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Computable General Equilibrium Modeling for Regional Analysis

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1 Introduction

1.1 Introduction

Partial equilibrium analysis illustrates results for one market at a time. However, there often exist market interactions and thus market feedbacks. As Nicholson suggests, pricing outcomes in one market usually have effects in other markets, and these effects, in turn, create ripples throughout the economy, perhaps even to the extent of affecting the price-quantity equilibrium in the original market. To represent this complex set of economic relationships, it is necessary to go beyond partial equilibrium analysis and construct a model that permits viewing many markets simultaneously. The general equilibrium model is a framework for analyzing linkages between markets and thus interactions between industries, factor resources and institutions.

de Melo and Tarr argue that inter-industry linkages are best captured in a general equilibrium framework. Although partial equilibrium may yield accurate estimates for particular sectors, estimates of aggregate costs of regional policies across sectors, for example, require a general equilibrium model to account for region-wide budget and resource constraints.

In the past, implementation of general equilibrium analysis was constrained by inadequate data and computational resources. Currently, however, the existence of large-capacity computer technology has made possible applications of such models to actual market situations. By recommending general equilibrium analysis, we do not mean that econometric estimates representing different sectors have little value. Rather, the two approaches should be viewed as complementary because it is neither feasible nor desirable to estimate, as a system of simultaneous equations, the full set of conditions describing a multisector economy model (de Melo and Tarr). In many cases, general equilibrium analysis borrows parameter estimates from partial equilibrium econometric studies.

1.2 General equilibrium economic models

Several approaches have been used to represent the regional macroeconomy interactions among sectors and, hence, the analysis of impacts of alternative policies. Most general equilibrium procedures are broadly categorized into fixed-price (multiplier) impact analysis and the endogenous price, quantity and income computable general equilibrium (CGE) methods. This section provides an overview and comparison of these model types and their variants.

Input-output analysis, attributed to Leontief, has been used for assessing the impact of a change in the demand conditions for a given sector of the economy. The basic relationship in these models is represented by

$$X_{ij} = a_{ij} X_j, \tag{1.1}$$

where X_{ij}, the amount of sector i's output required for the production of sector j's output, is assumed to be proportional to sector j's output X_j, and a_{ij} is the relevant input-output coefficient. Summing over sectors and adding final demand F_i to equation (1.1) produces the I-O model:

$$X_i = \sum_{j=1}^{\pi} a_{ij} X_j + F_i, \tag{1.2}$$

which is also assumed to hold in first-difference form (depicting changes in the variables). An increase in final demand in a particular sector by, say, ΔF_i will initially increase production for that sector, which in turn raises the intermediate demand for all sectors. To produce these intermediate inputs, however, more intermediate inputs are required. Although sectoral outputs keep on rising in several rounds, these increases become smaller and smaller such that their total always has a limit (Sadoulet and de Janvry). Equation (1.2) is often written in matrix notation:

$$X = (I - A)^{-1} F, \tag{1.3}$$

where X is the vector of outputs, F is the vector of final demands, A is the matrix of input-output coefficients, and I is the identity matrix (with ones on the diagonal and zeros elsewhere). The matrix $(I - A)^{-1}$ represents a multiplier used to calculate overall changes in sectoral outputs caused by changes in final demand. For a more complete discussion of input-output see the web text chapter by William A. Schaffer.

Input-output analysis hinges on the crucial assumption that sectoral production is completely demand-driven, implying that there is always excess capacity in all sectors that is capable of meeting increased demand with no price increase. Because this assumption is likely to be unrealistic, input-output models are more useful as guidelines to potential induced linkage effects, and as indicators of likely bottlenecks that may occur in a growing economy, than as predictive models (Sadoulet and de Janvry).

Further, I-O models assume a constant returns to scale production function with no substitution among the different inputs. Prices are also assumed constant, which is not a major problem as substitution among factors is expected to be induced only by nonexistent relative price movements.

Extension of the I-O model to a social accounting matrix (SAM) framework is performed by partitioning the accounts into endogenous and exogenous accounts and assuming that the column coefficients of the exogenous accounts are all constant. According to Sadoulet and de Janvry, endogenous accounts are those for which changes in the level of expenditure directly follow any change in income, while exogenous accounts are those for which we assume that the expenditures are set independently of income. In determining exogenous accounts, it is common practice to pick one or more among the government, capital, and the rest of the world accounts based on macroeconomic theory and the objectives of the study.

Although I-O and SAM models have typically been used for impact analyses, they do not consider the special case where productive capacity of a sector is curtailed or eliminated (Seung, et al.). This concern has led to the emergence of mixed exogenous/endogenous I-O models where the production capacity of a sector is exogenously reduced (Petkovich and Ching). To examine the impacts of timber production potentials on income distribution, Marcouiller, Schreiner and Lewis (1993) demonstrated an application of a SAM version of the mixed exogenous/endogenous model, the *supply-determined SAM (SDSAM) model,* to the analysis of forest products.

However, these mixed exogenous/endogenous models, though relatively easy to implement, have limitations similar to fixed-price models. These are fixity of prices and no factor substitution in production and no commodity substitution in consumption. Seung et al. contend that, by these restrictive assumptions, the SDSAM model lacks microtheoretic foundation. Thus, such models are internally inconsistent because outputs for some sectors are forced to be fixed and final demands for the same sectors are assumed endogenous.

To circumvent the limitations posed by the SDSAM model, regional economists have turned to using the more theoretically sound computable general equilibrium (CGE) models as a tool for policy and impact analyses. In CGE analysis, output in all sectors is endogenously determined and prices are assumed sufficiently flexible to clear the commodity and factor markets. An empirical comparison of the SDSAM and CGE approaches by Seung et al. indicates that, compared to the CGE model, the SDSAM model tends to overestimate the policy impacts and to estimate production decreases in sectors where production may not change or may increase. The authors conclude that a regional CGE model is theoretically more sound than mixed exogenous/endogenous fixed price models for impact analyses where productive capacity of sectors is curtailed or eliminated.

Partridge and Rickman argue that fixed-price regional models are limiting cases of the more general Walrasian general equilibrium system. In fixed-price models, which are characterized by perfectly elastic supply, the total change in the regional economy is always predicted to be proportionate to the exogenous change. The Walrasian general equilibrium procedure, which is grounded in neoclassical theory, specifies less than elastic supply with equilibration of demand and supply achieved through flexible prices. In these models, the total response in an economy to an exogenous change is not necessarily proportionate and depends upon the various elasticities of demand and supply.

2 Overview of CGE Analysis

The CGE framework offers an alternative for regional analysis. It encompasses both the I-O and SAM frameworks by making demand and supply of commodities and factors dependent on prices. A CGE model simulates the working of a market economy in which prices and quantities adjust to clear all markets. It specifies the behavior of optimizing consumers and producers while including the government as an agent and capturing all transactions in circular flow of income (Robinson, Kilkenny and Hanson).

In the Walrasian neoclassical general equilibrium approach, the main equations are derived from constrained optimization of the neoclassical production and consumption functions. Producers are assumed to choose their level of operation so as to maximize profits or minimize costs using constant returns to scale production technology. Production factors – labor, capital and land – are all paid in accordance with their respective marginal productivities. Consumers are assumed to choose their purchases to maximize their utility subject to budget constraints. At equilibrium, the model solution provides a set of prices that clears all commodity and factor markets and makes all the individual agent optimizations feasible and mutually consistent (Bandara).

CGE analysis has been applied to a wide range of policy issues, which include, among others, income distribution, trade policy, development strategy, taxes, long-term growth and structural change in both developed and less developed countries (LDCs). Dixon and Parmenter associate the proliferation of these models in LDCs with two major conditions. First, growing realization that CGE models, unlike a number of other types of economic models, allow the simulation of policy alternatives in a way which is readily understood and perceived to be both relevant and useful by policy makers. Second, vast progress in the development of user friendly, readily transferable high capacity computer software, which has greatly increased researchers' ability to handle models with considerable detail.

2.1 CGE analysis at national and regional levels

Most CGE models have been used to capture the effects of policies and economic shocks at the national level. Application of the technique to regions (such as states) is more recent. Examples of regional applications in Oklahoma include Koh, Lee, Budiyanti, and Amera.[1]

Regional CGE models differ from their national counterparts in several respects. Most of these differences stem from the fact that regions are relatively more open economies compared to nations. Because of regional openness, commodity trade and resource migration are more important in regional CGE models. For example, regional households and entrepreneurs would not invest within the region if other regions offered higher rates of return. Thus, while national CGE models require that savings be equal to investment, regional CGE models permit excess savings to flow out of the region and vice-versa. This is not to say that regional policymakers cannot influence rates of return to investments but that control over major components of monetary policy is mainly determined at the national level.

In general, CGE models require considerable data, which, in most cases, is difficult to obtain. This problem is more severe at the regional level, where data in most cases is virtually non-existent. In fact, one of the possible reasons for the relatively slow start of regional CGE modeling is the paucity of regional data, in addition to unresolved theoretical issues of regional specification [2] (Partridge and Rickman). Most of the limitations of regional CGE models are also inherent in alternative empirical regional modeling, such as I-O, SAM, and econometric.

Although regional CGE models have grown in popularity in recent years as an alternative method for examining regional economies and policy issues, their contribution has yet to be assessed. Partridge and Rickman present an extensive review of literature related to regional CGE modeling and conclude that regional CGE models, though still with unclear conclusions on issues of quantitative accuracy, represent a significant advancement in regional economic analysis. For details on the current state of the art of regional CGE modeling, readers are referred to Partridge and Rickman.

[1] These are all Ph.D. studies completed at Oklahoma State University. Results of the studies have been published in Koh, Schreiner and Shin; Schreiner, Lee, Koh, and Budiyanti; and Amera and Schreiner. Partridge and Rickman give an extensive review of many other regional studies.

[2] Issues of functional form, elasticity specification, closure rules, sensitivity analysis, market structure, and dynamics.

The greater openness of regional economies suggests some desired divergence in structure between national and regional CGE models. In spite of the differences between national and regional CGE models discussed above, the general formulation used in most studies is basically the same. While some studies have been designed to capture the added complexity, others have relied on the specifications common to the national CGE literature.

Most empirical applications of CGE models have been developed on the simplifying assumption of constant returns to scale production technology and perfectly competitive market structures. This has made these models fail to adequately represent industries with declining unit cost structures. Recently, de Melo and Tarr used the theory of duality to develop and apply a production modeling technique that accommodates imperfect competition in the U.S. auto and steel industries. Tembo has suggested and demonstrated an application of this technique to regional economies. Vargas and Schreiner show an application to monopsony markets in the regional timber industry.

The purpose of this chapter is to present and illustrate application of the salient features of the regional CGE model and to provide a step-by-step example of their empirical implementation. In this endeavor, the more traditional perfectly competitive constant returns to scale version of the CGE model is presented first. This is then followed by a variation that accommodates imperfect competition (see section 5.0).

2.2 Data and data organization

CGE models are very data intensive. Thus, the first step in implementation of a CGE model is identification and organization of data into a social accounting matrix (SAM). The SAM is a square matrix representing a series of accounts which describe flows between agents of commodity and factor markets and institutions. It is a double-entry book-keeping system capable of tracing monetary flows through debits and credits and constructed in such a way that expenditures (columns) and receipts (rows) balance. King distinguishes two objectives for the SAM: 1) to organize information about the economic and social structure of a country, region in a country, city or any other geographic unit of analysis; and 2) to provide a “fixed point” basis for the creation of a plausible model.

Regionalized economic datasets that can serve as a basis for regional CGE models are now available. This section describes the types of data required for building regional CGE models. These data needs include regional social accounts and parameters required for incorporating economic relationships among industries, in production and factor usage, among institutions, and in the generation of regional economic output. Each is addressed in-turn in the following sections.

2.2.1 Social accounting matrices

The base data upon which a regional CGE model is constructed relies on a static accounting for economic transactions taking place in a base year and specific to the region under examination. Input-output (I-O) tables provide one data framework but lack the comprehensive accounting of income flows. Base data on these income flows are necessary to address labor components, production structures, and government interaction necessary to conduct policy analysis. A more comprehensive accounting structure for regional economies is provided through an I-O extension known as a social accounting matrix (or SAM.) SAM extensions were initially developed during the late 1960’s and early 1970’s as a result of general dissatisfaction with the manner in which income flows were treated. A good overview of SAM development and analytical background for the interested reader can be found in Pyatt and Round and Hewings and Madden. SAMs as a basis for CGE models is addressed in Isard et al.

What is a SAM?

Like input-output accounts, social accounting matrices provide a comprehensive accounting structure of regional market-based productive activities and utilize similar double-counting book-keeping entries. Unlike input-output, however, social accounts focus on the household as the relevant unit of analysis and provide a comprehensive, and additional, set of accounts that track how household income is generated and distributed. Where input-output tables are focused on industries and their respective relationships with regional output, SAMs extend this into a more complete range of market mechanisms associated with generating household income. The relevant focus thus shifts from how regional output is produced to also address how regional income is generated and distributed. This comprehensive element is particularly important in regional CGE models that focus on both production processes and the economics of household factor supply, commodity demand, and government interaction.

How are SAM's useful for policy analysis?

Social accounting matrices have been employed in a wide array of situations arising in policy development to address key issues of economic structure and impact assessment. A good overview of SAM applications in policy analysis was written by Erik Thorbecke and found in the recent text by Isard et al. (pages 317-331.) Basically, SAMs are useful in assessments that require a more comprehensive accounting of circular flows of an economy.

Particularly useful for addressing issues of income distribution, SAMs have been widely employed in assessing development effectiveness in attaining equity-based outcomes of policy. Applications, however, are not limited to assessing redistributive income policies. This is particularly true in the United States as national and state level policies that support the redistribution of income to the poor are largely out of favor. Increasingly, welfare reform legislation has emphasized the role of private markets to provide for individual welfare. SAMs have been employed to assess the relative impacts of alternative market-based changes on the distribution of income within regions. Thus SAMs will continue to be relevant tools to address a wide array of policy situations and development issues.

The major strength of regional SAMs include accounting comprehensiveness. Although still widely used, it is important to note that SAMs have some rather serious theoretical shortcomings when used to model economic change. These modeling caveats have, in part, driven the movement toward developing more flexible modeling systems. As such their use in computable general equilibrium models remains the focus of this chapter.

How is a regional/state SAM constructed?

SAMs can be constructed in a variety of ways. The manner in which a SAM is specified is typically driven by the problem being addressed. A thorough assessment of the various types of SAM structures is beyond the scope of this chapter. Rather, for this discussion a generic SAM structure will be discussed illustrated by a modest empirical SAM constructed for the Oklahoma economy.

Data elements for constructing a SAM. An illustrative SAM framework is provided in Figure 2.1. From an input-output perspective, the rows and columns that correspond to industry and commodity are the focus. Whereas input-output is limited to this industrial perspective, social accounting matrices extend the dataset to more fully capture income distribution resulting from returns to primary factors of production (land, labor, and capital.) In this way, the circular flow of goods and services to households from firms and the corresponding factor market flows to firms from households are captured.

In the SAM, row totals and column totals are equal thus representing a regional economy in equilibrium. For example, total industry output just equals the outlay used in its production. Institutional income (to households for example) just equals the outlay required for the use of institutionally-owned land, labor, and capital in the factor markets. In general, total income equals total cost of inputs. SAM accounts are constructed to balance outputs with inputs.

Data sources for SAM building. Once again, the specific data requirements for constructing a regional SAM vary depending on the type of problems being addressed. However, some generalizations can be made. In addition to standard input-output data (industry production, interindustry transactions, final demands, factors of production and imports/exports), typical SAMs require additional data on total factor payments, total household income (by income category), total government expenditures and receipts (including intergovernmental transactions), institutional income distribution, and transfer payments (both to households and to production sectors.) SAMs are typically built as static snapshots of a region thus, data elements will need to be generally consistent in temporal and geographic specificity.

Figure 2.1 An Illustrative Social Accounting Matrix							
	Industry	Commodity	Factors	Institutions	Gov't	Trade	TOTAL
Industry (detail)		Make					Total Industry Output
Commodity (detail)	Use			Consumption		Exports Output	Total Commodity
Factors -land -labor -capital	Returns to Primary Factors (value added)					Exported Primary Factors (e.g. labor flow)	Total Factor Income
Institutions -households -other	Sales	Sales	Distribution of factor Income		Transfer Payments	Exports	Total Institutional Income
Government	Indirect Business Taxes	Sales Tax	Factor Taxes		Inter-governmental Transfers		Total Government Income
Trade	Imported Purchased Inputs	Imports	Imports			Trans-shipments	Total imports
TOTAL	Total Industry Outlay	Total Commodity Outlay	Total Factor Outlay	Total Institutional Outlay	Total Gov't Outlay	Total Exports	

2.2.2 Using IMPLAN to construct a SAM

For purposes of illustration, discussion will center on a readily available dataset for the initial regional static equilibrium. A good example of this base economic equilibrium data is found in the county-level files available from the Minnesota IMPLAN Group (or MIG.)[3] This consultancy group develops relational datasets built from secondary data available at the national, state, and county-level from the BEA REIS, BLS ES202, County Business Patterns and other sources. Specifically, this group first gathers data at the national level, converts it to a standardized format, derives national input-output tables and national tables for deflators, margins and regional purchase coefficients. State level data is gathered and controlled totaled to the national. County level data is gathered and controlled totaled to each state. County or regional-level input-output tables are derived using various data elements employed in the model development software embedded within IMPLAN Pro.

Over the course of development, the Minnesota IMPLAN Group has endeavored to adapt, expand, and extend datasets into more comprehensive accounting structures and regional modeling approaches. For example, a set of social accounts has been added to the county-level IMPLAN datasets. These accounts are available for use both in assessing inter-institutional transactions and in regional CGE modeling. The latter application has been under development for the past few years. Notable discussions of these developments can be found in Robinson and Sullivan, McCollum, and Alward.

Specific data incorporated into the IMPLAN SAM begins with standardized elements of the National Income and Product Accounts (NIPA.) Household transfer payments and distributional breakdowns come from the Census of Population, BEA REIS dataset and the BLS Consumer Expenditure Survey. Government data requirements originate from the Annual Survey of State and Local Government Expenditures. This data source provides state and local revenues and expenditures by detailed category.

[3] The IMPLAN Pro software and county/state datasets are available from the Minnesota IMPLAN Group (accessed on the world wide web at www.implan.com.) Their mailing address is 1725 Tower Drive West, Suite 140, Stillwater, MN 55082, Voice: 651/439-4421 Fax: 651/439-4813.

Generating a SAM from an IMPLAN model is rather straightforward given general knowledge of software and dataset operations. The SAMs generated from IMPLAN are not, however, without drawback. One key drawback of using the IMPLAN system to generate a social accounting matrix is the rather rigid categorization scheme used in dataset and model construction. For example, due to the manner in which the dataset was developed, value added remains in rather nebulous categories that match published secondary data sources. Instead of value added being separated into returns to land, labor, and capital, value added in IMPLAN is reported in categories that include employee compensation, other property type income, proprietary income, and indirect business taxes. One *ad hoc* method of conversion is to simply use employee compensation as a proxy for labor returns (which neglects proprietary income), other property type income as a proxy for land returns, and proprietary income as a proxy for capital returns (actually more a mixture of labor and capital returns). Although there exist procedures for disaggregating total value added into more standard categories of factor return, these methods tend to be data intensive and complex.

The aggregate SAM for Oklahoma

The number of sectors represented in the SAM and, hence, the number of markets in the CGE model depends to a large extent on the purpose of the study. Budiyanti, for example, aggregated the Oklahoma 1991 SAM to 14 industrial sectors of market goods, two sectors of non-market goods, three value-added sectors (capital, labor, and land), and three institutions (enterprises, households and government). The labor sector was further sub-divided into five skill levels. The household sector was also divided into low-, medium- and high-income classes. Government was represented by a state/local level and a federal level. Amera's 1993 Oklahoma SAM has 30 industrial sectors, three factor sectors, three household sectors, two government sectors, one enterprise sector, one investment sector, and a rest-of-the-world sector. For illustration purposes in this chapter, a highly aggregated (four-industrial sector) version of Amera's SAM is used as the data source (Table 2.1). This SAM also aggregates the household and government sectors into one sector each.

2.3 Determining parameter values

Once the economic agents are identified and their optimizing behavior specified by algebraic equations, the parameters in those equations must be evaluated. Data on endogenous and exogenous variables obtained at a snapshot point in time are typically used for this purpose. This process is referred to as calibration . Calibration or benchmarking determines the values of the normalizing (or free) parameters so as to replicate the observed flow values incorporated in the SAM (de Melo and Tarr). This process assumes that all equations describing market equilibriums in the system (model) are met in the benchmark period.

When dealing with flexible functional forms, such as the constant elasticity of substitution (CES) or the constant elasticity of transformation (CET), it is necessary to supplement the calibration process with these exogenously determined elasticities.[4] Other parameters obtained from literature (econometric studies) include income elasticities, migration elasticities, and price elasticities of export demand. These parameters are used to illustrate the calibration process of the various components of the regional CGE model.

The calibration process starts with choice of units. Because in CGE analysis only relative prices matter, all prices and factor rents are normalized to unity in the initial equilibrium. With prices normalized to one, then the flow "values" in the SAM (Table 2.1) may be interpreted as a physical index of quantity in the commodity (industry) and factor markets (click here for further explanation of normalized prices). Once all the parameters are specified, the model is solved to reproduce the benchmark data. The solution obtained with the benchmark data is referred to as the "replication" equilibrium, assuming the benchmark represents an equilibrium outcome, given existing exogenous conditions (Partridge and Rickman). In addition to providing a check on the accuracy of the calibration, the replication also shows that the complete circular flows of income and expenditures are balanced, which is referred to as microconsistency of the data. Counterfactual equilibria are obtained by introducing shocks to exogenous variables, changes in market conditions, or changes in any policy variable and rerunning the model. The general algebraic modeling system (GAMS) software is used for solving the regional CGE model. The following sections in this paper outline the general features

[4] Application of these elasticities in the CGE framework are discussed in chapter3.

of a regional CGE model and demonstrate the calibration and solution processes under both perfect and imperfect competition.

3 A Competitive Regional CGE Model

In a market economy there is generally a large number of homogeneous goods and services, which include not only consumption items but also factors used in production. Each of these goods and services has a market price, determined by the forces of supply and demand. Every market is assumed to clear at this set of prices. The perfectly competitive model further assumes zero transactions cost, a large number of price taking market participants (consumers and suppliers), and existence of perfect information, all of which support the law of one price (Nicholson).

Under these conditions, computable general equilibrium (CGE) models are similar to multimarket models, in which agents' decisions are price responsive and markets reconcile supply and demand. Because they also encompass macroeconomic components, such as investment and savings, balance of payments and government budget, they are best chosen for policy analysis when the socioeconomic structure, prices, and macroeconomic phenomena all prove important (Sadoulet and de Janvry). CGE models have been built to simulate the economic and social impacts of various scenarios. Examples of alternative scenarios include foreign trade shocks, changes in economic policies, and changes in domestic economic and social structure.

In a regional CGE model, production creates demand for value-added factors and goods and services used as intermediate inputs. Intermediate inputs consist of both imports and locally produced goods and services. Demand for value-added factors interacts with available factor supplies to determine factor prices. Margins, such as taxes and transportation costs, increase factor costs to firms, which in turn increase product prices. Factor rates of return and ownership of factor supplies determine personal income, which in turn influences demand for imports and locally produced goods and services. Equilibrium occurs at prices which equate the demands for goods and services with supplies, and the demands for factors with factor supplies.

Because the CGE model attempts to look at all adjustments simultaneously, it is inherently an extensive formulation. To enhance understanding by students and prospective users of CGE analysis, the model here is split into components and each component is explained separately. The components include commodity markets, factor markets, production systems, institutional agents, and welfare measures.

3.1 Production system

Unlike regional input-output and SAM models, which are based on Leontief technology, neoclassical theory guides specification of production in regional CGE models. In consequence, the CGE model does not represent factor demands as linear functions of output. Instead, factor demands depend on both output and relative prices. The only exception, however, is in relation to treatment of those goods and services that are used as intermediate inputs. The Leontief input-output production function is used to represent production of regional output with fixed proportions of composite primary factors and composite intermediate inputs.

The composite primary factors generally enter the production process in a manner allowing factor substitution. Thus, production is best described as a multi-level or nested production process. Note that all factors in a constant elasticity of substitution (CES) function have the same elasticity of substitution between any pair of factors. To allow for differing elasticities between sets of factors, multi-level or "nested" production function forms are used in CGE, with each level containing a different set of factors and their own corresponding elasticities of substitution. That is, the use of a multi-level structure allows for use of both fixed-coefficients and price responsiveness in the CES form.

3.1.1 Composite value-added and intermediate inputs

The Leontief input-output production function that represents the on- substitutability between intermediate and primary inputs constitutes the first level of the three-level production process characteristic of most CGE models. For a single industry/sector, the Leontief production function is presented as:

$$X_i = \min\left(\frac{VA_i}{a_{0i}}, \frac{V_i}{a_{1i}}\right) \tag{3.1.1}$$

where X_i is gross output of sector i, VA_i is composite factor (value-added) inputs of industry i *and* V_i is composite intermediate inputs of industry i. Constants a_{0i} and a_{1i} represent industry i's input-output coefficients for composite factor inputs and composite intermediate inputs.

By rearranging terms in equation (3.1.1), the (input-output coefficient) parameters of the Leontief production function are calibrated as follows:

$$a_{0i} = \frac{VA_i}{X_i}, \text{ and}$$
$$a_{1i} = \frac{V_i}{X_i} \tag{3.1.2}$$

For each calculation in equation (3.1.2), values of the variables on the right-hand-side (RHS) are given in the SAM. For the agricultural sector in Table 2.1, for example, total output $X_i = 4,344,160,000$ (the column or row total), composite factor inputs $VA_i = 1,713,668,000$, and composite intermediate inputs (locally produced plus imports) $V_i = 2,534,191,000$. Therefore, Leontief parameter values are $a_{0i} = 0.40$ and $a_{1i} = 0.58$ (click here for graphic presentation of Leontief production function). Although an industry is an aggregation of many producers, it is treated as a single firm in the CGE framework.

3.1.2 Substitution among primary factors of production

What generally distinguishes a regional CGE production structure from a simple input-output model is that value-added (primary) factor usage is responsive to factor costs, and imports of intermediate goods are price responsive (Partridge and Rickman). At the second level of production, nesting allows different treatment of intermediate goods from that of value-added factors.

Primary inputs and their demands

Cobb-Douglas (CD) or constant-elasticity-of-substitution (CES) functions are commonly specified to represent substitution among primary factors of production in a sector - land, labor, and capital. Here production technology is assumed to possess constant returns to scale (CRS). The CD function implicitly specifies unitary factor substitution elasticities, while the CES is a more general case that allows different from unitary elasticities of substitution. For simplicity, the Cobb-Douglas functional form is used to represent the second level of production:

$$VA_i = \phi_i^{VA} LAB_i^{\alpha_i^L} \cdot CAP_i^{\alpha_i^K} \cdot LAND_i^{\alpha_i^T}, \ (\alpha_i^L + \alpha_i^K + \alpha_i^T) = 1 \tag{3.1.3}$$

where LAB_i, CAP_i, and $LAND_i$ are labor, capital, and land inputs for industry i, respectively. Coefficient $\phi_i^{VA} > 0$ is the total factor efficiency parameter for composite primary factor inputs in sector i. Parameters α_i^L, α_i^K, and α_i^T are production elasticities (click here for CD production elasticities) and correspond to labor, capital and land, respectively. Constant returns to scale are imposed by assuming that the sum of the elasticities in equation (3.1.3) is equal to unity. Individually, the production parameters are also assumed to have values that lie between zero and one. By substituting and rearranging terms in equations (3.1.2) and (3.1.3), sectoral gross output (X_i) can be expressed in the Cobb-Douglas production function form:

$$X_i = \frac{\phi_i^{VA}}{\alpha_{0i}} LAB_i^{\alpha_i^L} \cdot CAP_i^{\alpha_i^K} \cdot LAND_i^{\alpha_i^T}, \tag{3.1.4}$$

or

$$X_i = \phi_i^X LAB_i^{\alpha_i^L} \cdot CAP_i^{\alpha_i^K} \cdot LAND_i^{\alpha_i^T}, \text{ where } \phi_i^X = \frac{\phi_i^{VA}}{\alpha_{0i}} \tag{3.1.5}$$

Assuming that labor, land, and capital are the only value-added (or primary) inputs in the production of sector i's output X_i, the sector's profit function is

$$\pi_i = PN_i \cdot X_i - PL \cdot LAB_i - PK_i \cdot CAP_i - PT \cdot LAND, \tag{3.1.6}$$

where π_i is profit (click here for example of profits) for sector i, PN_i is net price of output (i.e. output price less cost of intermediate inputs and indirect business taxes), PL is wage rate, PK_i is capital rent (assuming capital is fixed by sector), and PT is land rent.

Assuming all firms in the sector strive to maximize profits, differentiating equation (3.1.6) with respect to each of the inputs and equating the outcome to zero will give the first order conditions. Thus, the first order condition with respect to capital is:

$$\frac{\partial \pi_i}{\partial CAP_i} = PN_i \frac{\partial X_i}{\partial CAP_i} - PK_i = 0 \tag{3.1.7}$$

Rearranging terms in equation (3.1.7), the marginal product of capital is equal to the ratio of capital rent to output net price:

$$\frac{\partial X_i}{\partial CAP_i} = \frac{PK_i}{PN_i}. \tag{3.1.8}$$

Substituting equation (3.1.5) into equation (3.1.7) yields the following:

$$\frac{\partial \pi_i}{\partial CAP_i} = PN_i \frac{\partial(\phi_i^X LAB_i^{\alpha_i^L} \cdot CAP_i^{\alpha_i^K} \cdot LAND_i^{\alpha_i^T})}{\partial CAP_i} - PK_i = 0 \tag{3.1.9}$$

which translates into:

$$\frac{\partial \pi_i}{\partial CAP_i} = PN_i \frac{\alpha_i^K \phi_i^X LAB_i^{\alpha_i^L} \cdot CAP_i^{\alpha_i^K} \cdot LAND_i^{\alpha_i^T}}{CAP_i} - PK_i = 0 \tag{3.1.10}$$

Rearranging terms in equation (3.1.10) and substituting for X_i using equation (3.1.5), yields an expression for capital's share parameter in the Cobb-Douglas production function:

$$\alpha_i^K = \frac{PK_i}{PN_i} * \frac{CAP_i}{X_i} \text{ , or } \alpha_i^K = \frac{PK_i \cdot CAP_i}{PN_i \cdot X_i} \tag{3.1.11}$$

This is equivalent to multiplying capital's marginal product (see equation 3.1.8 above) by the ratio of capital to output, which is also the formula for elasticity. Therefore, expression (3.1.11) shows that factor shares are equal to production elasticities in a Cobb-Douglas function. Share parameters for labor and land are derived in a similar fashion. In equation (3.1.11), making CAP_i the subject of the formula yields the conditional demand (i.e. fixed output level) for capital in the industry, given by:

$$CAP_i = \frac{\alpha_i^K PN_i \cdot X_i}{PK_i} \tag{3.1.12}$$

Similarly, conditional demands for labor and land can be expressed as:

$$LAB_i = \frac{\alpha_i^L PN_i \cdot X_i}{PL_i} \text{ , and} \tag{3.1.13}$$

$$LAND_i = \frac{\alpha_i^T PN_i \cdot X_i}{PT} \tag{3.1.14}$$

Calibration of the Cobb-Douglas production equation (3.1.5), involves determining and evaluating two sets of parameters – share parameters and the efficiency parameter, where all prices are normalized to one. The numerator and denominator in equation (3.1.11) are provided in the SAM as total capital returns and total value-added, respectively. For the agricultural sector in the above SAM (Table 2.1), capital returns and total value-added are \$571,360,000 and \$1,713,668,000, respectively. Substituting these values into (3.1.11) yields $\alpha_i^K = 0.333$. Similarly, $\alpha_i^L = 0.253$ and $\alpha_i^T = 0.414$. The efficiency parameter for the Cobb-Douglas production function is calculated by rearranging equation (3.1.5):

$$\phi_i^{CX} = \frac{X_i}{LAB_i^{\alpha_i^L} \cdot CAP_i^{\alpha_i^K} \cdot LAND_i^{\alpha_i^T}} \tag{3.1.15}$$

Calibration of equation (3.1.15) proceeds by substituting the calibrated factor share parameters and the quantities for the factor variables obtained from the SAM. For the agricultural sector, $\phi_i^{CX} = 7.46$. Multiplying ϕ_i^{CX} by a_{oi} yields the value ϕ_i^{VA}. (Click here for graphic presentations of the calibrated production function and factor demands).

Intermediate inputs and their demands

By the Armington assumption (Armington), goods produced in different regions (and possibly countries) are assumed to be imperfect substitutes, usually specified as a constant elasticity of substitution (CES) function. These intermediate goods from different regions combine at the second level of production to form composite intermediate goods that enter the first level of production. The CES function representing the relationship between the two categories of intermediate inputs can be expressed as:

$$V_{ji} = \phi_{ji}^V[\delta_{ji}^V VM_{ji}^{\rho_j^V} + (1-\delta_{ji}^V)VR_{ji}^{\rho_j^V}]^{1/\rho_j^V} \;,\; \rho_j^V = \frac{\sigma_j^V - 1}{\sigma_j^V} \tag{3.1.16}$$

where $\phi_j^V > 0$ is the intermediate input efficiency parameter, $0 < \delta_{ji}^V < 1$ is the share parameter, VM_{ji} represents intermediate goods imported by sector i from sector j in the exporting region, VR_{ji} is regionally produced intermediate goods for sector i from sector j, σ_j^V is the elasticity of substitution for industry j, and ϕ_j^V is the substitution parameter. The value of σ_j^V depends on the degree of substitutability between the two sources of intermediate inputs. If $\sigma_j^V = \infty$, the two are perfect substitutes. If $\sigma_j^V = 0$, they are used in fixed proportions.

The following cost minimization problem is used to derive demand functions for regionally produced and imported intermediate inputs:

Minimize $PM_j \cdot VM_{ji} + PR_j \cdot VR_{ji}$

Subject to: $V_{ji} = \phi_{ji}^V[\delta_{ji}^V VM_{ji}^{\rho_1^V} + (1-\delta_{ji}^V)VR_{ji}^{\rho_1^V}]^{1/\rho_1^V}$,

where PM and PR represent, respectively, prices of imported and regionally produced intermediate inputs from sector j. Solving the first-order conditions of this problem and rearranging terms yields the following expression:

$$\frac{VR_{ji}}{VM_{ji}} = \left[\left(\frac{1-\delta_{ij}^V}{\delta_{ji}}\right)\left(\frac{PM_j}{PR_j}\right)\right]^{\sigma_i^V}. \tag{3.1.17}$$

Calibration of this equation requires knowledge of the elasticity of substitution σ_j^V and normalizing the two prices, PM and PR to one. As stated above, values of elasticities of substitution are obtained from other sources. For the Oklahoma agricultural sector, for example, manufacturing input has a value of 3.55 (Table 3.1). This leaves the share parameter δ_{ji}^V as the only unknown in equation (3.1.17). The value of δ_{ji}^V is calculated by substituting the elasticity of substitution and the base values for imported and regionally produced intermediate inputs (from SAM) in the rearranged form of equation (3.1.17):

$$\delta_{ji}^V = \left[\left(\frac{VR_{ji}}{VM_{ji}}\right)^{\frac{1}{\sigma_i^V}} + 1\right]^{-1}. \tag{3.1.18}$$

Table 3.1
Elasticities of Import Substitution

Sector	Parameter	Source
Agriculture	1.42	de Melo and Tarr
Mining	0.50	de Melo and Tarr
Manufacturing	3.55	de Melo and Tarr
Services	2.00	de Melo and Tarr

From the SAM, the known values for intermediate inputs from manufacturing to agriculture are $VR_{ji} =$ 159,671,000 and $VM_{ji} =$ 446,829,000. Thus, from equation (3.1.18), $\delta_{ji}^V = 0.359$. The efficiency parameter is computed by rearranging terms in the CES function (equation 3.1.16) and making the relevant substitutions:

$$\phi_{ji}^v = \frac{V_{ji}}{\left[\delta_{ji}^V VM_{ji}^{\rho_i^V} + (1-\delta_{ji}^V)VR_{ji}^{\rho_i^V}\right]^{1/\rho_i^V}}. \tag{3.1.19}$$

Total intermediate inputs from manufacturing to agriculture is, $V_{ji} = 606{,}500{,}000$ (see the SAM). Thus, evaluating equation (3.1.19) yields the value $\phi_{ji}^{V} = 1.931$ for the agricultural sector.(Click here for graphic presentations of the substitution between the two sources of intermediate inputs).

3.1.3 Substitution among types of factor inputs

A third level in the nested production process may represent substitution among labor skills within the overall labor input, among classes of land within the overall land input for agriculture, or types of capital inputs within the overall classification of capital. (The SAM presented in Table 2.1 does not show subcategories of primary inputs.) A common procedure is to consider the CES form of production which allows elasticities of substitution to differ among industries but requires the elasticity of substitution among any two subcategories (i.e. labor skills, land classes or types of capital) to be the same. Alternatively, subcategories could be grouped into two parts, such as production labor and all other, with one elasticity of substitution between the two and then two different classes of production labor with a different elasticity of substitution.

