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Measuring the Information Flow Received by High School Students to Solve a Simple Research Problem

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In the paper, we use a new approach to measure the information flow used for problem solving. The problem was taken from a high school course "Ecology and business" and presented as an interface program available on the Internet. The participants communicated with the program in a dialogue mode and received feedback only as the terms "right" or "wrong". The successions of correct moves were considered to give useful information to the participants. The information was measured according to Shannon's approach, as a logarithm of the probability of events. It was established that the minimum information required to solve the problem was within the range of 30-50 bits. Also, to provide a successful solution, the information received at one time by a participant had to exceed 10 bits at least once.

Keywords: information; problem solving; research task; phase portrait; Monte Carlo modeling

Introduction

The term "information" first appeared in mathematical and technical sciences in the first half of the 20th century. Since then, theories of signal transmitting and computer applications have depended on this term. However, ideas about information penetrated into less exact and more humanitarian spheres, such as intellect theories, learning, psychology and behavioral sciences. Newell and Simon (Newell & Simon, 1972), the founders of human problem solving theories,

wrote about a shift to understanding "a man as an active processor of symbolic information".

Some authors in evolutionary psychology discuss the concept of "information behavior" which is considered to be a socio-cognitive human ability. Information behavior includes seeking, organizing and using information to achieve ecological dominance. Humans are also able to create information (Spink & Cole, 2005, 2006, 2007). The notion of information is also a key point in discussing the profound

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analogies between biological evolution and human cognition. Both phenomena are seen as natural information processing systems (Sweller & Sweller, 2006).

The successful use of the concept of information in such scientific spheres as psychology, behavioral science and learning will eventually depend on how well research succeeds in the problem of measuring information. If it is treated too broadly and casually, the idea of information will be useless for these spheres.

It is widely known that there are three approaches to measuring information. The first one (combinatory) was suggested by Ralph Hartley, who in 1928 put forward the formula $H = \log_2 N$ where H is system entropy, and N stands for the number of equiprobable outcomes of an experiment. The Hartley formula allows one to estimate the information capacity of such simple events as tossing a coin or throwing dice. The second (probabilistic) approach was developed by Claude Shannon and was based on the idea that the quantity of information was a negative logarithm of the probability for some event to happen (1).

$$I = -\log_2 P_i \quad (1)$$

where I is the quantity of information resulting from some i -th event, the probability of which is P . The probabilistic approach allows one to estimate the information quantity in more complicated systems, where events happen with a variety of probabilities. As follows from the formula (1), a less probable event bears more information. The third (algorithmic) approach was suggested by Andrey Kolmogorov (Kolmogorov, 1965). As a rule, most real objects are so complicated and unique that they cannot be considered in terms of probability. In the algorithmic approach, the information capacity of an object can be

estimated as the length of a program to reproduce the object.

Generally speaking, for real objects and processes (e.g. the text of a fiction book), the combinatory and probabilistic approaches seem to be inapplicable. However, if researchers manage to distinguish elementary events and to estimate their probabilities, the measurement of information can be conducted through these approaches.

Problem solving is a kind of learning in that an individual finds (learns) a way of solving a problem. Like learning, problem solving is a process that involves a flow of information. Unlike traditional learning, problem solving often provides a structured and simplified learning environment. Therefore, one can distinguish events and estimate their probabilities within the process of problem solving. In a previous paper (Gavrikov & Khlebopros, 2010), we suggested a kind of a problem called a “research problem”. A typical research problem includes the need to understand, grasp or discover a principle of the functioning or logic of something that is yet unknown. The suggested research problem includes a method of solving that we called a “semi-binary dialogue”. The mode of such a dialogue allows the complicated process of problem solving to be divided into elementary events, to estimate their probabilities and, in principle, to measure information. The main focus here is on measuring information to estimate the threshold levels of incoming information which are necessary to solve the problem.

Details of the problem, its technical implementation and the way to measure information are given in the Methods section below.

Methods

Every academic course has a number of terms and ideas that determine the content of the

course. The research problem used in this study was taken from the fourth year course “Ecology and Business” for students at the Institute of Economics, Management and Environmental Studies (Siberian Federal University, Krasnoyarsk, Russia). The course’s professor divided all the terms into three categories: #1 “Climate change”, #2 “Environmental pollution” and #3 “Ecological penalties”. The stimuli for the participating students was a series of sets, each composed of nine terms, and only one set was used at a time. Within a set, a student had to make a move, i.e. choose any one of the terms. As a response to the move, the student received a message stating whether the choice was right or wrong. “Right” and “wrong” here had the conventional meaning and were determined by whether the term did or did not belong to the correct category. Also, the right category changed in succession. For example, the first right category was category #1, i.e. a term chosen from it was “right” and from any other was “wrong”. If the student chose a right term, a new set of 9 terms was offered in which a term from category #2 was considered right. Therefore, if a successful choice was made the “right” category was shifted according to the rule #1 #2 → #3 → #2 → #1 and so on. If a student made a wrong choice, another choice from the same set of terms was offered.

