

An Experimental Study of Thinking Solution of Problem in Children

著者	Kato Ken
journal or publication title	Tohoku psychologica folia
volume	9
number	1-4
page range	233-268
year	1942-01-30
URL	http://hdl.handle.net/10097/00130447

An Experimental Study of Thinking Solution of Problem in Children

By

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**To Professor Hakusi T. Chiba, dedicated in
gratitude and respect**

Contents

Introduction	233
I. On Experiment	234
II. The First Experiment	236
1) Experimental Procedures	236
2) Experimental Processes and Their Results	239
a) On Trace Performing Processes	239
b) On Judgment of Impossibility	246
3) Discussions	252
III. The Second Experiment	254
1) Finding The Law of Impossibility and its Application	254
2) Finding The Law of Starting-Point and its Application	261
3) Discussions	264
VI. Conclusions	268

Introduction : 'Unbroken tracing'

It is said that there were seven bridges on the river Pregel, which was flowing through Königsberg, the capital of East Prussia. These seven bridges connected both sides of the river by passing two isles, as the following figure indicates.

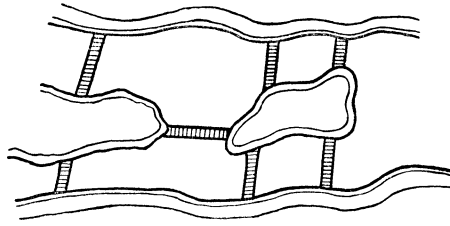


Fig. 1

A wondering of a citizen gave rise to a strange confusion among the citizens.

The problem is as follows: "Can you cross all the bridges in this city without passing the same bridge twice?"

This affair would have happened about two hundred years ago, because the solution of this problem is transmitted as an anecdote of L. Euler. We assume:— Some might have despised the problem itself as a matter of a trivial sort, and of no importance. And there might have been some others who had been lured on by the strangeness of the idea, and had worn away the soles of their shoes. And there one might have heard the following declaration, though without any particular reason, that is: "This should not be possible at all, I think."

Although the answers of the citizens, perhaps, would have tended to suggest that the problem was 'impossible', the citizens would have had a bit of feeling of uncertainty until they had heard of Euler's solution. The solution of this problem, i. e., the solution of the unbroken tracing problem, concerns, logically, a law, and, psychologically, productive thinking.

Now, the 'unbroken tracing' appears as a sort of child's play. Then we want to see the mental and behavioral processes of children in this play: How they will trace the figure of unbroken tracing? ; how they will grasp, and apply the law which controls the unbroken tracing problems? ; how these points in the unbroken tracing problem relate to the problem of learning processes, and, by the nature of these tasks, to the thinking processes in children?

I. On Experiment

Experiments by the trace performing processes in unbroken

tracing figure were carried out by Lindley¹ formerly, and by Bulbrook² recently. The former reached several important conclusions concerning the trace performing processes of the figure with such an exact and detailed observation as almost not to need to be supplemented. For example, he showed that (1) there exist the automatic repetition in the mental development and preference in selection of a starting-point on the outside of the figure with pupils of lower grades ; (2) with adults, he showed that there exist two types of tracing performance of the figure — i. e., the visual type and the motor type. And he discussed the factors of the solution of these problems. The latter, Bulbrook, not confining his observation to external aspects, carried out his experiments with more scientific consideration. He was directed by Prof. M. Bentley, and he discussed the factor of problem solution in his serial experiments. But they treated this problem using one or two figures. (These two figures are of a different kind of problem.) We also start from these unbroken tracing problems, but our set of problems is different from theirs in the following points.

1. The number of figures is not one or two. We use ten figures and more.

2. There are four figures in them which cannot be traced with an unbroken tracing.

3. Consequently the study of thinking according to law is also one of our purposes in this investigation.

If we use the above-mentioned series of figures, we shall be able to observe more exactly the several apparent aspects of unbroken tracing problem, and at the same time to study, especially, the following two problems : 1) the problem of the factor of learning, or problem solution, which is the disputed point especially between trial-and-success theory and field theory, and 2) the problem of development of logical thinking in children, which is the controversial problem raised by Schüssler against Meumann, and treated afterwards by Meier, Müller etc. especially concerning the development of reasoning.

Of course, the characteristic of the nature of this sort of problem lies in its concreteness. This suggests that these experimental sets

¹ Ernest H. Lindley, A Study of Puzzles with special reference to the psychology of mental adaptation. *Amer. J. Psych.*, **8**, 1897, 431-493.

² Mary E. Bulbrook, An experimental inquiry into the existence and nature of insight. *Amer. J. Psych.* **44**, 1932, 409-453.

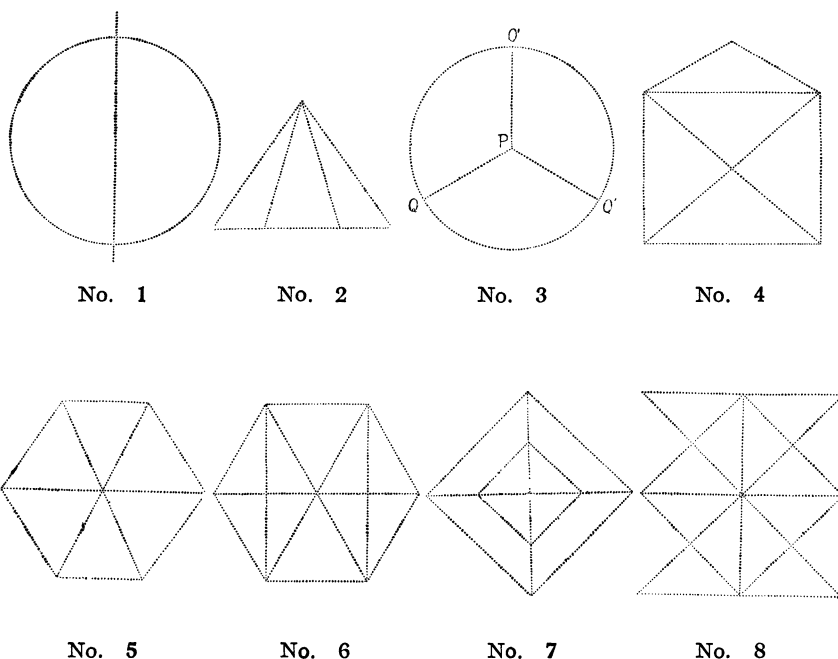
lack the so-called experimental clearness but, on the other hand, it can eliminate the abstractness which lacks the vividness of nature of the problem and the clue of discussion must be sought in the concrete behaviors or direct introspection of pupils. Although we shall later discuss these problems, we are going to treat the problems now by means of establishing the new situation.¹

II. The First Experiment

1) Experimental Procedures

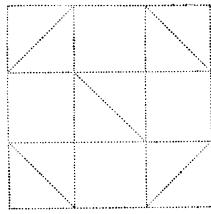
Ten figures are described in succession, proceeding from the simple figures to ones that are more complex.

Figures used in the first experiment

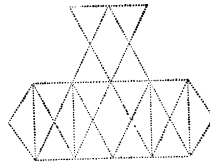


¹ These experiments were performed from July 3rd, 1939 to Oct. 21st. of the same year. The subjects were the pupils of the Miyagi Normal School from First Year to Seventh Year.

The total number of subjects were 111, 9 boys and 9 girls in each year, except First Year, in which 6 boys and 6 girls were chosen.