The elasticities of substitution for this level of the production process must come from other studies. (Click here for modeling substitution among labor skills). The studies by Koh and Budiyanti classified labor into five skill levels following work by Rose. They then assumed the Cobb-Douglas elasticity of substitution (equal to one) for all combinations of skill levels and for all industries. No sensitivity analysis was completed to test the results of varying these elasticities.

3.1.4 Net output price

Net output price in the competitive model is regional output price minus the unit cost of intermediate inputs and unit value of indirect business tax:

$$PN_i = PX_i - \sum_j a_{ji} P_j - ibt_i PX_i \tag{3.1.20}$$

where PN_i is commodity i's net price, PX_i is the composite regional output price, a_{ji} is the amount of the j^{th} commodity per unit output of the i^{th} commodity, P_j is the composite purchase price of the j^{th} comodity, and ibt_i is the indirect business tax per unit value of output. (See section 3.2.4 for explanation of composite regional output price and composite purchase price). The net output price is the per unit value of output available to compensate for primary factor use. Under conditions of constant returns to scale in production, the sum of the marginal value products for all primary factor use should exactly equal the commodity net price.

3.2 Commodity markets

Commodity trade involves both regional and export markets. Within the region, commodity supplies are obtained from regional sources (regional production sectors) as well as from out-of-region sources (imports). Though differentiated by source, these commodities are bought by industries (intermediate inputs), households and other institutions. Inter-industry commodity flows have been discussed in Section 3.1 as intermediate input demands. In this section, we discuss regional output markets and household commodity demand systems.

3.2.1 Market outlets for regional output

Each industry in the region produces a composite commodity that can be exported or sold in the regional market. Export markets include other regions within the country and international markets. In CGE analysis, exports and regionally sold products are assumed to be differentiated by market, with the relationship between them represented by a constant elasticity of transformation (CET) function. Price ratios and elasticities of transformation determine the levels of output exported and sold in the region. The substitution possibilities are, thus, represented as

$$X_i = \phi_i^X \left[\partial_i^X EXP_i^{\rho_i^X} + (1-\partial_i^X) R_i^{\rho_i^X} \right]^{1/\rho_i^X}, \ \rho_i^X = \frac{\sigma_i^X + 1}{\sigma_i^X} \tag{3.2.1}$$

where X_i is industry i's total output (as defined above), $\rho_i^X > 0$ is the output efficiency parameter, $0 < \partial_i^X < 1$ is the share parameter, EXP_i represents sector i's supply for export, R_i is the sector's output supply to the regional market, σ_i^X is the elasticity of transformation for industry i, and σ_i^X is the output substitution parameter. The value of σ_i^X depends on the degree of transformability between the two market outlets. If $\sigma_i^X = \infty$, the two are perfect in their transformation. If $\sigma_i^X = 0$, the two markets are not substitutable and further market behavior for each must be specified (see Berck et al. for an alternative to the CET).

Each firm allocates it's output between the regional and export markets so as to maximize revenue, subject to the CET function. Because the production process is assumed the same for each market, revenue maximization may be substituted for profit maximization. Thus, for given regional and export prices, the problem faced by the firm is to:

maximize $PE_i \bullet EXP_i + PR_i \bullet R_i$

subject to: $X_i = \phi_i^X \left[\partial_i^X EXP_i^{\rho_i^X} + (1-\partial_i^X) R_i^{\rho_i^X} \right]^{1/\rho_i^X}$,

where PE_i and PR_i are, respectively, prices of exported and regionally sold commodities from sector i. Solving the first-order conditions and rearranging terms yields the following:

$$\frac{R_i}{EXP_i} = \left[\left(\frac{1-\partial_i^X}{\partial_i^X} \right) \left(\frac{PE_i}{PR_i} \right) \right]^{-\sigma_i^X}. \tag{3.2.2}$$

Calibration of this equation requires knowledge of the elasticity of transformation σ_i^X, which is obtained from other sources, and normalizing the two prices, PE_i and PR_i to one. For the Oklahoma agricultural sector in Table 3.2, $\sigma_i^X = 3.90$. The value of ∂_i^X, the only unknown in equation (3.2.2), is calculated by substituting the elasticity of transformation and the benchmark values for exported and regionally sold commodities (from SAM) in the rearranged form of equation (3.2.2):

Table 3.2
Elasticities of Transformation

Sector	Parameter	Source
Agriculture	3.90	de Melo and Tarr
Mining	2.90	de Melo and Tarr
Manufacturing	2.90	de Melo and Tarr
Services	0.70	de Melo and Tarr

$$\partial_i^X = \left[\left(\frac{R_i}{EXP_i} \right)^{-\frac{1}{\sigma_i^X}} + 1 \right]^{-1} \tag{3.2.3}$$

For the agricultural sector (see SAM in Table 2.1), R_i = 1,752,557,000 and EXP_i = 2,591,603,000. Thus, from equation (3.2.3), $\partial_i^X = 0.47$. The efficiency parameter is computed by rearranging terms in the CET function (equation 3.2.1) and making the relevant substitutions:

$$\sigma_i^X = \frac{X_i}{\left[\partial_i^X EXP_i^{\rho_i^X} + (1-\partial_i^X) R_i^{\rho_i^X} \right]} \Bigg]^{1/\rho_i^X} \tag{3.2.4}$$

For the agricultural sector, X_i = 4,344,160,000 (see the SAM). Thus, evaluating equation (3.2.4) yields ϕ_i^X = 2.01 for the agricultural sector. (Click here for graphic presentation of the calibrated CET function for regional product and exports.)

3.2.2 Commodity consumption by households

Regional household income available for commodity expenditure is calculated as gross income minus government taxes, savings and, in this case, payments for labor employed by households. Equation (3.2.5) is an algebraic representation of this relationship:

$$HE_h = DY_h - HSAV_h - PL \cdot LH_h \tag{3.2.5}$$

where HE_h is household expenditure, DY_h is household disposable (minus government taxes) income, SAV_h represents household savings, PL is wage rate, and LH_h is labor employed directly by households. The subscript h represents household category (low, medium or high income). The current SAM (Table 2.1) shows only total households.

The regional consumption by households is nested in two levels. At the first level, households maximize utility from leisure and consumption of composite market commodities, subject to total time (work plus leisure), household budget constraints and prices. At the second level, they choose optimal combinations of imported and locally produced commodities, which are imperfect substitutes, so as to minimize their cost of purchasing predetermined amounts of market commodities. Substitution between these commodity groups is captured in a CES function. A detailed presentation of each of these levels of the household consumption follows below.

Household commodity demand systems

Several alternative formulations have been used to represent household demand systems in the literature. Examples include the almost ideal demand systems (AIDS) by Deaton and Muellbauer, the Rotterdam model by Theil, and Barten, and the linear expenditure system (LES) by Stone. In general, a theoretically consistent demand system permits imposition of the general restrictions of classical demand theory. These restrictions are a) adding-up: value of total demands equals total expenditure, b) homogeneity: demands are homogeneous of degree zero in total expenditure and prices, c) symmetry: cross-price derivatives of the Hicksian demands are symmetric, and d) negativity: direct substitution effects are negative for the Hicksian demands.

The linear expenditure system is the most commonly used in CGE analysis due, in part, to convention and because it allows representation of subsistence consumption, in addition to satisfying the above restrictions. In this subsection, we provide an overview of the LES demand system and its adaptation to the CGE framework. Readers interested in more detail about the LES and other demand systems are referred to Deaton and Muellbauer.

In the LES, demand equations are assumed to be linear in all prices and incomes and the set of demand functions is expressed in expenditure form:

$$p_i q_i = c_i + \sum_{j=1}^{\pi} a_{ij} p_j + \beta_i y, \tag{3.2.6}$$

where p_i is the price of the i^{th} commodity, q_i is the quantity of the commodity demanded, c_i is the i^{th} intercept, a_{ij} are the price parameters, β_i is the marginal budget share for the commodity, and y is the household's income. Empirically, the LES is derived from constrained maximization of the Klein-Rubin (also known as Stone-Geary) utility function, whose general form is

$$U = \sum \beta_i \ln(Q_i - \gamma_i), \sum \beta_i = 1 \tag{3.2.7}$$

where U is the utility level, Q_i is level of commodity i, β_i is as defined above, and γ_i, if positive, is subsistence minima as perceived by the consumer.

Given a fixed amount of household income that can be allocated to consumption, HE_h, the household faces the following constrained maximization problem:

Maximize $U(Q_{ih}) = \sum_{i=1}^{\pi} \beta_{ih} \ln(Q_{ih} - \gamma_h)$

subject to: $HE_h - \sum_{i=1}^{\pi} P_i \bullet Q_{ih} = 0,$

where the subscript h represents a particular category of households.[5] Solving the first order conditions of the Lagrangean to this problem produces the following results:

$$\frac{\beta_{ih}}{Q_{ih} - \gamma_{ih}} = \gamma P_i \text{ , and} \tag{3.2.8}$$

$$HE_h - \sum_{i=1}^{\pi} P_i \bullet Q_{ih} = 0. \tag{3.2.9}$$

Rearranging terms in (3.2.8), summing across i, and solving for the Lagrangean multiplier yields

$$\lambda = \frac{1}{HE_h - \sum_{i=1}^{\pi} P_i \bullet Q_{ih},} \tag{3.2.10}$$

where, as stated above, $\sum_{i=1}^{\pi} \beta_{ih} = 1$. Substituting (3.2.10) into (3.2.8) produces an expression for the expenditure on commodity i by household category h:

$$P_i \bullet Q_{ih} = P_i \gamma_{ih} + \beta_{ih} \left(HE_h - \sum_{j=1}^{\pi} P_j \gamma_{jh} \right). \tag{3.2.11}$$

As expected, the first derivative of equation (3.2.11) with respect to total expenditure HE_h is the marginal budget share, β_{ih}. The linear expenditure system (equation 3.2.12) is obtained by dividing equation (3.2.11) by P_i:

$$Q_{ih} = \gamma_{ih} + \frac{\beta_{ih}}{P_i} \left(HE_h - \sum_{j=1}^{\pi} P_j \gamma_{jh} \right). \tag{3.2.12}$$

To evaluate equation (3.2.12), we need values for γ_{ih} and β_{ih}, prices, and total consumption expenditure data from the SAM. Because γ_{ih} cannot be directly estimated from empirical data and because β_{ih} cannot be calculated from a one-period data set in the SAM, equation (3.2.12) is often implemented using a simplified version of the Stone-Geary LES. Rearranging equation (3.2.12) gives

$$\gamma_{ih} - \frac{\beta_{ih}}{P_i} \sum_{j=1}^{\pi} P_j \gamma_{jh} = \frac{HE_h}{P_i} \left(\frac{P_i \bullet Q_{ih}}{HE_h} - \beta_{ih} \right). \tag{3.2.13}$$

If we assume that the average budget share is equal to the marginal budget share, equation (3.2.13) implies the following:

$$\beta_{ih} = \frac{P_i \bullet Q_{ih}}{HE_h}, \text{ and} \tag{3.2.14}$$

$$\gamma_{ih} - \frac{\beta_{ih}}{P_i} \sum_{j=1}^{\pi} P_j \gamma_{jh} = 0. \tag{3.2.15}$$

Because $0 < \beta_{ih} < 1$ and $P_i > 0$, the relationship in equation (3.2.15) is guaranteed only if the minimum/subsistence consumption $\gamma = 0$ for all commodities. If this is the case, the LES demand function, equation (3.2.12), simplifies to:

$$Q_{ih} = \beta_{ih} \frac{HE_h}{P_i}. \tag{3.2.16}$$

Coefficients β_{ih} are calculated from equation (3.2.14) by using the benchmark data in the SAM. This process is accomplished by normalizing the prices to one, which transforms the expenditure results in the SAM to

[5] Often, households have been categorized into 'high income', 'medium income', and 'low income'. See, for example, Budiyanti. For simplicity, we assume here that all households are homogeneous ($h = 1$).

physical quantities. In our example (Table 2.1), total household expenditure on both imported and regionally produced commodities, HE_h = \$50,665,679,000 and expenditure on agricultural commodities is \$328,760,000. Thus, the marginal (equal to the average) budget share for agriculture is 0.0065. (Click here for a graphic presentation of the calibrated commodity demand.)

As you notice, equation (3.2.16) is based on very restrictive and somewhat unrealistic assumptions. It implies that income elasticities of demand are unitary for all commodities. Although the results are not appropriate for dynamic analysis, this assumption does not pose serious problems for comparative static analysis, particularly if expenditure patterns for several household income groups are embodied in the model. For the interested reader, click here for a more *general case of the LES* demand system, which provides for leisure, household labor supply, and varying commodity income elasticities.

Commodity substitution of imports for domestic product

The second level of household commodity demand involves determination of the minimum cost combination of regional and imported commodities. For each commodity i, substitution between the two sources is captured in the following CES function:

$$Q_{ih} = \phi_i^Q \left[\partial_i^Q QM_{ih}^{\rho_i^Q} + (1 - \partial_i^Q) QR_{ih}^{\rho_1^Q} \right]^{\frac{1}{\rho_1^Q}}, \rho_i^Q = \frac{\sigma_i^Q - 1}{\sigma_i^Q} \tag{3.2.17}$$

where $\phi_i^Q > 0$ is the household consumption efficiency parameter, $0 < \partial_i^Q < 1$ is the share parameter, QM_{ih} represents household demand for imports, QR_{ih} demand for regional products, σ_i^Q is the elasticity of substitution, and ρ_i^Q is the substitution parameter. The determination of the domestic (regional) and imported amounts of a fixed total household demand is the same as presented in equations (3.1.16) to (3.1.19). (Click here for a graphic presentation of *the substitution relationship between imported and regionally produced commodities* as shown in the form of a household indifference curve).

3.2.3 Institutional markets

Governments and capital formation are the two remaining commodity markets represented in the Oklahoma SAM. Quantity demanded is assumed exogenous for each of these markets. However, price is endogenous and, hence, expenditure by governments and for capital formation varies with price. Similar to intermediate commodity inputs and household commodity demands, imported and regionally produced commodities are imperfect substitutes in meeting the composite commodity demands. Exogenous commodity demand for governments (QG_i) and capital formation (QC_i) from the two sources (regional and imported) is given by the following CES function:

$$QX_i = \phi_i^{\delta c} \left[\partial_i^{\delta c} QXM_i^{\rho_i^{\delta c}} = \left(1 - \partial_i^{\delta c}\right) QXR_i^{\rho_i^{\delta c}} \right]^{\frac{1}{\rho_i^{\delta c}}}$$

$$\rho_i^{\delta c} = \frac{\sigma_i^{\delta c} - 1}{\sigma_i^{\delta c}} \tag{3.2.18}$$

where $QX_i = QG_i + QC_i, QXM_i$ is quantity imported and QXR_i is quantity domestically produced. All parameters are identified similar to those for equation (3.1.16). The elasticities of substitution $\sigma_i^{\delta c}$ are the same as for intermediate inputs and household demand (see Table 3.1). Solution to quantities imported and domestically produced is similar to equations (3.1.16) to (3.1.19).

3.2.4 Commodity prices

Composite purchase price

Commodity purchase prices are a composite of regional and import prices:

$$P_i = \frac{PR_i \cdot R_i + PM_i \cdot M_i}{R_i + M_i} \tag{3.2.19}$$

The composite purchase price (P_i) is the unit value for household consumption goods, intermediate inputs, and institutional purchases. PR_i is the regional purchase price and PM_i is the import price. R_i is the total amount of commodity regionally produced and consumed and M_i is the total amount of commodity imported:

$$R_i = TVR_i + TQR_i + QXR_i \tag{3.2.20}$$

$$M_i = TVM_i + TQM_i + QXM_i \tag{3.2.21}$$

The right hand side terms are as previously defined.

Composite output price

Commodity output prices are a composite of regional and export prices:

$$PX_i = \frac{PR_i R_i + PE_i + EXP_i}{R_i + EXP_i} \tag{3.2.22}$$

The composite output price (PX_i) is the weighted unit value of revenue received from regional and export sales. PR_i is the regional price and PE_i is the export price.R_i is the regional quantity and EXP_i is the export quantity.

3.2.5 Commodity market equilibrium

Total commodity demand is the sum of intermediate demand, institutional demand, and export demand. Total commodity supply is the sum of regional production and imports. Market equilibrium for commodity i is the following:

$$X_i + M_i = TV_i + TQ_i + QX_i + EXP_i \tag{3.2.23}$$

where X_i = regional production, M_i = imports, TV_i = total composite intermediate input demand, TQ_i = total composite household demand, QX_i = total composite exogenous commodity demands (governments plus capital formation), and EXP_i = export demand.

3.3 Factor markets and factor incomes

In section 3.1, we derived factor demands for a profit-maximizing firm. However, these industries are not the only participants on the demand side of the factor markets. Institutions such as governments and households demand factor services. In addition to discussing institutional demand for factors, this section also describes the supply side and equilibrium conditions for the factor markets.

In the CGE framework, market behavior for primary factors is studied from both short-run and long run perspectives. In the short run, capital is assumed to be fixed by sector while labor is assumed to be mobile between sectors and between regions. In the long run, both capital and labor are mobile between sectors and regions. Land is assumed fixed in both short- and long run.

Factors are assumed to migrate in search of interregional quantity-price equilibrium. Higher wage rates and capital rents relative to out-of-region levels encourage in-migration while lower rates induce out-migration. Few regonal CGE studies have attempted to incorporate interregional mobility in factor markets. In their national trade model, de Melo and Tarr derived an endogenous labor supply by incorporating leisure as a commodity in the household utility function. Lee endogenized labor supply by allowing the labor-leisure choice and labor migration through a labor migration elasticity in his Oklahoma regional CGE model. In modeling the U.S. economy, Rickman incorporated both labor and capital migration. Budiyanti adapted Lee's endogenous household labor supply and incorporated labor and capital migration in a regional CGE model.

For simplicity in the current exposition, initial institutional endowments and migration are assumed to influence factor supply. Equilibrium factor prices result when factor demands equal corresponding factor supplies. Endogenous labor supply (labor-leisure choice) is assumed to be insignificant and, hence, ignored. In the rest of this section, we present equilibrium conditions for the three primary factors - labor, capital and land - under conditions of no endogenous factor supplies. However, a detailed explanation of the modeling procedures required to address leisure-augmented household demand systems and endogenous labor supply is presented in this clickable. Most CGE models assume perfectly competitive factor markets, in which both firms (factor demanders) and households (factor suppliers) are treated as price takers. In the remainder of this section, we use the framework of perfect competition to discuss labor, capital and entrepreneurship, and land as factors and as sources of income.

3.3.1 The labor market

The labor market is in equilibrium when quantity supplied equals quantity demanded. Assuming all labor is homogeneous, equilibrium is expressed as:[2]

$$LSO + LMG = LDI + LDE \tag{3.3.1}$$

where LSO is total initial household labor, LMG is labor migration, $LDI = \sum_{i=1}^{\pi} LAB_i$ is total industry demand for labor, and LDE is exogenous demand for labor. LDE is equal to:

$$LDE = LDH + LDG \tag{3.3.2}$$

where LDH is labor demanded directly by households and LDG is labor demanded by all government agencies. The labor row total in the SAM (Table 2.1) shows that LSO = 37,489,772,000 and is equal to the sum of LDI (30,400,863,000) and LDE (7,088,909,000). This is true when the system is in benchmark equilibrium because LMG is then equal to zero.

As stated above, labor migration arises due to differences between regional and out-of-region wage rates. The degree of mobility depends on the labor migration elasticity. This relationship is:

$$LMG = LSO \bullet \delta^i \bullet \log\left(\frac{PL}{PLE}\right) \tag{3.3.3}$$

where LSO is initial labor supply, PL is regional wage rate, PLE is rest-of-the-world wage rate, and δ^i is labor migration elasticity. δ^i is obtained from external sources. For examples in this study, the parameter δ^i is (0.92) and is from Plaut.

Labor income

Total regional labor income (LY) is the sum of the product of labor demanded and the wage rate:

$$LY = PL \bullet (\Sigma_i LAB_i + LDH + LDG) \tag{3.3.4}$$

where PL is wage rate, LAB_i is labor demanded by industry i, LDH is labor demanded directly by households, and LDG is labor demanded by all government agencies. If the labor market is disaggregated by skill type, total labor income is determined by summing across all skills. Net labor income (NLY) is determined by subtracting payroll tax from total (or gross) labor income in equation (3.3.4):

$$NLY = LY(1 - ss\ tax) \tag{3.3.5}$$

where $ss\ tax$ is the labor payroll tax rate. All of net labor income (NLY = 31,363,057,000) is distributed to households (SAM, Table 2.1). Payroll tax rate is $ss\ tax = 0.164$.

[2] If labor is differentiated by skill,the relationships presented here would hold for each skill type.

3.3.2 The capital market

In the short run, when capital is assumed to be perfectly immobile, the capital market is in equilibrium when quantity demanded by each industry (CAP_i) is equal to that industry's initial capital stock (KSO_i):

$$CAP_i = KSO_i \tag{3.3.6}$$

If capital is mobile (the long run solution), the capital market is in equilibrium when total capital supply, which is the initial quantity plus migrated capital, equals total capital demand:

$$KMG + \sum_{i=1}^{n} KSO_i = \sum_{i=1}^{n} CAP_i \tag{3.3.7}$$

where KMG is capital supply from migration, and KSO_i and CAP_i are as defined above. Capital mobility ensures uniform capital rents across industries.

Like labor, capital migration arises due to differences between region and out-of-region rental prices:

$$KMG = \sum_{i=1}^{n} KSO_i \bullet \delta^i \bullet \log\left(\frac{PK}{PKE}\right) \tag{3.3.8}$$

where KSO_i is industry i's initial capital supply, PK is regional capital rent, PKE is rest-of-the-world capital rent, and δ^i is capital migration elasticity. The parameter δ^i is obtained from external sources. For examples in this study, the parameter δ^i is 0.92 and is taken from Plaut.

Capital income

Total capital income (KY) is the sum of the product of capital demanded and capital rent:

$$KY = \sum_i PK_i \bullet CAP_i \tag{3.3.9}$$

where PK_i is capital rent and CAP_i is the quantity of capital demanded by sector i.

In this formulation, capital is fixed with capital rents differentiated by industry. The overall capital rent is:

$$PK = \frac{\sum_i PK_i \bullet CAP_i}{\sum_i KSO_i} \tag{3.3.10}$$

When capital is mobile across sectors and regions, capital income is:

$$KY = PK \bullet CAP_i \tag{3.3.11}$$

where PK is the overall capital rent of the region.

Capital is owned by enterprises and households. Enterprise ownership is by corporations. Household ownership is by self-employed businesses including agriculture. Government subsidies are treated as an aggregate payment to capital. Thus net capital income (NKY) is the following:

$$NKY = (PK - gsub)KY \tag{3.3.12}$$

where PK is capital rent and $gsub$ is the government subsidy. From the Oklahoma SAM (Table 2.1), $gsub$ = 0.0494467, $ENTK$ = 12,510,953,000 and HHK = 7,848,069,000. Therefore, NKY = 19,352,336,000 when PK = 1.0. This is the same as the row and column totals for capital in the SAM.

Other accounting procedures and assumptions could be used in determining net capital income. In particular, business subsidies could be attributed directly to an industry.

3.3.3 The land market

Land is immobile and is assumed perfectly inelastic both in the short- and long run. Thus, the land market attains equilibrium when land use ($LAND_i$) is equal to initial quantity of land TSO_i:

$$LAND_i = TSO_i \tag{3.3.13}$$

Total land income (TY) is the sum of the product of quantity of land and land rent:

$$TY = \sum_i PT_i \cdot LAND_i \tag{3.3.14}$$

where PT_i is gross land rent and $LAND_i$ is the quantity of land demanded by sector i. For the Oklahoma SAM, agriculture is the only user of land. Net land income (NTY) is total land income less land tax:

$$NTY = (1 - t\ tax)(TY) \tag{3.3.15}$$

where $t\ tax$ is the land tax rate. From the Oklahoma SAM, $t\ tax = 0.0363379$ and $LAND = 709{,}066{,}000$. Therefore, $NTY = 683{,}300{,}000$. Because households own all land in the Oklahoma SAM, net land income accrues to households.

3.3.4 Enterprise income

The source of enterprise income ($ENTY$) is gross capital rents:

$$ENTY = PK \cdot ENTK \tag{3.3.16}$$

where PK is capital rent and $ENTK$ is the initial stock of enterprise capital.

Claims to enterprise income ($ENTY$) include regional households, governments and a broadly defined capital account. Governments receive revenues from corporate income taxation. The broadly defined capital account includes capital depreciation, retained earnings and capital payments to owners of capital (stock) outside of the region. Because the current regional CGE model is used as an analysis of comparative statics to marginal changes in the system, enterprise income is distributed to the three entities (regional households, governments and capital account) as fixed shares. This distribution of income may be realistic for households and governments but it is unrealistic for depreciation which is generally based on capital stock rather than capital income.

The assumed distribution is:

$$HENTY = h\ ENTY \tag{3.3.17}$$

$$GENTY = g\ ENTY \tag{3.3.18}$$

$$CENTY = c\ ENTY \tag{3.3.19}$$

where h, g, and c are shares of gross enterprise income distributed to households, governments and capital account, respectively. These shares are computed from the SAM and are $h = 0.1386$, $g = 0.1359$, and $c = 0.7255$.

3.3.5 Household income

Most household income comes from factor payments. As noted above, gross factor payments are subject to government taxes and capital depreciation. It is, thus, the total earnings less the applicable deductions that are available for distribution to owners of factors. Other sources of household income include inter-household transfers, government transfers, and net remittances from the rest-of-the-world.

Gathering these sources of income for households, gross household income (GHY) is:

$$GHY = NLY + PK \cdot HHK + NTY + HENTY + GOVTH + ROWTH \tag{3.3.20}$$

where NLY is net labor income, PK is capital rent, HHK is capital stock owned by households, NTY is net land income, $HENTY$ is household enterprise income, $GOVTH$ is government transfers to households, and $ROWTH$ is net transfers and remittances to households from rest-of-world. The latter two sources do not depend on regional resource ownership and factor prices. These sources are exogenous and assumed constant for the following analyses. All values may be read directly from the household row in the SAM.

Disposable household income (DHY) is:

$$DHY = (1 - hh\ tax) \cdot GHY \tag{3.3.21}$$

where ht is the household income tax rate. For the Oklahoma SAM, $hh\ tax$ = 0.1294835.

Household savings ($HSAV$) is:

$$HSAV = mps \cdot GHY) \tag{3.3.22}$$

where mps is the savings rate. Because this is negative in the Oklahoma SAM for 1993, it implies a negative savings rate for the aggregate of households. It is not uncommon for households to expend more than their income, particularly lower income households where inter-household transfers are large and expenditures are based on expected future earnings. In the Oklahoma SAM, because there is one household group, inter-household transfers are netted out of gross household income. In this case, mps = -0.0718137.

Because the model allows for labor and capital mobility, adjustments need to be made in factor compensations to households to assure that ownership of resources by households does not change with resource mobility. This is a major difference between regional and national CGE modeling. National models need not account for mobility of resources within the national boundary to hold original resource ownership constant by household group. For regions, households own labor, capital and land and receive transfers (inter-household, governments and rest-of-world). If labor moves, it is generally the household that relocates with its ownership rights to not only labor but also to capital and land. If resource adjustments are not made with labor mobility, changes in regional gross household income accounting may be the result of unintended changes in household resource ownership.

Consider household labor income with migration. Equation (3.3.1) shows regional labor market equilibrium with migration. Migration is shown in equation (3.3.3). Labor income (LY) for the benchmark (initial) regional households is the following:[3]

$$\begin{aligned} LY &= PL(LDI + LDE) \\ &\quad + PLE\left(\sqrt{LMG^2} - LMG\right)0.5 \\ &\quad - PL\left(\sqrt{LMG^2} + LMG\right)0.5 \end{aligned} \tag{3.3.23}$$

where all terms are as defined before. The first term on the right hand is regional gross labor compensation. The second term identifies out-migration and the compensation received when outmigrating. The third term identifies in-migration and the compensation received by immigrants. In-migration and out-migration are mutually exclusive as shown in the migration equation (3.3.3). Click here for two *hypothetical examples* of equation (3.3.23).

Household income from capital depends on household capital ownership and capital rents. Under the assumption of no capital mobility (short run with capital fixed by sector and region, i.e. equation 3.3.6), the initial regional households own capital resources equal to HHK = 7,848,069,000 and are compensated equal to $PK \cdot HHK$ where PK is the average regional price of capital.

Even though capital is immobile, with labor migration, households migrating out are assumed to take with them their proportion of capital rents which are further assumed to be spent out of the region. Those

[3]This formulationappears in the Amera and Schreiner regional CGE.

households remaining in the region will receive their proportionate share of capital compensation. Labor (household) in-migration is assumed to bring no other resource (capital and land) rents into the region. This assumption may be modified if further information is available.

Capital compensation to households is equal to:

$$YKH = PK \bullet HHK \tag{3.3.24}$$

The proportion of initial households associated with labor out-migration is:

$$aLMG = \frac{(\sqrt{LMG^2} - LMG)0.5}{LSO} \tag{3.3.25}$$

where $aLMG$ is used to show an adjustment amount to the following income variables. Only when LMG is negative (i.e. out-migration) will the numerator be greater than zero. When LMG is positive (i.e. in-migration) $aLMG$ will be zero. The capital compensation to households remaining in region is:

$$RYKY = (1 - aLMG)YKH \tag{3.3.26}$$

If $aLMG = 0$, then all of YKH remains in region.

With capital mobile, capital resources owned by the initial households are used in-region or out-of-region depending on the proportion of capital out-migration to initial capital stock. The proportion of capital migration to capital stock is:

$$aKMG = \frac{(\sqrt{KMG^2} - KMG)0.5}{\sum\limits_{1} KSO_i} \tag{3.3.27}$$

The assumption is that the same proportion of out-migration of capital applies equally to households and enterprises.

Capital compensation to households remaining in-region and with capital mobility is:

$$\begin{aligned} RYKH = {} & (1 - aLMG)(1 - aKMG)YKH \\ & + PKE \bullet aLMG \bullet HHK \end{aligned} \tag{3.3.28}$$

The first term on the right adjusts capital compensation to households (YKH) for out-migration of labor $(1 - aLMG)$ and out-migration of capital $(1 - aKMG)$. The second term adds back in the compensation for out-migration of capital but at a higher capital rent because $PKE > PK$.

Compensation for capital in-migration adds to gross regional (state) product but is assumed to flow back out-of-state because ownership resides out-of-state.

Household income from land depends on land ownership and land rents. All net land income (equation 3.3.15) accrues to households:

$$NTYH = (1 - t\ tax)TY \tag{3.3.29}$$

However, with labor out-migration, a proportion of $NTYH$ flows out of state. The proportion of $NTYH$ remaining in-state is:

$$RNTYH = (1 - aLMG)NTYH \tag{3.3.30}$$

where the argument is the same as for capital income given in equation(3.3.26).

Enterprise income, government transfers and rest-of-world remittances accruing to the initial regional households (equation 3.3.20) remaining in-region under conditions of labor out-migration is given as:

$$REYH = (1 - aLMG)(HENTY + GOVTH + ROWTH) \tag{3.3.31}$$

Benchmark data is in equilibrium with labor and capital migration equal to zero. However, changes in equilibriums under comparative statics should allow for mobility of labor (households) and capital. As a result, three possible household groups are identified, with their own sources of income and their own effects on regional variables including commodity demands, savings and taxation. Each household group is presented by a set of income accounting equations.

Regional households

This group of households is part of the initial set of regional households and remains in the region after resource mobility occurs and a new equilibrium is attained under comparative statics. It is this group that is of primary interest in measuring welfare change from a change in regional policy or regional structure. Income to regional households includes net labor income, gross capital income, net land income, enterprise income, government transfers and rest-of-world net remittances:

$$\begin{aligned} RHHY = {} & [LY(1 - ss\ tax) - PLE(\sqrt{LMG^2} - LMG)0.5] \\ & + RYKH + RNTYH \\ & + REYH \end{aligned} \tag{3.3.32}$$

The first term is household labor income adjusted for payroll taxes (equation 3.3.5) and labor out-migration (equation 3.3.23); the second term is household capital income adjusted for capital rents following labor migration (equation 3.3.26); the third term is net land income adjusted for land rents following labor migration (equation 3.3.29); and the fourth term is household enterprise income, government transfers and rest-of-world remittances, all adjusted for labor out-migration (equation 3.3.31). Under the conditions of capital mobility in addition to labor mobility, (equation 3.3.26) is replaced by (equation 3.3.28) and this becomes the second term in (equation 3.3.32).

Regional household expenditure for commodity demand is equal to:

$$RHE = (1 - hh\ tax\text{-}mps) * RHHYPL * (1 - aLMG)LDH \tag{3.3.33}$$

where $hh\ tax$ = household income tax rate, mps = household savings rate, and $PL \bullet (1 - aLMG)LDH$ is household payments directly to labor adjusted for out-migration. The latter is included because payments directly to labor are not part of the household demand (expenditure) system.

Labor out-migration households

Households associated with labor out-migration take with them the value of their labor plus their capital and land rents from the initial distribution of resource ownership. Similarly, the region has less government transfers and less rest-of-world remittances. These reductions translate into less expenditure in the region and less government tax revenue and regional savings.

Income of out-migration households is the following:

$$\begin{aligned} OMHHY = {} & PLE(\sqrt{LMG^2} - LMG)0.5 \\ & + aKNG \bullet YKH + aLMG \bullet RNTYH \\ & + aLMG \bullet H \bullet ENTY \end{aligned} \tag{3.3.34}$$

where the first term is the labor compensation received out of the region. Notice that payroll tax is not included because this tax would be paid in the region of employment. The second term is capital rents and the third term is net land rents associated with regional resource ownership of migrating households. Notice that capital subsidies flow out but that land tax remains within the region. The fourth term is enterprise income associated with out-migrating households. Because this income is from capital ownership, it is treated the same way as direct capital payments to households.

Although regions lose government income tax revenue on labor income, regions keep income tax revenue ($OMGR$) on capital and land rents and enterprise income:

$$OMGR = hh\ tax \bullet aLMG(YKH + RNTYH + H \bullet ENTY) \tag{3.3.35}$$

Labor in-migration households

Income associated with labor in-migration households is assumed limited to only their labor compensation:

$$IMHHY = PL(\sqrt{LMG^2} + LMG)0.5 \tag{3.3.36}$$

Regional expenditure associated with this income is equal to:

$$IMRE = (1 - hh\ tax\text{-}mps)IMHHY \tag{3.3.37}$$

It is this expenditure which accounts for the commodity demands of in-migrants in their linear expenditure system.

3.4 Measures of regional and household welfare

The primary purpose of CGE analysis is to evaluate policy and policy change. Policymakers frequently evaluate policy change using several criteria. Two broad criteria are presented here with each subdivided into more specific welfare measures. The first broad criteria is regional welfare and emphasizes policy change on regional macro-variables. Because of the openness of regions, these measures are prone to emphasize place prosperity (or growth) with little insight on how policy changes welfare of people. The second broad criteria is household welfare and emphasizes people prosperity irregardless of where people eventually reside. This criteria considers both income effects and price effects in evaluating welfare of households residing in the region.

3.4.1 Regional welfare

Gross regional product

The most comprehensive measure of regional change is gross regional product (GRP) or, if for a state, gross state product (GSP). This measure accounts for the quantity of primary factor inputs used and the compensation to each input. It generally includes the indirect business tax paid by industry. It includes total compensation for labor by industry including payroll taxes and employee benefits. It includes gross returns to capital (including profits) before depreciation.

GRP are payments to resources used (or employed) in the region irrespective of where resource owners reside. Thus, factor payments flow to resource owners located within the region and outside the region. It is not necessarily a good measure of welfare change of households residing within the region.

For Oklahoma, GSP is the sum of all factor payments ($57,551,174,000) plus indirect business tax ($5,268,195,000) for a total of ($62,819,369,000). The following variables account for GSP:

$$GSP = LY + KY + TY + \sum_i \sum ibt_i PR_i X_i \tag{3.4.1}$$

where the right hand terms are, respectively, gross labor income, gross capital income, gross land income, and indirect business tax. The following is the index of change in GSP:

$$IGSP = (GSP - GSPO)/GSPO \tag{3.4.2}$$

where $GSPO$ is the benchmark value of GSP.

Regional expenditure

Regional expenditures are defined here as aggregate expenditures by households, governments for consumption and businesses for capital formation. If regional expenditures are expanding, one would expect the state's economy to be growing. Expenditures as defined here are not adjusted for regional commodity imports. Presumably, households and governments have increasing incomes and revenues to support increasing expenditures, and investment opportunities are available to support increased capital formation.

Several caveats prevent this regional welfare measure from portraying viable economic growth. First, increased expenditure may be the result of increased commodity prices. A separate regional welfare measure accounts for the overall increase in price level. Second, expenditures may be financed from short term dissavings, government transfers, or out-of-region remittances. The negative savings ratio by households for Oklahoma in 1993 implies a dissavings for purposes of current consumption. Third, because governments were combined in the Oklahoma SAM, we can not view expenditure of only state and local governments. Federal government expenditures are more appropriately classified with regional exports. Fourth, double counting occurs because of government ransfers to households and household tax payments to governments. Fifth, for the current CGE model, government expenditures and capital formation are exogenous and change only as commodity prices change. Of course, other behavioral conditions can be modeled for describing these expenditures.

