engineering from Indian Institute of Engineering Science and Technology, Shibpur, West Bengal and M.Tech. (Specialization: Machine Design) in the Department of Mechanical Engineering from IIT Guwahati (India). He is pursuing his Ph.D from IIT Guwahati (India). His research area includes Carbon nanotubes-based composites, Fiber-reinforced composite, Finite element method, Fracture mechanics.