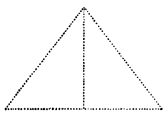


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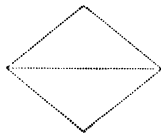


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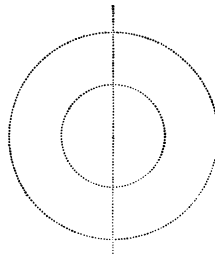
Supplementary figures



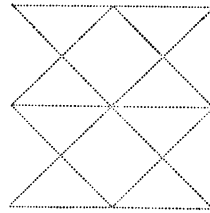
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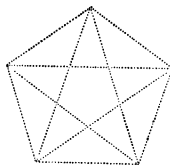
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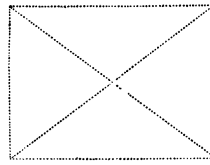
No. 13



No. 14



No. 15



No. 16

Fig. 2

1. The serial number of figures also shows the order of presentation.
2. In these figures Nos. (3), (5), (7), (8) are figures which cannot be traced with one unbroken tracing.
3. Besides these ten figures, we used figures especially for lower grade students.

Directions and Record

We used the following direction slightly changing terms according to the grade of the pupils.

“Now, you will be given a task and will be asked some easy questions, so please do it and answer the questions readily. The task is called one unbroken tracing (here I asked every subjects, whether he or she is already acquainted with the unbroken tracing problem). Suppose that there is a figure which is described with several lines, and the problem is to trace continuously, without retracing, every line of the figure.

Here are ten figures. But when I drew them, I did not care whether they could be traced continuously or not, so I fear that there may be some figures that cannot be traced continuously. When you find the figure which cannot be traced continuously, please state “this can not be traced.” When you can, trace, of course, to the end. You may happen on a case which cannot be traced to the end according you traced it unskilfully even in good figure. If so, use as many copies as you need.”

As we wanted to grasp the subjects' movements just as they happened, we endeavored to record not only the point of abstract aspects of thinking but also the whole picture of the subject in action.

When subjects ended their trials in each figure we asked them their introspections concerning the whole course of their trials, but paying special attention to the introspections concerning the motivation of selection and changing of starting-point, and the property of every intersecting point. And, at the same time, we tried to find out the existence of the guiding notion of starting-point and of the possibility, or impossibility in tracing. The reports of their introspections were required as well during their trials as when they had ended them, and after the dialogues concerning the guiding notion, we added: “Is there any way to determine whether it is possible or impossible? Please always keep this in mind.”

When a subject was confused, after a figure was presented, spending time tracing the figure with his eyes, without drawing with his crayon, I gave him the next instruction, for we thought that the effect of eye-tracing, when it is uncertain, has the same result with an unsuccessful trial. Subjects' questions and monologues during experiments were freely allowed and were recorded. This was done to find out the possibility of observation regarding the

solution movements with their feelings relieved.

Time Of Experiment

When the tracing performance or judgment of impossibility was not reached in about five minutes in each figure, I ordered, on the whole, the subject to stop his trial. But several times it was lengthened to a much longer time for I wanted the qualitative, not quantitative processes to be gone through. Consequently the whole time of the first experiment, including the time for reports of introspection and dialogue, ranged from one hour to one and a half.

2) Experimental processes and their results

I think that the quality of problem and other factors of this experiment were generally adequate for the elementary school pupils, considering the liveliness of their movements. Though the procedures and results in this experiment include many things, I shall confine the discussion to the problem solving processes and developmental viewpoint. It need not be said that the problem solving does never occur simply because of the thinking processes. For example, subjects' bodily and mental states at the time of the experiment will necessarily influence it. There will be needless to say, such occasions as the words of the dialogue between the subject and the leader of the experiment tend to influence the solution. Moreover we observed generally the next two points: One is that the movements and reports of subjects, as a whole, hit E's mind in each nuance, and we have no adequate method to record its differences; the other is the fact of what will be called the mental interrelated resistance between subject and leader. This fact occurred especially with the girls of middle-year classes (aged 11 and 12 years). The fact may be presumably explained with reference to their thinking processes, but I have no adequate method to grasp it and no decided reason to believe it, so I am obliged to eliminate these problems from the purpose of this study. But from the very lively solution movements I believe to be able to picture one aspect of the childrens' thinking processes and discuss the above-mentioned problems.

a) On trace performing processes (with traceable figure)

Trace performance of unbroken tracing figure sometimes can be attained by accident. So the factor of performance cannot be defined so as to be applied to all the figures in one meaning.

At first we want to see the results of completion of tracing.

Table 1 indicates the number of completion in each year grade.

Though this does not, of course, show the qualitative factor of completion, the general view of adaptability to the tracing of good figures may be acquired. Roughly speaking, the adaptability increases at the fourth- and third-year grade respectively in boys and girls classes.

Table

School year	Fig. Age	Number of Subjects							aver.
			1	2	4	6	9	10	
1	6—7	6	6	4	2	3	0	—	2.3
2	7—8	9	8	6	6	6	4	0	3.3
3	8—9	9	9	7	6	6	4	1	3.7
4	9—10	9	9	9	6	9	5	6	4.9
5	10—11	9	9	8	7	8	5	6	4.6
6	11—12	9	9	8	7	7	5	8	4.9
7	12—13	9	9	9	7	8	6	7	5.6

boys

Table 2 indicates the average number of copies used both in completion and in failure. This shows that generally the number of copies used with girls is smaller than with boys, and at the fourth- and third-year grade respectively with boys and girls, the number takes the direction of decrease.

Why did such differences appear in number of completion or in number of copies used ?

Table

School year	Fig. Number of Subjects							Total
		1	2	4	6	9	10	
1	6	2.2	4.5	5.0	4.8	4.0	—	20.5
2	9	1.9	3.4	3.6	3.2	5.1	6.0	22.5
3	9	1.6	4.8	4.1	4.8	4.5	4.6	24.2
4	9	1.6	3.9	2.1	1.4	3.1	3.8	15.9
5	9	1.0	2.2	2.1	2.0	3.0	3.7	14.0
6	9	1.0	2.6	2.6	2.9	3.2	4.1	16.5
7	9	1.0	1.9	1.9	2.2	2.6	2.6	12.1

boys

Now we wish to try to analyze the factors which seem to be utilized for trace performance of unbroken tracing figures. We can consider them under the following heads :

1. Choosing the starting-point.
2. Changing of starting-point.
3. Tracing course.

1

Number of Subjects	1	2	4	6	9	10	aver.
6	5	5	4	6	2	1	3.7
9	6	3	4	4	2	1	3.3
9	9	9	8	9	4	2	4.6
9	9	9	8	6	5	4	4.7
9	9	8	8	7	6	4	4.7
9	9	9	8	9	8	7	5.6

girls

1. Choosing the starting-point

Generally speaking, the tracing of unbroken tracing figure can be said to be accomplished by choosing the right starting-point. How is choosing the right starting-point be practised? We can mention the four methods as follows :

2

Number of Subjects	1	2	4	6	9	10	Total
6	3.5	2.5	2.3	2.3	2.8	4.0	17.4
9	1.5	3.3	2.3	3.8	4.0	6.0	20.9
9	1.8	2.9	1.7	2.3	4.6	4.3	17.3
9	1.2	1.8	2.9	1.7	2.0	3.0	12.7
9	1.2	2.9	2.9	2.2	2.9	5.7	17.8
9	1.0	2.3	2.1	2.1	2.7	3.4	13.6

girls

Type 1. (Random type) This is such a type as to place ones hand all of a sudden at a random point on the figure as soon as the figure is presented before the subjects. They are lacking the consciousness of possibility or impossibility of performance.

Type 2. (Transitional type) This type shows a sort of selection but not intentional. They do not give the methodical consideration on selection of starting-point. (In the subjects of this type, there are some who trace with the eye before placing their hand, but owing to the lack of methodical consciousness of selection, the eye-tracing cannot be utilized for the good tracing.)

Type 3. (Methodical type) The subjects of this type set their hand on the definite point under the conscious or unconscious consideration of the rule of starting-point or of selection of it after perfect or imperfect eye-tracing or with no eye-tracing.