The students were not aware that the terms they saw were grouped into categories. Nor did they know that the correctness of the terms had changed in any way. The only instruction given to the students before they began the test was that they should try to get only “right” responses.

First, the students had to recognize the number of available categories, second, recognize if a term belonged to the right category, and third, find an algorithm that ensured the right choices. We considered that those students who made six right moves in succession successfully

solved the problem, as had been indicated in the instructions.

To provide the interface for the problem, we wrote a special computer program RWR (right-wrong responder). The executing modules were uploaded to a server (<http://sandbox.kspu.ru/test2.html>) and were available from any local computer through the Internet. Students received the terms and notions as a 3 by 3 matrix, with a button under every term, which allowed them to choose the term.

A random procedure gathered the terms into one set, and each set contained one and only one right term. The messages for the students were “right choice”, “wrong choice”, or “6 right choices, you may stop”. The RWR program registered all the student’s moves and saved them to a file on the remote server.

From a technical point of view, the result of the problem solving is a sequence of units and zeros of the sort “...1011100101000...”, where 1 stands for an error and 0 stands for a right move. Every symbol in the sequence bears some information because it is a message from the program, its reaction to a human choice that has been consciously made. For this study, we decided to analyze only positive information, i.e. successions of right messages.

Because the probability of a single right move is known to be $1/9$, the probabilities of encountering various successions of right moves could be calculated. For practical reasons, the following successions were followed: “101” was a single right move, “1001” was a double right move, “10001” was a triple right move, etc. If moves are considered to be independent of each other, then it is possible to calculate the probabilities of the successions and therefore their informative capacities. For example, the probability for the succession “101” equals $(8/9) \times (1/9) \times (8/9) \approx 0,087$, and its informative capacity is $\log_2(0,087) \approx 3,52$ bits. This approach was applied to calculate

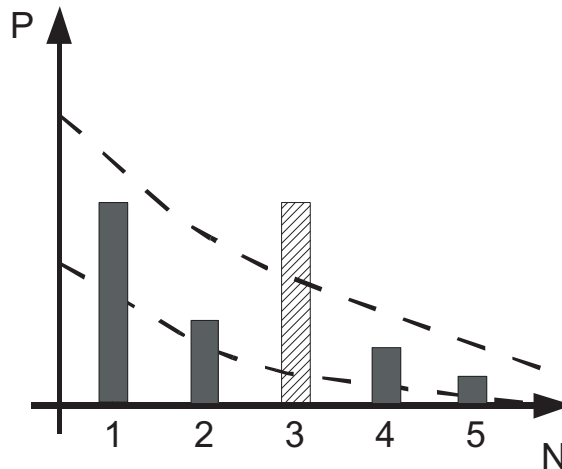


Fig. 1. The diagram that illustrates the principle of how to calculate the volume of information received by a participant in one attempt. N stands for lengths of right move successions that appear with the frequency P . The dashed lines delimit the Monte Carlo 95%-confidence intervals. In this fictitious example, successions of three right moves appear more often than they would in a random clicking and, therefore, bear a significant volume of information

the informative capacities for all other succession lengths, from 2 to 6.

In this study, we decided that not every succession of right moves bears some useful information for a participant. If a participant clicks quite randomly, successions of right moves will also appear. But it is reasonable to believe that the result of random clicking is information noise. We used Monte Carlo modeling to determine if the results of a participant significantly differed from information noise.

Monte Carlo methods is the general title of a group of numerical methods that use multiple realizations of stochastic processes.

To get the data for the confidence intervals, we modified the program so that it, and not a student, generated mouse clicks that randomly chose one of nine terms. In addition to the random clicking, the program treated the choice of the terms as if it was made by a human. Altogether, the program simulated the work of 100 people who made 400 random clicks each. The obtained data were used to calculate the 95% confidence interval.

If a participant clicks randomly, the observed frequencies of the successions of right moves will fall into definite intervals. If, however, the participant produces some successions greater than what would be randomly produced, then only those successions are considered to bear useful information. Fig. 1 shows how the approach works. All the information contained in such successions was summed up and this information was received by a participant as the result of his/her attempt.

