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Simultaneous Effects of Slip Conditions and Wall Properties on MHD Peristaltic Flow of a Maxwell Fluid with Heat Transfer

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The effects of both wall slip conditions and heat transfer on the magnetohydrodynamics (MHD) peristaltic flow of a Maxwell fluid in a porous planar channel with elastic wall properties have been studied. Mathematical formulation is based upon the modified Darcy's law. The analytical solution has been derived for the stream function and temperature under the assumptions of small wave number.The results obtained in the analysis have been discussed numerically and explained graphically.

Keywords: peristalsis, Maxwell fluid, modified Darcy's law, Brinkman number, Knudsen number, Heat transfer coefficient.

Introduction

Peristaltic flows are generated by the propagation of waves along the length of a distensible tube. It plays an indispensable role in transporting many physiological fluids in the body in various situations such as urine transport from kidney to bladder, movement of ovum in the fallopian tubes, the movement of enzyme in the gastrointestinal tract, swallowing of food through oesophagus and the vasomotion of small blood vessels. Some biomedical instruments, like the blood pumps in dialysis and the heart lung machine use the principle of peristaltic pumping to transport fluids without internal moving parts. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited and transport of a toxic liquid used in nuclear industry to avoid contamination of the outside environment.

The problem of the mechanism of peristaltic transport has attracted the attention of many investigators since the first investigation of Latham [1]. A number of analytical, numerical and experimental [2–15] studies have been conducted to understand peristaltic action for different kinds of fluids under different conditions with reference to physiological and mechanical situations. However the interaction of peristalsis and heat transfer has not received much attention which may become highly relevant and significant in several industrial processes. Also thermodynamical aspects of blood may become significant in processes like oxygenation and hemodialysis [16–18] when blood is drawn out of the body. Recently the combined effects of magnetohydrodynamics and heat transfer on the peristaltic transport of viscous fluid in a channel with compliant walls have been discussed by Mekheimer and Abd elmaboud and co-workers [19, 20]. Recently, Hayat et al. [21], Nadeem and Akram [22] developed the problem by considering slip conditions on the boundary of the channel

The study of fluid flows and heat transfer through porous medium has attracted much

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attention recently. It is well known that flow through a porous medium has practical applications especially in geophysical fluid dynamics. Examples of natural porous media are beach sand, sandstone, limestone, wood, the human lung, bile duct, gall bladder with stones and small blood vessels. In some pathological situations, the distribution of fatty cholesterol and artery clogging blood clots in the lumen of coronary artery can be considered as equivalent to a porous medium.Hayat et al. [23] have analyzed hall effects on peristaltic flow of a Maxwell fluid in a porous medium.Hayat et al. [24] have examined the effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space. Very recently, Hayat et al. [25] have investigated the influence of heat and mass transfer on MHD peristaltic flow of a Maxwell fluid with complaint walls which have not been discussed so far when no-slip condition is no longer valid. The present paper concentrate on this concept.

The main purpose of the present study is to highlight the importance of slip conditions and heat transfer on MHD peristaltic flow of a Maxwell fluid through a porous medium in planar channel with elastic wall properties. The perturbation method has been used for the analytic solution. The features of flow characteristics are analysed by plotting graphs. The significance of the present model over the existing models has been pointed out by comparing the results with other theories. The paper has been organized as follows. In section 2, the problem is first modeled and the non-dimensional governing equations are formulated. Section 3 includes the solutions of the problem.Numerical results and discussion are presented in section 4. The conclusions have been summarized in section 5.

1. Mathematical Formulation of the Problem

Consider the flow of an electrically conducting incompressible Maxwell fluid through a porous channel of uniform thickness in presence of a constant transverse magnetic field B_0 (see Fig. 1).