Besides these, there are some special cases.

For example : —

Type 1. S. S. (boy, 8 yrs.)

In a supplementary figure (Fig. 2, No. 12), he reversed the copy to figure surface slowly, glanced at the figure, soon took his eyes off it, fumbled with crayon which already had been given to him, and on turning his eyes to the figure again, set the crayon on point Q (the apex point). And thereafter he repeated the placing to the same point eight times. (Moreover his tracing course was also the same as the first !)

This point has been already noticed by Lindley¹, naming it 'automatic repetition', as we quoted before.

The number of subjects who seemed to belong to this type were seven (5 boys, 2 girls). But this placing, I think, is not an abnormal kind, for in preliminary experiments we have found frequently the young child who belongs to this type. Though we shall instruct them in question form that "if you repeat as such, will you be able to pass through here (the blank line)?" they will continue until we order them to stop. We think this type a natural one from the genetical viewpoint. The nature of this type is that determination of the placing point is definitely determined by the place on the figure. Not only the subjects of this type, but also the one of other types fix their eyes on the point which is conditioned by a figure-optic nature. But the difference from other types lies in the reflection of trace performance started from the point. For

verification of this fact, we tried with a supplementary figure (Fig 2, No. 11). Then all the subjects, as far as we have experimented, set their crayon on the point P (apex preference), and none ended in failure. This fact may be described by the words "being attracted." In case of the figure which has no apex point, the upper side point may be selected (upper preference), consequently we can say the Fig. 2, No. 4 has the difficulty in selection of starting-point.

In short, the placing of the type is determined by the figure-optic factor, containing no aspects of thinking.

Type 2. The characteristic of this type lies in the obscure consciousness of selection. The selection is executed by indistinct mental expectation, (as "If I start from this point, I shall be able to accomplish the whole tracing.") A feature of the type may be seen in the fact that just as he has set the crayon on some point, he removes to the other point, like hopping, and soon chooses it. Boys and girls of lower and middle grade of elementary schools mainly belong to this type.

Type 3. This type may be divided into two groups.

(a) Though this type apparently may be confused with type 2 it is distinct from it when it is considered in connection with the changing of starting-point. A characteristic movement of this type can be seen in the movement before placing. Some one may discover the main intersecting points on the figure and mark each one of them with crayon, and then follow them in order.

(b) Placing according to rule.

The expression "according to rule", here used, contains an analogy by common nature which should be divided when strictly applied.

For example, the starting-point of No. 4 had been selected by an analogy of No. 2, i. e., intersecting point of three lines. (H. F. boy, 13 yrs.)

The number of subjects of this type is not great in pupils of elementary school course, and the true regularly selected placing movement appears very rarely.

Special case.

The next case is the only one of placings in our subjects.

Before she set the crayon in No. 10, she divided the figure

as follows (Fig. 3), and succeeded in finding the right starting-point. (W.Y. girl, 13 yrs.) (When I designed the figure No. 10, I followed with the same plan as she did.)

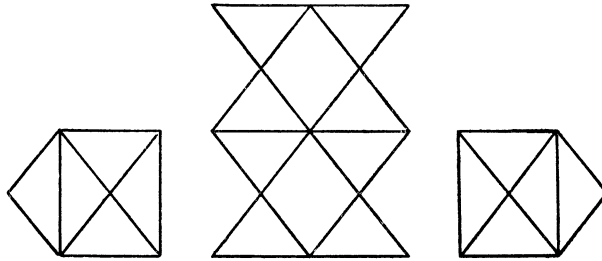


Fig. 3

2. Transfer of starting-point

Selection of right starting-point, as already stated, is the most important key to the trace performances of unbroken-tracings. And this right starting-point is mainly acquired by a transfer of trying point. It is difficult to determine the original form and motives of transfer of starting-point. If we take the introspection of subjects, in most cases, they reported that they 'tried', even when the points were chosen consciously. If we may be permitted to state our impressions from the observation of the whole course of behavior of subjects, the transfers of starting-point occur naturally in spite of the existence of methodical consciousness or of trying. We have stated, however, the persevering tendencies of starting-point. Then, how can the persevering tendencies and natural genesis of shifting of starting-point be explained harmoniously? It is unknown. But bodily and mental development of the child, I think, enlarges its behavioral range naturally, consequently the shifting also occurs there.

Next, there is another type of transfer, the 'retroaction' of starting-point. This means the retroaction from the tried starting-point to the right one by filling up the blank line, and it is one of the successful solving ways. This also can not be seen in early childhood. On the contrary, as the filling up of the blank line may be done from the same point in them, so there is necessarily produced the other blank line.

3. Course of tracing

The change in course of tracing as naturally occurs, I think, despite the persevering tendencies, as in the change of starting-point. But this course of tracing is not the decisive factor of success or failure of its tracing performance except in the case of some careless omission of tracing. On the problem of the course of tracing, it is interesting to pay some attentions to the form of course.

The form of the course of tracing may be divided roughly into two groups :

- 1) A case in which the neighbouring columns are connected as to be said to be adjacent.
- 2) Another case is, grasping the whole figure and analyzing it into some units of concluded line groups, to trace the figure as a composition of the units.

The relation of both cases also seems to be hierarchic, for the form tracings increase in number as the year grade proceeds, and the possibility of performance is greater in the latter than in the former. And the form grasping is carried out chiefly in the form of symmetrical grasping, and analysis of doubly-piled form.

Summary

1. The placing in an unbroken tracing is defined genetically by the place of point in the figure, especially where there exists the fact of apex preference.

Most pupils in lower-year grade essentially belong to this step.

2. The starting-point and the course of tracing, once selected primarily, have a persevering tendency. The tendency in the course is comparatively weak, but the starting-point once selected was frequently found to be constant in the middle-year grade pupils.

3. In selection and in transfer of starting-point, there can be seen the three hierarchic steps i. e., the random, the methodical, and the rational step.

4. In course of tracing, it seems to me to be true that the divert proximity is dominant in lower-year grades, but it tends gradually to the form grasping. Also in the case of repeated trials, there occurs gradually the form grasping. but in these points, there are very large individual differences.

5. The completion of unbroken-tracing is strongly influenced by the transfer of starting-point and the form grasping. The completion can be said to be in their functional relations.

b) On judging of impossibility (with the figures that cannot be traced in unbroken line)

Four in the ten figures used in the first experiment (the order of them are the third, fifth, seventh and eighth) require the judging of their impossibility.

At first we want to show generally the number of correct judgments of each figure in each year grade. (Table 3)

Table 3

School year	Fig.	boys					girls						
		3	5	7	8	Total	aver.	3	5	7	8	Total	aver.
1	1							0.5					
2		3.0	2.5	2.5	2.5	10.5	1.2	3.0	3.0	2.0	2.0	10.0	1.1
3		6.5	5.5	5	4	21.0	2.3	4.5	5	3.5	4.5	18.0	2.0
4		6	6.5	7	6	25.5	2.8	8	7	6.5	5.5	27.0	3.0
5		9	8	7	5.5	29.5	3.3	7	7.5	8	8	30.5	3.4
6		9	8.5	8	7	32.5	3.6	8	8.5	8.5	8	33.0	3.7
7		9	8	6.5	7	30.5	3.4						

We can observe following points in this table.

- 1) The number of 'confident' judgments increases from the third-year grade both with boys and girls, and the pupils below the second-year grade cannot judge of the impossibility except in a few cases.
- 2) The correct judgments of the seventh and eighth figures, which are comparatively complicated in construction, are not very difficult for the subjects who did the correct judgments 'confidently' in the third and fifth figures.
- 3) Although the correct judgments gradually increase as the year-grade proceeds, the individual differences are so large in them, that there remain some subjects who are unable to judge confidently among the seventh-year grade pupils.