In mathematical terms, $K = \sum I(i)$ only for those i that satisfy $X_i > M_i$, where X_i is the observed frequency of right moves' successions of the length i , M_i is the upper limit of the confidence interval for the same i , I is the Shannonian information, and K is the total information received by the participant in a given attempt.

A detailed analysis of the observed dynamics in one attempt may be carried out through studying the relationship $\dot{I}(I)$, i.e. the time derivative of information as a function of the information value. It closely resembles phase portraits, which are widely used in the qualitative

theory of different dynamic systems, from physics to ecological modeling.

However, in this study, the use of real time records was inapplicable. This is because no limitations were imposed on the participants' work schedule or behavior. Because of this, there were time gaps in the protocols, probably due to distractions or breaks which had no obvious relation to the problem solving. Therefore, for calculating the relationship $\dot{I}(I)$, we used the number of moves as a substitute for time as follows, $\dot{I} = (I_{i+1} - I_i) / (N_{i+1} - N_i)$. Here N_i represents information received by a participant with a succession that ends on the move N_i .

An assistant presented the program to the students in a regular computer class simply by giving them the Internet address of the program. Those students who did not solve the task at their first attempt could continue working with the program on the Internet at any convenient time. It is important to mention that participating in the program and solving the problem were totally voluntary on the students' part.

The students solved the task in May and November-December 2010. Altogether, 70 students took part in the study; their age ranged from 20 to 21 years, 19 of them were male and 51 were female. The successful solution was found in the protocols of 23 students.

Results and discussion

If an individual needs to solve a problem and doesn't know how to do so, then he/she has to get a certain amount of necessary information, obviously greater than zero. There is a definite minimum of information that is sufficient for solving the problem. A "sufficient minimum" means here that an excellent mind can solve the problem with that minimum at his/her disposal. On the other hand, if the amount of information is lower than this minimum, it should be recognized that the solution is not attainable.

Though a theory to predict such an information minimum (or one could say a threshold) is not available, the idea of such a threshold is nevertheless meaningful. If a participant solves a problem while having received too little information, this can be regarded as a "suspicious" case, in that unintended methods of solving may have been used.

All the participants' data were divided into two categories: those who solved the task (solvers) and those who did not (non-solvers). Fig. 2 shows the distribution of the participant results on the coordinates "total amount of information received" against "the total number of moves made". The number of moves is given in a logarithmic scale to make the chart more visually accessible because the individual numbers of moves could differ as many as tens of times.

The groups of solvers and non-solvers were compared with the help of a few popular non-parametric tests (Kolmogorov-Smirnov test, Mann-Whitney U test, Wald-Wolfowitz runs test). The groups differ very significantly ($p < 0.001$) in regard to both the average total amounts of received information (135 bits for solvers against 28 for non-solvers) and the amounts of information received in the last attempt (88 bits against 11 bits, respectively).

The solvers had higher total numbers of moves made (507) than did the non-solvers (382). Though not all tests show the significance of the mean difference, it seems plausible that on average the solvers' moves were more informative than those of the non-solvers.

In the data protocols of some participants, we found traces of a peculiar trait – a multiple pressing of an obviously wrong button up to five to ten or more times. Such "illogical" behavior can be interpreted as the participants expressing their emotional reaction to the problem's difficulty. The participants might thus disperse their irritation at being unable to penetrate the

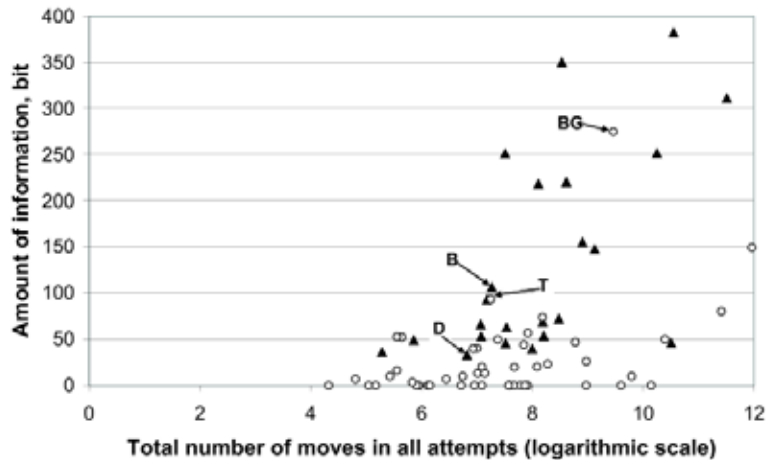


Fig. 2. Distribution of participants' results on the plane "total number of moves" against "total amount of information". Legends: ▲ stands for solvers, ○ stands for non-solvers. Letters B, D, T, BG denote results of individual participants

problem's logic. In some cases, this reaction was observed up to 8 to 11 times over the entire period of solving the problem. A direct comparison of the means of the emotional reactions for solvers and non-solvers is not correct. Still, it seems that non-solvers are more emotional (23%) than solvers (13%).