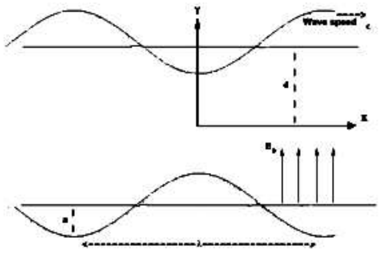

Fig. 1. Schematic diagram of the physical model

The induced magnetic field is assumed negligible for small magnetic Reynolds number. The walls of the channel are assumed to be flexible and are taken as a stretched membrane, on which travelling sinusoidal waves of moderate amplitude are imposed. The geometry of the channel wall is given by

$$y = \eta(x,t) = d + a\sin\frac{2\pi}{\lambda}(X - ct) \tag{1}$$

where d is the mean half width of the channel, a is the amplitude, λ is the wave length, t is the time, X is the direction of wave propagation, c is the phase speed of the wave.

The equations governing the motion of the present problem are [25]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-\sigma B_0^2 u+R_x, \tag{3}$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}+\frac{\partial S_{xy}}{\partial x}+\frac{\partial S_{yy}}{\partial y}+R_y, \tag{4}$$

$$\rho C_p\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=\kappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+S_{xx}\frac{\partial u}{\partial x}+S_{yy}\frac{\partial v}{\partial y}+S_{xy}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \tag{5}$$

where u, v are the velocities in the x and y directions respectively, p is the pressure, ρ is the density, μ is the coefficient of viscosity of fluid, σ is the electrical conductivity of the fluid, κ is the thermal conductivity, C_p is the specific heat at constant pressure, S_{xx}, S_{xy} and S_{yy} are the components of extra stress tensor S, R_x and R_y are the component of Darcy's resistance R and T is the temperature of the fluid. For a Maxwell fluid, the extra stress tensor S satisfies the following equation:

$$S+\Lambda_1\left(\frac{dS}{dt}-LS-SL^T\right)=\mu A_1 \tag{6}$$

Here $\Lambda_1(\geqslant 0)$ is the relaxation time. The espressions for velocity gradient L and first Rivlin-Erickson tensor A_1 are

$$L=\text{grad}V, \tag{7}$$

$$A_1=(\text{grad}V)+(\text{grad}V)^T \tag{8}$$

Since we are considering the slip on the wall, therefore, the corresponding boundary conditions for the present problem can be written as

$$u=\mp\beta\frac{\partial u}{\partial y} \quad at \quad y=\pm\eta, \tag{9}$$

$$T=T_0\mp\gamma\frac{\partial T}{\partial y} \quad at \quad y=\pm\eta \tag{10}$$

where T_0 is the temperature at the walls, β and γ are the dimensional slip parameters.
The governing equation of motion of the flexible wall may be expressed as :

$$L^*(\eta)=p-p_0 \tag{11}$$

where L^* is an operator, which is used to represent the motion of stretched membrane with viscosity damping force, flexural rigidity of the plate etc such that

$$L^*\equiv-\tau\frac{\partial^2}{\partial x^2}+m\frac{\partial^2}{\partial t^2}+d_1\frac{\partial}{\partial t}+B\frac{\partial^4}{\partial x^4}+H. \tag{12}$$

The continuity of stress at $y=\pm\eta$ and using x-momentum equation yield

$$\frac{\partial}{\partial x}L^*(\eta)=\frac{\partial p}{\partial x}=\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-\sigma B_0^2 u+R_x-\rho\frac{du}{dt} \quad at \quad y=\pm\eta \tag{13}$$

Here p_0 is the pressure on the outside surface of the wall due to the tension in the muscles, τ is the elastic tension in the membrane, m is the mass per unit area, d_1 is the coefficient of viscous damping forces, B is the flexural rigidity of the plate and H is the spring stiffness. Here we assumed $p_0=0$.

The Darcy's resistance R in Maxwell fluid can be obtained from the equation

$$\left(1+\lambda_1\frac{d}{dt}\right)R=-\frac{\mu}{k}V \tag{14}$$

where k is the permeability parameter, λ_1 is the relaxation parameter and V is the velocity of the fluid.