Next we observed the number of spontaneous statements

in judging. Though the spontaneous statement cannot be defined in one meaning, we shall be able to use it as a means to observe an aspect of confidence of their judgments. And in this table we obtained roughly the same distribution of correct judgments as in Table 3.

The judgments of impossibility are stated after the tracing trials of subjects. The number of tracing trials are counted in Table 4.

Table 4

School year	Fig.					Fig.				
	3	5	7	8	Total	3	5	7	8	Total
1	3.0					2.2				
2	5.0	6.3	5.9	4.8	21.3	2.7	6.8	5.8	5.3	19.3
3	3.8	5.8	5.4	4.9	19.6	3.0	3.4	3.2	2.4	12.1
4	3.2	3.9	4.9	4.1	16.1	1.2	2.4	3.7	3.7	10.8
5	1.4	1.9	2.7	3.7	9.7	3.2	2.9	2.9	3.0	12.0
6	1.1	2.6	3.6	4.2	11.6	1.2	2.0	2.2	2.2	7.6
7	0.9	2.0	2.7	2.2	7.8					

boys
girls

From this table we can observe the following points.

- 1) Compared with the number of copies used in good figures, the number of copies used in judging bad figures is very large especially in lower-year grades. But the number of the latter gradually decreases and both come closer in higher grades of elementary school pupils.
- 2) With higher grade pupils, there are some subjects who judge the impossibility by tracing with the eyes.
- 3) The number of trials with the third- and fourth-year grade boys is remarkably large (if we do not stop their trials we shall get larger and larger numbers), and it decreases decidedly in fifth-year grade. With girls the decrease exists in the third-year grade.

Attitudes of subjects in their judgments

After our subjects had stated their judgments of impossibility, we asked them the reasons of their judgments and tested the degrees of their confidence in their judgments. From these reasons and degrees, we can divide their attitudes toward impossibility into some stages.

1) A stage of lacking possible or impossible consciousness to bad figures.

This is the case in which the subjects cannot judge impossibility toward bad figures by any tracing trials, and there can be seen no spontaneous statements. If we ask them the possibility of tracing, they will necessarily answer that they are traceable. Incompleted trials are all supposed to be failures. Moreover there exist in their trials frequently the automatic repetitions of starting-point and that of tracing course, as already mentioned in good figures. Even among prudent and clever children, their judgments are 'possible'. Such being the case, we must think that there does not exist even the naive consciousness of possibility. They are as yet in the stage of pre-consciousness of judgment of possibility or impossibility.

Almost all the pupils in first- and second-year grades belong to this stage.

For example: Y. T. (boy, 8yrs.) Fig. 2, No. 3

He has ended eight trials during 8'25'' with slow but firm strokes. Whenever he turns to the next copy, he used such words as these. "I understood." "I erred these." "Strange. . . . Pass here, . . . pass here, Well I will try." etc. Even when he ended his eighth trial, I could get no expectation of his judgment. So, ordering him to stop his trial, I asked him the possibility of the figure, but he, instead of answering, took the next copy and began to trace.

2) A stage of lacking confidence in judgment.

When the pupils were asked to answer as to the possibility, those who were at this stage judged the impossibility of bad figures, at any rate, but their judgments were lacking in confidence. And when they were asked the reasons of their judgments, there appeared again, at times, a consciousness of possibility, it seems to me that their judgments were not based on the figures, but rather on their trials.

This stage seems to me to have a transitional meaning. Determination of this stage can be carried out by judgment of 'impossible', and at this point it is divided from Stage 1. But the lacking of confidence in judgment is the distinguishing point to be separated from the next Stage 3. And to see this stage as a transitional one, is due to the fact that the pupils at this stage are unsettl-

ed between the consciousness of possibility and impossibility.

For example :

(1) K. G. (boy, 10 yrs.), Fig. 2, No. 3

After seven trials (5'45''), I ordered him to stop and asked as the possibility. He answered. "This may not be traced." The cause of his answer was as follows. "I am inclined to be one line in excess (in this figure)." His confidence is perturbable.

(2) M. S. (boy, 11 yrs.), Fig. 2, No. 3

Eight trials (5'45''). Then he placed the crayon on the table and remained in that position until 6'30''. And when he was asked of its possibility, he answered this figure may be 'possible'.

No. 5. After his ninth trial, he stated. "This cannot be traced, . . . because this figure is constructed only with triangles." This reason of impossibility cannot be developed.

(3) T. Y. (girl, 12 yrs.), Fig. 2, No. 3

No stroke until 5 minutes. When she was asked, she answered. "This may be possible." But she never set her crayon on the figure. I asked. "Can this figure be traced through, if you passed through skilfully?" She answered. "Yes, this can be traced."

She did generally in the same manner in No. 5 as in No. 3.

No. 7. Ended three trials (4'5''). When she was asked, she answered that this was not possible, and it was because she tried many tracings from several points, and that she could not accomplish it. Her confidence was strong.

In No. 8, she has given generally the same answers. Her confidence was strong; but, after several questions concerning the reason of judgment, she could not be certain.

Although the above-mentioned three examples have a common characteristic, there are some differences between them, and it will be possible to say that those differences may be important and essential ones.

It will be plain that the first example is a transitional one which lies near the Stage 1 yet, the 'confidence' is very weak and judgment reaches the type of Stage 1 as the figure adds the external complexity. On the contrary, the last example is a transitional one which lies near the Stage 3.

Now the attitudes toward judgment of possibility and impossibility is, logically, the selection either of the former or of the latter. But, psychologically, the subjective conscious attitudes during some difinite time will confuse both cases. The subjects may ask them-

selves: "Which is right?" The consciousness of perplexity, which is produced by the selection of one from two, will appear when they already know the meaning of possibility and impossibility.

And an organized potentiality of this mental attitude will lead the subject to a definite direction, if there exists a preparation of receptiveness of a new perceiving tendency which seems to be produced by experience. So I think that establishing this stage may be adequate in classification of psychological stages.

3) A stage of confidence in concrete cases.

It need not be said that the arriving at correct and 'confident' judgment will be accomplished when it has been done in accordance with an abstract law; but there exists, besides above-mentioned cases, a sort of judgment, that is, a 'confident' judgment in concrete figures.

Examples of the cases which belong to this stage are as follows:

(1) "When I start from here, this remains. . . . And when start from here, then this remains. . . . So whatever point we choose as starting-point, we cannot trace this figure through." (H. Y., girl, 10 yrs.)

(2) "If a line was not traced twice, the remaining lines can never be traced." (H. T., girl, 10 yrs.)

(3) "If the lines were fewer than these, it could have been traced." (in Fig. 2, No. 3. N. S., boy, 10 yrs.)

(4) "There remains a line, for three lines converge." (This expression of reason of impossibility did not develop more than this. In Fig. 2, No. 3. K. I., boy, 10 yrs.)

(5) "There are many lines in the centre, so either PP' or PP'' remains." (in Fig. 2, No. 5. K. I., boy, 10 yrs.)

(6) "If we trace outside circle at first and then enter into the inner lines, some one of them remains. And if we trace two thirds of the circle and then enter into the inner lines, some one necessarily remains as before. If we enter into the inner lines after tracing one third of the circle, we have the same result. If we start from an intersecting point on the circumference, or from the centre point, there also remains an untraced line." (in Fig. 2, No. 3. M. K., boy, 13 yrs.)

(7) Though I tried to trace from several . . . from all four points, all ended in failure. So . . ." (in Fig. 2, No. 8. O. Y., boy, 13 yrs.)

(8) "This figure is made up of three triangles side by side.

Owing to this proximity this figure cannot be traced with an unbroken tracing." (in Fig. 2, No. 3. T. T., boy, 13 yrs.)