The following variables account for regional expenditures:

$$RE = HE + \sum_i P_i(QX_i) \tag{3.4.3}$$

where the right hand terms are total regional household expenditures and total exogenous commodity demand expenditures. The following is the index of change in RE:

$$IRE = (RE - REO)/REO \tag{3.4.4}$$

where REO is the benchmark regional expenditure.

Regional price level

Composite commodity prices are endogenous to the regional CGE. Therefore, growth in the monetary variables for the region may be because of quantity changes and/or price changes. Export and import commodity prices are exogenous but the composite price is endogenous because it is a weighted average of the domestic regional and import prices. The overall regional price level may be calculated as either a weighted index of the composite commodity prices or of regional output prices. The former is useful in measuring the effects of prices on regional expenditures. The latter is useful for comparing the overall regional price level to external price levels.

The price index presented here weights the price changes by the benchmark quantities. Other price indexes may be used to measure changes in the overall price level.

The composite commodity price level is the following:

$$P = \frac{\sum_i PR_i \bullet RO_i + \sum_i PM_i \cdot MO_i}{\sum_i (RO_i \bullet MO_i)} \tag{3.4.5}$$

where RO_i and MO_i are benchmark quantities of regional market supply and imports, respectively. The price level index relative to the benchmark price level (i.e. $PO = 1.0$) is the following

$$IP = 1 + \frac{P - PO}{PO} = P/PO \tag{3.4.6}$$

The regional output price level is the following:

$$PX = \frac{\sum_i PR_i \bullet RO_i + \sum_i PE_i \bullet EXPO_i}{\sum_i XO_i} \tag{3.4.7}$$

where XO_i is benchmark quantities of regional output. Presumably, with an increase in PX, regional output would be expanding and regional growth would occur. Similarly, with a PX less than one, regional output is decreasing and regional growth is contracting. The effects of this price level is particularly important when evaluating productivity changes in a region.

Net government revenue

Another important regional welfare measure is the change in net government revenue. An important policy question is whether a regional change in structure or policy adds more to regional government costs than is received in regional government revenue. This welfare measure is not considered here because of the aggregation of all government units (including federal) in the SAM. Several CGE studies are available that have disaggregated the governmental jurisdictions to trace government expenditures and revenues in considerable detail. One of the most detailed is a California study by Berck, et al. It also contains a review of the current literature in this area of application of CGE modeling.

Other regional measures of welfare

The rest-of-the world current trade account compares a region's exports to its imports. The importance of the balance of trade account is not so much that the aggregate of exports exceeds the aggregate value of imports as that the sources of exports and imports are identified. This assists in evaluation of the regional terms of trade, a comparison of the aggregate export price with the aggregate import price. Frequently, a region has a more limited array of export commodities compared to its basket of import commodities. This may lead to highly volatile terms of trade for some (especially small) regions. More diversified regions have less volatile terms of trade. Regions that have large export values compared to import values will have counter balancing monetary flows in the financial markets. Agriculturally related regions and older matured regions frequently have large monetary flows out of the region to counteract revenue inflows from exports. This generally means these regions have fewer investment opportunities compared to other regions. These results may be captured by constructing a balance of payments account for regions.

3.4.2 Household welfare

Household income

The most widely used measure of household welfare is household income. This measure is available in government documents for states and regions by time periods. However, to reproduce this measure from a CGE analysis after simulating a policy or impact change is not straight forward. In the regional CGE framework, households have an initial resource ownership with initial unit values. In addition, they have other sources of income such as government transfers and transfers from other households. In the typical comparative static analysis of policy or impact change, resource ownership and transfer income are held constant by household with emphasis on changes in unit values of resources and regional mobility of resources (labor and capital). The result is an accounting of income for three household groups after the policy change: (1) initial (benchmark) households remaining in the region, (2) initial households that migrate form the region, and (3) households added to the region through in-migration. Incomes for these three household groups are given in the equations of section 3.3.5.

Household incomes generated from regional CGE models are in nominal terms. To express in real terms, regional household incomes should be adjusted for changes in regional price level. One price index that may be used is the composite commodity price level calculated from equation (3.4.6). This adjusts regional household incomes by the purchasing value of commodities in the region.

Compensating and equivalent variation

Utility measures for individuals and households are the result of preferences expressed through markets. Similar measures are not available for regions. Policymakers express preferences for regions. Regional policymakers frequently choose preferences (goals) such as maximizing regional employment growth or maximizing gross regional product (GRP) or income. Such goals have little relevance when how they affect the welfare of individual households or groups of household is unknown (Levin). Maximizing employment growth may lead to trading many low paying jobs for fewer high paying jobs. Maximizing GRP may lead to emphasizing a regional structure of large corporate ownership of resources with high regional outflows of factor payments versus a regional structure of local ownership of resources with low regional outflows of factor payments.

An alternative goal is to increase welfare of one or more household groups within the region. Moving from one market result to another market result presumes a welfare change for most, if not all, household groups. To measure this change from a policy or program change, welfare must be measurable. Because utility is not directly measurable, an alternative measure must be chosen. An observable alternative for measuring the intensities of preferences of an individual for one situation versus another is the amount of money the individual is willing to pay or accept to move from one situation to another (Just, Hueth, and Schmitz, p. 10). The two most widely accepted willingness-to-pay measures are compensating and equivalent variations first proposed by Hicks. Compensating variation (CV) is the amount of money which, when taken away from an individual after an economic change, leaves the person just as well off as before. Equivalent variation (EV) is the amount of money which, if an economic change does not happen, leaves the individual just as well off as if the change had occurred (Just, Hueth, and Schmitz, pp. 10-11). Which welfare measure is employed depends on whether initial prices or new prices are used. The CV measure is based on new prices, and the EV measure is based on initial prices. Information on the distribution of welfare gains and losses among household groups should be useful to policymakers in making judgments on whether this policy result is inferior or superior to an alternative policy result.

Application of these criteria in national CGE models is available in de Melo and Tarr. Application to regional CGE models for Oklahoma are in studies by Lee, Budiyanti, and Amera. The equational forms for CV and EV are presented in Table 4.1.

4 Model Execution

In this section the procedure for implementing the regional CGE is presented as discussed in section 3.0. Because section 3.0 explains and derives the equations for a competitive regional CGE, there are more equations than needed for actual model execution. Section 4.1 puts in tabular form the actual model equations needed for execution (Table 4.1). Also, subscript notation is in Table 4.2; summary of endogenous variables is in Table 4.3; summary of exogenous variables is in Table 4.4; and summary of parameters is in Table 4.5.

A brief discussion of the General Algebraic Modeling System (GAMS) solution is presented in section 4.2 with reference to more detailed procedures. The actual model construction in GAMS is presented in section 4.3. The model itself can be downloaded for purposes of experimenting with model changes and model simulation. (Click here to *download GAMS input file*).

Results of a simulation of increased terms of trade for the region is presented in section 4.4.

4.1 Competitive CGE model equations

Table 4.1 Competitive CGE Model Equations

Table 4.2 Subscript Notation

Table 4.3 Summary of Endogenous Variables

Table 4.4 Summary of Exogenous Variables

Table 4.5 Summary of Parameters

4.2 GAMS Solution

A CGE Model is an integrated system of equations whose simultaneous solution determines values of endogenous variables. The underlying equations are derived from economic theory of the behavior of economicagents and markets – producers, institutions, factor markets, etc. Several approaches have been used to solve these models. Dervis, de Mello and Robinson have classified these algorithms into fixed point theorem based, tatonment process based and Jacobian approaches. Most recent CGE applications have used General Algebraic Modeling System (GAMS) whose solvers fall in the third category.

GAMS is a high-level modeling system consisting of a language compiler and a stable of integrated high-performance solvers. It is specifically designed for modeling linear, nonlinear and mixed integer optimization problems and is tailored for complex, large scale modeling applications. This permits building of large maintainable models that can be adapted quickly to new situations. One of the advantages of GAMS is that it is designed to accept equations in almost the same format as presented in Table 4.1. The use of subset notation allows implementation of different functional forms and closure rules for different subsets (in a variable vector) without having to introduce dummy variables. By eliminating the need to think about purely technical machine-specific problems such as address calculations, storage assignments, subroutine linkages, and input-output and flow control, GAMS increases the time available for conceptualizing and running the model, and analyzing the results. Detailed programming procedures are provided in Brooke, Kendrick and Meeraus.

Because of the presence of nonlinear functions in CGE model formulations, finding solutions requires use of nonlinear algorithms. Several such solution algorithms are present in GAMS. The syntax of optimization characteristic of GAMS requires an objective function with the rest of the CGE equations treated as constraints. Because none of the equations has an inequality sign in CGE, the model solution is invariant to choice of objective function. Therefore, any equation is eligible to be an objective function, as long as it is a scalar equation.

For empirical implementation, (two) positive slack variables are introduced in one of the equations (in our case, the production function). To equate the number of endogenous variables to the number of equations and, hence, to ensure full identification of the system, an extra equation is introduced which sums up the two slack variables. In the optimization process, the sum of the slack variables is minimized subject to all other equations (equality constraints). This ensures that the optimal solution is attained when the sum of the slack variables is equal to zero, a condition necessary to satisfy all the simultaneous equations in the model. With benchmark exogenous variable values, the program will replicate exactly the values of the endogenous variables contained in the SAM at the optimum. Thus, the introduction of slacks only facilitates the optimization process and does not affect the solution values. Brooke, Kendrick and Meeraus also recommend this (slack variable) technique in nonlinear optimization, arguing that it helps to address the infeasibility problem that frequently occurs during iterations in such models.

4.3 Model construction in GAMS

The model construction is presented in the syntax of the GAMS software program. For a guide to the GAMS-input-file click in this link *user's guide to GAMS-input-file*. To execute a GAMS program the reader must have the GAMS software program. For more information on how to obtain, install and run the GAMS software see this link {http://www.gams.com/}.

4.4 Model simulation

A change in regional terms of trade is used to show the results of a model simulation. Export prices for all commodities were increased five percent with import prices remaining at base level. This is similar (but in the opposite direction) to the impact of a decrease in agricultural commodity export prices during the mid 1980s, which contributed to considerable stress and change in rural Oklahoma. An overall price index in 1982 of 100 for agricultural commodities produced in the state was 89.0 by 1986 (Schreiner, Lee, Koh and Budiyanti, p 64). This implied about an 11 percent decrease in export prices of agricultural commodities during a relatively short period of time.

Simulation results of a five percent increase in terms of trade (5 % increase in all export prices), assuming long-term adjustment and capital mobility, are presented in Table 4.6 (below). These results are based on a recent paper by Tembo, Vargas and Schreiner presented at the 30$^{\text{th}}$ Mid-Continent Regional Science Association meetings, Minneapolis, MN, June 11-12, 1999.

Table 4.6 Effects of a 5% Change (Plus) in Terms of Trade, Oklahoma, 1993

Welfare index	Variable symbol	Base value	Simulation result
Total exports	EXP	1.0000	1.1375
Total imports	M	1.0000	1.0887
Composite price	P	1.0000	1.0161
Household income	RHHY	1.0000	1.0814
Gross state product	GSP	1.0000	1.1032

Total exports increase by 13.8% and imports increase by 8.9%. Remember that exports and imports are constrained by constant elasticities of transformation (CET) and constant elasticities of substitution (CES), respectively. This means that producers respond not only to a change in the price ratio of export markets to domestic markets but also to their willingness to substitute (transform their product) between the markets. Consumers in the regional (domestic) market must also adjust to higher regional prices (not shown for individual commodities in Table 4.6). On the import side, consumers are faced with higher regional prices and thus substitute imports (which are at the same price as before the simulation because they are exogenously set) for regional products, based on their willingness to substitute (i.e. the CES parameter). The small region effect is assumed here where regional output and regional demand do not change external prices.

The index of the composite price, a weighted average of regional and import prices, increases by 1.6%. This is the price regional (state) consumers pay for purchases within the region (state). Regional consumers include households, intermediate input buyers and governments. Incomes of regional households increase by 8.1% in nominal terms. By deflating nominal income by the composite price index, real income increases by 5.06%.

Gross state product (GSP) increases by 10.3%. GSP is the compensation for all resources employed in the state, no matter whether the resource owners reside in-state or out-of-state. GSP is the result of the changes in resource prices (wages and rents) and quantity of resources employed. In this simulation, wages and rents increase (not shown in Table 4.6) and quantities of labor and capital increase through migration. The latter increase because resource prices in the state are higher relative to prices out-of-state. Again, the small region effect is assumed where the regional demand for resources does not stimulate price increases out-of-state.

Results similar to Table 4.6 could be calculated to show the change in all endogenous variables (see Table 4.3). Because CGE emphasizes relative changes, results are generally expressed by index form showing the percent change from the base. Hence, an index of 1.1032 for GSP indicates a 10.32% increase over the base, whereas an index of 0.9600 would indicate a 4% decrease in GSP. Absolute changes are easily calculated by applying the percent change to the base level. For example the 10.32% change in GSP applied to the factor payments, $57,551,174,000, in Table 2.1 results in a change in GSP of $5,939,281,157 (excluding indirect business taxes paid to governments).

5 Increasing Returns and Imperfect Competition in Regional CGE Modeling

The CGE framework presented so far expands beyond the assumptions of input-output (I-O) based models. By relaxing the assumption of fixed prices, which in I-O models implies that increased demand is always met with no price increase due to excess production capacity and limitless supply of labor and other factors, we have a more realistic empirical model of regional analysis. The CGE framework allows demand and supply of commodities and resources to depend on prices. Furthermore, resources may be substitutable in production.

However, the competitive regional CGE modeling presented above has two important limitations. First, it does not consider the presence of imperfect competitive market structure and, second, it ignores production technologies characterized by increasing returns to scale (IRS). We present here regional CGE modeling of increasing returns to scale and imperfect competition. An introduction to the theoretical difficulties brought about by the inclusion of returns to scale to the competitive CGE framework is presented. The purpose is to introduce the reader to limitations of the modeling techniques presented above. But first, the case of forest product production and wood processing in Oklahoma is used to show the potential for increasing returns to scale and imperfect competition.

The wood-products manufacturing sector in Oklahoma has several highly concentrated industries. For example, in the sawmills and planning mills industry (SIC 242) 70% of total employees work for one multinational company. Similarly, the paper mills (SIC 262) and the paperboard mills (SIC 263) industries are represented by seven establishments of which 82.5% of total labor force works for two multinational companies. In addition to the high concentration of the industry, Oklahoma timber producers have limited options on where to sell their timber because of costly transportation and long distances between processing centers. All of this propitiates some kind of imperfect structure for the timber market (raw materials market) in which wood processing industries are capable of affecting the price paid for timber.

Once the price taking assumption is dropped, we face the challenges of modeling changes in the economic environment, government policies, technological advances, and external shocks. Researches have available to them considerable theoretical ground on how to model imperfect competition. Two approaches are partial equilibrium and general equilibrium. They differ in that the former considers regional wages and income of consumers to be determined outside of the model. As researchers in economics try to maximize their contributions to solving economic problems they are also constrained by time and data availability. CGE is more demanding on both time and data. Therefore, it is important for the profession to understand and contrast the benefits of using one approach over the other. Thus, using the Oklahoma's forest products industry (FPI) we contrast empirically the strengths of partial and general equilibrium approaches when modeling imperfect competition. We estimate the effects on household welfare, gross state product, employment, raw material prices, wage rate, returns to capital, and so on, for different imperfect market structures of Oklahoma's FPI.

5.1 Increasing returns, non-convexity, and competitive CGE models

The existence of increasing returns to scale (IRS) relies on the non-convexity of the production set. Non-convexity undermines the assumptions used to prove existence of general equilibrium. For the standard competitive general equilibrium, the equalization of prices and marginal rates of transformation is a necessary, and under the assumption of convex preferences and choice sets, a sufficient condition for optimality. This is not the case when non-convexity is present. To understand why, we may use the following line of thought. The presence of IRS leads to large-scale firms because at some price ρ^0 above minimum average cost, profits increase indefinitely with the scale of operation. This is a direct result of average cost always being greater than marginal cost under IRS. Thus, as firms increase the scale of operation the market becomes more and more concentrated which in turn leads to fewer and fewer firms (even one) in the industry and possible collusion of prices. Theoretically, the price mechanism loses its efficiency characteristics and the optimality and efficiency dichotomy that attracts us to competitive general equilibrium (Villar). Indeed, firms with IRS are not consistent with the hypothesis of perfect competitive markets.

The presence of IRS is not the only case that precludes the benefits of competitive equilibrium. Imperfect competition, for example, may be a direct consequence of limitations to entering the market or of a firm's exclusive right to use a resource granted by the regional, federal, or local government. We concentrate in modeling increasing returns and imperfect competition while motivating the reader to investigate the extensions of our modeling description.[1]

5.2 Modeling increasing returns and imperfect competition

Harris' work is considered by many as the first successful and compelling general equilibrium model to incorporate both imperfect competition and increasing returns to scale. His work deals with a small open economy and formulates for the first time the modeling of IRS using the dual approach (see below). After Harris's work, imperfect competitive general equilibrium models have been extensively used, especially in trade liberalization issues.

Imperfect market structures that characterize the product side of the production system have been the major focus of the majority of theoretical and empirical work. Monopolistic competition and oligopolistic competition, for example, have extensively been applied in trade models. However, market imperfections related to the factor (input) side of the production system remain unexplored. The reason, at least in the opinion of these authors, is the international trade focus of most national CGE models where factor market imperfections are of less concern: i.e., how strong is the case for monopsony modeling when commodities are traded nationally and internationally?

However, at the regional level and particularly for agriculture and other natural resource based sectors, one may argue for modeling input side market distortions , i.e. monopsony and cooperative behavior (see Rogers and Sexton). Thus, the state of the art of CGE is very promising for output distortions of markets but less promising for distortions of input markets.

5.2.1 Increasing returns -- the dual approach

The modeling of IRS at regional levels is adopted from literature on international trade and national CGE formulations. Its implementation/adaptation to regional CGE models has been limited with few exceptions identified by Partridge and Rickman. Harris' basic approach is used here. The main characteristic of the approach is the use of the dual formulation of increasing returns to scale. Duality is less restrictive in modeling and allows treatment of the assumption of convex input requirement sets as compared to the primal approach.

Under constant returns to scale, marginal costs are assumed to be constant and equal to average variable cost(VC_i/X_i, where VC_i is variable costs and X_i is output for the i^{th} sector). Under increasing returns to

[1] Several issues are still not totally clear on theoretical grounds. First, the selection of the numeraire has no implication for the competitive CGE framework; however, this issue is still controversial when imperfect competition is involved (see, Ginsburgh). Furthermore, the possibility of non-uniqueness of equilibrium is "a potentially serious problem" for applied general equilibrium models with imperfect competition and economies of scale (Mercenier). Finally, regional CGE modeling has adapted concepts and specifications from the national and/or trade CGE literature; however, the implications of its implementation at the regional level has been greatly criticized (i.e., the Armington assumption on product differentiation).

scale, average cost is a monotonically decreasing function[2].

$$AC = \frac{FC}{X} + MC \tag{5.1}$$

where FC is fixed costs and MC and AC are marginal and average cost, respectively. We assume that marginal costs are governed by the preferred constant returns to scale production function, but a subset of inputs are committed *a priori* to production and these costs must be covered regardless of the output level. Thus, increasing returns to scale takes the form of unrealized economies of scale in production. There is no customary procedure in defining fixed costs. Fixed costs may involve the same mix of inputs as marginal costs or, alternatively, fixed costs may be assumed to involve a different set of inputs. However, the specification of the fixed costs has important consequences for the calibration procedure (to be discussed).

As a measure of unrealized scale economies it is customary to use the concept of cost disadvantage ratio (CDR). The CDR provides an estimate of unrealized economies of scale (de Melo and Tarr). Depending on the value of this ratio, an industry may be facing economies/diseconomies of scale or it may be operating at the minimum efficient scale. The CDR is calculated as:

$$CDR = 1 - \frac{1}{S} \tag{5.2}$$

where

$$S = \frac{AC}{MC}$$

and AC and MC are average cost and marginal cost, respectively. Thus, If $CDR > 0$, there are *Economies of Scale*; if $CDR < 0$, there are *Diseconomies of Scale*; and if $CDR = 0$, the firm is operating at the *Minimum Efficient Scale*[3].

5.2.2 Increasing returns -- the primal approach

The primal approach in modeling increasing returns to scale has been infrequently used by CGE modelers. The reason is the indeterminacy under increasing returns to scale. Kilkenny, however, argues that "when factor markets are geographically segmented and the pool of labor is limited" factor costs will rise for an industry which is expanding operation using unexploited increasing returns to scale. Thus, existence of an optimal output level is thus obtained.

In the primal approach, increasing returns to scale are much easier to model. We adjust, for example, the coefficients of a Cobb-Douglas production function to exhibit increasing returns to scale: making $\sum_f \alpha_f > 1$ where f states for factor index and α is the exponential (share) parameters in the Cobb-Douglas technology specification.

5.2.3 Market power

Before modeling market power we require specification of the degree of product differentiation used in the model. We assume Armington preferences at the regional level. Thus, substitution in purchases is allowed between domestically produced consumer goods and out-of-region produced consumer goods. Traded goods are imperfect substitutes by origin and goods produced domestically are imperfect substitutes for imports. Also, goods supplied on the domestic regional market are imperfect substitutes for goods supplied for export. Armington specifications also apply to sectors with IRS. In those sectors, goods are produced by N_t identical firms implying goods produced for domestic sales in these sectors are perfect substitutes.

[2] An alternative specification states average cost as $AC = X^{\phi-1} f(\omega)$ where $f(\omega)$ represents the cost function for a homogenous bundle of primary and intermediate inputs. This alternative formulation is used to specify scale economies due to returns from specialization.

[3] For multi-product scale economies we carry out the following modification: $S = \frac{C(Y)}{\Sigma Y_i \cdot dC/dY_i}$ where C and Y_i are, respectively, cost and output of the $i^t h$ product.

Contestable pricing

Two pricing hypotheses are considered for the IRS sectors. First, we assume low-cost entry and exit such that the threat of entry forces firms to price at average cost. This is called the contestable pricing behavior:

$$PX = AC \tag{5.3}$$

where PX is the weighted sum of the unit sales prices on the regional (PR) and export (PE) markets. Firms in a perfectly contestable market will be forced to operate as efficiently as possible, and to charge as low a price as long-run financial survival permits.

This pricing rule represents only a small departure from the competitive pricing rule because price also equals average cost in the long-run equilibrium of the competitive model (de Melo and Tarr). Another advantage of contestable pricing is that it is easy to calibrate. According to de Melo and Tarr, the calibration process is complete by just equating output price to average cost.

From monopoly to oligopoly

In the second alternative, we assume that each (identical) firm behaves in the regional market as if it is facing a downward-sloping demand curve. The equilibrium condition for each firm is given by:

$$\frac{PR - MC}{PR} = \frac{1+\theta}{N \bullet \delta} \tag{5.4}$$

where δ is the endogenous elasticity of aggregate sectoral demand, N is the number of firms, and θ is the representative firm's conjecture about the response of competitors to its output decision. This alternative is the conjectural variation specification where one may or may not have entry/exit assumptions.

In long-run equilibrium, entry/exit ensures zero profits. If N represents the number of firms, then as $N \to \infty$ we expect $\theta \to 0$; thus, firms behave competitively. Why should the representative firm's conjecture banish as the number of firms increase? Two explanations are given. First, collusion is difficult if more firms arrive to the market, and second, more firms imply greater availability of varieties. A conjectures formulation that accounts for both product variety and effects on collusion of firms is given by:

$$\theta = \frac{\Delta Q_{-1}}{\Delta Q_1} = N^{-1} \tag{5.5}$$

where ΔQ_{-1} is the change in aggregate output of other firms due to a change in the j^{th} firm, and N is an arbitrary number normalized to unity in the calibration.

On the other hand, with barriers to entry it is possible to have supernormal profit because firms sell in the domestic regional market at a price $\tilde{P}R > PR$. If we define an exogenous rate of profit (Ψ) per unit of regional sales, then the mark-up pricing equation (5.3) is replaced by:

$$PX \bullet (\tilde{P}R, PE) = AC \bullet (1 + \Psi) \tag{5.6}$$

This equation is the same for contestable market scenario when $\Psi = 0$. In the conjectural variation case, we have $\pi = \Psi$.

Our empirical example applies all of these modeling techniques to the Oklahoma region. For example, high concentration in the pulp manufacturing industry increases the likelihood of lower outputs and higher price than under a competitive structure. As well, its actions may distort the timber market, thus affecting the welfare of both forest land owners and consumers (Tillman).

5.3 5.3 Calibration

We calibrate our model using a modified social accounting matrix that identifies the forest complex. We have considered the forest complex to constitute the forestry sector and the forest product industry (FPI).

Our calibration procedure depends on assumptions of market structure and strategic behavior by firms. Calibrating parameters of the model utilizes the information obtained from econometric work and/or economic theory.

Depending on the price rule and the exit/entry assumption, each alternative entails a different model calibration. In the case of normal initial profits ($\Psi = 0$), we reduce the primary variable cost component of total costs by the amount of fixed costs. For the monopolistic case, equation (5.4) is solved to yield the value of the conjecture θ parameter.

In the case of supernormal profits, we allocate fixed costs as before. Then, given the profit rate, Ψ, and all quantities and out-of-region prices, we solve for the region (domestic) price $\tilde{P}R$ which satisfies the firm's profitability constraint. Finally, θ is solved from (5.4) but with the new set of regional prices.

Modeling IRS and imperfect competition requires additional parameters, mainly estimates of the following elasticities: elasticity of capital/labor substitution; import price elasticities of demand; and export supply price elasticities. Finally, the calibrated price elasticity of demand, ε, will depend on the functional form selected to represent import demand and export supply.

An example of the application of CGE to imperfect markets in the forest product industry is available in a paper presented by Vargas and Schreiner at the Mid-Continent Regional Science Association meetings, Minneapolis, MN, June 11-12, 1999 and published in *The Journal of Regional Analysis and Policy* (Vol. 29,2: 51-74, 1999)(see website `http://www.jrap-journal.org/pastvolumes/1990/v29/29-2-3.pdf`).

6 Policy Applications and Summary and Conclusions

Regional development is a field of study requiring policy decisions. In a purely competitive economy, markets determine what is produced and what is consumed. Seldom do we permit markets in regions to operate completely unregulated. Externalities of production, public good nature of infrastructure, missing markets for amenities, and the importance of the distribution of benefits of economic growth enter into the political process of guiding regional development. Some policymakers view area development as an end in itself, irrespective of the results on measures of welfare for area populations.

In this chapter we presented a framework for analysis of regional development programs and policies. Markets were defined in terms of structure and behavior. Economic behavior of producers and consumers was specified, and ownership of resources was identified. The economic model is a form of regional general equilibrium where prices, quantities, and incomes are endogenous and changes in regional welfare are measured.

Examples of policy applications of regional general equilibrium studies completed in Oklahoma are presented in the next section. Examples of other studies are referenced in Partridge and Rickman. The last section is a summary and conclusion for this chapter of the webtext.

6.1 Policy applications

This section summarizes studies of regional welfare change associated with development issues by means of regional CGE. The first study is a state-level analysis of welfare losses due to agricultural export price decreases in the 1980s. Results explain why state policymakers were anxious to replace regional welfare losses. The second study is an effort to show the state economic impacts of potential damages a change in surface water quality may have on sport fishing in Oklahoma. The models are not presented, but are available in references cited.

6.1.1 Agricultural export prices[4]

Agricultural commodity prices showed a sizable decrease during the mid1980s. Farm foreclosures and bankruptcies were several times higher than normal for the state. Low agricultural commodity prices and depressed energy prices decreased income and employment levels throughout the state, particularly in rural areas.

From 1982 to 1986 there was about an 11 percent decrease in export prices of agricultural commodities. In this context, a counterfactual experiment of a 10 percent decrease in export (national) prices of agricultural commodities was simulated for the Oklahoma economy focusing on welfare changes by household income group.

Welfare changes in terms of compensating variation (CV) amounted to a state loss of about $123,702,000 at the 1990 price level. Welfare losses equaled $83,525,000 for the high income household group and $51,281,000 for the middle income household group. Low income households showed a slight welfare gain of $11,104,000. The latter is a result of lower commodity prices, particularly for nontradable commodities. When compared to the initial level of expenditure for each household income group, welfare change for high income households was –0.86 percent, middle income households was -0.26 percent, and low income households was +0.10 percent.

Most policymakers seek strategies that are short- to intermediate-term. Such strategies have limited success because most regional development issues are structural and require long-term changes in comparative advantage. When Oklahoma lost aggregate income and employment because of the decrease in agricultural prices, policymakers sought to replace the loss as quickly as possible. The strategies proposed, however, were long-term. Investments in value-added activities, international trade development, and development of alternative crop and livestock enterprises require long-term commitment – results of such development strategies are not felt immediately. Rural development research has not adequately recognized the differences

[4]This section draws on the methods and results presented in Koh and Lee. The basic regional general equilibrium model is available in Koh, Schreiner, and Shin but was modified by Lee to include a labor migration elasticity, the labor-leisure relationship, and measures of welfare change. The basic social accounting matrix also was updated to 1990.

between proposed development strategies and policy expectations. In part this is because rural development research has not focused on how factor and commodity markets work in rural regions in the short to intermediate term versus the long term.

The regional equilibrium model developed and applied at the state level in this study simulates the short-run conditions for markets by holding land and capital fixed by sector. Labor is assumed mobile between sectors and between regions. Hence, simulation results approach the short- to intermediate-term effects that correspond with expectations of policymakers.

A major conclusion of the study is that resource owners have a large stake in the re-establishment of economic activity. Land owners had a 20.9 percent reduction in land rents, and capital owners had a reduction of capital rents ranging from 20.9 percent in agriculture to 0.5 percent in services. Labor compensation was reduced 0.5 percent which, because of mobility between sectors and regions, was significantly less than the losses by land and capital resource owners. Labor that migrated had the lowest loss in resource compensation.

6.1.2 Sport fishing trip demand[5]

Growth in sport fishing and the associated increase in angler expenditures have heightened the need for understanding how variations in expenditure can affect a regional economy and the welfare of economic participants. In Oklahoma the number of anglers increased 14 percent from 1980 to 1990, compared to a 20 percent increase nationally. Fedler and Nickum estimate that angler expenditure in Oklahoma was $387.3 million or 0.6 percent of gross state product in 1991. The fixed price multiplier impact was estimated by Fedler and Nickum to equal $793.5 million in output for all Oklahoma sectors, $202.2 million in job earnings, and 11,606 in employment. But what are the general equilibrium results, when both price and quantity are endogenous, from a change in the demand for sport fishing trips? Such general equilibrium results are important for measuring policy implications of changes in quality of sport fishing and subsequent changes in trip demands.

Agriculture accounted for about 5 percent of gross state product (GSP) in Oklahoma in 1992. Scifres and Osborn estimate that 15.4 percent of GSP is associated directly and indirectly with agriculture. Natural resource systems provide valuable services in support of agricultural production and sport fishing activities. Boosting agricultural production by applying more fertilizer and other chemical products could substantially affect the quality of water in natural resource systems and negatively impact sport fishing.

This study utilizes information on sport fishing trips and sport fishing expenditures in Oklahoma to measure welfare gains/losses due to a change in trip demand. The National Survey of Fishing, Hunting and Wildlife Associated Recreation shows that 803,700 U.S. anglers fished in Oklahoma during 1991 with a total angler expenditure of $387,326,000.

Model experiments focused on decreased trip demands. The premise is that if quality of sport fishing decreases, trip demand decreases. Quality of sport fishing is hypothesized to be associated with number of fish caught per trip. The number of fish caught per trip is hypothesized to be associated with fish population which, in turn, is hypothesized to be associated with water quality. Hence, a decrease in water quality (i.e., an increase in chemical discharge) reduces fish populations which reduces fish caught per trip and thus decreases the quality of sport fishing and number of trips in Oklahoma. Presumably anglers have alternative sites outside of the state at which they can replace their desire for sport fishing. These experiments begin to address the general equilibrium policy implications of the fixed price impact analyses by Fedler and Nickum for sport fishing expenditures and by Scifres and Osborn for cash receipts of agriculture.

Two scenarios were analyzed: a 10 percent and 50 percent quality tax imposed on the price (costs) of in-state trips. This increases the cost of in-state relative to out-of–state trips. Regional welfare was measured by gross state product and household welfare. Loss in GSP with a 10 percent quality tax on in-state fishing trips was estimated at $14,910,000 and at $55,670,000 with a 50 percent quality tax. These loses are due to outmigration of resources and lower resource returns (wage rate and capital and land rents).

[5] This section draws on methods presented in a paper for the Sixth International CGE Modeling Conference (Budiyanti, Schreiner, and Li) and on modeling methods in Lee. Results are from Budiyanti.

A more revealing welfare measure was the compensating variation loss to households. This loss is a measure of the income it would take to bring households back to their original level of welfare before the fishing trip quality tax. The distribution of welfare loss showed that high income households had the greatest percentage loss when compared to the before quality tax income level. Low income households had a higher percentage loss compared to middle income households. The 50 percent quality tax had about four times the percentage welfare loss compared to the 10 percent quality tax. If a dollar of welfare loss is valued equally across the household income groups, then for all households in the state the welfare loss was $16,556,000 with the 10 percent quality tax and $64,070,000 with the 50 percent quality tax.

6.2 Summary and conclusions

We need better analyses of regional development programs and policies as they impact the welfare of households. We need better and more integrated policy frameworks in which to perform analyses. We need better analytical models to evaluate programs and policies that allow prices, quantities, and incomes to be endogenously determined for regions. We need more and better regional data including estimates of structural parameters.

Regional economies are characterized by complex variable interdependencies and market interactions. This makes the general equilibrium framework a more appropriate analytical method compared to partial equilibrium methods. In this chapter, an attempt was made to present the salient features and a step-by-step illustration of the implementation of a regional computable general equilibrium (CGE) model. Because of the rigid nonsubstitution assumptions and absence of the role of price in alternative general equilibrium models, such as input-output and SAM multiplier models, we argue that the more flexible and theoretically sound CGE approach is the more appropriate framework of analysis.

A typical CGE model incorporates the core of neoclassical features of a well functioning economy that is characterized by perfectly competitive markets and constant returns to scale production technologies. This chapter has demonstrated CGE modeling for such an economy, using Oklahoma's 1993 social accounting matrix (SAM). Because this material is intended for a wide range of readership – including upper-class undergraduate students, graduate students, and practitioners – several additional assumptions have been adopted to simplify the scope and size of the model. For example, the Oklahoma regional economy is aggregated into four industrial sectors, and a single household income group. Also, local, state and federal governments are all represented by a single government institution. Household commodity demand functions are assumed to be derived from a non-leisure-augmented Stone-Geary (linear expenditure system) utility function. The aggregated Oklahoma CGE model was used to simulate an increase in terms of trade. Results of the simulation were used to interpret the workings of regional CGE.

While the assumptions adopted in this model help to reduce the scope and complexity of the model to levels that are relatively easy to comprehend, such ideal economies seldom exist in reality. Relaxing some of the assumptions of this basic structure is likely to result in a more representative picture of the economy. However, the general modeling techniques remain the same. In section 5.0 of this chapter, a monopsonistic market structure was proposed for the forest products industry and the model was respecified. We encourage the readers to extend the basic model in ways to address real world regional impact and policy problems.

Although a CGE model is generally theoretically sound, it is not clear whether its quantitative predictions are superior to alternative models. Most of the specification problems in CGE analysis emanate from its reliance on one year of data implied by the calibration process. This tends to make the system underidentified, making it imperative for the researcher to use external parameters, which in most cases are estimated in a framework that is inconsistent with general equilibrium analysis.