The distribution of solvers and non-solvers shown in Fig. 2 suggests that the empirical boundary of the minimal amount of information required to solve the problem lies in the range of 30 to 50 bits. However, this estimation requires several remarks. First, the estimation should be treated in the way established for measuring information (see 2. Methods section). Second, the obtained data allow us to differentiate between a *problem solution* and the *full realization of its logic*.

The usual successful scenario seen in the majority of cases follows a pattern. The participant makes one or, more often, several attempts, and at the end of the last one he/she achieves success i.e. makes six right moves in succession. Any other activity thereafter is not registered, which means that the participant no longer tries to work with the program.

Infrequently though (in two out of 23 cases) we observed another scenario. For example, participant D solved the problem by the end of the second attempt, with the total information of the attempt being 33 bits. Still in two hours, she did made an attempt that lasted 173 moves and finally ended with the success. In this attempt, the total information received was 277 bits. Participant B reached the solution in his first attempt (106 bits), but on the next day he made another attempt (26 bits) that also ended with the solution.

Therefore, the minimal amount of information sufficient for a single solution can be estimated as 30-50 bits. But a profound understanding of the task can require two to four times more information. Unfortunately, due to few examples of such behavior, it is difficult to make more definite assessments.

An analysis of the participant's individual results is provided by the phase portraits of their attempts. In particular, the phase portrait's form can give a hint of what the level of the information threshold might be. According to theoretical considerations (Gavrikov & Khlebopros, 2009), such a threshold separates two steady states of an individual's learning: ignorance and knowledge.

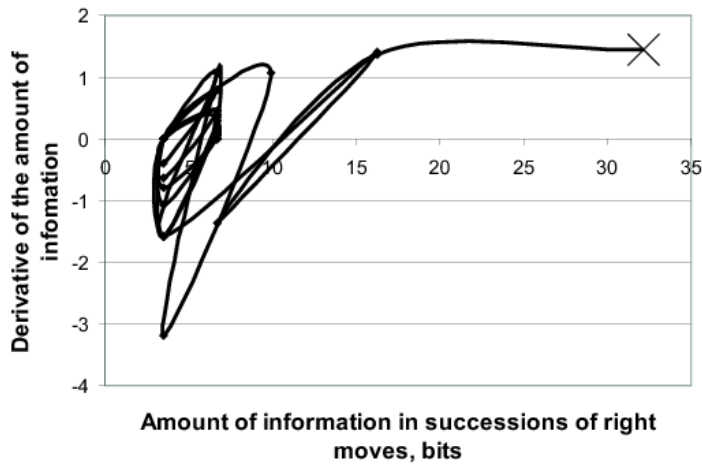


Fig. 3. Phase portrait of the first attempt of the participant B (see Fig. 2). The symbol \times denotes the final succession of right moves

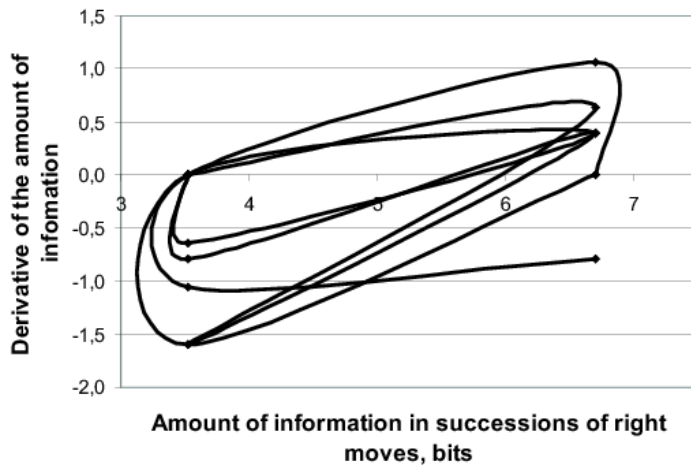


Fig. 4. Phase portrait of the second (and the last) attempt of participant T (see Fig. 2)

An individual stays in a state of ignorance until he/she crosses the threshold as a result of, for example, efforts to learn.