(9) "I tried to trace from the two points, R' and P, and both trials ended in failure. So" So I asked: "Then, from the point Q?" He answered. "It is needless to try from that point, for it has failed in the trial from P'." (in Fig. 2, No 7. H. T., boy, 14 yrs.)

These examples will show the confident judgments in concrete figures. In these statements of their reasons of judgment, we see a tendency for the statements grow gradually more logical and argumentative. And, at last when they have been attracted with abstract nature, there appears the judgments which were done according to rule.

4) A stage of judgment using abstract natures of points.

The subjects who belong to this stage were very few.

For example :

H. F. (boy, 13 yrs.)

He has judged the impossibility of Fig. 2, No. 3 in 22 seconds. Starting-points tried with eyes by him were Q' and P. He has stated the reason of his judgment as follows. "The group of lines shaped Y cannot be drawn with an unbroken line, and so" And he judged the Fig. 2, No. 7 in 7 seconds. But he did not arrive at the law of impossibility of unbroken tracing by the end of first experiment.

Among those who belong to this stage, there were some subjects who arrived spontaneously at the conclusion of the law of impossibility. So we intend to describe them together with the results of the second experiment.

Analysis of factors used in Judgment

Now we have mentioned the reasons of judgments in bad figures expressed by our subjects, and settled an attempt of classification of their developmental stages. We can see that these four stages develop gradually as the year-grade proceeds. Then, what factors exist in these development? The answer to this problem, I think, lies in starting-point and in grasping of shape. Those points were treated in trace performance of good figures. And the traits described already appear in the judgment of bad figures. So we do not treat them here in detail again. If we only look at

some points of selection and transfer of starting-points and of the grasping of shape on the whole aspects of the behavior of subjects, we shall understand these statements.

Summary

1) The judgment of impossibility in unbroken tracing figures begins from about third-year grade both with boys and girls. And there can be hardly seen the confident judgment below the second-year grade except in rare special cases.

2) There exists a stage of 'confident' judgment in concrete figures, and a germ of spontaneous judgment according to rule can be seen above sixth-year grade. And there appeared one subject who accomplished spontaneous production of the law among the seventh-year grade boys. With girls, we cannot determine the time in which they execute their spontaneous production of the law.

3) In execution of judging, the selection and transfer of starting-point and the grasping of shape seem to be most important methodical factors.

3) Discussions of the first experiment

In this first experiment, are contained the problems to accomplish the tracings of six figures and to judge the impossibility of four figures. From the experimental results, we shall be able to attempt to consider the problem solving processes.

(1) It is clear that the accomplishments of tracing in good figures can be obtained by finding a correct starting-point, and tracing a suitable course, as we have already observed those factors from some aspects.

In these tracing 'accomplishments', it is possible that there are such as were obtained by accident. These cannot, however, be called true solutions. For these solutions will never influence the solution of other figures in the meaning of the understanding of problems, which are constructed under the same principle.

That the simple repetitions of accidental accomplishments which will be explained by the law of practices and of effects, can become the factor of learning, is not the correct interpretation of learning; true solution becomes complete when someone does a

treatment according to rule toward all the figures. Then the interpretation of solving processes will be acquired on the ground of the observation of whole processes of this true solution.¹

If we grasp the factors of solution on its appearance, it must be abrupt. Needless to say that the appearance of true solution is intuitive.² However, the appearance of solution and the whole course of solution are different things.

At the trials of tracing in unbroken tracing figure, we have seen already that there exist persevering tendencies of starting-point and of tracing courses. The behavior under these tendencies is not connected with the completion of tracing. But, when there is a consciousness of the problem, there occurs necessarily the transfer of starting-point and change of course. These changes are at first accidental, but they grow selective, discriminative, and methodical in the end. Primarily they have no direct relation with the problem solution. It is essentially an accidental sort of thing and its nature can be seen in expression of surprise or disappointment in the case of failure. But, when they grow to a methodical one, the failure is no failure, and it has a positive meaning of analysis of incorrect trials. These methodical treatments are intuitive behavior. But only this methodical treatment as itself does not seem to determine the solving behavior. I am inclined to take the arbitrary stage as an important part of the learning processes.

Then we must proceed to the next problem. What is the determining factor in the case of the genesis of this sort of insight? Of course this question cannot be answered now from an experimental or positivistic point of view. Here I venture a conception of perceptive maturing. This idea is neither perfectly defined nor cautiously analyzed yet. The functional aspects of these problems are to be studied in future.

(2) Next we shall be able to point out the fact of perceptive maturing more distinctly in judgment of impossibility. This judgment of impossibility also may be perfectly executed by means of ideas according to rule, but it is also a fact that there exists the confident judgment of impossibility in concrete figures.

¹ Ref. E. L. Thorndike, *Educational Psychology*. 1914.

² Ref. W. Köhler, *Intelligenzprüfungen an Menschenaffen*. 1934 ; *Psychologische Probleme*. 1933. 67.

K. Koffka, *Principles of gsetalt psychology* 1935. 535.

Here is one simple judgment problem: "Can you go to two places at one and the time?" This problem is self-evident. If we show this problem with linear drawing, it will become as Fig. 4.

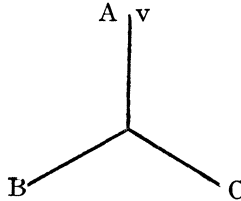


Fig. 4

No one, starting from A, can go to B and C at the same time. And the principle of Fig. 2, No. 3 is entirely the same as this. But the aspects of this judgment of impossibility vary its degrees at some stage, as already stated. These differences come about from the differences of complexity of the figure. But the principle itself contains no essential differences. There we may expect the occurrence of discrimination in the whole situation. There I expect the perceptive maturing. The perceptive maturing is not the word concerning perceptive ability which is distinguished from the whole situation, but the perceptive maturing appears in thinking to be closely connected with perception. Judgment is arrived at in the situation in which the perception and the consciousness of relation exist in converging condition.

III. The Second Experiment

We intend to observe the producing processes of the law of impossibility and the law of starting-point of figures drawn in unbroken tracing, and then to observe the applying processes of the law there obtained. In short, it is our purpose to study the processes of learning of thought problem in children.

1) Finding the law of impossibility

To see the formative processes of the law of impossibility, we divided them into following groups.

(A) Spontaneous finding :

During the first performance and judgment experiment, some subjects found the law by themselves. We called the cases the spontaneous finding.

(B) Findings resulting from instruction :

These findings were divided into five stages according to the abstractness of instruction.

Stage i. At first a subject is given the name and the nature of the odd point and even point, by means of using the Fig. A. Applying this knowledge to some bad figures the law may be discovered.

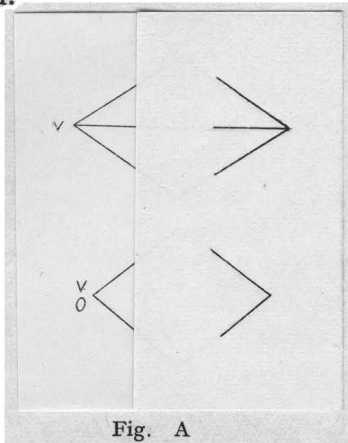


Fig. A

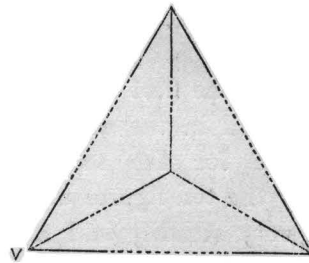


Fig. B

Stage ii. In supplementary Fig. B, the understandings of the meaning and method of discrimination of points on the figure were arrived at by instruction. Using this discrimination of points, the law may be discovered.

Stage iii. After the learning of discrimination of points, Fig. B is traced concretely by the subject, and there the law may be discovered.