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List of Tables and Figures

Table 4.2
Subscript Notation

INDEX	Description
i, j	Sectors and Commodities
	AGR agriculture MIN mining MAN manufacture SER services
ag	Agricultural sectors AGR
nag	Nonagricultural sectors MIN, SER, MAN
f	Factors of production L labor K capital T land
fl	Factors not land L, K

Table 4.3 **Summary of Endogenous Variables**		
VARIABLE	**DESCRIPTION**	**NUMBER**
Z	Objective Function Value	1
PL	Wage Rate	1
PK	Capital rate in short-run	n
PKL	Capital rate in the long run	1
PT	Land rent	1
PN	Net price	n
PR	Regional price	n
P	Composite price faced by consumers	n
PX	Composite price faced by producers	n
LAB	Labor demand	n
CAP	Capital demand	n
LAND	Land demand	1
TCAP	Total Capital Demand	1
TLAB	Total Labor Demand	1
LS	Labor supply	1
LMIG	Labor migration	1
KMIG	Capital migration	1
VA	Value added	n
V	Composite intermediate good demand	nXn
VM	Imported int good demand	nXn
VR	Reg int good demand	nXn
R	Regional supply	n
X	Outpost	n
EXP	Export	n
M	Import	n
TVM	Imported int good total demand	n
TVR	Reg int good total demand	n
TV	Composite intermediate good total demand	n
adjL	Labot adjustment	1
LY	Labor income (original hhs)	1
ALY	Adjusted labor income (staying + in-migtation)	1
KY	Capital income (original capiyal stock)	1
TY	Land income	1
YENTt	Enterprise income	1
RETENT	Retained Earnings by enterprises	1
YH	Income of hh staying in the region (including in-migrants	1
DYH	Disposable hh income (staying in the region + inmigra)	1
HSAV	Household saving (staying + inmigrat)	1
SAV	Total saving	1
INV	Investment	1
YGOV	gov revenue	1
IBTX	Indirect business tax	n
GRP	Gross region product	1
AHEXP	Adjusted household expenditure (spent within the region)	1
Q	Demand for comp consump good	n
QM	Demand for imp consump good	n
QR	Demand for reg consump good	n
GOVEXP	gov expend	1
QGOV	gov demand for comp good	n

Table 4.3 Summary of Endogenous Variables		
VARIABLE	**DESCRIPTION**	**NUMBER**
QGOVM	gov demand for imported good	n
QGOVR	gov demand for reg good	n
QINV	Invest gov demand for comp good	n
QINVM	Invest gov demand for imported good	n
QINVR	Invest gov demand for reg good	n
SLACK	Slack variable	n
SLACK2	Slack variable	n

Table 4.4
Summary of Exogenous Variables

VARIABLE	DESCRIPTION	NUMBER
PLROC0	Wage rate of rest-of-country	1
PKROC0	Cap rate of rest-of-country	1
PE0	Export price	n
PM0	Import price	n
LS0	Labor supply by regional household	1
LHHH0	Labor employed by household grouop	1
LGOV0	Labor employed by gov	
KS0	Supply of pri capital (short-run)	1
TS0	Supply of land	1
ROWSAV0	Saving from res-of-world	1
TRGOV0	Gov transfer to hh	1
REMIT0	Remittance from outside the region to household	
GOVITR0	Inter gov transfer	1
GOVBOR0	Government borrowing	1
QGOV0	government demand for comp good	n
Qinv0	Invest demand for comp good	n

Table 4.5		
Summary of Parameters		
VARIABLE	**DESCRIPTION**	**NUMBER**
a_{oi}	composite primary factor input coefficient	n
a_{ji}	intermediate input coefficient	nXn
α_i^f	factor production of elasticity for product i	nXn
ϕ_i^{VA}	value added shift parameter sector i	n
ρ_i^V	intermediate input substitution parameter	n
δ_{ji}^V	intermediate input import share parameter	nXn
ϕ_{ji}^V	Intermediate input imort-domestic substitution efficiency parameter	nXn
ρ_i^X	market substitution parameter	n
δ_i^X	output share parameter	n
ϕ_i^X	output shift parameter	n
ρ_i^{GOV}	government substitution parameter	n
δ_i^{GOV}	government output share parameter	n
ϕ_i^{GOV}	government output shift parameter	n
ρ_i^{INV}	investment substitution parameter	n
δ_i^{INV}	investment share parameter	n
ϕ_i^{INV}	investment shift parameter	n
δ^L	Labor migration elasticity	l
δ^K	Capital migration elasticity	1
$ktax$	capital tax rate	1
$sstax$	factor income tax rate for labor	1
$ttax$	factpr omcp,e tax rate for land	1
$retr$	rate of retained earnings fr ent inc	1
et	enterprise tax rate	1
$hhtax$	income tax rate for hh	1
mps	saving rate	1
$ibtax$	indirect business tax	n
$beta$	houosehold budget shares	n

List of acronyms

Computable General Equilibrium Modeling for Regional Analysis

Eliécer Vargas, Dean Schreiner, Gelson Tembo, and David Marcouiller

List of Acronyms	
AIDS	Almost Ideal Demand System
BEA	Bureau of Economic Analysis
BLS	Bureau of Labor Statistics
CD	Cobb-Douglas
CES	Constant Elasticity of Substitution
CET	Constant Elasticity of Transformation
CGE	Computable General Equilibrium
CRS	Constant Returns to Scale
GAMS	General Algebraic Modeling System
GRP	Gross Regional Product
GSP	Gross State Product
IMPLAN	IMpact analysis for PLANning
I-O	Input-output
LES	Linear Expenditure System
NIPA	National Income and Product Accounts
REIS	Regional Economic Information System
SAM	Social Accounting Matrix
SDSAM	Supply-determined SAM
VA	Value-added

Glossary of Terms

Armington assumption: Allows domestically produced and foreign produced goods to be imperfect substitutes in use, making the consumption of quantities of domestically produced and imported variants of the commodity to enter the representative consumer's utility function as distinct elements. In empirical CGE formulations, this assumption helps to overcome the "specialization" problem (de Melo and Tarr).

Calibration: The process by which values of the normalizing (or free) parameters are determined so as to replicate the observed flow values incorporated in the social accounting matrix (SAM), assuming all the equations describing the equilibrium in the system (model) are met in the benchmark period. This process is augmented by literature search (and on occasion econometric estimation) for key model parameters, whose values are required before the calibration can proceed. In practice, due to the wide spread use of CES functions in applied models, "key" parameters are more or less synonymous with elasticities (Mansur and Whalley).

Cobb-Douglas production function: A production function in which the elasticity of factor substitution is constant and equal to unity. In general, this function has the form $f(x, y) = Ax^{\alpha}y^{\beta}$, where A is an efficiency parameter, x and y are the inputs and α and β are their coefficients (which, for this function, are equal to elasticities).

Compensating variation: An estimate in money terms of the amount households would require as compensation in order to remain as well off after an exogenous shock as they were before the shock. This welfare measure is based on new equilibrium prices.

Computable General Equilibrium (CGE) model: An integrated system of equations (or general equilibrium model), derived from economic theory of the behavior of all economic agents, whose simultaneous solution uses a numerical database to determine values of the endogenous variables. By simulating the effects of policy, structural or market changes, a well-defined CGE model is a useful tool for economic impact analysis.

Duality: Duality in neoclassical microeconomics refers to the existence, under appropriate regularity conditions, of indirect (or dual) functions which embody the same essential information on preferences or technology as the more familiar direct (or primal) functions such as production and utility functions. Dual functions contain information about both optimal behavior and structure of the underlying technology or preferences, whereas the primal functions describe only the latter. Many relationships that are difficult to understand when looked at directly become simple, or even trivial, when looked at using the tools of duality (Varian).

Equivalent variation: Defined the same as compensating variation except that this welfare measure is based on initial equilibrium prices rather than new equilibrium prices.

ES202: A federal-state program summarizing employment, wage and contribution data from employers subject to state unemployment laws, as well as workers covered by unemployment compensation for federal employees (UCFE). The ES202 program is also called Covered Employment and Payrolls (CEP) program and involves the Bureau of Labor Statistics (BLS) of the U.S. Department of Labor and the State Employment Security Agencies (SESAs).

Externality: Side effects of an action that influence the well-being of nonconsenting parties. The nonconsenting parties may be either helped (by external benefits) or harmed (by external costs). For example, the effect of an industry's output on the total costs of each firm and/or other participants in the economy.

Frisch parameter: Marginal utility of income with respect to income (de Melo and Tarr).

General equilibrium model: A model of an economy that portrays the operation of many markets simultaneously.

Hicksian demands: Demand functions that are derived from cost minimization, commonly referred to as the dual problem in demand analysis. These functions tell us how quantity is affected by prices with utility held constant. Primal to these demands are the *Marshallian demands*, which are derived from maximizing utility holding income constant.

Imperfect competition: Any market structure in which firms do not exhibit the characteristics of perfect competition.

Lagrangian: A mathematical technique used to find values of variables that minimize or maximize an objective function while satisfying equality constraints.

Law of one price: The assumption that the price of a commodity differs between any two levels of the marketing channel by no more than the transfer costs. For example, by this law, the price is expected to differ between any two locations by no more than transportation costs. Implicit in this law is the assumption of extreme specialization and perfect substitution between domestic and foreign commodities.

Leontief production function: (see Leontief technology below)

Leontief technology: A production technology in which inputs always enter in fixed proportions to produce a unit of output (zero elasticity of factor substitution). Thus, the input that poses a binding constraint determines the amount of output to be produced. Mathematically, Leontief technology is presented as: $f(x, y) = \min(ax, by)$, where x and y are the inputs and a and b are their fixed coefficients.

Market distortion: Market failure that is caused by deliberate policy intervention, such as imposition of a tax or a subsidy. [see definition of market failure below]

Market failure: Failure of the market system to attain hypothetically ideal allocative efficiency. This means that potential gain exists that (for some reason) has not been captured.

Missing markets: Absence or incompleteness of markets for some goods and services, which renders prices for such commodities nonexistent. For example, an agent (e.g. firm) may care about an externality (e.g. pollution) generated by another agent but have no way to influence it.

Monopoly: A market structure characterized by a single seller of a well-defined commodity for which there are no good substitutes and by high barriers to the entry of other firms into the market for that commodity.

Monotonically decreasing: A function $g(-)$ is said to be monotonically decreasing if, for any $x > y$, $g(x) < g(y)$.

Monotonically increasing: A function $g(-)$ is said to be monotonically increasing if, for any $x > y$, $g(x) > g(y)$.

Non-convexity: Non-convexity is explained by the incapacity of the *additivity* and *divisibility* hypotheses on production to hold. The additivity assumption says that if two production plans are technologically feasible, a new production plan consisting of the sum of these two will also be possible. Divisibility, on the other hand, states that if a production plan is feasible, then any production plan consisting of a reduction in scale will also be feasible. Failure of the *divisibility* assumption is argued as the main source of non-convexities in production.

Oligopoly: A market structure in which there are only a few sellers of a commodity (competition among the few).

Partial equilibrium model: A model of a single market that ignores repercussions in other markets.

Perfect competition: A widely used economic model (market structure), where it is assumed that there is a large number of buyers and sellers for any commodity and each agent is a price taker.

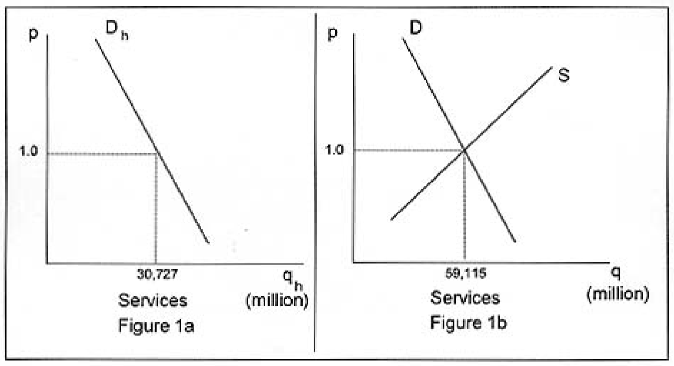
Returns to scale: The term returns to scale refers to the response of output when proportional increases in all inputs are carried out (scale of operation). If output increases by a smaller proportion, then the technology is said to exhibit decreasing returns to scale (diseconomies), but if it increases by a greater proportion than the inputs it exhibits increasing returns to scale (economies). If output increases by the same proportion as the inputs, we refer to this technology as constant returns to scale. Mathematically, if $f(mX) = m^i f(X), k > 1$, implies increasing returns, $k < 1$ decreasing returns, and $k = 1$ constant returns when X is a vector of inputs, $f(X)$ is the production technology, and m is a scalar.

Specialization problem: The Law of One Price implies extreme specialization in an economy where goods are produced under CRS and the number of commodities exceeds the number of factors of production.

Normalized prices in the commodity and factor markets

As an example, consider the services (industry) row in Table 2.1 (i.e. regionally produced services consumed in the region or exported). The values in this row are purchases by industry, institutions and buyers outside the region (exports). Each of these values is expressed as $p \bullet q = R$, or price times quantity equals expenditure (revenue). The only value we know in this expression is R (the value in the SAM). But if we normalize (code) $p = 1.0$ then $(1.0) \bullet q = R$ and R can be interpreted as a quantity index of output, q.

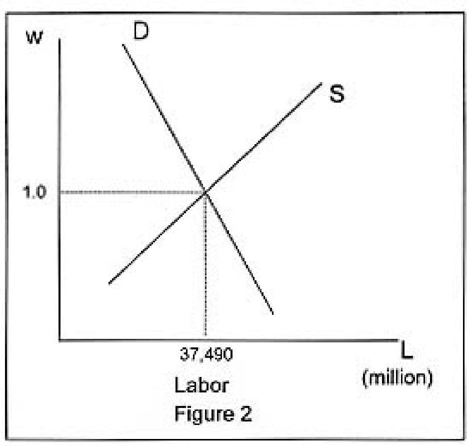
Consider household demand (D_h) for services. Because we know all markets are in equilibrium we can specify the market result for household demand as in Figure 1a. The shape of the demand curve has not been specified yet but we know that the equilibrium price-quantity combination lies on the demand curve. The quantity is taken from Table 2.1 and equals 30,727 million units. Each of the other sources of demand (industry, governments, capital and exports)



Services
Figure 1a
Services
Figure 1b

for services will also be in market equilibrium. Aggregate demand for regionally produced services is shown in Figure 1b along with regional supply. Again, in equilibrium we know that regional price is 1.0 and quantity is 59,115 million physical units (index). The shape of the supply curve has not yet been specified but we know this equilibrium point is on the curve.

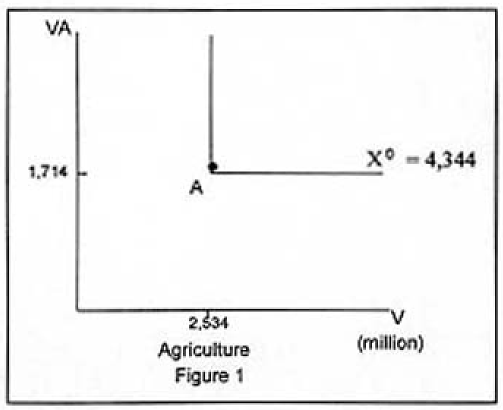
Similarly, the labor row in Table 2.1 shows the compensation for labor from industry and institutions. Because there is only one labor market (more labor markets may be identified if labor is segmented by skill level), the wage bill (labor compensation) is equal to wage rate times quantity of labor, $w \bullet L = W$. The only value we know in this expression is W (from Table 2.1). If we normalize $w = 1.0$ then $(1.0) \bullet L = W$ and, again, W is a quantity index of labor, L. Because the labor market is in benchmark equilibrium, we show this equilibrium point in Figure 2 at $w = 1.0$ and $L = 37,490$ quantity index (million). The shape of the demand and supply curves for labor have not yet been specified.



Labor
Figure 2

The Leontief Production Function

Remember that the SAM values in monetary units ($) can be interpreted as physical units (index) when prices are normalized to one (click here for normalized prices). For agriculture, the isoquant relationship showing the combination of composite factor inputs to composite intermediate inputs in attaining X^o output is shown in Figure 1. From equation (3.1.1), producers would always choose the minimum input to obtain the X^o output. To choose any point on the X^o isoquant other than point A would increase the quantity of a costly input without increasing output. Furthermore, producers would always combine the inputs in the ratios of $\frac{VA}{a_0}$ and $\frac{V}{a_\ell}$ no matter the price of the inputs. In other words, because the inputs are not substitutable in obtaining X^o, any price ratio between the inputs will always go through point A.



Agriculture
Figure 1

Production Elasticities

In the Cobb-Douglas (CD) production function (equation 3.1.3) we define the production elasticity for labor as:

$$\frac{\partial VA}{\partial LAB} \bullet \frac{LAB}{VA} = \alpha^L \phi^{VA} LAB^{(\mu^L - 1.0)} \bullet CAP^{\mu^Z} \bullet LAND^{\mu^T} \bullet \frac{LAB}{VA}$$

and simplifying,

$$\frac{\partial VA}{\partial LAB} \bullet \frac{LAB}{VA} = \alpha^L$$

Therefore, the production elasticity is constant and equal to the production parameter α^L. Thus, a one percent change in the quantity of labor (*ceterus paribus*, capital and land constant) used in production for the i^{th} sector results in an α^L percent change in net output.

Similarly, the production elasticities for capital and land in producing net output for the i^{th} sector are α_i^L and α_i^T, respectively.

Profits

Profit is defined as total revenue (TR) minus total cost (TC). In the competitive model, there are zero profits. Using previous information on normalized price, TR for agriculture (sector 1) (Table 2.1) is $PX_1 \bullet X_1 = \$4,344,000,000$. Total cost ($TC$) for agriculture is:

$$TC_1 = \sum_{j=1}^{4} P_j a_{j1} X_1 + PL \bullet LAB_1 + PK_1 \cdot CAP_1 + PT \bullet LAND_1 + ibt_1 PX_1 \bullet X_1 \tag{1}$$

where P_j is the price of the intermediate input from sector j, $a_{jl}X_l$ is the agricultural sector's requirement for intermediate inputs from sector j to produce one unit of output X_l (note the relationship in equation 3.1.2), PL is the wage rate, LAB_1 is labor in agriculture, PK_1 is the unit capital return in agriculture, CAP_1 is capital in agriculture, PT is land rent, $LAND_1$ is land in agriculture and ibt_1 is per unit indirect business tax rate in agriculture (value).

Because intermediate inputs and indirect business tax are fixed ratios of sector output (equation 3.1.2), a net price of output can be defined as:

$$PN_i = PX_i - \sum_{1} P_j a_{j1} - ibt_i \bullet PX_i \tag{2}$$

Net price is interpreted as the unit value of output available for compensating primary factors of production. For agriculture, the net price is: $PN_1 = 1.0 - 0.40 - 0.02 = 0.58$ (using benchmark data).

The profit function is:

$$\pi_1 = TR_1 - TC_1 \tag{3}$$

By substituition,

$$\pi_1 = (PN_1 + \sum_{1} P_j a_{1j} + ibt_1 PX_1)X_1 - \sum_{1} P_j a_{1j} X_1 - PL \bullet LAB_1 - PK_1 \bullet CAP_1 - PT \bullet LAND_1 - ibt_1 \bullet PX_1 \bullet X_1$$

$$= PN_1 \bullet X_1 - PL \bullet LAB_1 - PK_1 \bullet CAP_1 - PT \bullet LAND_1$$

This is the same as equation (3.1.6) in the text.

Graphic Presentations of the Calibrated CD Production Function and Factor Demands

After calibrating the production function (equation 3.1.5) and factor demands (equations 3.1.12 to 3.1.14), we can visualize the form of these functions. With calibrations, the Cobb-Douglas value-added (net product) production function (3.1.15) becomes:

$$X_i = 7.46 \cdot LAB_i^{0.253} \cdot CAP_1^{0.333} \cdot LAND_1^{0.414}$$

The figure below shows the demand functions for the three factors – labor, land and capital – for the agricultural sector.

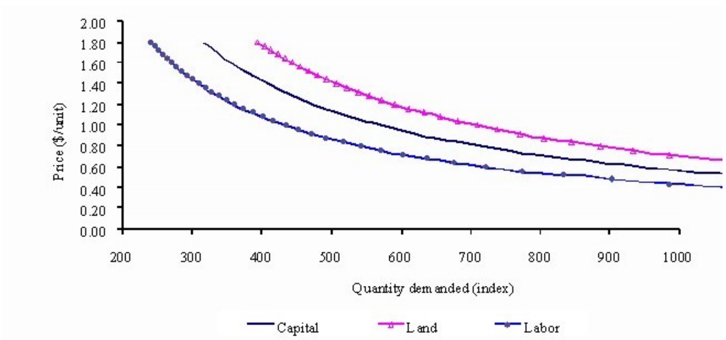


Figure: Factor demands for agriculture

The process of generating the function from the calibrated equations also helps to validate the calibration process. If there are no mistakes in the process of determining the parameters, initial values from the SAM are obtained when initial values of the variables are used. For example, in the above figure, the three demand functions should display the initial SAM levels for factor demands when the (normalized) unitary price is used. This should be true for all components of the CGE model. If this is not true, then accuracy in the calibration of the parameters should be checked.

The figure below shows two production functions relating labor to output in agriculture while holding capital and land constant. In the first schedule, the base-year value of each parameter is assumed. In the second schedule, technical efficiency improvement is assumed.

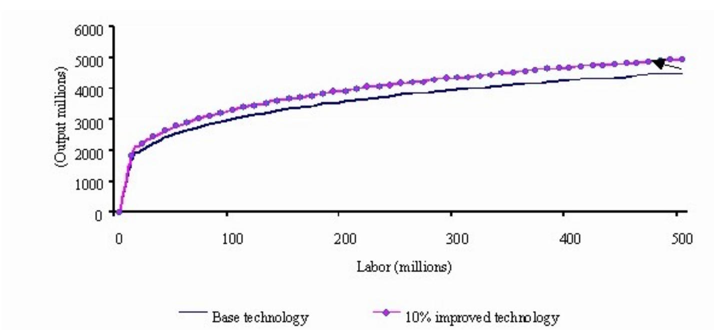


Figure: Production functions for agriculture with capital and land constant

By using the benchmark schedule (base technology), the base year (SAM) value of output (4,344,160,000) is obtainable by reading on the vertical axis the value corresponding to the benchmark level of labor (433,242,000). Because of truncating the calibrated parameters in the production function the output value may not be exactly the same as in the SAM. However, when calibrating the parameters for the algorithm solution, the parameter values will not be truncated and results will be more consistent with initial SAM values.

Improved technology (neutral with respect to inputs) was introduced into the above production function by multiplying the efficiency parameter (ϕ^{CX}) by 1.1. As expected, this alteration caused the curve to shift upward. Note that other conditions can be expressed using the partial equilibrium analysis. For example, one can introduce change in the share parameters or the elasticities of substitution.

Graphic Presentation of the Calibrated CES Function for Domestic and Imported Intermediate Inputs

Using the calibrated values for manufacturing inputs to the agricultural sector, $\delta_{ji}^V = 0.57, \phi^V = 1.93$; and $P_j^V = 0.72$ the following CES function; $V_{ji} = \phi_{ji}^V[\delta_{ji}^V VM_{ji}^{\phi_1^V} + (1-\delta_{ji}^V)VR_{ji}^{\rho_j^r}]^{1/\rho_1^V}$ is graphed (figure below).

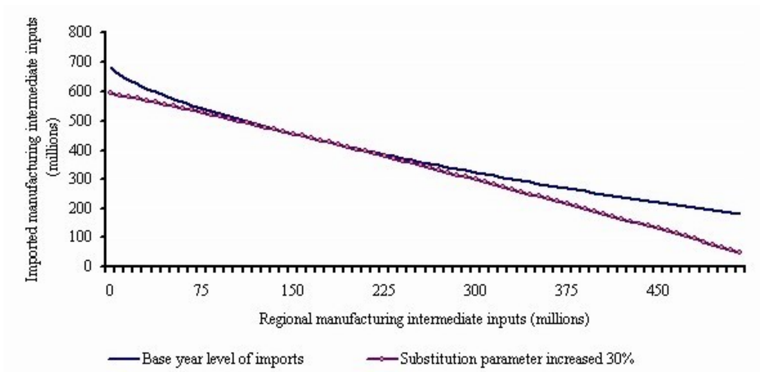


Figure: Substitution between regionally produced and imported manufacturing intermediate inputs into the agricultural industry

Increasing the substitution parameter by 30 percent in the above figure decreases the curvature of the relationship. This is the *a priori* expectation, that the greater the degree of substitution, the more linear the result. The two schedules in the figure are tangent at the base quantities of imported and regionally produced intermediate inputs, indicating that the calibration was accurate.

Modeling substitution among labor skills

The labor skill CES function for industry i is given as:

$$LAB_i = \phi_i^{LAB}\left[\sum_s \partial_{is}^{LAB} LD_{is}^{\rho_1^{LAB}}\right]^{\sigma_1^{\frac{1}{LAB}}}, \rho_i^{LAB} = \frac{\sigma_i^{LAB}-1}{\sigma_i^{LAB}} \tag{1}$$

where $\rho\frac{LAB}{i} \neq 0$ is the substitution parameter among labor skills, $\phi_i^{LAB} > 0$ is the labor efficiency parameter, $\partial_{is}^{LAB}(0 < \partial_{is}^{LAB} < 1, \sum \partial_{is}^{LAB} = 1)$ is the share parameter for labor with skill s, LD_{is} is the quantity demanded of that skill type, and σ_i^{LAB} is the elasticity of substitution among labor skills.

The derived demand for labor skill s in industry i is based on cost minimization to satisfy the aggregate labor requirement in the industry. Thus, the producers want to choose the level of LD_{is} so as to minimize $\sum_s PLS_s \cdot LD_{is}$

subject to: $LAB_i = \phi_i^{LAB}\left[\sum_s \partial_{is}^{LAB} LD_{is}^{\rho_1^{LAB}}\right]^{\rho_1^{\frac{1}{LAB}}}$

where $\sum_s PLS_s \cdot LD_{is}$ is the total wage bill, PLS_s is the wage rate of labor of skill s. First order conditions of this problem result in

$$PLS_s = \phi_i^{LAB}\left[\sum_s \partial_{is}^{LAB} \cdot LD_{is}^{\rho_1^{LAB}}\right]^{\frac{1-\sigma_1^{LAB}}{\sigma_1^{LAB}}} \partial_{is}^{LAB} \cdot LD_{is}^{-(1-\rho_i^{LAB}})) \tag{2}$$

From equation (2), the ratio of wages for two labor skills of type s and t can be expressed as:

$$\frac{PLS_t}{PLS_s} = \frac{\partial_{it}^{LAB} \bullet LD_{it}^{-(1-\rho_1^{LAB})}}{\partial_{is}^{LAB} \bullet LD_{is}^{-(1-\rho_1^{LAB})}} \text{ , or} \tag{3}$$

$$LD_{is} = \left(\frac{PLS_s \bullet \partial_{it}^{LAB}}{PLS_t \bullet \partial_{is}^{LAB}}\right)^{\frac{1}{1+\rho_1^{LAB}}} \tag{4}$$

By substituting equation (4) into equation (1), the skill-augmented labor demand equations become

$$LAB_i = \phi_i^{LAB} \left[\sum_s \partial_{is}^{LAB} \left(\frac{PLS_s \partial_{it}^{LAB}}{PLS_t \partial_{is}^{LAB}} \right)^{\frac{\rho_i^{LAB}}{(1-\rho_1^{LAB})}} LD_{it} \right]^{\frac{1}{\rho_1^{LAB}}} \tag{5}$$

Thus, by using equations (4) and (5), it can be shown that the demand for labor of type s has the form

$$LAB_i = \frac{LD_{is}}{\phi_i^{LAB}} \left[\sum_s \partial_{is}^{LAB} \left(\frac{PLS_s \partial_{it}^{LAB}}{PLS_t \partial_{is}^{LAB}} \right)^{\frac{\rho_1^{LAB}}{(1-\rho_1^{LAB})}} \right]^{\frac{1}{\rho_1^{LAB}}} \tag{6}$$

Calibration of the labor substitution relationships discussed in this section involves obtaining each sector's estimate of the elasticity of substitution from external sources. Equation (4) can be used to compute the value of the labor skill share parameters by normalizing the prices to one and rearranging the terms. The base year quantities of the labor skills, if available, are presented in the SAM. The SAM used in this paper, however, does not differentiate labor by skill. We will leave calibration of this section to the interested reader. An example of a social accounting matrix that differentiates labor by skill type is in Budiyanti.

Graphic Presentation of the Calibrated CET Function for Regional Product and Exports

The figure below shows an example of a market possibility frontier (for agriculture sector) for regional and export markets, using benchmark and counter factor parameter data.

By increasing the substitution parameter by 60 percent, the market possibilities frontier becomes more concave, which is in accordance with *a priori* expectations. The two schedules in the figure are tangent at the same benchmark (SAM) quantity exported and quantity marketed regionally, an indication that the calibration was accurate.

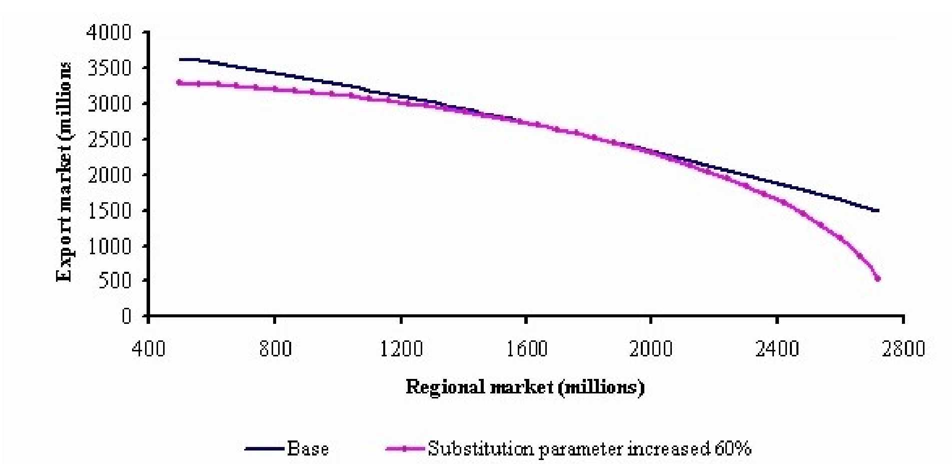


Figure: Market possibility frontier for the agricultural sector

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Graphic Presentation of the Calibrated Commodity Demand

Using the calibrated budget share for agriculture, the demand schedule is obtained by varying the price and computing the corresponding quantities using equation (3.2.16). The figure below shows the relationship for household demand for agricultural commodities.

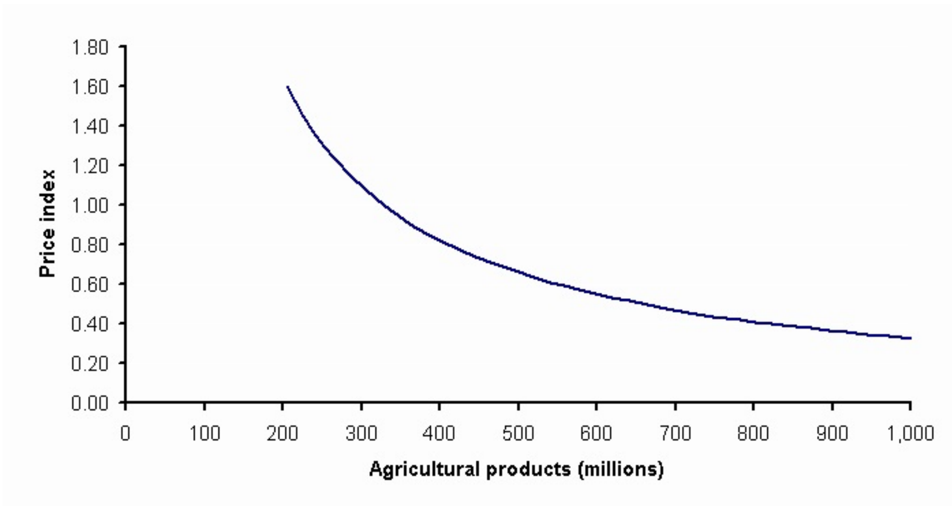

Figure: Household demand schedule for agricultural commodities.

The LES with Leisure and Positive Minimum Consumption

Following de Melo and Tarr, the Klein-Rubin utility function can be modified to incorporate leisure for each household group (M = composite market goods composed of imports and regionally produced):

$$U = \beta_{0h} \ln(Q_{0h} - \gamma_{\sigma h}) + \sum_{ieM} \beta_{ih} \ln(Q_{ih} - \gamma_{ih}) \tag{1}$$

In equation (1), the worker-consumer is assumed to purchase a combination of leisure (Q_{0h}) and composite market commodities (Q_{ih}). To derive the LES, equation (1) is maximized subject to the household's full income (FY_h), which is equal to non-labor income plus imputed value of time (equation 2):

$$FY_h = Y_{NL} + \omega T_h = Y_h + \omega Q_{0h} \tag{2}$$

where $Y_h = \sum_i^m p_i Q_{ih}$ is household money income, which is also equal to HE_h; Y_{NL} is non-labor income; and ωT_h is the imputed value of time (ω is the imputed unit value of time and T_h is total time available to the household for work and leisure). The subscript m represents the number of commodities. Because of considerations for leisure and non-market commodities in equation (1), the resulting LES is augmented as follows:

$$Q_{0h} = \gamma_{0h} + \left(\frac{\beta_{0h}}{\omega}\right)\left(FY_h - \sum_{j=0}^{n} P_j \gamma_{jh}\right) \tag{3}$$

$$Q_{ih} = \gamma_{ih} + \left(\frac{\beta_{ih}}{P_i}\right)\left(FY_h - \sum_{j=0}^{n} P_j \gamma_{jh}\right) \tag{4}$$

where P_0 is the wage rate (ω). Expenditure on the i^{th} commodity consists of expenditure on the minimum required quantity for that commodity plus the proportion of the budget which is left over after paying for all minimum requirements. This proportion, β_i , is the marginal budget share that determines the allocation of supernumerary income. If leisure is ignored, only composite commodities would be purchased with money (not full) income.

Substitution of equation (2) into equation (3) and rearranging terms results in the household labor supply function (see de Melo and Tarr and Lee for details of the derivations):

$$LS_h = MT_h - \left(\frac{\beta_{0h}}{\omega}\right)\left(\frac{HE_h - \sum_{j=1}^{n} P_j \gamma_{jh}}{1 - \beta_{0h}}\right) \tag{5}$$

where $LS_h + Q_{0h} = T_h, MT_h = T_h - \gamma_{0h}$ is the maximum work time available to the household, $HE_h = Y_h$, and $\omega = P_0$. Similarly, substituting equation (2) into (3) and rearranging terms provides demand functions for market commodities by the household group, h:

$$Q_{ih} = \gamma_{ih} + \left(\frac{\beta_{ih}}{(1 - \beta_{0h})P_i}\right)\left(HE_h - \sum_{j=1}^{n} P_j \gamma_{jh}\right) \tag{6}$$

Equations (5) to (6) are the final formulations that constitute the household demand system.

To evaluate these equations, the parameters β_{0h}, β_{ih}, and γ_{ih} need to be calibrated using data from the social accounting matrix (Table 2.1), a Frisch parameter, income elaticities of demand, and income elasticity of labor supply.

First, β_{0h} is calculated from the formula for the income elasticity of labor supply $\frac{\partial LS_h}{\partial HE_h} \cdot \frac{HE_h}{LS_h}$ and applying to equation (5):

$$\delta_h^{LH} = \frac{-\beta_{0h} HE_h}{(1 - \beta_{0h}) w LS_h} \tag{7}$$

where δ_h^{LY} is available from other studies. Like other prices, w is normalized to one in the base. The variables HE_h and LS_h are obtained from the SAM. Rearranging terms in (7) yields the expression used to obtain the value of the marginal budget share for leisure, β_{0h}:

$$\beta_{0h} = \frac{\omega LS_h \delta_h^{LY}}{\omega LS_h \delta_h^{LY} - HE_h} \tag{8}$$

In the 1993 SAM (Table 2.1), for example, $HE_h = 50,665,679,000$, which is the households column/expenditure total (53,880,000,000) less government taxes (6,976,571,000), less households' payment for labor services (107,070,000 and less household savings –(3,869,320); and $LS_h = 37,489,772,000$ – the labor row/income total. Using these data and a household labor supply elasticity of –0.18 (negative, implying leisure is a normal good) equation (8) is used to calculate $\beta_{0h} = 0.1175$. As usual, w and other prices are normalized to one in the base year.

From the commodity demand equation (6), the elasticity of demand for commodity i with respect to income, whose value is also obtained from previous studies, is given by:

$$\delta_h^Y = \frac{\beta_{ih} HE_h}{(1 - \beta_{0h}) P_i \cdot Q_{ih}}, \sum \beta_{ih} = 1 \tag{9}$$

Given the value of β_{0h} from (8), the only unknown in (9) is β_{ih}, the marginal budget share for commodity i. P_i is normalized to one and the remaining values in the equation are obtained from the SAM. By rearranging (9), we obtain an expression used to calibrate the value of the unknown,

$$\beta_{ih} = \frac{(1 - \beta_{0h}) P_i \cdot Q_{ih} \delta_h^Y}{HE_h} \tag{10}$$

An exogenously determined *Frisch* parameter is used to compute the minimum subsistence requirement for commodity i. The Frisch parameter, which is a measure of the marginal utility of income, is given as

$$Frisch = \frac{HE_h}{HE_h - \sum_j P_i \gamma_{jh}} \tag{11}$$

Substituting (11) into (6) and rearranging terms gives an expression for calibrating γ_{ih}, the minimum subsistence requirement:

$$\gamma_{ih} = Q_{ih} + \left(\frac{\beta_{ih}}{(1-\beta_{0h})P_i}\right)\left(\frac{HE_h}{Frisch_h}\right) \tag{12}$$

The maximum number of hours available for work MT_h, which is equal to total time endowment T_h (24 hours minus time necessary for sleeping and other minimal maintenance tasks) less minimum requirement for leisure, is calibrated by rearranging (5):

$$MT_h = LS_h + \left(\frac{\beta_{0h}}{\omega}\right)\left(\frac{HE_h - \sum_{j=1}^{n} P_j\gamma_{jh}}{1-\beta_{0h}}\right) \tag{13}$$

Sector	Income elasticity of demand	Source
Agriculture	0.30	deMelo and Tarr
Mining	0.89	deMelo and Tarr
Manufacturing	1.06	deMelo and Tarr
Services	1.05	deMelo and Tarr
Income elasticity of labor supply	(-0.18):	Abbot and Ashenfelter
Frisch parameter	(-1.60):	Lluck, Powell and Williams

Graphic Presentation of the Calibrated Household Indifference Curve Between Regionally Produced and Imported Commodities

A graphical presentation of the substitution relationship between imported and regionally produced commodities is shown in the form of a household indifference curve. The figure below illustrates this relationship for agricultural commodities.