Figures 3 and 4 show the attempts by participants B and T respectively. The participants received a comparable amount of information (see Fig. 2). Fig. 3 gives a picture of typical successful problem solving. The form of the trajectory suggests that there is an attraction area in the range of 5 bits where a quasi-cyclical dynamic takes place. The level of information

obviously corresponds to the state of ignorance. The trajectory shows that the threshold may lie at about 10 bits. Crossing the threshold may lead to another steady state that is the point of the maximum possible information (problem solution).

Fig. 4 shows a typical unsuccessful attempt. The trajectory oscillates about a range of 5 bits because the majority of the participant's right moves are single (~3.5 bits) and double (~6.7 bits) successions. No crossing of the threshold is

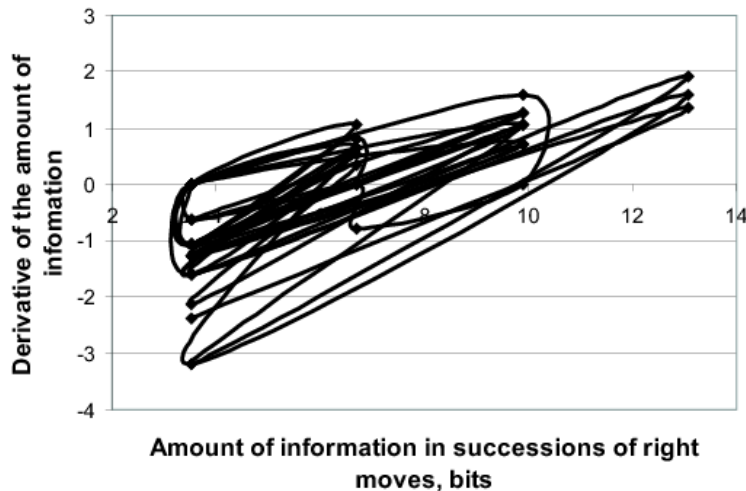


Fig. 5. Phase portrait of the last of six attempts did in one day by the participant BG

observed, so there is no progress in the problem solving.

However, it is necessary to mention some “strange” cases when the participants received more than solvers’ amounts of information but were still unable to solve the problem. For example, a non-solver participant BG (see Fig. 2) received as much as 218 bits in one attempt, which looks more impressive than solver B’s result (106 bits). The phase portrait of the most promising attempt by BG shows that she crossed the level of 10 bits several times but did not solve the task (Fig. 5). There may be a few explanations for this, but obviously BG could not perceive or properly interpret the received information, possibly from fatigue. The attempt by BG is the sixth attempt made in over an hour of attempted problem solving. It seems quite plausible that, first, the threshold is a matter of individual variability and that, second, the current physiological (fatigue) or psychological (fear, irritation) state of the participants can shift the threshold’s value.

Conclusion

In this paper, we have tried to measure the information flow used in problem solving.

Through a dialogue mode the participants received information and could use it to successfully solve the problem. We also explored a hypothesis that there is a threshold in the amount of received information which separates “no solution” and “solution” steady states. From an empirical viewpoint, the threshold seems to lie in the range of 30 to 50 bits of total received information. A detailed analysis using phase portraits shows that not only the total amount of received information, but also the peak values, play an important role. If the participant receives more than 10 bits at one time, he/she will almost certainly solve the problem. In some individual cases, 10 bits may be insufficient for success. The causes of such failures are not known but may be the result of some cognitive deficiencies.

It is necessary to remember that the estimates of information values depend on the way in which they are measured. Still, it seems important to use the units accepted in information theory whenever possible in psychological research. Psychological studies that use their own units or no measurements at all cannot produce results comparable with others in a broad, often international, scientific context. At the same time,

the theory of information as a solid natural science provides such a basis for wide comparisons.

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Поток информации при решении студентами вуза простой исследовательской задачи

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В статье анализируется характеристика получения информации человеком, решающим простую исследовательскую задачу на распознавание категорий в понятийном аппарате учебного курса. Технически задача была реализована как сетевая интерактивная программа, доступная через Интернет. Работая в режиме диалога с программой, испытуемые получали информацию о ходе решения в виде сообщений «правильно» и «неправильно». Информация измерялась как функция от вероятности случайного появления серий правильных ходов разной длины. Согласно полученным данным, для решения предложенной задачи необходимо, чтобы испытуемый суммарно получил как минимум 30-50 бит информации. Анализ индивидуальной динамики решения показывает, что решение задачи возможно, если разовая порция получаемой информации составляет более 10 бит.

Ключевые слова: информация; решение проблем; исследовательская задача; фазовый портрет; Монте-Карло-моделирование.