Stage iv. Concerning the number of odd points, a suggestion was given.

Stage v. The law which is prepared is presented, and the subject understands it.

If some one did not understand yet after the above-mentioned procedures, we took him to be unable to understand.

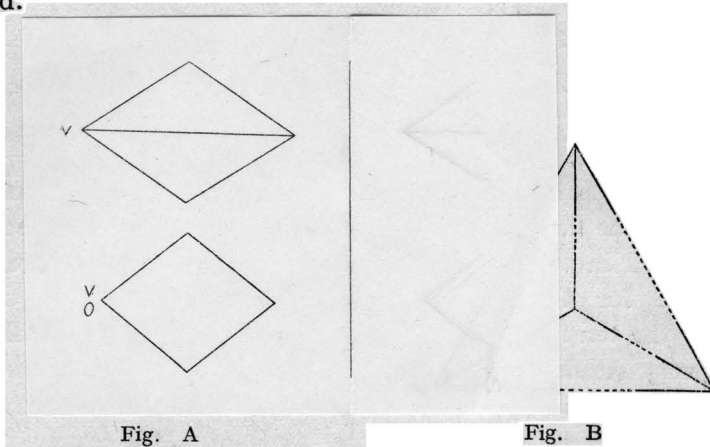
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Stage ii. In supplementary Fig. B, the understandings of the meaning and method of discrimination of points on the figure were arrived at by instruction. Using this discrimination of points, the law may be discovered.

Stage iii. After the learning of discrimination of points, Fig. B is traced concretely by the subject, and there the law may be discovered.

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Processes of finding the law.

(A) Of spontaneous finding

There were only two subjects who arrived at finding the law spontaneously.

For example :

(1) A. K. (boy, 14 yrs.)


Fig. 2, No. 3 Correct judgment (21s.). He stated the reason as follows. "There is a line in excess . . . Besides, all the intersecting points are the points of odd numbers."



Fig. 2, No. 5 Correct judgment (16 s.). On the statement of judgment, he declared. "Is not there a reason in these ?" (I think this word correctly indicates that he is in the direction of the finding the law.)

And there the law was found. Some days after he brought a sheet of paper on which sixteen figures were newly designed by himself, and the sixteen figures were classified according to the nature of figure.

(2) H. T. (boy, 14 yrs.)

He did not responded so well as in the former example in judging of the first experiment. And at the end of it he seemed to remain still in the confident stage in concrete figures. But when he came for the second experiment, and he was required to classify the ten figures used in the first experiment, he finished the work very steadily and rapidly. Then I asked the standard of classification, and he stated the law concerning unbroken tracing in general. Here we missed the moment of discovery, but he presented the next introspections. According his expressions, he

picked out the figure  from No. 7, and tried its tracings numeral times. From these trials he had found that there exist two cases in which the tracing courses are brought to an impasse.

They are  and . He said, "When we enter into the point

where three lines converge we are either obliged to stay there even when there remain other lines, or we are unable to return unless we trace the same line twice." So he guessed that, "the meeting point of three lines is doubtful." Then he has drawn many figures and traced them in many ways, and while he was doing the tasks he arrived at the moment to believe in his idea of rule. And here

we shall be able to point out the fact that his progress in finding the law came after he had acquired the idea of the odd point, and he has accomplished, if we speak formally, his task of law formation by inductive method, under some accidental idea, rather than deductive one under the nature of odd point and even point.

(B) Instructive finding

Stage i.

For example :

(1) S. A. (girl, 11 yrs.)

After instruction of the nature of points, she did as follows. She took up the Fig. 2, No. 5 and said. "If we enter into Q'' from Q', and go out from Q'' and again enter into Q'', we cannot any longer go out." (This may probably be called the behavior form of Stage iii). So I asked. "If you would end all the remaining lines before you came back to Q''?" She answered. "If all the other lines had been ended, we shall be stopped at some point before returning to Q''." Here the law was accomplished.

(2) Strictly speaking, the above example tells us that it has a concrete limitation of abstract natures. Here is a true example.

W. Y. (girl, 12 yrs.)

Stage i. In Fig. 2, No. 3, she said. "There are three odd points (pointing Q, Q', Q''), and two of them are stop-points. So this cannot be traced with an unbroken tracing."

Applying these procedures in Fig. 2, No. 5, and No. 7, she found out the law.

Stage ii.

For example :

(1) I. Y. (boy, 12 yrs.)

Stage i. His idea was not evident.

Stage ii. Fig. B. has been presented. Discrimination of each point was prominently distinct, and at the moment of the ending of this work, he said as follows, as if he was waiting to say. "The entering corner must be one, is not that so? Then the law was accomplished.

(2) S. H. (boy, 10 yrs.)

Stage i. The understanding of nature of points was clear, but the reasons of judgment in Fig. 2, No. 3 remained concrete,

and no connection of this task appeared.

Stage ii. The reasons of judgments remained concrete. So I asked him, "Then, can you distinguish the bad figures without any tracing?" He answered nothing, but pointed Q' and Q'' with crayon, as if there was no need to answer.

In Fig. 2, No. 5, he answered in the same manner. But in his concrete expression there was a steadiness which seemed not to be found in Stage iii. So I asked again, "Then, how do you distinguish with a new figure?" He answered, "If there are three points converging three lines in the figure, it is impossible."

Stage iii.

For example :

N. Y. (girl, 9 yrs.)

Although, in her first experiment, she traced very cautiously and gave me some impressions of intuitive behavior, there appeared no clear expression of judgment of impossibility.

Stage i. No ideas appeared.

Stage ii. The discrimination of points was cautious and it required considerable time. And the results were all correct. But these were never unified in one idea.

Stage iii. In Fig. 2, No. 5, she gave the reason of judgment as there are two entering points. When she was asked about those points, she pointed to them in the order Q''' , Q'''' , Q''''' , Q' , Q'' , and corrected her error, saying "five."

And the reason of judgment was given at last in these words. "If we enter into one of them, we shall never be able to enter into the other, So this figure is impossible."

Trying her judgment in No. 7 made her arrival at a law.

The characteristic of this example is her remoteness from the idea of rule. It could be seen that she was reluctantly and indifferently doing her work for she was compelled to understand it. Generally the pupils in this year-grade are essentially still indifferent to the idea of law, or logical causal relationships. In conclusion, I would say, they are still in the stage of simple tracing in the problem of unbroken tracing figure.

Stage iv.

Those who belong to this stage are in parallel to those of stage iii.

For example :

(1) U. Y. (boy, 11 yrs.)

He remains concrete till the stage iii.

Stage iv. In Fig. 2, No. 3, I told him that there are three entering points among the points meeting three lines. He answered readily, "Yes!" His attitude is different from before; there occurred, so to speak, the centralization of thinking. Thus he arrived at the finding of law. The applications of this law were correct.

(2) O. M. (girl, 9 yrs.)

The answers until stage iii was either concrete or passive.

Stage iv. I have suggested the number of odd entering points. With Fig. 2, No. 3, she said. "At first I decide on a starting-point. And if there are two entering points of opposite ends, it is impossible." With these childish expressions, however, she had carried out confident application. This shows, no doubt, that there is the understanding of properties of entering point. But this understanding of the properties of entering point has a characteristic to be associated always with the course of tracing.

Stage v.

It will be needless to say that there were those subjects who understood the law by means of the presentation of it. The ranges of years extended from 8 yrs. to 11 yrs.

Note : Those who are unable to understand the law.

Pupils below the second-year grade (with girls) and the third-year (with boys) generally belong to the type. And there can be seen no one who belongs to this type above sixth with boys and fifth with girls.