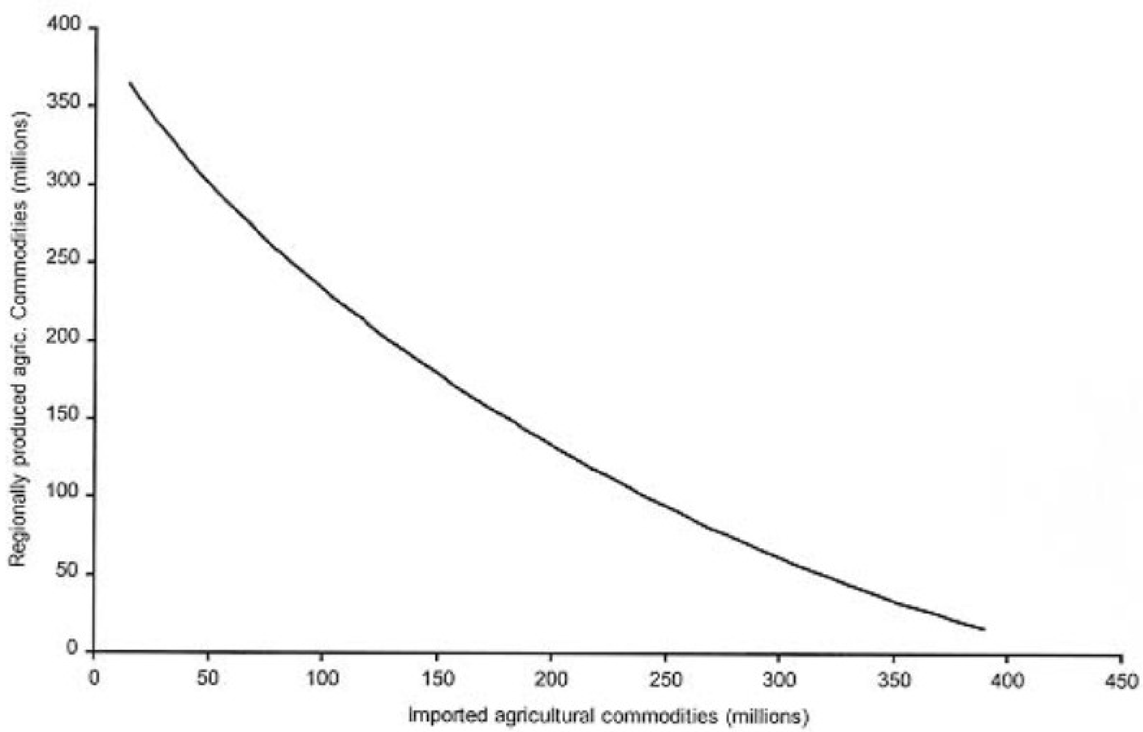

Figure: Household indifference curve between regionally produced and imported agricultural commodities

Hypothetical Examples of Labor Migration

Results of equation (3.3.23) are shown for two hypothetical examples. The first shows outmigration where the results of equation (3.3.3) is LMG = -9. Let us assume that $LSO = 30$, regional labor demand is $(LDI + LDE) = 21$, $PLE = 1.0$ and $PL = 0.9$. Results of equatioin (3.3.23) are:

$$LY = 0.9(21) + 1.0\left[\left(\sqrt{(-9)^2} - (-9)\right)0.5\right] - 0.9\left[\left(\sqrt{(-9)^2} + (-9)\right)0.5\right] = 27.9$$

The first term on the right is regional labor compensation and is equal to 18.9. The second term is compensation for labor that migrated out of the region and equals 9.0. The third term identifies inmigration and is equal to zero. The original regional households had a labor supply of $LSO = 30$ of which 21 units were compensated at the regional price of 0.9 and 9 units were compensated at the out-of-region price of 1.0. Therefore the original households had labor income of 27.9 of which 18.9 was earned in the region and 9 was earned outside the region after outmigration.

The second example shows inmigration where $LMG = 9$. Again we assume that $LSO = 30$, regional labor demand is $(LDI + LDE) = 39$, $PLE = 1.0$ and $PL = 1.1$. Results of equation (3.3.23) are:

$$LY = 1.1(39) = 1.0\left(\sqrt{+9^2} - (+9)\right)0.5 - 1.1\left[\left(\sqrt{+9^2} + 9\right).5\right] = 33.0$$

Regional labor compensation is 42.9 There is no outmigration and compensation to inmigrants is 9.9. The original households with labor of 30 units were compensated at the regional wage rate of 1.1 or 33 monetary units. Results of equation (3.3.23) show that it holds for both outmigration and inmigration of labor. Equation (3.3.3) is the deciding factor of migration.

Table 2.1: Oklahoma Social Accounting Matrix, 1993 in thousands of dollars.

	EXPENDITURES																	
	INDUSTRY							FACTORS				INSTITUTIONS						ROW
	Agri-culture	Mining	Forestry	FPI	Manufacturing	Services	Total	Labor	Capital	Land	Total	Enterprise	Households	Governments	Capital	Total	Exports	TOTAL
INDUSTRY																		
Agriculture	870862	8116	4936	2668	820923	34800	1542305						147210	12863	9780	169053	2591601	4303759
Mining	122579	2180942	891	85705	1192412	881343	4443872						1587998	231250	19097	1838345	5807568	12089785
Forest Complex																		
Forestry				40400			40400											40400
FPI	9839	72630	1264	6026	179590	206456	475805						138648	96782	248026	483456	826339	1785600
Manufacturing	147584	1318071	984	109769	3299584	3746744	8622736						2517437	1757284	4503431	8778152	15003939	32404827
Services	379945	1317332	1597	275941	4996845	9752027	16723087						30727365	1477994	557652	32763011	9629092	59115191
Sub-total industry	1330809	4897091	9672	499909	10489354	14621370	31848205						35118658	3576173	5337986	44032817	33858539	109739561
FACTORS																		
Labor	426998	1622808	6244	188400	7389027	20767388	30400863						107070	6981839		7088909		37489772
Capital	566973	2713109	4387	525780	3499379	12042709	19352337											19352337
Land	701385		7681				708066											709066
Sub-total	1695356	4335915	18312	714180	10888406	32810097	50462266						107070	6981839		7088909		57551175
INSTITUTIONS																		
Enterprises									12510953		12510953							12510953
Households								31363057	7848069	683300	39894426	1734234		11490516		13224750	7600825	53880001
Governments	95405	866971	896	85720	101159	4318042	5268193	6126715	-1006686	25766	5145795	1699623	11490516	8477813		17154007	4375095	31943090
Capital												9077096	3869320			5207776	2789519	7997295
Sub-total Institutions	35405	666971	896	85720	101159	4318042	5268193	37489772	19352238	709068	57551174	12510953	3107251	19968329		35586533	7925439	106331339
IMPORTS																		
Agriculture	574915	5160	4955	1230	377192	41300	1004752						181550	20097	10447	212094		1216846
Mining	11222	1274869	628	16247	294847	385272	1983085						141562	29912	15759	187333		2170418
Forest Complex																		
Forestry																		
FPI	27620	23552	2742	19243	436987	143637	653781						299232	43146	128560	470938		1124719
Manufacturing	414296	427425	2171	350537	8028706	2606708	11829843						5414473	780700	2326243	8521416		20351259
Services	154136	458802	1024	998534	1788176	4188764	8689436						9510103	542893	178299	10231295		16920731
Sub-total imports	1182189	2189808	11520	485791	10985908	7365681	22160897						15547020	1416748	2659308	19623076		41783973
COLUMN TOTAL	4303759	12089785	40400	1785600	32404827	59115190	109739561	37489772	19352336	709066	57551174	12510953	53879999	3193089	7997294	106331335	41783978	315406048

Table 2: Short-run Pro-competitive Simulation Results: Indexes for Selected Endogenous Variables

Sector	IP (1)	IPR (2)	IPX (3)	IPL (4)	IPK (5)	IPT (6)	IX (7)	IR (8)	IEXP (9)	IIMP (10)	IQ (11)	IQR (12)	IQM (13)	IVA (14)	IL (15)	IK (16)	IT (17)
AGR	1.00001	1.00001	1.00000	1.00015	0.99998	0.99998	0.99996	0.99999	0.99994	1.00001	0.99974	0.99973	0.99975	0.99996	0.99984	1.00000	1.00000
MIIN	1.00005	1.00007	1.00003	1.00015	1.00002		0.99995	1.00004	0.99985	1.00006	0.99970	0.99970	0.99973	0.99995	0.99987	1.00000	
RM		1.02550	1.02550	1.00015	1.03953	1.03953	1.01326							1.01326	1.03938	1.00000	1.00000
FPI	0.99802	0.99573	0.99772	1.00015	1.04920		1.01326	1.00742	1.02001	0.99465	1.00173	1.01217	0.99690	1.01326	1.04904	1.00000	
MAN	1.00002	1.00003	1.00002	1.00015	1.00003		0.99992	0.99997	0.99987	1.00020	0.99973	0.99965	0.99977	0.99992	0.99989	1.00000	
SER	1.00005	1.00007	1.00006	1.00015	0.99999		0.99990	0.99991	0.99986	1.00005	0.99970	0.99967	0.99980	0.99990	0.99984	1.00000	

where P, PR, PX, PL, PK, PT are composite price, regional price, output price, wage rate, capital price, and land price; X, R, EXP, IMP, Q, QR, and QM are regional output, regional production consumed in region, export, import, composite household demand, household demand from regional production, and household import demand; VA, L, K, T are composite value added, labor demand, capital demand, and demand for land. The I before each variable indicates index.

Table 3: Long-run Pro-competitive Simulation Results: Indexes for Selected Endogenous Variables

Sector	IP (1)	IPR (2)	IPX (3)	IPL (4)	IPK (5)	IPT (6)	IX (7)	IR (8)	IEXP (9)	IIMP (10)	IQ (11)	IQR (12)	IQM (13)	IVA (14)	IL (15)	IK (16)	IT (17)
AGR	1.00015	1.00025	1.00010	1.00045	1.00142	0.99897	0.99881	0.99940	0.99842	0.99986	0.99975	0.99956	0.99991	0.99881	0.99852	0.99756	1.00000
MIIN	1.00097	1.00131	1.00068	1.00045	1.00142		0.99795	0.99976	0.99599	0.99979	0.99893	0.99888	0.99953	0.99795	0.99856	0.99759	
RM		1.06612	1.06612	1.00045	1.00142	1.17142	1.09566							1.09566	1.17089	1.16976	1.00000
FPI	0.98584	0.97064	0.98486	1.00045	1.00142		1.09566	1.05139	1.14630	0.96320	1.01426	1.08934	0.97998	1.09566	1.09643	1.09537	
MAN	1.00017	1.00038	1.00020	1.00045	1.00142		0.99912	0.99963	0.99854	1.00180	0.99972	0.99881	1.00015	0.99912	0.99943	0.99847	
SER	1.00050	1.00067	1.00056	1.00045	1.00142		0.99981	0.99989	0.99942	1.00125	0.99940	0.99908	1.00042	0.99981	1.00017	0.99920	

where P, PR, PX, PL, PK, PT are composite price, regional price, output price, wage rate, capital price, and land price; X, R, EXP, IMP, Q, QR, and QM are regional output, regional production consumed in region, export, import, composite household demand, household demand from regional production, and household import demand; VA, L, K, T are composite value added, labor demand, capital demand, and demand for land. The I before each variable indicates index.

Table 4: Regional Indexes for the Pro-Competitive Simulations, Selected Variables

Variable	Short-run	Long-run	Weighting
Quantities			
Output	1.00014	1.00096	base prices
Export	1.00036	1.00195	base prices
Import	0.99998	1.00038	base prices
Labor demand	1.00013	1.00041	NA
Capital demand	1.00000	1.00146	NA
Land demand	1.00000	1.00000	NA
Prices			
Output	1.00002	1.00022	base quantities
Export	1.00000	1.00000	NA
Import	1.00000	1.00000	NA
Labor	1.00015	1.00045	NA
Capital	1.00135	1.00142	SR (base quantities)
Land	1.00041	1.00084	base quantities

Table 5: Regional Impacts on Factor Payments from the Pro-Competitive Simulations

Factor	Units	Short-run	Long-run
Labor compensation	$ thousand	37,500,224	37,522,232
Change	$ thousand	10,652	32,460
Index	base = 1.0	1.00028	1.00087
Capital compensation	$ thousand	19,378,463	19,407,836
Change	$ thousand	26,126	55,499
Index	base = 1.0	1.00135	1.00287
Land compensation	$ thousand	709,355	709,662
Change	$ thousand	289	595
Index	base = 1.0	1.00041	1.00084
Value added	$ thousand	57,588,042	57,639,730
Change	$ thousand	36,867	88,554
Index	base = 1.0	1.00064	1.00154

Table 6: Regional Household Income Impact from the Pro-Competitive Shock

Item	Units	Short-run	Long-run
Household nominal			
Income	$ thousand	53,897,921	53,908,295
Change	$ thousand	17,920	28,295
Index	base = 1.0	1.00033	1.00053
Composite price			
Index	base = 1.0	1.00003	1.00034
Household real			
Income	$ thousand	53,896,304	53,889,972
Change	$ thousand	16,303	9,972
Index	base = 1.0	1.00030	1.00019

Table 2.1: Oklahoma Social Accounting Matrix, 1993 in thousands of dollars.

	EXPENDITURES																	
	INDUSTRY							FACTORS				INSTITUTIONS						ROW
	Agri-culture	Mining	Forestry	FPI	Manufacturing	Services	Total	Labor	Capital	Land	Total	Enterprise	Households	Governments	Capital	Total	Exports	TOTAL
INDUSTRY																		
Agriculture	870862	8116	4936	2668	820923	34800	1542305						147210	12863	9780	169053	2591601	4303759
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Forest Complex																		
Forestry				40400			40400											40400
FPI	9839	72630	1264	6026	179590	206456	475805						138648	96782	248026	483456	826339	1785600
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Services	379945	1317332	1597	275941	4996845	9752027	16723087						30727365	1477994	557652	32763011	9629092	59115191
Sub-total industry	1330809	4897091	9672	499909	10489354	14621370	31848205						35118658	3576173	5337986	44032817	33858539	109739561
FACTORS																		
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Capital	566973	2713109	4387	525780	3499379	12042709	19352337											19352337
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Sub-total	1695356	4335915	18312	714180	10888406	32810097	50462266						107070	6981839		7088909		57551175
INSTITUTIONS																		
Enterprises									12510953		12510953							12510953
Households								31363057	7848069	683300	39894426	1734234		11490516		13224750	7600825	53880001
Governments	95405	866971	896	85720	101159	4318042	5268193	6126715	-1006686	25766	5145795	1699623	11490516	8477813		17154007	4375095	31943090
Capital												9077096	3869320			5207776	2789519	7997295
Sub-total Institutions	35405	666971	896	85720	101159	4318042	5268193	37489772	19352238	709068	57551174	12510953	3107251	19968329		35586533	7925439	106331339
IMPORTS																		
Agriculture	574915	5160	4955	1230	377192	41300	1004752						181550	20097	10447	212094		1216846
Mining	11222	1274869	628	16247	294847	385272	1983085						141562	29912	15759	187333		2170418
Forest Complex																		
Forestry																		
FPI	27620	23552	2742	19243	436987	143637	653781						299232	43146	128560	470938		1124719
Manufacturing	414296	427425	2171	350537	8028706	2606708	11829843						5414473	780700	2326243	8521416		20351259
Services	154136	458802	1024	998534	1788176	4188764	8689436						9510103	542893	178299	10231295		16920731
Sub-total imports	1182189	2189808	11520	485791	10985908	7365681	22160897						15547020	1416748	2659308	19623076		41783973
COLUMN TOTAL	4303759	12089785	40400	1785600	32404827	59115190	109739561	37489772	19352336	709066	57551174	12510953	53879999	3193089	7997294	106331335	41783978	315406048

```
$TITLE REGIONAL CGE MODEL FOR OKLAHOMA (1993)(CRS.GMS)
$OFFSYMLIST OFFSYMXREF OFFUPPER

SETS
i       Sectors            /Agr  agriculture
                            Min  mining
                            Man  manufacture
                       SER services/
ag(i)   Agricultural sectors            / AGR/
nag(i)  Nonagricultural market sectors   / MIN, SER, MAN/
f       Factors          /L    labor, K    capital, T    land/
fl(f)   Factors not land / L, K/
ALIAS(i,j);


*#####-- DECLARATION OF BASE YEAR VARIABLES (AS PARAMENTERS)

PARAMETERS


*@Price block
  PL0            Wage rate
  PLROC0         Wage rate of rest-of-country
  PKROC0         Cap rate of rest-of-country
  PK0(i)         cap rate
  PT0(ag)        Land rent
  PE0(i)         Export price
  PM0(i)         Import price
  PR0(i)         Reg price
  P0(i)          Composite price
  PN0            Net output price or value-added price of sector i
  PX0(i)     Composite price face for producers

*@Production block
  L0(i)          Labor demand
  LS0            Labor supply by hh
  TLS0           Total labor supply
  LHHH0          Labor employed by household group
  LGOV0          Labor employed by gov
  K0(i)          capital demand
  T0(i)          Land demand
  KS0            Supply of pri capital
  TKS0           Total pri capital supply
  TS0            Supply of land
  VA0(i)         Value added
  V0(j,i)        Composite intermediate good demand
  TV0(i)         Composite intermediate good total demand
  VR0(j,i)       Reg int good demand
  VM0(j,i)       Imported int good demand
  TVR0(i)        Reg int good total demand
  TVM0(i)        Imported int good total demand
  IBT0(I)    Indirect business taxes
  X0(i)          Sector output
  E0(i)          Export of reg product
  M0(i)          Import
  R0(i)          Reg supply of reg product
```

```
*@Income block
  LY0            Labor income
  KY0            capital income
  TY0            Land income
  YENT0          Gross Enterprise income
  YH0            Household income
  DYH0           Disposable hh income
  HSAV0          Household saving
  SAV0           Total saving
  ROWSAV0        Saving from rest-of-world
  TRGOV0         Gov transfer to hh
  REMIT0         Remittance from outside the region to household
  YGOV0          Gov revenue
  ENTY0          Enterprise income distrib to hhs
  GOVITR0        Inter gov transfer
  GOVBOR0        Government Borrowing
  GRP0           Gross regional product

*@Expenditure block
  HEXP0          Household expend
  QR0(i)         Demand for reg consump good
  QM0(i)         Demand for imp consump good
  Q0(i)          Demand for comp consump good
  GOVEXP0        government expenditure
  QGOVR0(i)      government demand for reg good
  QGOVM0(i)      government demand for imported good
  QGOV0(i)       government demand for comp good
  QInvR0(i)      Invest demand for reg good
  QInvM0(i)      Invest demand for imported good
  QInv0(i)       Invest demand for comp good
  INV0           Total invest

*The following variables are defined as  "logical variables". A logical
*variable takes the value of 1 if the condition stated is true and "0"
*if not.

*************************************
*Regional         x     x     0     0     0=zero, x=not zero
*Import                 x     0     x     0
*
*NZV              T     F     F     F     T=TRUE,  F=FALSE
*ZVR              F     F     T     F
*ZVM              F     T     F     T
*************************************

  ZVM(i,J)  non imported intermediate demand with-or-without regional interm.
demand
  ZVR(i,J)  only imported intermediate demand
  NZV(i,J)  both imported intermediate demand and regional demand

  ZQM(i)    non imported final demand and either none or some regional final
demand
  ZQR(i)    only imported final demand
  NZQ(i)    both imported final demand and regional final demand

  ZGOVM(i)
  ZGOVR(i)
```

```
  NZGOV(i)

  ZInvM(i)
  ZInvR(i)
  NZInv(i)


*#####-- DECLARATION OF PARAMETERS TO BE CALIBRATED.

PARAMETERS

*This parameters are those specified in Table 5.5.

*@Production block
  a0(i)          composite value added req per unit of output i
  a(j,i)         req of interm good j per unit of good i
  Alpha(i,f)     value added share param
  Ava(i)         value added shift param
  RHOv(i)        interm input subs param
  deltav1(j,i)
  deltav(j,i)    interm input share param
  Av(j,i)        interm input shift param
  RHOx(i)        output transformation param
  deltax1(i)
  deltax(i)      output share param
  Ax(i)          output shift param

*@Income block
  ktax           capital tax rate
  sstax          factor income tax rate for labor
  ttax           factor income tax rate for land
  retr           rate of retained earnings fr ent inc
  et         enterprise tax rate
  hhtax          income tax rate for hh
  ltr            Household Income Transfer Coefficient
  mps            saving rate
  ibtax(i)       indirect business tax
  beta(i)        param calc fr elast of comm demand wrt inc

*@Expenditure block
  RHOq           consumer demand subs param
  deltaq1(i)
  deltaq(i)      consumer demand share param
  Aq(i)          consumer demand constant eff param
  RHOgov         gov demand subs param
  deltagov1
  deltagov       gov demand share param
  Agov           gov demand constant eff param
  RHOinv         inv gov demand subs param
  deltainv1
  deltainv       inv gov demand share param
  Ainv           inv gov demand constant eff param
  ;

*### DATA: Data come from our SAM (Table 2.1)

Table      IOR(i,j) Input-output regional matrix
```

```
      AGR   MIN           MAN  SER
AGR   675.798     8.115     863.991 34.800
MIN   123.47      2180.942    1258.117    881.343
MAN   159.671     1390.701    3594.97     3953.2
SER   381.542     1317.332    5272.186    9752.027


;
Table    IOM(i,j)   Input-output import matrix

      AGR   MIN        MAN            SER
AGR   579.870     5.160  378.422    41.300
MIN   11.850      1274.869 311.094  385.272
MAN   446.830     450.977      8835.472   2750.345
SER   155.160     458.802      1886.710   4188.764

;

Table VAD(i,f)  Value added matrix
            L           K               T
AGR          433.242    571.360        709.066
MIN         1622.806    2713.109
MAN         7577.427    4025.159
SER        20767.388    12042.708
;


Table HHCONR(i,*)  Household consumption demand for regional goods

        HOUSE
AGR      147.210
MIN     1587.998
MAN     2656.085
SER     30727.366
;

Table HHCONM(i,*)  Household consumption demand for imported goods
        HOUSE
AGR     181.550
MIN     141.662
MAN    5713.705
SER    9510.103
;

Table GOVCONR(i,*)  Government consumption demand for regional goods


         GOV
AGR       12.863
MIN      231.250
MAN     1854.066
SER     1477.995
;

Table GOVCONM(i,*) Government consumption demand for imported goods
         GOV
```

```
AGR      20.097
MIN      29.912
MAN     823.846
SER     542.893
;

Table FYDIST(*,f)  Factor income distribution to hhs
             L                 K             T
HH          31363.057         0.00          683.300
;

TABLE ParamA(*,i)   BASE YEAR VALUES FOR INDUSTRY
                  AGR            MIN          MAN              SER

PT0              1.00           1.00          1.00             1.00
PK0              1.00           1.00          1.00             1.00
PR0              1.00           1.00          1.00             1.00
P0               1.00           1.00          1.00             1.00
PM0              1.00           1.00          1.00             1.00
PE0              1.00           1.00          1.00             1.00
X0              4344.160      12089.784     34190.427      59115.190
R0              1752.557       6282.217     18360.150         49486.101
E0                2591.603       5807.567    15830.277          9629.089
M0              1216.846      2170.418        21475.978         16920.731
IBT0             96.301       666.971       186.879       4318.043
QINVR0           9.780          19.097          4751.457          557.653
QINVM0           10.447         15.759          2454.803           178.299
SIGMAv           1.42           0.50           3.55            2.00
SIGMAx           3.90           2.90           2.90            0.70
SIGMAq           1.42           0.50           3.55            2.00
SIGMAgov         1.42           0.50           3.55            2.00
SIGMAinv         1.42           0.50           3.55            2.00

;

TABLE ParamB(f,*)   BASE YEAR VALUES FOR FACTORS
     WAGE0   WAGEROC0     FTAX0          RETENT0       CAP0   CAPROC0
L     1.0       1.0     6126.715            0
K                       -1006.686      9077.096           1       1
T                        25.766             0
;

TABLE ParamC (*,*)   BASE YEAR VALUES FOR HH GROUPS

             HTAX0           HSAV0           TRGOV0
HOUSE        6976.571    -3869.320        11490.516

+           REMIT0        ENTYDIS0
HOUSE       760.824       9582.303
;

TABLE ParamD(*,*)   BASE YEAR VALUES FOR GOVTS
             BOR0         GOVDR0    GOVDM0
GOV            0.0       3576.174   1416.748
;
SCALAR LHHH0     Labor used by high inc hh  /  107.070/;
SCALAR LGOV0     Labor used by government   / 6981.839/;
```

```
SCALAR GOVITR0   Inter-government transfer  / 8477.813/;
SCALAR YENT0     Enterprise income          /20359.022/;
SCALAR ENTTAX0   Enterprise taxes       / 1699.623/;
SCALAR ROWGOV0   Rest of world trans.to gov. /4375.094/;
SCALAR ROWSAV0   Saving from ROW               /2789.519/;
SCALAR QINVMSUM0 Inv demand for imported goods / 2659.308/;
SCALAR etaL      Labor migr elasticity         /     0.92 /;
SCALAR etaK      Capital migr elasticity       /     0.92 /;
Scalar KMobil    Capital Mobility              /     1.0  /;

*@Production block
  L0(i)        =VAD(i,"L");
  K0(i)        =VAD(i,"K");
  T0(i)        =VAD(i,"T");
  VA0(i)       =sum(f,VAD(i,f));
  V0(j,i)      =IOR(j,i)+IOM(j,i);
  TV0(i)       =sum(j,V0(i,j));
  VM0(j,i)     =IOM(j,i);
  VR0(j,i)     =IOR(j,i);
  TVM0(i)      =sum(j,VM0(i,j));
  TVR0(i)      =sum(j,VR0(i,j));
  LHHH0        =LHHH0;
  LGOV0        =LGOV0;
  LS0          =sum(i,VAD(i,"L"))+LHHH0+LGOV0;
  X0(i)        =ParamA("X0",i);
  E0(i)        =ParamA("E0",i);
  R0(i)        =ParamA("R0",i);
  KS0(i)       =VAD(i,"K");
  TKS0         =sum(i,KS0(i));
  TS0(i)       =VAD(i,"T");
  IBT0(I)      =PARAMA("IBT0",I);

*@Income block
  TRGOV0       =ParamC ("HOUSE","TRGOV0");
  LY0          =sum(i,VAD(i,"L"))+LHHH0+LGOV0;
  KY0          =sum(i,VAD(i,"K"));
  TY0          =sum(i,VAD(i,"T"));
  YENT0        =YENT0;
  REMIT0       =ParamC ("HOUSE","REMIT0");
  YH0          =sum(f,FYDIST("HH",f))+ParamC("HOUSE","ENTYDis0")+TRGOV0
                +REMIT0;
  DYH0         =YH0 -ParamC ("HOUSE","HTAX0");
  HSAV0        =ParamC ("HOUSE","HSAV0");
  HEXP0        =DYH0-HSAV0-LHHH0;
  SAV0         =ParamB("K","RETENT0")+ ParamC ("HOUSE","HSAV0")+ROWSAV0;
  ROWSAV0      =ROWSAV0;
  YGOV0        =sum(i,ParamA("IBT0",i))+sum(f,ParamB(f,"FTAX0"))
                +ParamC("HOUSE","HTAX0")+ENTTAX0+ROWGOV0+GOVITR0;
  ENTY0        =ParamC("HOUSE","ENTYDis0");
  GOVBOR0       =ParamD("GOV","BOR0");
  GRP0         =LY0+KY0+TY0+sum(i,ParamA("IBT0",i));

*@Expenditure block
  QR0(i)       =HHCONR(i,"HOUSE");
  QM0(i)       =HHCONM(i,"HOUSE");
  Q0(i)        =QM0(i)+QR0(i);
  GOVEXP0      =ParamD("GOV","GOVDR0")+ParamD("GOV","GOVDM0")
```

```
                +ParamC("HOUSE","TRGOV0")+LGOV0+GOVITR0;
  QGOVR0(i)    =GOVCONR(i,"GOV");
  QGOVM0(i)    =GOVCONM(i,"GOV");
  QGOV0(i)     =QGOVM0(i)+QGOVR0(i);
  QINVR0(i)    =ParamA("QINVR0",i);
  QINVM0(i)    =ParamA("QINVM0",i);
  QINV0(i)     =QINVM0(i)+QINVR0(i);
  INV0         =sum(i,QINV0(i));
  M0(i)        =ParamA("M0",i);

*@Price block
  PL0          =ParamB("L","WAGE0");
  PK0(i)       =ParamA("PK0",i);
  PLROC0       =ParamB("L","WAGEROC0");
  PKROC0       =ParamB("K","CAPROC0");
  PT0(ag)       =ParamA("PT0",ag);
  PE0(i)       =ParamA("PE0",i);
  PM0(i)       =ParamA("PM0",i);
  PR0(i)      =ParamA("PR0",i);
  P0(i)        =ParamA("P0",i);
  PX0(i)      =(PR0(i)*R0(i)+PM0(i)*M0(i))/(R0(i)+M0(i));

*-----------------------------------------------------
* Regional     x  x  0  0     0=zero, x=not zero
* Import       x  0  x  0
*
* NZV          T  F  F  F     T=True, F=False
* ZVR          F  F  T  F
* ZVM          F  T  F  T
*-----------------------------------------------------

  ZVM(i,j)     =(VM0(i,j) eq 0);
  ZVR(i,j)     =(VR0(i,j) eq 0) and (VM0(i,j) ne 0);
  NZV(i,j)     =(VR0(i,j) ne 0) and (VM0(i,j) ne 0);

  ZQM(i)    =(QM0(i) eq 0);
  ZQR(i)    =(QR0(i) eq 0) and (QM0(i) ne 0);
  NZQ(i)    =(QR0(i) ne 0) and (QM0(i) ne 0);

  ZGOVM(i)      =(QGOVM0(i) eq 0);
  ZGOVR(i)      =(QGOVR0(i) eq 0) and (QGOVM0(i) ne 0);
  NZGOV(i)      =(QGOVR0(i) ne 0) and (QGOVM0(i) ne 0);

  ZInvM(i)     =(QInvM0(i) eq 0);
  ZInvR(i)     =(QInvR0(i) eq 0) and (QInvM0(i) ne 0);
  NZInv(i)     =(QInvR0(i) ne 0) and (QInvM0(i) ne 0);


PARAMETER SAM1 SOCIAL ACOUNTING MATRIX -BASE YEAR PRICES-;
SAM1(I,"PK")=PK0(I);
SAM1(ag,"PT")=PT0(ag);
SAM1(I,"PE0")=PE0(I);
SAM1(I,"PM0")=PM0(I);
SAM1(I,"PR0")=PR0(I);
SAM1(I,"P0")=P0(I);
SAM1(I,"PR0")=PR0(I);
```

```
PARAMETER SAM2  SOCIAL ACCOUNTING MATRIX -BASE YEAR DATA-;
SAM2(I,"L0")=L0(I);
SAM2(I,"K0")=K0(I);
SAM2(I,"KS0")=KS0(I);
SAM2(I,"T0")=T0(I);
SAM2(I,"TS0")=TS0(I);
SAM2(I,"VA0")=VA0(I);
SAM2(I,"TVR0")=TVR0(I);
SAM2(I,"TVM0")=TVM0(I);
SAM2(I,"TV0")=TV0(I);
SAM2(I,"IBT0")=IBT0(I);
SAM2(I,"X0")=X0(I);
SAM2(I,"M0")=M0(I);
SAM2(I,"R0")=R0(I);
SAM2(I,"E0")=E0(I);
SAM2(I,"Q0")=Q0(I);
SAM2(I,"QR0")=QR0(I);
SAM2(I,"QM0")=QM0(I);
SAM2(I,"QGOV0")=QGOV0(I);
SAM2(I,"QGOVR0")=QGOVR0(I);
SAM2(I,"QGOVM0")=QGOVM0(I);
SAM2(I,"QINV0")=QINV0(I);
SAM2(I,"QINVR0")=QINVR0(I);
SAM2(I,"QINVM0")=QINVM0(I);

OPTION DECIMALS=0;
DISPLAY SAM1;

OPTION DECIMALS=3;
DISPLAY SAM2;

DISPLAY V0,VM0,VR0,LS0,PL0, PLROC0,LHHH0,LGOV0,LY0,KY0,TY0,
YENT0,REMIT0,YH0,DYH0,YGOV0,GRP0,HSAV0,HEXP0,GOVEXP0,SAV0,ROWSAV0,
TRGOV0,ENTY0,ENTTAX0,GOVBOR0;


*##########################################################*
*                                                          *
*                   PARAMETER CALIBRATION                  *
*                                                          *
*##########################################################*


*#####-- CALIBRATION

*@Production block

  a0(i)           =VA0(i)/X0(i);
  a(j,i)          =V0(j,i)/X0(i);
  alpha(ag,"K")   =VAD(ag,"K")/VA0(ag);
  alpha(ag,"T")   =VAD(ag,"T")/VA0(ag);
  alpha(ag,"L")   =1-alpha(ag,"K")-alpha(ag,"T");
  alpha(nag,"K")  =VAD(nag,"K")/VA0(nag);
  alpha(nag,"L")  =1-alpha(nag,"K");
  Ava(ag)         =VA0(ag)/Prod(f,VAD(ag,f)**alpha(ag,f));
  Ava(nag)        =VA0(nag)/PROD(fl,VAD(nag,fl)**alpha(nag,fl));
  RHOv(i)         =1-1/ParamA("SIGMAv",i);
```

```
  deltav1(j,i)
    $(NZV(j,i))   =(VR0(j,i)/VM0(j,i))**(1-RHOv(j))*(PR0(j)/PM0(j));
  deltav(j,i)
    $(NZV(j,i))   =1/(1+deltav1(j,i));
  Av(j,i)
    $(NZV(j,i))   =V0(j,i)/(deltav(j,i)*VM0(j,i)**RHOv(j)
                    +(1-deltav(j,i))
                    *VR0(j,i)**RHOv(j))**(1/RHOv(j));
  RHOx(i)        =1+1/ParamA("SIGMAx",i);
  deltax1(i)     =(R0(i)/E0(i))**(1-RHOx(i))*(PR0(i)/PE0(i));
  deltax(i)      =1/(1+deltax1(i));
  Ax(i)          =X0(i)/(deltax(i)*E0(i)**RHOx(i)+(1-deltax(i))
                    *R0(i)**RHOx(i))**(1/RHOx(i));

*@Income block
  sstax         =ParamB("L","FTAX0")/LY0;
  ktax          =ParamB("K","FTAX0")/KY0;
  ttax          =ParamB("T","FTAX0")/TY0;
  retr          =ParamB("K","RETENT0")/sum(i,VAD(i,"K"));
  ibtax(i)      =ParamA("IBT0",i)/(PR0(i)*X0(i));
  et        =ENTTAX0/KY0;
  hhtax         =ParamC("HOUSE","HTAX0")/YH0 ;
  ltr           =1;
  mps           =ParamC("HOUSE","HSAV0")/YH0 ;

*@Expenditure block

  RHOq(i) = 1-1/ParamA("SIGMAq",i);
  deltaq1(i)$NZQ(i) = (QR0(i)/QM0(i))**(1-RHOq(i))*(PR0(i)/PM0(i));
  deltaq(i)$NZQ(i) =1/(1+deltaq1(i));
  Aq(i)$NzQ(i) = Q0(i)/(deltaq(i)*QM0(i)**RHOq(i)+(1-
deltaq(i))*QR0(i)**RHOq(i))**(1/RHOq(i));
  RHOgov(i) = 1-1/ParamA("SIGMAgov",i);
  deltagov1(i)$NZGOV(i)  =(QGOVR0(i)/QGOVM0(i))**(1-RHOgov(i))*(PR0(i)/PM0(i));
  deltagov(i)$NZGOV(i) = 1/(1+deltagov1(i));
  Agov(i)$NZGOV(i) = QGOV0(i)/(deltagov(i)*QGOVM0(i)**RHOgov(i)+(1-
deltagov(i))*QGOVR0(i)**RHOgov(i))**(1/RHOgov(i));
  RHOinv(i)= 1-1/ParamA("SIGMAinv",i);
  deltainv1(i)$NZInv(i) = (QINVR0(i)/QINVM0(i))**(1-RHOinv(i))*(PR0(i)/PM0(i));
  deltainv(i)$NZInv(i) = 1/(1+deltainv1(i));
  Ainv(i)$NZInv(i) = QINV0(i)/(deltainv(i)*QINVM0(i)**RHOinv(i)+(1-
deltainv(i))*QINVR0(i)**RHOinv(i))**(1/RHOinv(i));
  beta(i) = Q0(i)*P0(i)/HEXP0;

PARAMETER CALIBR PARAMETER CALIBRATED;
CALIBR(I,"A0")=A0(I);
CALIBR(I,"AVA")=AVA(I);
CALIBR(I,"RHOV")=RHOv(I);
CALIBR(I,"RHOQ")=RHOQ(I);
CALIBR(I,"DELTAQ")=DELTAQ(I);
CALIBR(I,"AQ")=AQ(I);
CALIBR(I,"IBTAX")=IBTAX(I);
CALIBR(I,"RHOGOV")=RHOGOV(I);
CALIBR(I,"DELTAGOV")=DELTAGOV(I);
CALIBR(I,"AGOV")=AGOV(I);
CALIBR(I,"RHOINV")=RHOINV(I);
CALIBR(I,"AINV")=AINV(i);
```

```
CALIBR(I,"RHOX")=RHOX(i);
CALIBR(I,"DELTAX")=DELTAX(i);
CALIBR(I,"AX")=AX(i);
CALIBR(I,"BETA")=BETA(i);
DISPLAY CALIBR;
DISPLAY a,Av,deltav,alpha,
ktax,sstax,ttax,retr,et,mps,hhtax;

*##########################################################*
*                                                          *
*      VARIABLE DECLARATION                                *
*                                                          *
*##########################################################*

* ENDOGENOUS VARIABLES

VARIABLES
  Z             Objective Function Value

*@Price block
  PL                Wage rate
  PK(i)             Capital rate
  PKL                Capital rate in the long run
  PT(ag)            Land rent
  PN(i)             Net price
  PR(i)             Regional price
  P(i)              Composite price
  PX(i)             Composite price faced by consumers

*@Production block
  LAB(i)            Labor demand
  CAP(i)            Capital demand
  LAND(ag)           Land demand
  TCAP              Total Capital Demand
  TLAB              Total Labor Demand
  LS                Labor supply
  LMIG              Labor migration
  KMIG              Capital migration
  VA(i)             Value added
  V(j,i)            Composite intermediate good demand
  VM(j,i)           Imported int good demand
  VR(j,i)           Reg int good demand
  R(i)              Regional supply
  X(i)              Output
  EXP(i)            Export
  M(i)              Import
  TVM(i)            Imported int good total demand
  TVR(i)            Reg int good total demand
  TV(i)             Composite intermediate good total demand
  adjL              Labor adjustment

*@Income block
  LY                Labor income (original hhs)
  ALY               Adjusted labor income (staying + in-migrating)
  KY                capital income (original capital stock)
  TY                Land income
  YENT              Enterprise income
```

```
  RETENT            Retained Earnings by enterprises
  YH                Income of hh staying in the region (including in-migrants)
  DYH               Disposable hh income (staying in the region + inmigra)
  HSAV              Household saving (staying +inmigrat)
  SAV               Total saving
  INV               Investment
  YGOV              gov revenue
  IBTX              Indirect business tax
  GRP               Gross region product

*### Expenditure block
  AHEXP             Adjusted household expenditure (spent within the region)
  Q(i)              Demand for comp consump good
  QM(i)             Demand for imp consump good
  QR(i)             Demand for reg consump good
  GOVEXP            gov expend
  QGOV(i)           gov demand for comp good
  QGOVM(i)          gov demand for imported good
  QGOVR(i)          gov demand for reg good
  QINV(i)           Invest gov demand for comp good
  QINVM(i)          Invest gov demand for imported good
  QINVR(i)          Invest gov demand for reg good
  SLACK(i)
  SLACK2(i)

POSITIVE VARIABLE   SLACK, SLACK2;

*###########################################################*
*                                                           *
*      EQUATION DECLARATION                                 *
*                                                           *
*###########################################################*

*This section declares the equations of the model
*which are those presented in table 5.1

EQUATIONS
  EQZ               objective function

*@Price block
  NETprice(i)      net price
  Price(i)         composite price
  Price1(i)

*@Production block
  Ldemand(i)       labor demand
  KdemandSR(i)       capital demand
  KdemandLR(i)
  Tdemand(ag)       land demand
  TLdem             total labor demand
  TKdem             total capital demand
  VAdemand(i)       value added demand
  Vdemand(j,i)      intermediate demand
  VAprod1(nag)      value added prod fc
  VAprod2(ag)       value added prod fc
  Vces(j,i)         ces fc for int demand
  TVdemand(i)       intermediate total demand
```

```
  TVRdemand(i)      int reg total demand
  TVMdemand(i)      int imp total demand
  VRdem(j,i)        demand for reg int good
  VRdem0(j,i)       demand for reg int good for goods with zero import
  VMDem0(j,i)       demand for imp int good for goods with zero import
  Xcet(i)           cet fc for reg product
  Rsupply(i)        reg supply of reg product
  LSupply           labor supply
  LMIGrat           labor migration
  adjustL           labor migration adjustment
  KMIGrat           capital migration
  KMIGrat1


*@Income block
  LYincome          labor income
  ALYincome         adjusted labor income
  KYincomeSR          capital income
  KYincomeLR
  TYincome          land income
  YENTincome        enterprise income
  RETearn           Retained earning by enterprises
  YHincome          household income
  DHYincome         disposable income
  HSAVings          household savings
  SAVings           total savings
  INVest            total investment
  YGOVincome        Government income
  INDtax            Indirect business tax
  GRProduct         gross region product

*@Expenditure block
  AHEXPLow          adj. household expenditure
  Qces              ces fc for consumption
  Qdemand           cons demand for composite good
  QRdem0            cons demand for reg goods
  QRdem1
  QRdem2
  QMdem1
  QMdem2
  GOVEXPend         Gov expenditure
  QGOVces           ces for st and loc gov demand
  QGOVdemand        st and loc gov cons
  QGOVRdem0         st and loc gov reg cons
  QGOVRDem1
  QGOVRDem2
  QGOVMDem1
  QGOVMDem2
  QINVces           ces for invest gov demand
  QINVemand         invest gov cons
  QINVRdem0         invest gov reg cons
  QInvRdem1
  QInvRdem2
  QInvMdem1
  QInvMdem2
  Mimports(i)      import
```

```
*@Equilibrium
  COMMequil(i)      comm market equilibrium
  Lequil            labor market equilibrium
  Kequil(i)         cap market equilibrium
  Kequil1
  Tequil(ag)         land market equilibrium;

*###########################################################*
*                                                           *
*      EQUATION DEFINITION                                  *
*                                                           *
*###########################################################*

*All equations are defined following the algebraic structure
*on table 5.1.

  EQZ..            Z       =e= sum(i,SLACK(i)+SLACK2(i));

*@Price block
  NETprice(i)..   PN(i)   =e= PX(i)-sum(j,A(j,i)*P(j))-ibtax(i)*PX(i);
  Price(i)..      P(i)   =e= (PR(i)*R(i)+PM0(i)*M(i))/(R(i)+M(i));
  Price1(i)..     PX(i)  =e= (PR(i)*R(i)+PE0(i)*Exp(i))/(R(i)+Exp(i));

*@Production block
  Ldemand(i)..    LAB(i) =e= alpha(i,"L") *PN(i)*X(i)/PL;
  KdemandSR(i)$(Not Kmobil)..    CAP(i) =e= alpha(i,"K")*PN(i)*X(i)/PK(i);
  KdemandLR(i)$(Kmobil)..    CAP(i) =e= alpha(i,"K")*PN(i)*X(i)/PKL;
  Tdemand(ag)..   LAND(ag)=e= alpha(ag,"T") *PN(ag)*X(ag)/PT(ag);
  TLdem..         TLAB  =e= Sum(i,LAB(i));
  TKdem..         TCAP  =e= Sum(i,CAP(i));
  LSupply ..      LS    =e= LS0;
  LMIGrat ..      LMIG  =e= etaL*LS0*LOG(PL/PLROC0);
  adjustL..       adjL  =e= (LS0+LMIg)/LS0;
  KMIGrat$(KMobil)..  KMIG  =e=etaK*(SUM(i,K0(i))*LOG(PKL/PKROC0));
  KMIGrat1$(not KMobil).. KMIG       =e= 0;
  VAdemand(i)..    VA(i)+SLACK(i)+SLACK2(i)=e= a0(i)*X(i);
  VAprod1(nag)..  VA(nag)  =e= Ava(nag)*LAB(nag)**alpha(nag,"L")*CAP(nag)**
                                alpha(nag,"K");
  VAprod2(ag)..    VA(ag)     =e= Ava(ag)*LAB(ag)**alpha(ag,"L")*CAP(ag)**
                                alpha(ag,"K")*LAND(ag)**alpha(ag,"T");
  Vdemand(j,i)..   V(j,i)  =e= a(j,i)*X(i);
  Vces(j,i)..      V(j,i)  =e= Av(j,i)*(deltav(j,i)*VM(j,i)
                              **RHOv(j)+(1-deltav(j,i))
                              *VR(j,i)**RHOv(j))**(1/RHOv(j));
  TVdemand(i)..    TV(i)   =e= sum(j,V(i,j));
  VRdem(j,i)$NZV(j,i)..
                   VR(j,i) =e= VM(j,i)*((1-deltav(j,i))/
                                deltav(j,i)*
                                PM0(j)/PR(j))**(1/(1-RHOv(j)));
  VRdem0(j,i)$ZVM(j,i)..   VR(j,i) =e= V(j,i);
  VMdem0(j,i)$ZVM(j,i)..   VM(j,i) =e= 0;
  TVRdemand(i)..   TVR(i)  =e= sum(j,VR(i,j));
  TVMdemand(i)..   TVM(i)  =e= sum(j,VM(i,j));
  Xcet(i)..        X(i)    =e= Ax(i)*(deltax(i)*EXP(i)**RHOx(i)+(1-
                              deltax(i))*R(i)**RHOx(i))
                              **(1/RHOx(i));
  Rsupply(i)..     R(i)    =e= EXP(i)*((1-DELTAx(i))/DELTAx(i)
```

```
                                *PE0(i)/PR(i))**(1/(1-RHOx(i)));
  INDtax..          IBTX    =E= Sum(i,ibtax(i)*X(i));
  GRProduct..       GRP     =e= ALY + KY + TY + IBTX;


*@Income block

*ALY is defined for all labor; LY is defined for original household
  ALYincome..      ALY      =e=  PL*(TLAB+LHHH0+LGOV0);
  LYincome..       LY       =e=  ALY+PLROC0*(SQRT(LMig**2)-LMig)*0.5
                                 -PL*(SQRT(LMig**2)+LMig)*0.5;
  KYincomeSR$(not kmobil)..   KY   =e= sum(i,PK(i)*CAP(i));
  KYincomeLR$(kmobil)..   KY   =e= sum(i,PKL*CAP(i))+PKROC0*(SQRT(KMIG**2)-KMIG)
                                        *0.5-PKL*(SQRT(KMIG**2)+KMIG)*0.5;
  RETearn..        RETENT  =e= retr*KY;
  TYincome..       TY      =e= sum(ag,PT(ag)*LAND(ag));
  YENTincome..     YENT    =e= KY*(1-ktax);
  YHincome ..      YH      =e= ALY*(1-sstax)
                               +TY*(1-ttax)+(YENT-RETENT-et*KY)
                               +REMIT0+adjL*TRGOV0
                               -((SQRT((adjL-1)**2)-(adjL-1))*0.5)
                               *(TY*(1-ttax)+(YENT-RETENT-et*KY)
                               +REMIT0);
  DHYincome ..   DYH   =e= YH *(1-hhtax );
  HSAVings ..    HSAV  =e= mps *YH ;
  SAVings..        SAV     =e= HSAV+RETENT+ROWSAV0;
  INVest..         INV     =e= sum(i,P(i)*QINV(i));
  YGOVincome..     YGOV     =e= Sum(i,ibtax(i)*PX(i)*X(i))
                                +sstax*ALY
                                +ktax*KY+et*KY
                                +ttax*TY
                                +hhtax *YH+GOVBOR0+GOVITR0;
*@Expenditure block
  AHEXPLow..   AHEXP  =e= DYH-HSAV-PL*LHHH0;
  Qdemand(i)..  Q(i)   =e= beta(i)*AHEXP/P(i);
  Qces(i)$NZQ(i).. Q(i)  =e= Aq(i)*(deltaq(i)*QM(i)
               **RHOq(i)+(1-deltaq(i))*QR(i)**RHOq(i))
                                  **(1/RHOq(i));
  QRdem0(i)$NZQ(i).. QR(i) =e= QM(i)*((1-deltaq(i))/deltaq(i)
                            *PM0(i)/PR(i))**(1/(1-RHOq(i)));
  QRdem1(i)$ZQM(i).. QM(i) =e= 0;
  QMdem1(i)$ZQM(i).. QR(i) =e= Q(i);
  QRdem2(i)$ZQR(i).. QR(i) =e= 0;
  QMdem2(i)$ZQR(i).. QM(i) =e= Q(i);
  GOVEXPend..        GOVEXP     =e= sum(i,P(i)*QGOV(i))+adjL*
                                   TRGOV0+PL*LGOV0+GOVITR0;
  QGOVdemand(i)..   QGOV(i)   =e= QGOV0(i);
  QGOVces(i)$NZGOV(i)..     QGOV(i)  =e= Agov(i)*(deltagov(i)
                               *QGOVM(i)**RHOgov(i)+(1-deltagov(i))
                               *QGOVR(i)**RHOgov(i))**(1/RHOgov(i));
  QGOVRdem0(i)$NZGOV(i)..   QGOVR(i) =e=QGOVM(i)*((1-deltagov(i))
                 /deltagov(i)*PM0(i)/PR(i))**(1/(1-RHOgov(i)));
  QGOVRdem1(i)$ZGOVM(i)..   QGOVM(i) =e= 0;
  QGOVMdem1(i)$ZGOVM(i)..   QGOVR(i) =e= QGOV(i);
  QGOVRdem2(i)$ZGOVR(i)..   QGOVR(i) =e= 0;
  QGOVMdem2(i)$ZGOVR(i)..   QGOVM(i) =e= QGOV(i);
  QINVemand(i)..   QINV(i)  =e= QINV0(i);
```

```
  QINVces(i)$NZInv(i)..   QINV(i) =e=Ainv(i)*(deltainv(i)*QINVM(i)
                                  **RHOinv(i)+(1-
deltainv(i))*QINVR(i)**RHOinv(i))
                                  **(1/RHOinv(i));
  QINVRdem0(i)$NZInv(i).. QINVR(i)=e= QINVM(i)*((1-deltainv(i))
                         /deltainv(i)*PM0(i)/PR(i))**(1/(1-RHOinv(i)));
  QInvRDem1(i)$ZInvM(i).. QInvM(i)=e= 0;
  QInvMDem1(i)$ZInvM(i).. QInvR(i)=e= QInv(i);
  QInvRDem2(i)$ZInvR(i).. QInvR(i)=e= 0;
  QInvMDem2(i)$ZInvR(i).. QInvM(i)=e= QInv(i);
  Mimports(i)..    M(i)     =e= TVM(i)+QM(i)+QGOVM(i)+QINVM(i);

*@Equilibrium
  COMMequil(i).. X(i)+M(i)=e=TV(i)+Q(i)+QGOV(i)+QINV(i)+EXP(i);
  Lequil..          sum(i,LAB(i))+LHHH0+LGOV0 =e= LS0+LMIG;
  Kequil1$(KMobil)..        KMig =e= Sum(i,CAP(i)-KS0(i));
  Kequil(i)$(not KMobil).. CAP(i)          =e= KS0(i);
  Tequil(ag)..      LAND(ag)  =e= T0(ag);

*###########################################################*
*                                                           *
*      INITIALIZATION OR STARTING VALUES                    *
*                                                           *
*###########################################################*
*@Price block                  *@Income block
  PL.L       =PL0       ;
  PKL.L       =1;
  PK.L(i)   =PK0(i)     ;
  PT.L(ag)    =PT0(ag)    ;          HSAV.L   =HSAV0           ;
  PR.L(i)    =PR0(i)    ;          YGOV.L      =YGOV0        ;
  P.L(i)     =P0(i)     ;
  PX.L(i)   = PX0(i)    ;
  PN.L(i)      = PX0(i)-sum(j,A(j,i)*P0(j))-ibtax(i)*PX0(i);

*@Production block
  SLACK.L(i) =0;              SLACK2.L(i)   =0;
  LAB.L(i)  =L0(i)    ;     INV.L      =INV0;
  CAP.L(i) =K0(i)   ;       GRP.L      =GRP0;
*                            *@Expenditure block
  LAND.L(ag) =T0(ag)    ;

  LS.L     = LS0;
  LMIG.L   =0;
  KMIG.L    =0;
  VA.L(i)   =VA0(i)   ;
  VM.L(j,i)  =VM0(j,i)  ;
  VR.L(j,i)  =VR0(j,i)  ;          QM.L(i) =QM0(i)   ;
  V.L(j,i)   =V0(j,i)   ;
  TVM.L(i)   =TVM0(i)   ;
  TVR.L(i)   =TVR0(i)   ;          GOVEXP.L    =GOVEXP0      ;
  TV.L(i)    =TV0(i)    ;          QGOV.L(i)  =QGOV0(i)    ;
  R.L(i)    =R0(i)    ;          QGOVM.L(i) =QGOVM0(i)   ;
  X.L(i)     =X0(i)     ;          QGOVR.L(i) =QGOVR0(i)   ;
  EXP.L(i)  =E0(i)    ;
  M.L(i)    =M0(i)    ;
  Q.L(i)     =beta(i)*HEXP0/PX0(i);
  QR.L(i) =QR0(i)   ;
```

```
*@Income block
  LY.L       =LY0       ;
  KY.L      =KY0        ;
  TY.L       =TY0       ;
  adjL.L      =1     ;
  YENT.L     =YENT0     ;
  YH.L     =YH0     ;
  SAV.L      =SAV0      ;
  DYH.L    =DYH0    ;
  QINVM.L(i) =QINVM0(i) ;
  QINVR.L(i) =QINVR0(i) ;
  QINV.L(i)  =QINV0(i)  ;

*##########################################################*
*                                                          *
*      VARIABLE BOUNDS                                     *
*                                                          *
*##########################################################*

  PL.LO       = 0.000001;
  PT.LO(ag)    = 0.000001;
  PK.LO(i)   =  0.000001;
  PR.LO(i)    = 0.000001;
  PN.LO(i)   = 0.000001;
  P.LO(i)     = 0.000001;
  R.LO(i)    = 0.000001;
  PX.LO(i)   =  0.000001;
  QM.LO(i)$(QM0(i) ne 0)  = 0.000001;
  QR.LO(i)$(QR0(i) ne 0)  = 0.000001;
  Q.LO(i)$(Q0(i) ne 0)    = 0.000001;
  QM.LO(i)$(QM0(i) eq 0)  = 0;
  QR.LO(i)$(QR0(i) eq 0)  = 0;
  Q.LO(i)$(Q0(i) eq 0)    = 0;
  VR.LO(i,j)$(VR0(i,j) ne 0)  = 0.000001;
  VM.LO(i,j)$(VM0(i,j) ne 0)  = 0.000001;
  V.LO(i,j)$(V0(i,j) ne 0)    = 0.000001;
  VR.LO(i,j)$(VR0(i,j) eq 0)  = 0;
  VM.LO(i,j)$(VM0(i,j) eq 0)  = 0;
  V.LO(i,j)$(V0(i,j) eq 0)    = 0;



OPTIONS ITERLIM=5000, LIMROW=0, LIMCOL=0, SOLPRINT=OFF;
*-- MODEL DEFINITION AND SOLVE STATEMENT

MODEL OKLAHOMA /ALL/;
SOLVE OKLAHOMA MINIMIZING Z USING NLP;

*-- SOLUTION DISPLAY STATEMENT
*-- SOLUTION VALUES OF ENDOGENOUS VARIABLES


PARAMETER VALID VARIABLES FOR THE VALIDATION OF THE MODEL;
VALID(i,"SLACK1") = SLACK.L(i);
VALID(i,"SLACK2") = SLACK2.L(i);
VALID(i,"PR") = PR.L(i);
```

```
VALID(i,"P") = P.L(i);
VALID(i,"PN") = PN.L(i);
VALID(i,"PK") = PK.L(i);
VALID(ag,"PT") = PT.L(ag);
VALID(i,"PX") = PX.L(i);
VALID(i,"PE") = PE0(i);
VALID(i,"X") = X.L(i);
VALID(i,"R") = R.L(i);
VALID(i,"EXP") =EXP.L(i);
VALID(i,"M") = M.L(i);
VALID(i,"VA") = VA.L(i);
VALID(i,"LAB") =LAB.L(i);
VALID(i,"CAP") =CAP.L(i);
VALID(ag,"LAND") =LAND.L(ag);
VALID(i,"TVR") =TVR.L(i);
VALID(i,"TVM") =TVM.L(i);
VALID(i,"TV") =TV.L(i);
VALID(i,"Q") =Q.L(i);
VALID(i,"QR") =QR.L(i);
VALID(i,"QM") =QM.L(i);
VALID(i,"QGOV") =QGOV.L(i);
VALID(i,"QGOVR") =QGOVR.L(i);
VALID(i,"QGOVM") =QGOVM.L(i);
VALID(i,"QINV") =QINV.L(i);
VALID(i,"QINVR") =QINVR.L(i);
VALID(i,"QINVM") =QINVM.L(i);

PARAMETER VALID2  -INTERMEDIATE USE MATRIX-;
VALID2(I,"AGR","V")=V.L(I,"AGR");
VALID2(I,"MIN","V")=V.L(I,"MIN");
VALID2(I,"MAN","V")=V.L(I,"MAN");
VALID2(I,"SER","V")=V.L(I,"SER");
VALID2(I,"AGR","VR")=VR.L(I,"AGR");
VALID2(I,"MIN","VR")=VR.L(I,"MIN");
VALID2(I,"MAN","VR")=VR.L(I,"MAN");
VALID2(I,"SER","VR")=VR.L(I,"SER");
VALID2(I,"AGR","VM")=VM.L(I,"AGR");
VALID2(I,"MIN","VM")=VM.L(I,"MIN");
VALID2(I,"MAN","VM")=VM.L(I,"MAN");
VALID2(I,"SER","VM")=VM.L(I,"SER");

PARAMETER VALID3 -VALIDATION OF THE MODEL-;
VALID3("OBJECTIVE") = Z.L;
VALID3("PL") = PL.L;
VALID3("LMIG")=LMIG.L;
VALID3("KMIG")=KMIG.L;
VALID3("TCAP")=TCAP.L;
VALID3("TLAB")=TLAB.L;
VALID3("LS")=LS.L;
VALID3("LMIG")=LMIG.L;
VALID3("ADJL") = ADJL.L;
VALID3("LY")=LY.L;
VALID3("ALY")=ALY.L;
VALID3("KY")=KY.L;
VALID3("TY")=TY.L;
VALID3("YENT") = YENT.L;
VALID3("RETENT")=RETENT.L;
```

```
VALID3("YH")=YH.L;
VALID3("PL") = PL.L;
VALID3("DYH")=DYH.L;
VALID3("HSAV")=HSAV.L;
VALID3("SAV")=SAV.L;
VALID3("INV") = INV.L;
VALID3("YGOV")=YGOV.L;
VALID3("GOVEXP")=GOVEXP.L;
VALID3("IBTX")=IBTX.L;
VALID3("GRP")=GRP.L;
VALID3("AHEMP")=AHEXP.L;

option decimals=3;
DISPLAY VALID,VALID2,VALID3;

*######## SIMULATION ############*

PE0(i)=1.1;
model simul1 /all/;
solve simul1 minimizing z using nlp;
OPTION SOLPRINT=OFF;

*-- SOLUTION DISPLAY STATEMENT
*-- SOLUTION VALUES OF ENDOGENOUS VARIABLES


PARAMETER PRICES MARKET CLEARING PRICES;
PRICES(i,"SLACK1") = SLACK.L(i);
PRICES(i,"SLACK2") = SLACK2.L(i);
PRICES(i,"PR") = PR.L(i);
PRICES(i,"P") = P.L(i);
PRICES(i,"PN") = PN.L(i);
PRICES(i,"PK") = PK.L(i);
PRICES(ag,"PT") = PT.L(ag);
PRICES(i,"PX") = PX.L(i);
PRICES(i,"PE") = PE0(i);

PARAMETER PROD1 MARKET CLEARING PRODUCTION VARIABLES;
PROD1(i,"X") = X.L(i);
PROD1(i,"R") = R.L(i);
PROD1(i,"EXP") =EXP.L(i);
PROD1(i,"M") = M.L(i);
PROD1(i,"VA") = VA.L(i);
PROD1(i,"LAB") =LAB.L(i);
PROD1(i,"CAP") =CAP.L(i);
PROD1(ag,"LAND") =LAND.L(ag);

PARAMETER TRADE1 MARKET CLEARING PRODUCTION VARIABLES;
TRADE1(i,"TVR") =TVR.L(i);
TRADE1(i,"TVM") =TVM.L(i);
TRADE1(i,"TV") =TV.L(i);
TRADE1(i,"Q") =Q.L(i);
TRADE1(i,"QR") =QR.L(i);
TRADE1(i,"QM") =QM.L(i);
TRADE1(i,"QGOV") =QGOV.L(i);
TRADE1(i,"QGOVR") =QGOVR.L(i);
TRADE1(i,"QGOVM") =QGOVM.L(i);
```

```
TRADE1(i,"QINV") =QINV.L(i);
TRADE1(i,"QINVR") =QINVR.L(i);
TRADE1(i,"QINVM") =QINVM.L(i);

PARAMETER PRODUCT2  -PRODUCTION SYSTEMS VARIABLES-;
PRODUCT2(I,"AGR","V")=V.L(I,"AGR");
PRODUCT2(I,"MIN","V")=V.L(I,"MIN");
PRODUCT2(I,"MAN","V")=V.L(I,"MAN");
PRODUCT2(I,"SER","V")=V.L(I,"SER");
PRODUCT2(I,"AGR","VR")=VR.L(I,"AGR");
PRODUCT2(I,"MIN","VR")=VR.L(I,"MIN");
PRODUCT2(I,"MAN","VR")=VR.L(I,"MAN");
PRODUCT2(I,"SER","VR")=VR.L(I,"SER");
PRODUCT2(I,"AGR","VM")=VM.L(I,"AGR");
PRODUCT2(I,"MIN","VM")=VM.L(I,"MIN");
PRODUCT2(I,"MAN","VM")=VM.L(I,"MAN");
PRODUCT2(I,"SER","VM")=VM.L(I,"SER");

PARAMETER OTHER1 MARKET CLEARING VALEUES OF VARIABLES;
OTHER1("OBJECTIVE") = Z.L;
OTHER1("PL") = PL.L;
OTHER1("LMIG")=LMIG.L;
OTHER1("KMIG")=KMIG.L;
OTHER1("TCAP")=TCAP.L;
OTHER1("TLAB")=TLAB.L;
OTHER1("LS")=LS.L;
OTHER1("LMIG")=LMIG.L;
OTHER1("ADJL") = ADJL.L;
OTHER1("LY")=LY.L;
OTHER1("ALY")=ALY.L;
OTHER1("KY")=KY.L;
OTHER1("TY")=TY.L;
OTHER1("YENT") = YENT.L;
OTHER1("RETENT")=RETENT.L;
OTHER1("YH")=YH.L;
OTHER1("PL") = PL.L;
OTHER1("DYH")=DYH.L;
OTHER1("HSAV")=HSAV.L;
OTHER1("SAV")=SAV.L;
OTHER1("INV") = INV.L;
OTHER1("YGOV")=YGOV.L;
OTHER1("GOVEXP")=GOVEXP.L;
OTHER1("IBTX")=IBTX.L;
OTHER1("GRP")=GRP.L;
OTHER1("AHEMP")=AHEXP.L;

option decimals=3;

DISPLAY PROD1, TRADE1,PRODUCT2;

OPTION DECIMALS = 8;
DISPLAY OTHER1, PRICES;

* Parameters AS INDEX WITH 1993=1.000
PARAMETERS
* -- Price block
  IPL               Wage rate index
```

```
  IPK(i)            Rent to  capital index
  IPT(ag)            Land rent index
  IPR(i)            Regional price index
  IP(i)             Composite price index
  IPG               General composite price index
* -- Production block
  IL(i)             Labor demand index
  ITL               Total labor demand index
  ILS               Labor supply index
  IK(i)             capital demand index
  ITK               Total Capital use index
  ITT               Total Land use index
  IT(ag)             Land demand index
  IVA(i)            Value added index
  IX(i)             Output index
  ITVA            Total Value added index
  ITX             Total Output index
  ITE             Total Export index
  ITR             Total Reg. supply index
  ITM             Total Import index
  IVM(j,i)          Imported interm demand index
  IVR(j,i)          Regional interm demand index
  IR(i)             Regional supply index
  IE(i)             Export index
  IM(i)             Import index

* -- Income block

  IYH             Household (in the region) income index
  YHch            Change in hh income
  IDYH            Disposable income index
  IHSAV           Household saving index
  IYGOV             Government revenue index
  NETGOV            Net Revenue for government
  IGRP              Gross region product index
  GRPch             Change in Gross regional product
  CapComp           Capital Compensation
  LandComp          Land Compensation
  Rconsup           Resident angler consumer surplus loss
  NRconsup          NonResident angler consumer surplus loss
* -- Expenditure block
  IAHEXP          adj. Household expenditure index
  IGOVEXP         Government expenditure index
  IQ(i)           Commodity demand index
  IQM(i)          Imported commodity demand index
  IQR(i)          Regional commodity demand index
;

*-- EQUATIONS FOR CALCULATION OF INDEX WITH 1993=1.000

*### Price block
 IPL        = PL.L/PL0;
 IPK(i)     = PK.L(i)/PK0(i);
 IPT(ag) = PT.L(ag)/PT0(ag);
 IPR(i)     = PR.L(i)/PR0(i);
 IP(i)      = P.L(i)/P0(i);
 IPG        =SUM(i, (PR.L(i)*R0(i)+PM0(i)*M0(i))/(R0(i)+M0(i)))/4;
```

```
*#*  Production block
  IL(i)     = LAB.L(i)/L0(i);
  ITL       = (Sum(i,LAB.L(i))+(LHHH0+LGOV0))
                      /(Sum(i,L0(i))+LHHH0+LGOV0);
  ILS       = LS.L /LS0 ;
  IK(i)    = CAP.L(i)/K0(i);
  ITK       = Sum(i,PK.L(i)*CAP.L(i))/Sum(i,K0(i));
  IT("Agr") = LAND.L("Agr")/T0("Agr");
  ITT       = PT.L("Agr")*LAND.L("Agr")/T0("Agr");
  IVA(i)   = VA.L(i)/Va0(i);
  ITVA      = Sum(i,VA.L(i))/Sum(i,Va0(i));
  IX(i)     = X.L(i)/X0(i);
  ITX       =Sum(i,X.L(i))/Sum(i,X0(i));
  ITR       =Sum(i,R.L(i))/Sum(i,R0(i));
  ITM       =Sum(i,M.L(i))/Sum(i,M0(i));
  IVM(j,i)= VM.L(j,i)/VM0(j,i);
  IVR(j,i)= VR.L(j,i)/VR0(j,i);
  IR(i)    = R.L(i)/R0(i);
  IE(i)    = EXP.L(i)/E0(i);
  ITE        =Sum(i,EXP.L(i))/Sum(i,E0(i));
*## Income block
  IYH       = YH.L /YH0 ;
  IDYH      = DYH.L /DYH0 ;
  IHSAV     = HSAV.L /HSAV0 ;
  IGRP      = GRP.L/GRP0;
  GRPch     = GRP.L-GRP0;
*#Expenditure block
  IAHEXP    = AHEXP.L /HEXP0 ;
  IQ(i)    = Q.L(i)/Q0(i);
  IQM(i)   = QM.L(i)/QM0(i);
  IQR(i)   = QR.L(i)/QR0(i);
  IM(i)    = M.L(i)/M0(i);
  YHch      = YH.L -adjL.L*YH0 ;
  IYGOV     = YGOV.L/YGOV0;
  IGOVEXP   = GOVEXP.L/GOVEXP0;
  NETGOV    = YGOV.L-GOVEXP.L;

*##- SOLUTION VALUES OF INDEX
option decimals=5;

PARAMETER INDEX INDEXES FOR THE SIMULATION;
INDEX(I,"IPR")=IPR(I);
INDEX(I,"IX")=IX(I);
INDEX(I,"IE")=IE(I);
INDEX(I,"IL")=IL(I);
INDEX(I,"IK")=IK(I);
INDEX(I,"IPK")=IPK(I);
INDEX(ag,"IPT")=IPT(ag);
INDEX(ag,"IT")=IT(ag);
INDEX(I,"IVA")=IVA(I);
INDEX(I,"IR")=IR(I);
INDEX(I,"IM")=IM(I);
INDEX(I,"IQ")=IQ(I);
INDEX(I,"IQR")=IQR(I);
INDEX(I,"IQM")=IQM(I);
INDEX(I,"IPR")=IPR(I);
INDEX(I,"IPR")=IPR(I);
```

```
DISPLAY INDEX;
DISPLAY ITX,ITE,ITL,IPL,
         ITK,ITT,
         IGRP,GRPch,ITVA,ITR,ITM, YHch,
         IYH, IYGOV,IGOVEXP,NETGOV,
         ILS,IDYH,IHSAV,IAHEXP,
         IVM,IVR;

DISPLAY IGRP,IPG,IYH,ITE,ITM;
```
1

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Table 4.1

Competitive CGE Model Equations

	Equation	Description Equations	No. of Equations	Endogenous Variables	Exogenous Variables	Parameters
0.	$Z = \sum_i (SLACK_i + SLACK2_i)$	*Objective function*	*1*	Z $SLACK_i$ $SLACK2_i$		
PRODUCTION SYSTEM						
1.	$LAB_i = \frac{\alpha_i^L PN_i X_i}{PL}$	*Labor demand*	*n*	LAB_i PN_i PL X_i		α_i^L
2a.	$CAP_i = \frac{\alpha_i^K PN_i X_i}{PK_i}$	*Capital demand SR*	*n*	CAP_i PN_i PK_i X_i		α_i^K
2b.	$CAP_i = \frac{\alpha_i^K PN_i X_i}{PK}$	*Capital demand LR*	*n*	CAP_I PN_i PK X_i		α_i^K
3.	$LAND_{ag} = \frac{\alpha_{ag}^T PN_{ag} X_{ag}}{PT_{ag}}$	*Land demand*	*n*	$LAND_{ag}$ PN_{ag} PT_{ag} X_{ag}		α_i^T
4.	$VA_i = a_{0i} X_i$	*Composite factor demand*	*n*	VA_i X_i		a_{0i}
5.	$V_{ji} = a_{ji} X_i$	*Intermediate input demand*	*n* x *n*	V_{ji} X_i		a_{ji}

Table 4.1 (Continued)

	Equation	Description Equations	No. of Equations	Endogenous Variables	Exogenous Variables	Parameters
6a.	$VA_{ag} = \phi_{ag}^{VA} LAB_{ag}^{\alpha_{ag}^{L}} CAP_{ag}^{\alpha_{ag}^{K}} LAND_{ag}^{\alpha_{ag}^{T}}$	*Net product production function sector with land*	*1*	$VA_{ag}\ LAB_{ag}\ CAP_{ag}\ LAND_{ag}$		$\phi_{ag}^{VA}\ \alpha_{ag}^{L}\ \alpha_{ag}^{K}\ \alpha_{ag}^{T}$
6b.	$VA_{nag} = \phi_{nag}^{VA} LAB_{nag}^{\alpha_{nag}^{L}} CAP_{nag}^{\alpha_{nag}^{K}}$	*Net product production function sector without land*	*n-1*	$VA_{nag}\ LAB_{nag}\ CAP_{nag}$		$\phi_{nag}^{VA}\ \alpha_{nag}^{L}\ \alpha_{nag}^{K}$
7.	$\mathrm{V}_{\mathrm{ji}} = \phi_{\mathrm{ji}}^{\mathrm{V}} \left[\delta_{\mathrm{ji}}^{\mathrm{V}} \mathrm{VM}_{ji}^{\rho_{\mathrm{j}}^{\mathrm{V}}} + (1-\delta_{\mathrm{ji}}^{\mathrm{V}}) \mathrm{VR}_{ji}^{\rho_{\mathrm{j}}^{\mathrm{V}}} \right]^{\frac{1}{\rho_{\mathrm{j}}^{\mathrm{V}}}},\ \sigma_{\mathrm{j}}^{\mathrm{v}} = \frac{1}{1-\rho_{j}^{v}}$	*CES for intermediate input demand*	n x n	$V_{ji}\ VM_{ji}\ VR_{ji}$		$\phi_{ji}^{V}\ \delta_{ji}^{V}\ \rho_{j}^{V}\ \sigma_{j}^{V}$
8.	$TV_i = \sum_j V_{ij}$	*Total composite intermediate demand*	n	$TV_i\ V_{ij}$		
9.	$VR_{ji} = VM_{ji} \left[\left(\frac{1-\delta_{ji}^{V}}{\delta_{ji}^{V}} \right) \left(\frac{PM0_j}{PR_j} \right) \right]^{\sigma_j^V}$	*Regional produced intermediate input demand*	n^2	$VM_{ji}\ VR_{ji}\ PR_j$	$PM0_j$	$\delta_{ji}^{V}\ \sigma_{j}^{V}$
10.	$TVR_i = \sum_j VR_{ji}$	*Total intermediate regional demand*	n	$TVR_i\ VR_{ji}$		
11.	$TVM_i = \sum_j VM_{ji}$	*Total intermediate imported demand*	n	$TVM_i\ VM_{ji}$		

Table 4.1 (Continued)