Application of the law of impossibility

Procedures

For the test of the application of the law of impossibility, the next five figures (Fig. C) were successively presented before each subject, and the determinations by eye inspection of their possibility or impossibility were asked. The construction of these five figures involves two impossible ones (their orders of presentation were respectively 2 and 4), and the other three figures were good ones, but the No. 3 was different.

Processes and results

An application of law lies in the general ordinance of concrete figures according to the abstract nature of law. In our experiment it must be executed confidently without concrete tracing. Observations of the manner of application of the law are divided into the next four groups.

(1) A case in which the application of law is impossible.

(2) A case in which the attitude of judging was guided by the concrete tracing and thereafter the application was executed indifferently.

(3) A case in which the application of law was accompanied by the concrete tracing.

(4) A case of pure application of law.

We shall take some instances in each cases.

(1) Case 1.

N. T. (boy, 8 yrs.)

Notwithstanding that the law is presented, he does trial tracings in each figure.

Moreover there are such statements as these. (in No. 2) "It is strange. Though I had accomplished it now." In No. 4, "I have doen." This time I gave him a crayon, and made him trace actually.

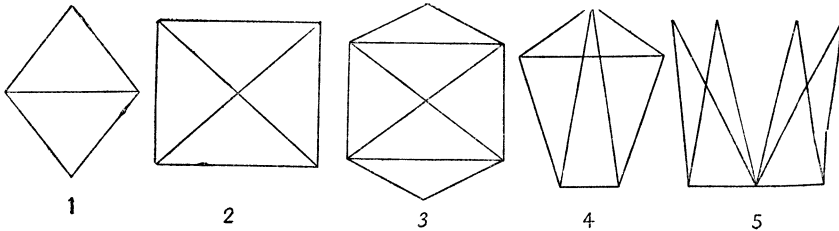


Fig. C

Then he said. "It is strange." His judgments were always "possible."

(2) Case 2.

K. M. (girl, 10 yrs.)

Her judgments were all right, but she tried tracing in each figure. Being asked the reason of her judgment, she could not answer readily, for she referred to the law presented, after her tracing trials.

(3) Case 3.

K. R. (boy, 11 yrs.)

In Fig. 2, he judged correctly (23 s). And he stated the reason as follows. "There are four entering places". I asked. "Are there four?" He answered. "Ah! three." Then I asked again. "How did you determined that this is impossible?" He answered. "I traced the outside square at first, and went into the centre. And then, if I go to Q, I must stop there. And there remain two points."

In the case 4, they will examine the points, counting the number of lines meeting at each point, and will execute the 'confident' judgement. There will be no need to illustrate the example.

Summary

1. In the pupils below the second-year grade, the understanding of the law of impossibility of unbroken tracing may not occur necessarily.

2. There exists a stage of concrete confidence of judgment before the understanding of the law of impossibility occurs.

3. An abstract thinking which is connected in a unified whole with concrete trial seems to be carried out purely in above the sixth-year grade.

4. An application of the law of impossibility in unbroken tracing appears in about sixth-year grade in its pure form, and above about third-year grade there exists the application of law accompanying the concrete tracing.

2) Finding the laws of starting-point and its application

Finding the laws of starting-point

Experimental procedures.— Experimental procedures of finding the laws of starting point are almost the same as in the finding the law of impossibility.

Experimental processes and their results.— In these points also there appeared almost the same as in finding the law of impossibility. So we intend hereafter to state some new points.

(1) For the preparation of getting the idea concerning starting-points, subjects were made to trace two figures. These figures are two of the five figures used in the application of the law of impossibility.

In these trials almost all the subjects immediately accomplished their tracings. We suppose from this fact that the learning of the law of impossibility is in the same relationship to the learning of the nature of starting-points.

(2) Ideas obtained concerning starting-points

The ideas of subjects obtained in the above-mentioned trials were asked. And they will roughly be divided into two groups.

1) The ideas which contained no meaning of odd point. For example, the pupils in lower-grade naively pointed out the point in concrete figures. "Here and here" is the expression of this case.

There are many other ideas as follows.

"From the place where there are a stick."

(A stick indicates the line pp') (girl, 8 yrs.)

"From a corner" (boy, 9 yrs.)

and "From an end." (boy, 9 yrs.).

There can be seen no idea of this group in pupils above sixth-year grade, but there were three boys who did not express their idea at all.

2) The ideas which contained the meaning of odd points.

Some examples removed to the point of three lines.

For example :

(1) S. S. (boy, 9 yrs.)

At first he pointed out P and P' , saying 'here and here'. I asked him what point it is. But, as he remained long with no answer, I urged him to answer. His answer was that "from this point of three lines." And his reason with which he selected the points was that by starting from the certain point he would be able to trace many lines.

(2) D. M. (boy, 10 yrs.)

At first he said the point at the end of a line to be possible. And the idea of three lines was found first in third-year grade.

But the idea of the point of three lines develops towards the odd point in general, and then it becomes a completed law.

The idea of odd point in general was found in fourth-year grade with boys and there was only one among all the girl subjects in fifth-year grade.

Summary

(1) In the ideas obtained, there are many ideas of the three

lines point, but, by contrast, the idea of the odd point in general is very rare.

(2) The idea of the three lines point appears in second-year grade but it increases in third-year grade.

(3) Both with boys and girls, pupils above sixth-year grade showed the ideas of the odd point (containing the three lines point).

(4) The foundations of their ideas are, on the whole, concrete, but in seventh-year grade, it was found out rationally.

Finding stages of the laws of starting-points and application of the laws.

Generally the processes of finding the laws of starting-points were studied under the same scheme as in the finding the law of impossibility. But there have existed already some differences in their experimental conditions. For example, the subjects have already been learned the law of impossibility and have understood the natures of abstract points. So in these findings of the laws of starting-points, we must say that we can not observe the spontaneous finding in its pure form. In other words, the methodical experiences must have influenced these processes. And there have been obtained on the whole the similar distribution of subjects in each stage. So we can omit the description in detail.

And at the same time we have tested the application of these laws for all the subjects. In these applications there have appeared also almost the same results as in the application of the laws of impossibility.

So we will give only a summary of these finding processes and the results of their application here.

1. With pupils below the second-year grade, the understanding of the laws of starting-points in unbroken tracing may not occur necessarily.

2. The understanding of the laws of starting-points presupposes the concrete and confident grasping of unbroken tracing. But the understanding of law directly requires the analysis of abstract nature. In these finding processes, there has existed the fact that the discrimination of points presuming an incorrect point as its starting-point is more difficult than that from the correct starting-point.

3. An abstract thinking which is connected in unity with a

concrete trial seems to be executed purely above the sixth-year grade.

4. An application of the laws of starting-points in unbroken tracing appears in about sixth-year grade in its pure form, and above about third-year grade there exist the application of the law accompanying the concrete tracing.

3) Discussions

What is the logical construction of the problem of finding the laws of impossibility and of starting-points in unbroken tracing ?

Its logical construction may be considered as dividing into two parts from the viewpoint of finding processes, i. e., 1) a case of spontaneous finding 2) another case of instructive finding. We can see the essential differences existing in these two parts on the true finding of the properties of abstract points only in the forms, even though in the latter these must be understood. Now, let us suppose that there is given an abstract nature, i. e., the general proposition. When we connect it with a concrete affair, there is formed, it may be said, a logical inference.

If we consider it with the judgment problem in unbroken tracing, it will be demonstrated as follows :

{	One cannot go to two places at a time.
	This figure (for example, Fig. B) has two (or more) points which require going to at a time.
	Therefore this figure is impossible to trace in an unbroken tracing.

Thereupon, the unbroken tracing problem is constructed with finding of an abstract nature and an inference of a kind, when we analyse it formally.

Even in our instructive finding, the major premise is not given in the sentence form of itself, so the subject must find it by some suggestions which have been given in five stages as stated before. But the actual fact of the major premise is very simple. Although it is not possible to compare with the a fact selected by W. Köhler, it might easily be understood by all the subjects if they have a perception of time and place. The fact is that one cannot be at two places at a time. It is an interesting problem how a child constructs the major premise in his syllogisms. And as the tasks are rarely given in the form of syllogisms in the world of actual facts, the major premises must always be constructed. At the same time the

sylogistical definition of conceptions is not done strictly in daily affairs. So it seems to me very significant to observe the processes of inference in children from these constructive aspects.

There are several studies which tested the children's thinking by syllogism. That is, Meumann, Scüssler, Stern, Meyer, Müller, etc. Then, by contrast with the formal inferences, we shall call, for the time being, the inference which contains the construction of premises, a constructive inference. If we compare with both results, we shall be able to obtain a ground to interpret the Meumann's assertion, I think.

Next, the application of the laws there obtained is in the same relation to syllogisms, and, in this case, as there is some room for concrete trials, we shall be able to obtain some suggestions concerning the interests of children in the law, and the problem of learning of law in children.

As the judgment of possibility or impossibility or the selection of correct starting-point in unbroken tracing acquires first the generality when they have been executed rationally, so, only in the judgment according to law may be said to be true judgments. Until the judgment according to law in unbroken tracing was accomplished, there could be seen certain stages in our subjects.

1. A stage in which consciousness of judgment is lacking.

To the subjects who belong to this stage, every figure is judged to be traceable, and it is so especially even when they started from any of points on the figure. In other words, they are trying with indifference to possibility or impossibility. Most pupils in first- and second-year grades in elementary schools are in this stage, eliminating a few exceptions.

2. A stage in which the concrete trials are the foundation of their judgments.

The subjects who belong to this stage arrive at a judgment, a connection with a confined abstract nature, as a result of some trials for the concrete problems which are directed by the nature of tasks.

Although their judgments are confident in individual cases, they are still entertain misgivings towards ideas according to law.

We saw that they were overwhelmingly in the range from third- to five-year grade in elementary school pupils.

3. The third stage is that in which abstract thinking are carried out in its pure form.

In unbroken tracing, the natures of the points on the figure are connected functionally as an abstract itself, and the abstract relations between points are grasped in concrete figure. Though there are accompanied by the concrete trials, these seem to produce the stage of application of the abstracts.

This type of thinking seems to occur in its pure form in sixth- or seventh-year grade in elementary school pupils.

Let us consider these ideas in reference to the results of application of the law.

Most of the pupils who arrived at the understanding of the law by means of some concrete trials, the instruction of discrimination of the natures of points, or the presentation of the law itself, made concrete tracing trials at first on its application, and thereafter applied it when they were asked its application. This fact seems to show that the maturity of interest to ideas according to law is not as yet sufficient. That there exist the test trials after the application of the law in the pupils of the more advanced years, seems also to show that there exist not yet the unification of concreteness and generalities.

The law proceeds from the concrete affairs. However, the law once established dominates the concretes. This dominance by law on the concrete facts is acquired essentially, I think, in the attainment of pure abstract thinking.

If this idea is allowed, I think that the claims of Meumann¹ contain some agreeable points and at the same time the assertion of Schüssler² also has a point. The essential point in this problem lies in the question, how to grasp the essential qualities of thinking.

Now we shall turn to the problem of qualitative consideration concerning the solution of the thinking problem.

As the construction of problem in this experiment contains the concrete trial in some way, that the optic moments or kinaesthetic moments which are accompanied by stroke, will attribute to solution in any degree, seems to be the nature of this task.

I feel from the results of this experiment that the existence of confidence in judgment acquired optically or acquired kinaesthe-

¹ E. Meumann, *Vorlesungen zur Einführung in die experimentelle Pädagogik und ihre psychologischen Grundlagen*, Bd. 1, 1916, 549.

² H. Schüssler, *Ist die Behauptung Meumanns richtig: Kinder können im allgemeinen vor dem 14 Lebensjahre nicht logisch schließen?* *Z. angew. Psych.* Bd. 11, 1916.

tic show their important meaning as a factor of solution.

But it is an important problem how the relation of the perception and the judgment itself will be.

The constant selection of the same starting-point and the same tracing course have no relation to the judgment of possibility of figure and there can be found no solution.

However, there occur naturally the changing of starting-points and tracing courses. They are never established on rational foundations, but they become rational in the end. The judgments are concretely or optically confident in the stage of the concrete confidence, and they are distinguished from the simple repetitions of trials. Hereupon we ought to suppose a mental aspect in development of thinking processes and this stage was described exactly by Müller.¹

Now we must think of the relation of this concrete thinking and pure abstract thinking.

We have already seen the facts of understanding the laws by means of concrete trials and the abstract finding of it. Also we have seen the application of the laws accompanying tracing and its pure application. In these facts we see the unified whole processes of the sensuous and the intellectual. For the intellectuals are always accompanied by the sensuous, and the intellectuals are withdrawn from the sensuous.

But they are not the same. The sensuous are related only to the concrete figure, and the intellectuals dominate the concretes. So I think it is enough that there exists the thinking consciousness which includes the sensuous in it and is in higher dimension than sensuous.

Here I remember the confrontation of sensationalism and intellectualism. The sensationalism as an explaining principle of thought processes adopted by Associationists, had been exploded by the theory of thought of Würzburg school.

But, as the Gestalt theory rose into power, it came to stand in opposition to the Würzburg school. And there exist some authors² who call the Gestalt theory a sensationalism (though the cause of it is not the same as to the Associationism.).

In contrast with the Gestalt theory in which the mental organization is considered as a unified whole, Willwoll, who has been dire-

¹ C. V. Müller, Experimentelle Untersuchungen über kindliche Schlussprozesse mit besonderer Berücksichtigung der Vorgänge der Repräsentation. Arch. Psych. 78, 1930 u. 79, 1931.

² K. Nakamura, Der Wert und das Denken. 1939, 107-8.

cted by Bühler, has introduced the concept of "Beziehungseinsicht" and demonstrated the following opinion in his studies of "Begriffsbildung." He says. „Das Denken musste sich ‚beziehungen-erfassend‘ und die anschaulichen Bildelemente den logischen Beziehungen der Aufgabe gemäss umgestaltend und ordnend am Anschauungsstoff betätigt haben. Es wäre gewiss übertrieben, wollten wir schlechthin jedes Auftauchen etwa aufgabedienlicher Umgruppierung der Vorstellungsinhalte oder anschaulicher Schemata als Beweis für das Walten reflex bewusster Beziehungseinsichten buchen. Aber restlich bleiben beide ohne diese unverständlich.“¹

When we examine the processes of the law production and its application in our experiments of unbroken tracing, we understand the Willwoll's opinion as a suggestive and interesting one.

IV. Conclusions

Through the experiments we shall be able to reach the following points concerning the thinking processes of children.

1) Thinking solutions of the problem of unbroken tracing figure, both of tracing and of judging, appear mainly with perceptive maturing in children.

2) The perceptive maturing begins at first in the behaviors of children which have apparently a nature of some arbitrary trials, but it develops to concrete thinking and, as the abstract nature occupies the dominance in thinking, there establishes the genuine abstract thinking.

3) Although the mode of appearance of solution in thinking problem is intuitive, we should recognize the meaning of trials in developmental meaning.

4) The thinking solutions of logical problems is yet in germ in children of lower-year grade in elementary schools, and generally in middle-year grade it develops to the confident but concrete stage. And voluntary abstract thinking solution appears in higher-year grade.

5) But the individual differences in the development of thinking behavior are very large. So the presentation of a thinking problem in school tasks ought to be thought to have a different kind of purpose according to the developmental stages of children.

(Received 4. X, 1941)

¹ A. Willwoll, Begriffsbildung. 1926, 142.