Equation	Description Equations	No. of Equations	Endogenous Variables	Exogenous Variables	Parameters
12. $X_i = \phi_i^x \left[\delta_i^x EXP_i^{\rho_i^x} + (1-\delta_i^x) R_i^{\rho_i^x} \right]^{\frac{1}{\rho_i^x}}, \sigma_i^X = \frac{1}{\rho_i^X - 1}$	*CET for regional and export markets*	*n*	X_i EXP_i R_i		ϕ_i^x δ_i^x ρ_i^x σ_i^x
13. $R_i = EXP_i \left[\left(\frac{1-\delta_i^x}{\delta_i^x} \right) \left(\frac{PE0_i}{PR_i} \right) \right]^{-\sigma_i^x}$	*Regional supply for regional demand*	*n*	R_i EXP_i PR_i	$PE0_i$	ϕ_i^x δ_i^x σ_i^x
<u>COMMODITY MARKETS</u>					
14. $Q_i = \left(\frac{\beta_i}{P_i} \cdot AHEXP \right)$	*Composite household demand*	*n*	Q_i P_i $AHEXP_i$		β_i
15. $Q_i = \phi_i^Q \left[\delta_i^Q QM_i^{\rho_i^Q} + (1-\delta_i^Q) QR_i^{\rho_i^Q} \right]^{\frac{1}{\rho_i^Q}}, \sigma_i^Q = \frac{1}{1-\rho_i^Q}$	*CES for household demand*	*n*	Q_i QM_i QR_i		ϕ_i^Q δ_i^Q ρ_i^Q σ_i^Q
16. $QR_i = QM_i \left[\left(\frac{1-\delta_i^Q}{\delta_i^Q} \right) \left(\frac{PM0_i}{PR_i} \right) \right]^{\frac{1}{1-\rho_i^Q}}$	*Regionally produced household demand*	*n*	QR_i QM_i PR_i	$PM0_j$	δ_i^Q ρ_i^Q
17. $QGOV_i = QGOV0_i$	*State / Local gov commodity demand*	*n*	$QGOV_i$	$QGOV0_i$	

Table 4.1 (Continued)

	Equation	Description Equations	No. of Equations	Endogenous Variables	Exogenous Variables	Parameters
18.	$QGOV_i = \phi_i^{GOV} * \left(\delta_i^{GOV} QGOVM_i^{\rho_i^{GOV}} + \left(1-\delta_i^{GOV}\right)\cdot QGOVR_i^{\rho_i^{GOV}}\right)^{\frac{1}{\rho_i^{GOV}}}$	*CES for government domestic and import demand*	*n*	$QGOV_i$ $QGOVM_i$ $QGOVR_i$		ϕ_i^{GOV} δ_i^{GOV} ρ_i^{GOV} σ_i^{GOV}
19.	$QGOVR_i = QGOVM_i \left[\left(\frac{1-\delta_i^{GOV}}{\delta_i^{GOV}}\right)\cdot\left(\frac{PM0_i}{PR_i}\right)\right]^{\frac{1}{1-\rho_i^{GOV}}}$	*State / Local government demand for regional good*	*n*	$QGOVR_i$ $QGOVM_i$ PR_i	$PM0_i$	δ_i^{GOV} ρ_i^{GOV}
20.	$QINV_i = QINV0_i$	*Investment demand*	*n*	$QINV_i$	$QINV0_i$	
21.	$\mathrm{QINV}_i = \phi_i^{\mathrm{INV}}\left[\delta_i^{\mathrm{INV}}\mathrm{QINVM}_i^{\rho_i^{\mathrm{INV}}} + \left(1-\delta_i^{\mathrm{INV}}\right)\mathrm{QINVR}_i^{\rho_i^{\mathrm{INV}}}\right]^{\frac{1}{\rho_i^{\mathrm{INV}}}}$	*CES for investment domestic and import demand*	*n*	$QINVi$ $QINVMi$ $QINVRi$		ϕ_i^{INV} δ_i^{INV} ρ_i^{INV} σ_i^{INV}
22.	$\mathrm{QINVR}_i = \mathrm{QINVM}_i\left[\left(\frac{1-\delta_i^{\mathrm{INV}}}{\delta_i^{INV}}\right)\cdot\left(\frac{\mathrm{PM0}_i}{\mathrm{PR}_i}\right)\right]^{\frac{1}{1-\rho_i^{\mathrm{INV}}}}$	*Investment demand for regional good*	*n*	QINVR_i QINVM_i PR_i	$PM0_i$	δ_i^{INV} ρ_i^{INV}

Table 4.1 (Continued)

Equation	Description Equations	No. of Equations	Endogenous Variables	Exogenous Variables	Parameters
FACTOR MARKETS					
23. $LS = LS0$	*Household labor supply*	1	LS	$LS0$	
24. $TLAB = \sum_i LAB_i$	*Total labor demand*	1	$TLAB\ LAB_i$		
25. $LY = ALY + \mathrm{PLROC0} \cdot \left(\sqrt{\mathrm{LMIG}^2} - \mathrm{LMIG}\right) \cdot 0.5 - PL \cdot \left(\sqrt{\mathrm{LMIG}^2} + \mathrm{LMIG}\right) \cdot 0.5$	*Labor income*	1	$ALY\ \ PL\ \ LMIG$	$PLROC0$	
26. $LMIG = LS0 \cdot log\left(\frac{PL}{PLROC0}\right) \cdot \eta^L$	*Labor migration*	1	$LMIG\ \ PL$	$LSO\ PLROC0$	η^L
27. $ALY = PL \cdot \left(\sum_i LAB_i + LHHH0 + LGOV0\right)$	*Adjusted labor income*	1	$AYL,\ PL,\ LAB_i$	$LHHH0$ $LGOV0$	
28. $adjL = \frac{LS0 + LMIG}{LS0}$	*Household adjustment factor*	1	$adjL\ \ LMIG$	$LS0$	
29. $TCAP = \sum_i CAP_i$	*Total capital demand*	1	$TCAP\ \ CAP_i$		

Table 4.1 (Continued)

Equation		Description Equations	No. of Equations	Endogenous Variables	Exogenous Variables	Parameters
30a.	$KY = \sum_i PK_i \cdot CAP_i$	*Capital income short-run*	*1*	PK_i KY CAP_i		
30b.	$KY = \sum_i CAP_i PKL + \mathrm{PKROC0} \cdot \left(\sqrt{\mathrm{KMIG}^2} - \mathrm{KMIG} \right) \cdot 0.5 - PKL \cdot \left(\sqrt{\mathrm{KMIG}^2} + \mathrm{KMIG} \right) \cdot 0.5$	*Capital income long run*	*1*	KY $CAPi$ PKL $KMIG$	$PKROC0$	
31a.	$KMIG = 0$	*Capital migration (Short run equilibrium)*	*1*	$KMIG$		
31b.	$KMIG = \sum_i KSO_i \cdot \log\left(\frac{\mathrm{PKL}}{PKROC0} \right) \cdot \eta^{\mathrm{K}}$	*Capital migration (long run equilibrium)*	*1*	$KMIG$ PK	$KS0_i$ $PRROC0$	η^K
32.	$TY = \sum_{ag} LAND_{ag} PT_{ag}$	*Land income*	*1*	TY $LAND_{ga}$ PT_{ag}		
INSTITUTIONAL ACCOUNTS						
33.	$YENT = KY \cdot (1 - Ktax)$	*Enterprise income*	*1*	$YENT$ KY		*ktax*
34.	$RETENT = ret \cdot KY$	*Retained earnings*	*1*	$RETENT$ KY		*retr*

Table 4.1 (Continued)

	Equation	Description Equations	No. of Equations	Endogenous Variables	Exogenous Variables	Parameters
35.	$YH = ALY \cdot (1 - sstax) + TY \cdot (1 - ttax) + (YENT - RETENT\text{-}etKY) + REMIT0 + adjL \cdot TRGOV0 - \left(\sqrt{(adjL - 1)^2} - (adjL - 1) \right) \cdot 0.5 * [TY \cdot (1 - ttax) + (YENT\text{-}RETENT - et \cdot KY) + REMIT0]$	*Household income*	*1*	*YH ALY TY YENT RETENT KY adjL*	*REMIT0 TRGOV0*	*et sstax ttax*
36.	$DYH = YH \cdot (1 - hhtax)$	*Disposable income*	*1*	*DYH YH*		*hhtax*
37.	$HSAV = mps \cdot YH$	*Household saving*	*1*	*HSAV YH*		*mps*
38.	$INV = \sum_i QINV_i P_i$	*Total investment*	*1*	$INV\ QINV_i\ P_I$		
39.	$AHEXP = DYH - HSAV\text{-} PL \cdot LHHH0$	*Household expenditure*	*1*	*AHEXP DYH HSAV PL*	*LHHH0*	
40.	$GRP = LY + KY + TY + IBTX$	*Gross regional product*	*1*	*GRP LY KY TY IBTX*		
41.	$IBTX = \sum_i ibtax_i \cdot X_i$	*Indirect business tax*	*1*	$IBTX\ X_I$		$ibtax_i$
42.	$GOVEXP = \sum_i QGOV_i \cdot P_i + adjL \cdot TRGOV0 + PL \cdot LGOV0 + GOVITR0$	*Government expenditures*	*1*	$QGOV_i$, *GOVEXP* P_i *PL*	*LGOV0 TRGOV0 GOVITR0*	

Table 4.1 (Continued)

Equation	Description Equations	No. of Equations	Endogenous Variables	Exogenous Variables	Parameters
43. $YGOV = \left(\sum_i ibtax_i PX_i X_i\right) + (sstax \cdot ALY)$ $+ (ktax \cdot KY) + et \cdot KY + ttax \cdot TY + hhtax \cdot YH$ $+ GOVBOR0 + GOVITR0 + ROWGOV0$	*Government revenue*	*1*	$YGOV$ PX_i X_i ALY KY TY YH	*GOVBOR0* *GOVITRO* *ROWGOV0*	$ibtax_i$ $sstax$ $ktax$ $ttax$ $hhtax$
44. $SAV = HSAV + RETENT + ROWSAV0$	*Total Saving*	*1*	SAV $HSAV$ $RETENT$	$ROWSAV0$	
EQUILIBRIUM OF MARKETS					
45. $M_i = TVM_i + QM_i + QGOVM_i + QINVM_i$	*Total regional imports*	*n*	M_i TVM_i QM_i $QGOVM_i$ $QINVM_i$		
46. $X_i + M_i = TV_i + Q_i$ $+ QGOV_i + QINV_i + EXP_i$	*Commodity market equilibrium*	*n*	X_i M_i TV_i Q_i $QGOV_i$ $QINV_i$ EXP_i		
47. $\sum_i LAB_i + LHHHO + LGOV0 = LS + LMIG$	*Labor market equilibrium*	*l*	LS $LMIG$ LAB_i	*LHHH0* *LGOV0*	
48a. $CAP_i = KS0_i$	*Capital market equilibrium (short run equilibrium)*	*n*	CAP_i	$KS0_i$	
48b. $\sum_i CAP_i = \sum_i KS0_i + KMIG$	*Capital market equilibrium (long run equilibrium)*	*1*	CAP_i $KMIG$	$KS0_i$	

Table 4.1 (Continued)

Equation	Description Equations	No. of Equations	Endogenous Variables	Exogenous Variables	Parameters
49. $LAND_i = TS0_i$	*Land market equilibrium*	*1*	$LAND_i$	$TS0_i$	
<u>EQUILIBRIUM PRICES</u>					
50. $PN_i = PX_i - \sum_j a_{ji} P_j - ibtax_i PX_i$	*Net price*	*n*	PN_i , PX_i , P_i		a_{ij} , $ibtax_i$
51. $P_i = \frac{PR_i R_i + PM0_i M_i}{R_i + M_i}$	*Composite commodity price*	*n*	P_i PR_i R_i M_i	$PM0_i$	
52. $PX_i = \frac{PR_i R_i + PEO_i EXP_i}{R_i + EXP_i}$	*Composite price faced by producer*	*n*	PR PX_i R_i EXP_i	PEO_i	

WELFARE MEASURE

Compensating Variation:

$$CV_h = \left(\frac{1}{1-\beta_{0h}}\right)\left[\left(AHEXP_h - adjL\sum_j PX_j \gamma_{jh}\right) - \left(adjL_h HEXP0_h - adjL\sum_j P0_j \gamma_{jh}\right) \prod_i \left(\frac{PX_i}{P0_i}\right)^{\beta_{ih}} \left(\frac{PL}{PL0}\right)^{\beta_{0h}}\right], \; ij \varepsilon M, NR,$$

Changes in Compensating Variation by Household h *Income Group*

Equivalent Variation:

$$EV_h = \left(\frac{1}{1-\beta_{0h}}\right)\left[\left(AHEXP_h - adjL\sum_j PX_j \gamma_{jh}\right) \prod_i \left(\frac{P0_i}{PX_i}\right)^{\beta_{ih}} \left(\frac{PL0}{PL}\right)^{\beta_{0h}} - \left(adjL_h HEXP0_h\right) - adjL\sum_j P0_j \gamma_j\right], \; ij \varepsilon M, NR,$$

Changes in Equivalent Variation by Household h *Income Group*

A Guide to the GAMS-input-file

This is a user's guide to the GAMS-input-file of the regional CGE model described throughout section 3. It transforms the mathematical program specified in section 4.1 into an executable computer program based in GAMS (the acronym stands for General Algebraic Modeling System). To make this chapter self-contained we reproduce some introductory material on the construction of a CGE-model in GAMS, however we recommend the reading of GAMS tutorial (Brooke et al., Chap. 2).

We provide the complete GAMS-PROGRAM in this link {click here to download CRS2.GMS}. It's clearly only a prototype and the numerical values of the parameters and initial values were explained in section 3.2. For a collection of models with similar specification, but somehow more sophisticated, we offer the following link: {Oklahoma State University; Department of Agricultural Economics; RCGE}

In what follows, the GAMS-Program input file is presented and explained in its major components.

Index sets

The application starts with a definition of the main index sets and subsets. A set declaration consists of declaring and specifying the index to be used. Sets should be declared before their subsets. Every declaration consists of a logical name, a label field, followed by a list of elements of the index set. As such, it is the same as indexes used in the equations of the model. They correspond to the subscription notation of table 4.2{click here to go to table 4.2}.

```
$TITLE REGIONAL CGE MODEL FOR OKLAHOMA (1993)(CRS.GMS)
$OFFSYMLIST OFFSYMXREF OFFUPPER
SETS
i       Sectors             /Agr  agriculture
                             Min  mining
                             Man  manufacture
                    SER services/
ag(i)   Agricultural sectors            / AGR/
nag(i)  Nonagricultural market sectors  / MIN, SER, MAN/
f       Factors          /L       labor,  K       capital,  T
land/
fl(f)   Factors not land / L, K/
ALIAS(i,j);
```

A $-sign at the beginning of the program, is used for special commands, i.e., $TITLE, where we introduce the title of the model. All GAMS-statements end with a semicolon. The ALIAS-statement defines an alternative name for an index set (subscript).

BASE YEAR DATA

Base year variables are based upon the Social Accounting Matrix (SAM) and are distinguished by "0" as a suffix in their names, i.e., L0(i) states base year labor. GAMS requires a DECLARATION and ASSIGNMENT of each variable or parameter. Here, we declare the base year variables as parameters. GAMS offers flexible arrangements for introducing the parameters (variables). We recommend first to declare (initiate) all the parameters, then use tables to enter data and finally, assign the values.

To provide better readability, parameters are declared by blocks: prices, production, income and expenditure blocks. In GAMS, comment-lines and text in general are introduce by "*" in the first column of a row.

```
*#####-- DECLARATION OF BASE YEAR VARIABLES (AS PARAMENTERS)

PARAMETERS


*@Price block
  PL0             Wage rate
  PLROC0          Wage rate of rest-of-country
  PKROC0          Cap rate of rest-of-country
  PK0(i)          cap rate
  PT0(ag)         Land rent
  PE0(i)          Export price
  PM0(i)          Import price
  PR0(i)          Reg price
  P0(i)           Composite price
  PN0               Net output price or value-added price of
sector i
  PX0(i)      Composite price face for producers

*@Production block
  L0(i)           Labor demand
  LS0             Labor supply by hh
  TLS0            Total labor supply
  LHHH0           Labor employed by household group
  LGOV0           Labor employed by gov
  K0(i)           capital demand
  T0(i)           Land demand
  KS0             Supply of pri capital
  TKS0            Total pri capital supply
  TS0             Supply of land
  VA0(i)          Value added
  V0(j,i)         Composite intermediate good demand
  TV0(i)          Composite intermediate good total demand
  VR0(j,i)        Reg int good demand
  VM0(j,i)        Imported int good demand
  TVR0(i)         Reg int good total demand
  TVM0(i)         Imported int good total demand
  IBT0(I)     Indirect business taxes
  X0(i)           Sector output
```

```
E0(i)           Export of reg product
M0(i)           Import
R0(i)           Reg supply of reg product

*@Income block
LY0             Labor income
KY0             capital income
TY0             Land income
YENT0           Gross Enterprise income
YH0             Household income
DYH0            Disposable hh income
HSAV0           Household saving
SAV0            Total saving
ROWSAV0         Saving from rest-of-world
TRGOV0          Gov transfer to hh
REMIT0              Remittance from outside the region to household
YGOV0           Gov revenue
ENTY0           Enterprise income distrib to hhs
GOVITR0         Inter gov transfer
GOVBOR0         Government Borrowing
GRP0            Gross regional product

*@Expenditure block
HEXP0           Household expend
QR0(i)          Demand for reg consump good
QM0(i)          Demand for imp consump good
Q0(i)           Demand for comp consump good
GOVEXP0         government expenditure
QGOVR0(i)       government demand for reg good
QGOVM0(i)       government demand for imported good
QGOV0(i)        government demand for comp good
QInvR0(i)       Invest demand for reg good
QInvM0(i)       Invest demand for imported good
QInv0(i)        Invest demand for comp good
INV0            Total invest
```

The following variables are defined as "logical variables". A logical variable takes the value of 1 if the condition stated is true and "0" if not. We use these variables when defining an equation or for assigning value to a particular variable depending on the "true" or "false" condition of a specific condition, i.e., variable NZV takes the value of “1” if both regional and imported intermediate input are used, according to the following graph.

```
*************************************
*Regional         x     x     0     0     0=zero, x=not zero
*Import           x     0     x     0
*
*NZV              T     F     F     F     T=TRUE,  F=FALSE
*ZVR              F     F     T     F
*ZVM              F     T     F     T
*************************************
```

```
ZVM(i,J)  non imported intermediate demand with-or-without regional interm. demand
ZVR(i,J)  only imported intermediate demand
NZV(i,J)  both imported intermediate demand and regional demand

ZQM(i)    non imported final demand and either none or some regional final demand for household
```

```
ZQR(i)    only imported final demand for household
NZQ(i)    both imported final demand and regional final demand for households

ZGOVM(i)
ZGOVR(i)
NZGOV(i)

ZInvM(i)
ZInvR(i)
NZInv(i)
```

DECLARATION OF PARAMETERS TO BE CALIBRATED

These parameters are those specified in Table 4.5. {Click here to see table 4.5}. They are declared in the following segment of the application but they will be initialized later.

```
*#####-- DECLARATION OF PARAMETERS TO BE CALIBRATED.

PARAMETERS

*This parameters are those specified in Table 5.5.

*@Production block
  a0(i)              composite value added req per unit of output i
  a(j,i)          req of interm good j per unit of good i
  Alpha(i,f)      value added share param
  Ava(i)          value added shift param
  RHOv(i)         interm input subs param
  deltav1(j,i)
  deltav(j,i)     interm input share param
  Av(j,i)         interm input shift param
  RHOx(i)         output transformation param
  deltax1(i)
  deltax(i)       output share param
  Ax(i)           output shift param

*@Income block
  ktax            capital tax rate
  sstax           factor income tax rate for labor
  ttax            factor income tax rate for land
  retr            rate of retained earnings fr ent inc
  et          enterprise tax rate
  hhtax           income tax rate for hh
  ltr             Household Income Transfer Coefficient
  mps             saving rate
  ibtax(i)        indirect business tax
  beta(i)         param calc fr elast of comm demand wrt inc

*@Expenditure block
  RHOq            consumer demand subs param
  deltaq1(i)
  deltaq(i)       consumer demand share param
  Aq(i)           consumer demand constant eff param
  RHOgov          gov demand subs param
  deltagov1
  deltagov        gov demand share param
  Agov            gov demand constant eff param
  RHOinv          inv gov demand subs param
```

```
    deltainv1
    deltainv       inv gov demand share param
    Ainv           inv gov demand constant eff param
    ;
```

DATA

Data comes from our SAM (Table 2.1). {Click here to see table 2.1} You should note that values from our SAM are scaled to millions of dollars instead of thousands. Though the scaling of our data is not a “must” for solving the model, we strongly recommend scaling. Scaling problems have been found to create more serious problems in more disaggregated models.

```
Table      IOR(i,j) Input-output regional matrix

AGR   MIN          MAN        SER
AGR   675.798     8.115       863.991          34.800
MIN   123.47      2180.942    1258.117    881.343
MAN   159.671     1390.701    3594.97          3953.2
SER   381.542     1317.332    5272.186    9752.027


;
Table    IOM(i,j)   Input-output import matrix

      AGR   MIN          MAN    SER
AGR   579.870     5.160 378.422     41.300
MIN   11.850      1274.869    311.094     385.272
MAN   446.830     450.977     8835.472    2750.345
SER   155.160     458.802     1886.710    4188.764

;

Table VAD(i,f)  Value added matrix
            L           K               T
AGR        433.242    571.360        709.066
MIN       1622.806    2713.109
MAN       7577.427    4025.159
SER      20767.388    12042.708
;
```

```
Table HHCONR(i,*)  Household consumption demand for regional goods

        HOUSE
AGR      147.210
MIN     1587.998
MAN     2656.085
SER     30727.366
;

Table HHCONM(i,*)  Household consumption demand for imported goods
        HOUSE
AGR     181.550
MIN     141.662
MAN    5713.705
SER    9510.103
;

Table  GOVCONR(i,*)    Government  consumption  demand  for regional goods
```

```
                 GOV
      AGR      12.863
      MIN     231.250
      MAN    1854.066
      SER    1477.995
      ;

      Table   GOVCONM(i,*)   Government   consumption   demand   for
imported goods
                 GOV
      AGR     20.097
      MIN     29.912
      MAN    823.846
      SER    542.893
      ;

      Table FYDIST(*,f)  Factor income distribution to households
                       L          K             T
      HH          31363.057         0.00          683.300
      ;

      TABLE ParamA(*,i)   BASE YEAR VALUES FOR INDUSTRY
                          AGR                    MIN               MAN
SER

      PT0                    1.00               1.00              1.00
1.00
      PK0                    1.00               1.00              1.00
1.00
      PR0                    1.00               1.00              1.00
1.00
      P0                     1.00               1.00              1.00
1.00
      PM0                    1.00               1.00              1.00
1.00
      PE0                    1.00               1.00              1.00
1.00
      X0                   4344.160        12089.784       34190.427
59115.190
      R0               1752.557       6282.217       18360.150
49486.101
      E0               2591.603       5807.567       15830.277
9629.089
      M0               1216.846      2170.418         21475.978
16920.731
      IBT0              96.301  666.971         186.879
4318.043
      QINVR0            9.780     19.097        4751.457
557.653
      QINVM0            10.447    15.759        2454.803
178.299
      SIGMAp              1.00001          1.00001          1.00001
1.00001
      SIGMAv              1.42              0.5                3.55
4
      SIGMAx               3.90              2.90              2.90
0.70
      SIGMAq               1.42              0.50              3.55
2.00
      SIGMAgov            1.42              0.50              3.55
2.00
```

```
      SIGMAinv           1.42             0.50             3.55
2.00

      ;

      TABLE ParamB(f,*)   BASE YEAR VALUES FOR FACTORS
           WAGE0   WAGEROC0     FTAX0         RETENT0       CAP0
CAPROC0
      L     1.0       1.0     6126.715           0
      K                        -1006.686      9077.096           1
1
      T                        25.766             0
      ;

      TABLE ParamC (*,*)   BASE YEAR VALUES FOR HOUSEHOLD GROUPS

                   HTAX0           HSAV0           TRGOV0
      HOUSE        6976.571    -3869.320        11490.516

      +           REMIT0        ENTYDIS0
      HOUSE       760.824       9582.303
      ;

      TABLE ParamD(g,*)   BASE YEAR VALUES FOR GOVTS
                   BOR0         GOVDR0    GOVDM0
      GOV            0.0       3576.174   1416.748
      ;
      SCALAR LHHH0      Labor used by household /  107.070/;
      SCALAR LGOV0      Labor used by government      /6981.839/;
      SCALAR GOVITR0    Inter-government transfers    /8477.813/;
      SCALAR YENT0      Enterprise income             /20359.022/;
      SCALAR ENTTAX0    Enterprise taxes              /1699.623/;
      SCALAR ROWGOV0    Rest  of  world  transfers  to  government
/4375.094/;
      SCALAR ROWSAV0    Saving from ROW               /2789.519/;
      SCALAR QINVMSUM0  Investment  demand  for  imported  goods  /
2659.308/;
      SCALAR etaL       Labor migration elasticity     /      .92
/;
      SCALAR etaK       Capital migration elasticity   /      .92
/;
      Scalar KMobil     Capital Mobility              /       1.0
/;
```

ASSIGNING VALUES: Initialization of Parameters

Here, we assign a value to each of the base year variables declared previously. This assigning of values should correspond to our SAM.

```
*@Production block
  L0(i)        =VAD(i,"L");
  K0(i)        =VAD(i,"K");
  T0(i)        =VAD(i,"T");
  VA0(i)       =sum(f,VAD(i,f));
  V0(j,i)      =IOR(j,i)+IOM(j,i);
  TV0(i)       =sum(j,V0(i,j));
  VM0(j,i)     =IOM(j,i);
  VR0(j,i)     =IOR(j,i);
  TVM0(i)      =sum(j,VM0(i,j));
  TVR0(i)      =sum(j,VR0(i,j));
  LHHH0        =LHHH0;
```

```
        LGOV0          =LGOV0;
        LS0            =sum(i,VAD(i,"L"))+LHHH0+LGOV0;
        X0(i)          =ParamA("X0",i);
        E0(i)          =ParamA("E0",i);
        R0(i)          =ParamA("R0",i);
        KS0(i)         =VAD(i,"K");
        TKS0           =sum(i,KS0(i));
        TS0(i)         =VAD(i,"T");
        IBT0(I)        =PARAMA("IBT0",I);

      *@Income block
        TRGOV0         =ParamC ("HOUSE","TRGOV0");
        LY0            =sum(i,VAD(i,"L"))+LHHH0+LGOV0;
        KY0            =sum(i,VAD(i,"K"));
        TY0            =sum(i,VAD(i,"T"));
        YENT0          =YENT0;
        REMIT0         =ParamC ("HOUSE","REMIT0");
        YH0
=sum(f,FYDIST("HH",f))+ParamC("HOUSE","ENTYDis0")+TRGOV0
                        +REMIT0;
        DYH0           =YH0 -ParamC ("HOUSE","HTAX0");
        HSAV0          =ParamC ("HOUSE","HSAV0");
        HEXP0          =DYH0-HSAV0-LHHH0;
        SAV0                         =ParamB("K","RETENT0")+   ParamC
("HOUSE","HSAV0")+ROWSAV0;
        ROWSAV0        =ROWSAV0;
        YGOV0
=sum(i,ParamA("IBT0",i))+sum(f,ParamB(f,"FTAX0"))

+ParamC("HOUSE","HTAX0")+ENTTAX0+ROWGOV0+GOVITR0;
        ENTY0          =ParamC("HOUSE","ENTYDis0");
        GOVBOR0         =ParamD("GOV","BOR0");
        GRP0           =LY0+KY0+TY0+sum(i,ParamA("IBT0",i));

      *@Expenditure block
        QR0(i)         =HHCONR(i,"HOUSE");
        QM0(i)         =HHCONM(i,"HOUSE");
        Q0(i)          =QM0(i)+QR0(i);
        GOVEXP0
=ParamD("GOV","GOVDR0")+ParamD("GOV","GOVDM0")
                        +ParamC("HOUSE","TRGOV0")+LGOV0+GOVITR0;
        QGOVR0(i)      =GOVCONR(i,"GOV");
        QGOVM0(i)      =GOVCONM(i,"GOV");
        QGOV0(i)       =QGOVM0(i)+QGOVR0(i);
        QINVR0(i)      =ParamA("QINVR0",i);
        QINVM0(i)      =ParamA("QINVM0",i);
        QINV0(i)       =QINVM0(i)+QINVR0(i);
        INV0           =sum(i,QINV0(i));
        M0(i)          =ParamA("M0",i);

      *@Price block
        PL0            =ParamB("L","WAGE0");
        PK0(i)         =ParamA("PK0",i);
        PLROC0         =ParamB("L","WAGEROC0");
        PKROC0         =ParamB("K","CAPROC0");
        PT0(ag)         =ParamA("PT0",ag);
        PE0(i)         =ParamA("PE0",i);
        PM0(i)         =ParamA("PM0",i);
        PR0(i)        =ParamA("PR0",i);
        P0(i)          =ParamA("P0",i);
        PX0(i)        =(PR0(i)*R0(i)+PM0(i)*M0(i))/(R0(i)+M0(i));

      *----------------------------------------------------
      * Regional     x  x  0  0     0=zero, x=not zero
```

```
* Import       x  0  x  0
*
* NZV          T  F  F  F     T=True, F=False
* ZVR          F  F  T  F
* ZVM          F  T  F  T
*-----------------------------------------------------

  ZVM(i,j)     =(VM0(i,j) eq 0);
  ZVR(i,j)     =(VR0(i,j) eq 0) and (VM0(i,j) ne 0);
  NZV(i,j)     =(VR0(i,j) ne 0) and (VM0(i,j) ne 0);

  ZQM(i)    =(QM0(i) eq 0);
  ZQR(i)    =(QR0(i) eq 0) and (QM0(i) ne 0);
  NZQ(i)    =(QR0(i) ne 0) and (QM0(i) ne 0);

  ZGOVM(i)      =(QGOVM0(i) eq 0);
  ZGOVR(i)      =(QGOVR0(i) eq 0) and (QGOVM0(i) ne 0);
  NZGOV(i)      =(QGOVR0(i) ne 0) and (QGOVM0(i) ne 0);

  ZInvM(i)     =(QInvM0(i) eq 0);
  ZInvR(i)     =(QInvR0(i) eq 0) and (QInvM0(i) ne 0);
  NZInv(i)     =(QInvR0(i) ne 0) and (QInvM0(i) ne 0);
```

So far, we have assigned values to our base year variables (parameters). Has GAMS read the assignments correctly? Next, we define new parameter to check for accuracy of our assignment statements. If correct, we should get our SAM and a block of unity prices. Though, the DISPLAY statement of GAMS allows the modeler to easily see the assignment results with statements like

```
DISPLAY PK0, PT0, L0, K0,TSO;
```

we prefer to define new parameters, so the output will be easier to read and presented in table format. The advantage of this procedure may not be appreciated in small CGE models, but definitely are greatly appreciated in much bigger models.

```
PARAMETER SAM1 SOCIAL ACOUNTING MATRIX -BASE YEAR PRICES-;
SAM1(I,"PK")=PK0(I);
SAM1(ag,"PT")=PT0(ag);
SAM1(I,"PE0")=PE0(I);
SAM1(I,"PM0")=PM0(I);
SAM1(I,"PR0")=PR0(I);
SAM1(I,"P0")=P0(I);
SAM1(I,"PR0")=PR0(I);

PARAMETER SAM2  SOCIAL ACCOUNTING MATRIX -BASE YEAR DATA-;
SAM2(I,"L0")=L0(I);
SAM2(I,"K0")=K0(I);
SAM2(I,"KS0")=KS0(I);
SAM2(I,"T0")=T0(I);
SAM2(I,"TS0")=TS0(I);
SAM2(I,"VA0")=VA0(I);
SAM2(I,"TVR0")=TVR0(I);
SAM2(I,"TVM0")=TVM0(I);
SAM2(I,"TV0")=TV0(I);
SAM2(I,"IBT0")=IBT0(I);
SAM2(I,"X0")=X0(I);
SAM2(I,"M0")=M0(I);
SAM2(I,"R0")=R0(I);
SAM2(I,"E0")=E0(I);
SAM2(I,"Q0")=Q0(I);
SAM2(I,"QR0")=QR0(I);
SAM2(I,"QM0")=QM0(I);
SAM2(I,"QGOV0")=QGOV0(I);
```

```
SAM2(I,"QGOVR0")=QGOVR0(I);
SAM2(I,"QGOVM0")=QGOVM0(I);
SAM2(I,"QINV0")=QINV0(I);
SAM2(I,"QINVR0")=QINVR0(I);
SAM2(I,"QINVM0")=QINVM0(I);

OPTION DECIMALS=0;
DISPLAY SAM1;

OPTION DECIMALS=3;
DISPLAY SAM2;

DISPLAY V0,VM0,VR0,LS0,PL0, PLROC0,LHHH0,LGOV0,LY0,KY0,TY0,
YENT0,REMIT0,YH0,DYH0,YGOV0,GRP0,HSAV0,HEXP0,GOVEXP0,SAV0,RO
WSAV0,
TRGOV0,ENTY0,ENTTAX0,GOVBOR0;
```

PARAMETER CALIBRATION

Calibration is the setting of model parameters in order to make the equilibrium solution fit the data of a given base year (our SAM). The way to perform this adjustment in GAMS is to solve at fixed (consistent) values of observed variables, treating some of the parameters as variables. The solution will then fit the model to the data.

The calibration procedure was introduced in section 2.3. We have linked the text equation that is used in the calibration with each of our definitions; i.e., clicking over the definition

```
a0(i)   = VA0(i)/X0(i);
```

takes you to equation 3.1.2 in our text. Once again, remember that our base year variables are identified by a “0” suffix in the name.

```
*##########################################################*
*                                                          *
*                  PARAMETER CALIBRATION                   *
*                                                          *
*##########################################################*


*#####-- CALIBRATION

*@Production block

  a0(i)           =VA0(i)/X0(i);
  a(j,i)          =V0(j,i)/X0(i);
  alpha(ag,"K")   =VAD(ag,"K")/VA0(ag);
  alpha(ag,"T")   =VAD(ag,"T")/VA0(ag);
  alpha(ag,"L")   =1-alpha(ag,"K")-alpha(ag,"T");
  alpha(nag,"K")  =VAD(nag,"K")/VA0(nag);
  alpha(nag,"L")  =1-alpha(nag,"K");
  Ava(ag)         =VA0(ag)/Prod(f,VAD(ag,f)**alpha(ag,f));
  Ava(nag)
=VA0(nag)/PROD(fl,VAD(nag,fl)**alpha(nag,fl));
  RHOv(i)         =1-1/ParamA("SIGMAv",i);
  deltav1(j,i)
    $(NZV(j,i))                      =(VR0(j,i)/VM0(j,i))**(1-
RHOv(j))*(PR0(j)/PM0(j));
  deltav(j,i)
    $(NZV(j,i))   =1/(1+deltav1(j,i));
```

```
    Av(j,i)
      $(NZV(j,i))   =V0(j,i)/(deltav(j,i)*VM0(j,i)**RHOv(j)
                     +(1-deltav(j,i))
                     *VR0(j,i)**RHOv(j))**(1/RHOv(j));
    RHOx(i)        =1+1/ParamA("SIGMAx",i);
    deltax1(i)                             =(R0(i)/E0(i))**(1-
RHOx(i))*(PR0(i)/PE0(i));
    deltax(i)      =1/(1+deltax1(i));
    Ax(i)                 =X0(i)/(deltax(i)*E0(i)**RHOx(i)+(1-
deltax(i))
                     *R0(i)**RHOx(i))**(1/RHOx(i));

  *@Income block
    sstax         =ParamB("L","FTAX0")/LY0;
    ktax          =ParamB("K","FTAX0")/KY0;
    ttax          =ParamB("T","FTAX0")/TY0;
    retr          =ParamB("K","RETENT0")/sum(i,VAD(i,"K"));
    ibtax(i)      =ParamA("IBT0",i)/(PR0(i)*X0(i));
    et        =ENTTAX0/KY0;
    hhtax         =ParamC("HOUSE","HTAX0")/YH0 ;
    ltr           =1;
    mps           =ParamC("HOUSE","HSAV0")/YH0 ;

  *@Expenditure block

    RHOq(i) = 1-1/ParamA("SIGMAq",i);
    deltaq1(i)$NZQ(i)         =         (QR0(i)/QM0(i))**(1-
RHOq(i))*(PR0(i)/PM0(i));
    deltaq(i)$NZQ(i) =1/(1+deltaq1(i));
    Aq(i)$NzQ(i)    =    Q0(i)/(deltaq(i)*QM0(i)**RHOq(i)+(1-
deltaq(i))*QR0(i)**RHOq(i))**(1/RHOq(i));
    RHOgov(i) = 1-1/ParamA("SIGMAgov",i);
    deltagov1(i)$NZGOV(i)         =(QGOVR0(i)/QGOVM0(i))**(1-
RHOgov(i))*(PR0(i)/PM0(i));
    deltagov(i)$NZGOV(i) = 1/(1+deltagov1(i));
    Agov(i)$NZGOV(i)                                         =
QGOV0(i)/(deltagov(i)*QGOVM0(i)**RHOgov(i)+(1-
deltagov(i))*QGOVR0(i)**RHOgov(i))**(1/RHOgov(i));
    RHOinv(i)= 1-1/ParamA("SIGMAinv",i);
    deltainv1(i)$NZInv(i)    =    (QINVR0(i)/QINVM0(i))**(1-
RHOinv(i))*(PR0(i)/PM0(i));
    deltainv(i)$NZInv(i) = 1/(1+deltainv1(i));
    Ainv(i)$NZInv(i)                                         =
QINV0(i)/(deltainv(i)*QINVM0(i)**RHOinv(i)+(1-
deltainv(i))*QINVR0(i)**RHOinv(i))**(1/RHOinv(i));
    beta(i) = Q0(i)*P0(i)/HEXP0;
```

To check values for the calibration we use the following parameters which allow us to display the results of calibration in a table-like display.

```
PARAMETER CALIBR PARAMETER CALIBRATED;
CALIBR(I,"A0")=A0(I);
CALIBR(I,"AVA")=AVA(I);
CALIBR(I,"RHOV")=RHOv(I);
CALIBR(I,"RHOQ")=RHOQ(I);
CALIBR(I,"DELTAQ")=DELTAQ(I);
CALIBR(I,"AQ")=AQ(I);
CALIBR(I,"IBTAX")=IBTAX(I);
CALIBR(I,"RHOGOV")=RHOGOV(I);
CALIBR(I,"DELTAGOV")=DELTAGOV(I);
CALIBR(I,"AGOV")=AGOV(I);
CALIBR(I,"RHOINV")=RHOINV(I);
```

```
CALIBR(I,"AINV")=AINV(i);
CALIBR(I,"RHOX")=RHOX(i);
CALIBR(I,"DELTAX")=DELTAX(i);
CALIBR(I,"AX")=AX(i);
CALIBR(I,"BETA")=BETA(i);
DISPLAY CALIBR;
DISPLAY a,Av,deltav,alpha,
ktax,sstax,ttax,retr,et,mps,hhtax;
```

VARIABLE DECLARATION

All symbols belonging to the list of choice variables in the mathematical program should be declared as VARIABLES, not as PARAMETERS. Endogenous variables are given in table 4.3. {Click here to see table 4.3} Every endogenous variable declaration has a logical name followed by a label field (optional).

```
*##########################################################*
*                                                          *
*      VARIABLE DECLARATION                                *
*                                                          *
*##########################################################*

* ENDOGENOUS VARIABLES

VARIABLES
  Z             Objective Function Value

*@Price block
  PL                 Wage rate
  PK(i)              Capital rate
  PKL                 Capital rate in the long run
  PT(ag)             Land rent
  PN(i)              Net price
  PR(i)              Regional price
  P(i)               Composite price
  PX(i)              Composite price faced by consumers

*@Production block
  LAB(i)             Labor demand
  CAP(i)             Capital demand
  LAND(ag)            Land demand
  TCAP               Total Capital Demand
  TLAB               Total Labor Demand
  LS                 Labor supply
  LMIG               Labor migration
  KMIG               Capital migration
  VA(i)              Value added
  V(j,i)             Composite intermediate good demand
  VM(j,i)            Imported int good demand
  VR(j,i)            Reg int good demand
  R(i)               Regional supply
  X(i)               Output
  EXP(i)             Export
  M(i)               Import
  TVM(i)             Imported int good total demand
  TVR(i)             Reg int good total demand
  TV(i)              Composite intermediate good total demand
  adjL               Labor adjustment

*@Income block
```

```
        LY                  Labor income (original hhs)
        ALY                    Adjusted labor income (staying + in-
migrating)
        KY                  capital income (original capital stock)
        TY                  Land income
        YENT                Enterprise income
        RETENT              Retained Earnings by enterprises
        YH                      Income of hh staying in the region
(including in-migrants)
        DYH                    Disposable hh income (staying in the
region + inmigra)
        HSAV                Household saving (staying +inmigrat)
        SAV                 Total saving
        INV                 Investment
        YGOV                gov revenue
        IBTX                Indirect business tax
        GRP                 Gross region product

      *### Expenditure block
        AHEXP                 Adjusted household expenditure (spent
within the region)
        Q(i)                Demand for comp consump good
        QM(i)               Demand for imp consump good
        QR(i)               Demand for reg consump good
        GOVEXP              gov expend
        QGOV(i)             gov demand for comp good
        QGOVM(i)            gov demand for imported good
        QGOVR(i)            gov demand for reg good
        QINV(i)             Invest gov demand for comp good
        QINVM(i)            Invest gov demand for imported good
        QINVR(i)            Invest gov demand for reg good
        SLACK(i)
        SLACK2(i)
```

The following statement ensures that we are working with positive variables. All variables may be assigned as positive variables except the “Z” variable which we use in the optimization statement.

```
      POSITIVE VARIABLE   SLACK, SLACK2;
```

Equation Declaration

This section declares the equations of the model which are those presented in Table 4.1. {Click here for table 4.1} Equations are also denoted by symbols. Hence, every equation can be referred to by its logical name.

```
*###########################################################*
*                                                           *
*      EQUATION DECLARATION                                 *
*                                                           *
*###########################################################*

*This section declares the equations of the model
*which are those presented in table 5.1

EQUATIONS
  EQZ                 objective function

*@Price block
  NETprice(i)       net price
  Price(i)          composite price
  Price1(i)
```

```
*@Production block
  Ldemand(i)       labor demand
  KdemandSR(i)       capital demand
  KdemandLR(i)
  Tdemand(ag)       land demand
  TLdem             total labor demand
  TKdem             total capital demand
  VAdemand(i)       value added demand
  Vdemand(j,i)      intermediate demand
  VAprod1(nag)      value added prod fc
  VAprod2(ag)       value added prod fc
  Vces(j,i)         ces fc for int demand
  TVdemand(i)       intermediate total demand
  TVRdemand(i)      int reg total demand
  TVMdemand(i)      int imp total demand
  VRdem(j,i)        demand for reg int good
  VRdem0(j,i)        demand for reg int good for goods with
zero import
  VMDem0(j,i)        demand for imp int good for goods with
zero import
  Xcet(i)           cet fc for reg product
  Rsupply(i)        reg supply of reg product
  LSupply           labor supply
  LMIGrat           labor migration
  adjustL           labor migration adjustment
  KMIGrat           capital migration
  KMIGrat1


*@Income block
  LYincome          labor income
  ALYincome         adjusted labor income
  KYincomeSR          capital income
  KYincomeLR
  TYincome          land income
  YENTincome        enterprise income
  RETearn           Retained earning by enterprises
  YHincome          household income
  DHYincome         disposable income
  HSAVings          household savings
  SAVings           total savings
  INVest            total investment
  YGOVincome        Government income
  INDtax            Indirect business tax
  GRProduct         gross region product

*@Expenditure block
  AHEXPLow          adj. household expenditure
  Qces              ces fc for consumption
  Qdemand           cons demand for composite good
  QRdem0            cons demand for reg goods
  QRdem1
  QRdem2
  QMdem1
  QMdem2
  GOVEXPend         Gov expenditure
  QGOVces           ces for st and loc gov demand
  QGOVdemand        st and loc gov cons
  QGOVRdem0         st and loc gov reg cons
  QGOVRDem1
  QGOVRDem2
  QGOVMDem1
  QGOVMDem2
```

```
  QINVces          ces for invest gov demand
  QINVemand        invest gov cons
  QINVRdem0        invest gov reg cons
  QInvRdem1
  QInvRdem2
  QInvMdem1
  QInvMdem2
  Mimports(i)     import

*@Equilibrium
  COMMequil(i)     comm market equilibrium
  Lequil           labor market equilibrium
  Kequil(i)        cap market equilibrium
  Kequil1
  Tequil(ag)        land market equilibrium;
```

EQUATION DEFINITION

All equations are defined following the algebraic specification given in Table 4.1. {Click here to see table 4.1.} This section requires special attention and intense scrutiny. To help the reader, we have linked each equation definition. Thus, it is possible to move from GAMS-specification format to its algebraic specification. Furthermore, each algebraic equation in Table 4.1 is itself linked to the part of the text where derivation takes place.

In the equation definition, the “=E=” represents an equality-sign; the greater-or-equal sign is written as =G=, and smaller-or-equal as =L=. Thus, by comparing with Table 5.1 algebraic specification, the meaning of each equation is straightforward. Exceptions are equations involving a dollar expression; i.e., QCES(CI)$NZQ(CI). A dollar expression indicates that the value of the variable (i.e., the equation QCES) should be considered only if the expression that follows is true.

```
*############################################################*
      *                                                          *
      *      EQUATION DEFINITION                                 *
      *                                                          *
      *##########################################################*

      *All equations are defined following the algebraic structure
      *on table 5.1.

        EQZ..            Z       =e= sum(i,SLACK(i)+SLACK2(i));

      *@Price block
        NETprice(i)..     PN(i)     =e=  PX(i)-sum(j,A(j,i)*P(j))-
ibtax(i)*PX(i);
        Price(i)..                          P(i)             =e=
(PR(i)*R(i)+PM0(i)*M(i))/(R(i)+M(i));
        Price1(i)..                          PX(i)           =e=
(PR(i)*R(i)+PE0(i)*Exp(i))/(R(i)+Exp(i));

      *@Production block
        Ldemand(i)..    LAB(i) =e= alpha(i,"L") *PN(i)*X(i)/PL;
        KdemandSR(i)$(Not    Kmobil)..              CAP(i)    =e=
alpha(i,"K")*PN(i)*X(i)/PK(i);
        KdemandLR(i)$(Kmobil)..                    CAP(i)     =e=
alpha(i,"K")*PN(i)*X(i)/PKL;
        Tdemand(ag)..              LAND(ag)=e=     alpha(ag,"T")
*PN(ag)*X(ag)/PT(ag);
```

```
        TLdem..          TLAB  =e= Sum(i,LAB(i));
        TKdem..          TCAP  =e= Sum(i,CAP(i));
        LSupply ..       LS    =e= LS0;
        LMIGrat ..       LMIG  =e= etaL*LS0*LOG(PL/PLROC0);
        adjustL..        adjL  =e= (LS0+LMIg)/LS0;
        KMIGrat$(KMobil)..                                    KMIG
=e=etaK*(SUM(i,K0(i))*LOG(PKL/PKROC0));
        KMIGrat1$(not KMobil).. KMIG       =e= 0;
        VAdemand(i)..    VA(i)+SLACK(i)+SLACK2(i)=e= a0(i)*X(i);
        VAprod1(nag)..              VA(nag)                   =e=
Ava(nag)*LAB(nag)**alpha(nag,"L")*CAP(nag)**
                                   alpha(nag,"K");
        VAprod2(ag)..              VA(ag)                     =e=
Ava(ag)*LAB(ag)**alpha(ag,"L")*CAP(ag)**

alpha(ag,"K")*LAND(ag)**alpha(ag,"T");
        Vdemand(j,i)..   V(j,i)  =e= a(j,i)*X(i);
        Vces(j,i)..      V(j,i)  =e= Av(j,i)*(deltav(j,i)*VM(j,i)
                                    **RHOv(j)+(1-deltav(j,i))

*VR(j,i)**RHOv(j))**(1/RHOv(j));
        TVdemand(i)..    TV(i)   =e= sum(j,V(i,j));
        VRdem(j,i)$NZV(j,i)..
                         VR(j,i) =e= VM(j,i)*((1-deltav(j,i))/
                                      deltav(j,i)*
                                      PM0(j)/PR(j))**(1/(1-
RHOv(j)));
        VRdem0(j,i)$ZVM(j,i)..   VR(j,i) =e= V(j,i);
        VMdem0(j,i)$ZVM(j,i)..   VM(j,i) =e= 0;
        TVRdemand(i)..   TVR(i)  =e= sum(j,VR(i,j));
        TVMdemand(i)..   TVM(i)  =e= sum(j,VM(i,j));
        Xcet(i)..                        X(i)                 =e=
Ax(i)*(deltax(i)*EXP(i)**RHOx(i)+(1-
                                    deltax(i))*R(i)**RHOx(i))
                                    **(1/RHOx(i));
        Rsupply(i)..              R(i)           =e=   EXP(i)*((1-
DELTAx(i))/DELTAx(i)
                                    *PE0(i)/PR(i))**(1/(1-
RHOx(i)));
        INDtax..         IBTX    =E= Sum(i,ibtax(i)*X(i));
        GRProduct..      GRP     =e= ALY + KY + TY + IBTX;


      *@Income block

      *ALY is defined for all labor; LY is defined for original
household
        ALYincome..      ALY      =e=  PL*(TLAB+LHHH0+LGOV0);
        LYincome..        LY        =e=  ALY+PLROC0*(SQRT(LMig**2)-
LMig)*0.5
                                    -
PL*(SQRT(LMig**2)+LMig)*0.5;
        KYincomeSR$(not kmobil)..   KY   =e= sum(i,PK(i)*CAP(i));
        KYincomeLR$(kmobil)..            KY                   =e=
sum(i,PKL*CAP(i))+PKROC0*(SQRT(KMIG**2)-KMIG)
                                           *0.5-
PKL*(SQRT(KMIG**2)+KMIG)*0.5;
        RETearn..        RETENT  =e= retr*KY;
        TYincome..       TY      =e= sum(ag,PT(ag)*LAND(ag));
        YENTincome..     YENT    =e= KY*(1-ktax);
        YHincome ..      YH      =e= ALY*(1-sstax)
                                   +TY*(1-ttax)+(YENT-RETENT-
et*KY)
                                   +REMIT0+adjL*TRGOV0
```

```
                                    -((SQRT((adjL-1)**2)-(adjL-
1))*0.5)
                                    *(TY*(1-ttax)+(YENT-RETENT-
et*KY)
                                    +REMIT0);
        DHYincome ..   DYH   =e= YH *(1-hhtax );
        HSAVings ..    HSAV  =e= mps *YH ;
        SAVings..        SAV     =e= HSAV+RETENT+ROWSAV0;
        INVest..         INV     =e= sum(i,P(i)*QINV(i));
        YGOVincome..     YGOV     =e= Sum(i,ibtax(i)*PX(i)*X(i))
                                     +sstax*ALY
                                     +ktax*KY+et*KY
                                     +ttax*TY
                                     +hhtax *YH+GOVBOR0+GOVITR0;
      *@Expenditure block
        AHEXPLow..   AHEXP  =e= DYH-HSAV-PL*LHHH0;
        Qdemand(i)..  Q(i)   =e= beta(i)*AHEXP/P(i);
        Qces(i)$NZQ(i).. Q(i)  =e= Aq(i)*(deltaq(i)*QM(i)
                     **RHOq(i)+(1-deltaq(i))*QR(i)**RHOq(i))
                                      **(1/RHOq(i));
        QRdem0(i)$NZQ(i)..       QR(i)       =e=       QM(i)*((1-
deltaq(i))/deltaq(i)
                                  *PM0(i)/PR(i))**(1/(1-RHOq(i)));
        QRdem1(i)$ZQM(i).. QM(i) =e= 0;
        QMdem1(i)$ZQM(i).. QR(i) =e= Q(i);
        QRdem2(i)$ZQR(i).. QR(i) =e= 0;
        QMdem2(i)$ZQR(i).. QM(i) =e= Q(i);
        GOVEXPend..                         GOVEXP              =e=
sum(i,P(i)*QGOV(i))+adjL*
                                        TRGOV0+PL*LGOV0+GOVITR0;
        QGOVdemand(i)..   QGOV(i)   =e= QGOV0(i);
        QGOVces(i)$NZGOV(i)..                  QGOV(i)        =e=
Agov(i)*(deltagov(i)
                                    *QGOVM(i)**RHOgov(i)+(1-
deltagov(i))

*QGOVR(i)**RHOgov(i))**(1/RHOgov(i));
        QGOVRdem0(i)$NZGOV(i)..        QGOVR(i)   =e=QGOVM(i)*((1-
deltagov(i))
                    /deltagov(i)*PM0(i)/PR(i))**(1/(1-
RHOgov(i)));
        QGOVRdem1(i)$ZGOVM(i)..   QGOVM(i) =e= 0;
        QGOVMdem1(i)$ZGOVM(i)..   QGOVR(i) =e= QGOV(i);
        QGOVRdem2(i)$ZGOVR(i)..   QGOVR(i) =e= 0;
        QGOVMdem2(i)$ZGOVR(i)..   QGOVM(i) =e= QGOV(i);
        QINVemand(i)..   QINV(i)  =e= QINV0(i);
        QINVces(i)$NZInv(i)..                               QINV(i)
=e=Ainv(i)*(deltainv(i)*QINVM(i)
                                        **RHOinv(i)+(1-
deltainv(i))*QINVR(i)**RHOinv(i))
                                        **(1/RHOinv(i));
        QINVRdem0(i)$NZInv(i)..      QINVR(i)=e=      QINVM(i)*((1-
deltainv(i))
                            /deltainv(i)*PM0(i)/PR(i))**(1/(1-
RHOinv(i)));
        QInvRDem1(i)$ZInvM(i).. QInvM(i)=e= 0;
        QInvMDem1(i)$ZInvM(i).. QInvR(i)=e= QInv(i);
        QInvRDem2(i)$ZInvR(i).. QInvR(i)=e= 0;
        QInvMDem2(i)$ZInvR(i).. QInvM(i)=e= QInv(i);
        Mimports(i)..                  M(i)                 =e=
TVM(i)+QM(i)+QGOVM(i)+QINVM(i);

      *@Equilibrium
```

```
        COMMequil(i)..
X(i)+M(i)=e=TV(i)+Q(i)+QGOV(i)+QINV(i)+EXP(i);
        Lequil..          sum(i,LAB(i))+LHHH0+LGOV0 =e= LS0+LMIG;
        Kequil1$(KMobil)..        KMig =e= Sum(i,CAP(i)-KS0(i));
        Kequil(i)$(not KMobil).. CAP(i)          =e= KS0(i);
        Tequil(ag)..      LAND(ag)  =e= T0(ag);
```

STARTING VALUES and BOUNDS

Before a model is solved, it is necessary to initialize all relevant bounds. Bounds are treated in the same way as parameters. Here, we introduce GAMS language to characterize a variable. A GAMS-variable is characterized by a suffix:

.L current level of the variable
.M shadow price on the bound
.LO lower bound
.UP upper bound
.FX fixed (lower bound=upper bound).

The variables (.L-values) keep their ***level*** value from one solution to the next assignment. Unassigned upper bounds are set at plus infinity, non-initialized lower bounds at minus infinity. In direct assignments, variables should be referenced with their suffices. The initialization is at arbitrary values, in order to test the computational procedure. However, in empirical applications it is recommended to initialize the variables at their SAM-values.

```
*###########################################################*
*                                                           *
*      INITIALIZATION OR STARTING VALUES                    *
*                                                           *
*###########################################################*
*@Price block                   *@Income block
  PL.L       =PL0       ;
  PKL.L       =1;
  PK.L(i)   =PK0(i)     ;
  PT.L(ag)      =PT0(ag)     ;           HSAV.L    =HSAV0
;
  PR.L(i)     =PR0(i)     ;           YGOV.L        =YGOV0
;
  P.L(i)     =P0(i)     ;
  PX.L(i)   = PX0(i)    ;
  PN.L(i)      = PX0(i)-sum(j,A(j,i)*P0(j))-ibtax(i)*PX0(i);

*@Production block
  SLACK.L(i) =0;             SLACK2.L(i)   =0;
  LAB.L(i)  =L0(i)    ;     INV.L      =INV0;
  CAP.L(i) =K0(i)   ;       GRP.L      =GRP0;
*                            *@Expenditure block
  LAND.L(ag) =T0(ag)    ;

  LS.L     = LS0;
  LMIG.L   =0;
  KMIG.L    =0;
  VA.L(i)   =VA0(i)   ;
  VM.L(j,i)  =VM0(j,i)  ;
  VR.L(j,i)  =VR0(j,i)  ;         QM.L(i) =QM0(i)   ;
  V.L(j,i)   =V0(j,i)   ;
  TVM.L(i)   =TVM0(i)   ;
  TVR.L(i)    =TVR0(i)    ;           GOVEXP.L     =GOVEXP0
;
  TV.L(i)    =TV0(i)    ;         QGOV.L(i)  =QGOV0(i)    ;
  R.L(i)    =R0(i)    ;         QGOVM.L(i) =QGOVM0(i)   ;
```

```
  X.L(i)     =X0(i)     ;          QGOVR.L(i) =QGOVR0(i)   ;
  EXP.L(i)  =E0(i)    ;
  M.L(i)    =M0(i)    ;
  Q.L(i)     =beta(i)*HEXP0/PX0(i);
  QR.L(i) =QR0(i)   ;

*@Income block
  LY.L       =LY0       ;
  KY.L      =KY0        ;
  TY.L       =TY0       ;
  adjL.L      =1     ;
  YENT.L     =YENT0     ;
  YH.L     =YH0     ;
  SAV.L      =SAV0      ;
  DYH.L    =DYH0    ;
  QINVM.L(i) =QINVM0(i) ;
  QINVR.L(i) =QINVR0(i) ;
  QINV.L(i)  =QINV0(i)  ;

*##########################################################*
*                                                          *
*      VARIABLE BOUNDS                                     *
*                                                          *
*##########################################################*

  PL.LO       = 0.000001;
  PT.LO(ag)    = 0.000001;
  PK.LO(i)   =  0.000001;
  PR.LO(i)    = 0.000001;
  PN.LO(i)   = 0.000001;
  P.LO(i)     = 0.000001;
  R.LO(i)    = 0.000001;
  PX.LO(i)   =  0.000001;
  QM.LO(i)$(QM0(i) ne 0)  = 0.000001;
  QR.LO(i)$(QR0(i) ne 0)  = 0.000001;
  Q.LO(i)$(Q0(i) ne 0)    = 0.000001;
  QM.LO(i)$(QM0(i) eq 0)  = 0;
  QR.LO(i)$(QR0(i) eq 0)  = 0;
  Q.LO(i)$(Q0(i) eq 0)    = 0;
  VR.LO(i,j)$(VR0(i,j) ne 0)  = 0.000001;
  VM.LO(i,j)$(VM0(i,j) ne 0)  = 0.000001;
  V.LO(i,j)$(V0(i,j) ne 0)    = 0.000001;
  VR.LO(i,j)$(VR0(i,j) eq 0)  = 0;
  VM.LO(i,j)$(VM0(i,j) eq 0)  = 0;
  V.LO(i,j)$(V0(i,j) eq 0)    = 0;
```

The follow statement uses GAMS-Options to reduce the amount of output and computer time assigned to solve the model. This is not recommended for beginners who may do better by getting more output from GAMS. Especially, for those having problems obtaining a “zero error message”. Iterlim, limrow, lincol and solprint, will limit the number of iterations, suppress the printing of equations, suppress the printing of columns, and suppress the list of the solution, respectively. Although this saves paper, we do not recommend it unless you understand your model very well and have your model running without error messages.

```
OPTIONS ITERLIM=5000, LIMROW=0, LIMCOL=0, SOLPRINT=OFF;
```

MODEL and SOLVE statements

A group of equations constitute a mathematical model. GAMS uses the statement **MODEL** to allow us to specify which equations should be considered as part of our mathematical model. In addition, we need to give a name to our model, i.e.; our Model is called OKLAHOMA.

```
*-- MODEL DEFINITION AND SOLVE STATEMENT

MODEL OKLAHOMA /ALL/;
```

We use in our example all the declared equations, if that would not be the case, instead of the word “ALL” we would have written each equation needed.

Equilibrium is found by minimizing the objective function ***EQZ*** that calculates the absolute sum of deviations (Slack variables). This process was introduced in section 4.2. {Click here to review section 4.2} The GAMS-statement to solve the mathematical program defined by the model OKLAHOMA with objective Z, using the MINOS5 non-linear programming algorithm NLP, reads as:

```
SOLVE OKLAHOMA MINIMIZING Z USING NLP;
```

REPORTING VALIDATION OF THE MODEL

When an equilibrium solution has been computed, the results are sorted in tabulation format. We define tables for commodity balances, prices, consumer budgets, etc.. These tables give the level of the endogenous variables of OKLAHOMA model. If they are correct, the values of these tables validate with those of our base year (SAM values). We call this process the validation of the model.

```
*-- SOLUTION DISPLAY STATEMENT
*-- SOLUTION VALUES OF ENDOGENOUS VARIABLES


PARAMETER VALID VARIABLES FOR THE VALIDATION OF THE MODEL;
VALID(i,"SLACK1") = SLACK.L(i);
VALID(i,"SLACK2") = SLACK2.L(i);
VALID(i,"PR") = PR.L(i);
VALID(i,"P") = P.L(i);
VALID(i,"PN") = PN.L(i);
VALID(i,"PK") = PK.L(i);
VALID(ag,"PT") = PT.L(ag);
VALID(i,"PX") = PX.L(i);
VALID(i,"PE") = PE0(i);
VALID(i,"X") = X.L(i);
VALID(i,"R") = R.L(i);
VALID(i,"EXP") =EXP.L(i);
VALID(i,"M") = M.L(i);
VALID(i,"VA") = VA.L(i);
VALID(i,"LAB") =LAB.L(i);
VALID(i,"CAP") =CAP.L(i);
VALID(ag,"LAND") =LAND.L(ag);
VALID(i,"TVR") =TVR.L(i);
VALID(i,"TVM") =TVM.L(i);
VALID(i,"TV") =TV.L(i);
VALID(i,"Q") =Q.L(i);
VALID(i,"QR") =QR.L(i);
```

```
VALID(i,"QM") =QM.L(i);
VALID(i,"QGOV") =QGOV.L(i);
VALID(i,"QGOVR") =QGOVR.L(i);
VALID(i,"QGOVM") =QGOVM.L(i);
VALID(i,"QINV") =QINV.L(i);
VALID(i,"QINVR") =QINVR.L(i);
VALID(i,"QINVM") =QINVM.L(i);

PARAMETER VALID2  -INTERMEDIATE USE MATRIX-;
VALID2(I,"AGR","V")=V.L(I,"AGR");
VALID2(I,"MIN","V")=V.L(I,"MIN");
VALID2(I,"MAN","V")=V.L(I,"MAN");
VALID2(I,"SER","V")=V.L(I,"SER");
VALID2(I,"AGR","VR")=VR.L(I,"AGR");
VALID2(I,"MIN","VR")=VR.L(I,"MIN");
VALID2(I,"MAN","VR")=VR.L(I,"MAN");
VALID2(I,"SER","VR")=VR.L(I,"SER");
VALID2(I,"AGR","VM")=VM.L(I,"AGR");
VALID2(I,"MIN","VM")=VM.L(I,"MIN");
VALID2(I,"MAN","VM")=VM.L(I,"MAN");
VALID2(I,"SER","VM")=VM.L(I,"SER");

PARAMETER VALID3 -VALIDATION OF THE MODEL-;
VALID3("OBJECTIVE") = Z.L;
VALID3("PL") = PL.L;
VALID3("LMIG")=LMIG.L;
VALID3("KMIG")=KMIG.L;
VALID3("TCAP")=TCAP.L;
VALID3("TLAB")=TLAB.L;
VALID3("LS")=LS.L;
VALID3("LMIG")=LMIG.L;
VALID3("ADJL") = ADJL.L;
VALID3("LY")=LY.L;
VALID3("ALY")=ALY.L;
VALID3("KY")=KY.L;
VALID3("TY")=TY.L;
VALID3("YENT") = YENT.L;
VALID3("RETENT")=RETENT.L;
VALID3("YH")=YH.L;
VALID3("PL") = PL.L;
VALID3("DYH")=DYH.L;
VALID3("HSAV")=HSAV.L;
VALID3("SAV")=SAV.L;
VALID3("INV") = INV.L;
VALID3("YGOV")=YGOV.L;
VALID3("GOVEXP")=GOVEXP.L;
VALID3("IBTX")=IBTX.L;
VALID3("GRP")=GRP.L;
VALID3("AHEMP")=AHEXP.L;

option decimals=3;
DISPLAY VALID,VALID2,VALID3;
```

SIMULATION

Before starting a simulation run, one should specify the name of the scenario (here, simul1). The last step in preparing the model is to define the index sets and parameters for reporting. We define post-equilibrium variables that we use in constructing indexes for the relevant variables.

```
*######## SIMULATION ############*
```

```
PE0(i)=1.1;
model simul1 /all/;
solve simul1 minimizing z using nlp;
OPTION SOLPRINT=OFF;

*-- SOLUTION DISPLAY STATEMENT
*-- SOLUTION VALUES OF ENDOGENOUS VARIABLES


PARAMETER PRICES MARKET CLEARING PRICES;
PRICES(i,"SLACK1") = SLACK.L(i);
PRICES(i,"SLACK2") = SLACK2.L(i);
PRICES(i,"PR") = PR.L(i);
PRICES(i,"P") = P.L(i);
PRICES(i,"PN") = PN.L(i);
PRICES(i,"PK") = PK.L(i);
PRICES(ag,"PT") = PT.L(ag);
PRICES(i,"PX") = PX.L(i);
PRICES(i,"PE") = PE0(i);

PARAMETER PROD1 MARKET CLEARING PRODUCTION VARIABLES;
PROD1(i,"X") = X.L(i);
PROD1(i,"R") = R.L(i);
PROD1(i,"EXP") =EXP.L(i);
PROD1(i,"M") = M.L(i);
PROD1(i,"VA") = VA.L(i);
PROD1(i,"LAB") =LAB.L(i);
PROD1(i,"CAP") =CAP.L(i);
PROD1(ag,"LAND") =LAND.L(ag);

PARAMETER TRADE1 MARKET CLEARING PRODUCTION VARIABLES;
TRADE1(i,"TVR") =TVR.L(i);
TRADE1(i,"TVM") =TVM.L(i);
TRADE1(i,"TV") =TV.L(i);
TRADE1(i,"Q") =Q.L(i);
TRADE1(i,"QR") =QR.L(i);
TRADE1(i,"QM") =QM.L(i);
TRADE1(i,"QGOV") =QGOV.L(i);
TRADE1(i,"QGOVR") =QGOVR.L(i);
TRADE1(i,"QGOVM") =QGOVM.L(i);
TRADE1(i,"QINV") =QINV.L(i);
TRADE1(i,"QINVR") =QINVR.L(i);
TRADE1(i,"QINVM") =QINVM.L(i);

PARAMETER PRODUCT2  -PRODUCTION SYSTEMS VARIABLES-;
PRODUCT2(I,"AGR","V")=V.L(I,"AGR");
PRODUCT2(I,"MIN","V")=V.L(I,"MIN");
PRODUCT2(I,"MAN","V")=V.L(I,"MAN");
PRODUCT2(I,"SER","V")=V.L(I,"SER");
PRODUCT2(I,"AGR","VR")=VR.L(I,"AGR");
PRODUCT2(I,"MIN","VR")=VR.L(I,"MIN");
PRODUCT2(I,"MAN","VR")=VR.L(I,"MAN");
PRODUCT2(I,"SER","VR")=VR.L(I,"SER");
PRODUCT2(I,"AGR","VM")=VM.L(I,"AGR");
PRODUCT2(I,"MIN","VM")=VM.L(I,"MIN");
PRODUCT2(I,"MAN","VM")=VM.L(I,"MAN");
PRODUCT2(I,"SER","VM")=VM.L(I,"SER");

PARAMETER OTHER1 MARKET CLEARING VALEUES OF VARIABLES;
OTHER1("OBJECTIVE") = Z.L;
OTHER1("PL") = PL.L;
OTHER1("LMIG")=LMIG.L;
OTHER1("KMIG")=KMIG.L;
OTHER1("TCAP")=TCAP.L;
```

```
OTHER1("TLAB")=TLAB.L;
OTHER1("LS")=LS.L;
OTHER1("LMIG")=LMIG.L;
OTHER1("ADJL") = ADJL.L;
OTHER1("LY")=LY.L;
OTHER1("ALY")=ALY.L;
OTHER1("KY")=KY.L;
OTHER1("TY")=TY.L;
OTHER1("YENT") = YENT.L;
OTHER1("RETENT")=RETENT.L;
OTHER1("YH")=YH.L;
OTHER1("PL") = PL.L;
OTHER1("DYH")=DYH.L;
OTHER1("HSAV")=HSAV.L;
OTHER1("SAV")=SAV.L;
OTHER1("INV") = INV.L;
OTHER1("YGOV")=YGOV.L;
OTHER1("GOVEXP")=GOVEXP.L;
OTHER1("IBTX")=IBTX.L;
OTHER1("GRP")=GRP.L;
OTHER1("AHEMP")=AHEXP.L;

option decimals=3;

DISPLAY PROD1, TRADE1,PRODUCT2;

OPTION DECIMALS = 8;
DISPLAY OTHER1, PRICES;

* Parameters AS INDEX WITH 1993=1.000
PARAMETERS
* -- Price block
  IPL                 Wage rate index
  IPK(i)              Rent to  capital index
  IPT(ag)              Land rent index
  IPR(i)              Regional price index
  IP(i)               Composite price index
  IPG                 General composite price index
* -- Production block
  IL(i)               Labor demand index
  ITL                 Total labor demand index
  ILS                 Labor supply index
  IK(i)               capital demand index
  ITK                 Total Capital use index
  ITT                 Total Land use index
  IT(ag)               Land demand index
  IVA(i)              Value added index
  IX(i)               Output index
  ITVA              Total Value added index
  ITX               Total Output index
  ITE               Total Export index
  ITR               Total Reg. supply index
  ITM               Total Import index
  IVM(j,i)            Imported interm demand index
  IVR(j,i)            Regional interm demand index
  IR(i)               Regional supply index
  IE(i)               Export index
  IM(i)               Import index

* -- Income block

  IYH               Household (in the region) income index
  YHch              Change in hh income
  IDYH              Disposable income index
```

```
  IHSAV           Household saving index
  IYGOV             Government revenue index
  NETGOV            Net Revenue for government
  IGRP              Gross region product index
  GRPch             Change in Gross regional product
  CapComp           Capital Compensation
  LandComp          Land Compensation
  Rconsup           Resident angler consumer surplus loss
  NRconsup          NonResident angler consumer surplus loss
* -- Expenditure block
  IAHEXP          adj. Household expenditure index
  IGOVEXP         Government expenditure index
  IQ(i)           Commodity demand index
  IQM(i)          Imported commodity demand index
  IQR(i)          Regional commodity demand index
;

*-- EQUATIONS FOR CALCULATION OF INDEX WITH 1993=1.000

*### Price block
 IPL        = PL.L/PL0;
 IPK(i)     = PK.L(i)/PK0(i);
 IPT(ag) = PT.L(ag)/PT0(ag);
 IPR(i)     = PR.L(i)/PR0(i);
 IP(i)      = P.L(i)/P0(i);
 IPG                                                    =SUM(i,
(PR.L(i)*R0(i)+PM0(i)*M0(i))/(R0(i)+M0(i)))/4;
*#*  Production block
  IL(i)     = LAB.L(i)/L0(i);
  ITL       = (Sum(i,LAB.L(i))+(LHHH0+LGOV0))
                      /(Sum(i,L0(i))+LHHH0+LGOV0);
  ILS       = LS.L /LS0 ;
  IK(i)    = CAP.L(i)/K0(i);
  ITK       = Sum(i,PK.L(i)*CAP.L(i))/Sum(i,K0(i));
  IT("Agr") = LAND.L("Agr")/T0("Agr");
  ITT       = PT.L("Agr")*LAND.L("Agr")/T0("Agr");
  IVA(i)   = VA.L(i)/Va0(i);
  ITVA      = Sum(i,VA.L(i))/Sum(i,Va0(i));
  IX(i)     = X.L(i)/X0(i);
  ITX       =Sum(i,X.L(i))/Sum(i,X0(i));
  ITR       =Sum(i,R.L(i))/Sum(i,R0(i));
  ITM       =Sum(i,M.L(i))/Sum(i,M0(i));
  IVM(j,i)= VM.L(j,i)/VM0(j,i);
  IVR(j,i)= VR.L(j,i)/VR0(j,i);
  IR(i)    = R.L(i)/R0(i);
  IE(i)    = EXP.L(i)/E0(i);
  ITE        =Sum(i,EXP.L(i))/Sum(i,E0(i));
*## Income block
  IYH       = YH.L /YH0 ;
  IDYH      = DYH.L /DYH0 ;
  IHSAV     = HSAV.L /HSAV0 ;
  IGRP      = GRP.L/GRP0;
  GRPch     = GRP.L-GRP0;
*#Expenditure block
  IAHEXP    = AHEXP.L /HEXP0 ;
  IQ(i)    = Q.L(i)/Q0(i);
  IQM(i)   = QM.L(i)/QM0(i);
  IQR(i)   = QR.L(i)/QR0(i);
  IM(i)    = M.L(i)/M0(i);
  YHch      = YH.L -adjL.L*YH0 ;
  IYGOV     = YGOV.L/YGOV0;
  IGOVEXP   = GOVEXP.L/GOVEXP0;
  NETGOV    = YGOV.L-GOVEXP.L;
```

```
*##- SOLUTION VALUES OF INDEX
option decimals=5;

PARAMETER INDEX INDEXES FOR THE SIMULATION;
INDEX(I,"IPR")=IPR(I);
INDEX(I,"IX")=IX(I);
INDEX(I,"IE")=IE(I);
INDEX(I,"IL")=IL(I);
INDEX(I,"IK")=IK(I);
INDEX(I,"IPK")=IPK(I);
INDEX(ag,"IPT")=IPT(ag);
INDEX(ag,"IT")=IT(ag);
INDEX(I,"IVA")=IVA(I);
INDEX(I,"IR")=IR(I);
INDEX(I,"IM")=IM(I);
INDEX(I,"IQ")=IQ(I);
INDEX(I,"IQR")=IQR(I);
INDEX(I,"IQM")=IQM(I);
INDEX(I,"IPR")=IPR(I);
INDEX(I,"IPR")=IPR(I);

DISPLAY INDEX;
DISPLAY ITX,ITE,ITL,IPL,
         ITK,ITT,
         IGRP,GRPch,ITVA,ITR,ITM, YHch,
         IYH, IYGOV,IGOVEXP,NETGOV,
         ILS,IDYH,IHSAV,IAHEXP,
         IVM,IVR;

DISPLAY IGRP,IPG,IYH,ITE,ITM;
```