Introducing ψ such $u = d\dfrac{\partial\psi}{\partial y}$, $v = -\dfrac{\partial\psi}{\partial x}$ and the following non-dimensional quantities :

$$\begin{cases} \bar{\psi} = \dfrac{\psi}{cd};\ \bar{x} = \dfrac{x}{d};\bar{y} = \dfrac{y}{d};\ \bar{\eta} = \dfrac{\eta}{d};\varepsilon = \dfrac{a}{d};\delta = \dfrac{d}{\lambda};\bar{p} = \dfrac{d^2p}{c\mu\lambda};Re = \dfrac{cd\rho}{\mu};\theta = \dfrac{T-T_0}{T_0};\ \bar{\beta} = \dfrac{\beta}{d}; \\ \bar{\gamma} = \dfrac{\gamma}{d};\ P_r = \dfrac{\rho\nu C_p}{\kappa};\ \bar{S}_{ij} = \dfrac{dS_{ij}}{c\mu};\ \bar{\lambda}_1 = \dfrac{\lambda_1 c}{d};\ \bar{t} = \dfrac{ct}{\lambda};\ M = \sqrt{\dfrac{\sigma}{\mu}}B_0 d;\ E_c = \dfrac{c^2}{C_pT_0}; \\ \bar{K} = \dfrac{k}{d^2};\ E_1 = -\dfrac{\tau d^3}{\lambda^3\mu c};\ E_2 = \dfrac{mcd^3}{\lambda^3\mu};\ E_3 = \dfrac{d_1d^3}{\lambda^2\mu};\ E_4 = \dfrac{Bd^3}{\lambda^3\mu c};\ E_5 = \dfrac{Hd^3}{\lambda\mu c}; \end{cases} \tag{15}$$

in (3)–(5) and using (14), we finally get (after dropping bars)

$$\begin{aligned} &\delta Re\left[1+\delta\lambda_1\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\right]\left[\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\left(\delta^2\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)\right]= \\ &=-\frac{1}{K}\left(\delta^2\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+\left[1+\delta\lambda_1\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\right]\left[\left(\frac{\partial^2}{\partial y^2}-\delta^2\frac{\partial^2}{\partial x^2}\right)S_{xy}+\right. \\ &\left.+\delta\frac{\partial^2}{\partial x\partial y}\left(S_{xx}-S_{yy}\right)-M^2\frac{\partial^2\psi}{\partial y^2}\right], \end{aligned} \tag{16}$$

$$\begin{aligned} &\delta RePr\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\theta=\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+Br\left[\delta(S_{xx}-S_{yy})\frac{\partial^2\psi}{\partial x\partial y}+\right. \\ &\left.+\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)S_{xy}\right], \end{aligned} \tag{17}$$

in which the components of extra stress tensor can be obtained through Eqs. (6)–(8) and are given by

$$S_{xx}+\lambda_1\left[\delta\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)S_{xx}-2(\delta S_{xx}\psi_{xy}+S_{xy}\psi_{yy})\right]=2\delta\psi_{xy}, \tag{18}$$

$$S_{yy}+\lambda_1\left[\delta\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)S_{yy}+2(\delta^2 S_{xy}\psi_{xx}+\delta S_{yy}\psi_{xy})\right]=-2\delta\psi_{xy}, \tag{19}$$

$$S_{xy}+\lambda_1\left[\delta\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)S_{xy}+(\delta^2 S_{xx}\psi_{xx}-S_{yy}\psi_{yy})\right]=\psi_{yy}-\delta^2\psi_{xx}, \tag{20}$$

where Re is tha Reynolds number, δ is the wave number, M is the Hartman number, P_r is the Prandtl number, E_c is the Eckert number, $B_r(=E_cP_r\ )$is the Brinkman number, E_1, E_2, E_3, E_4 and E_5 are non-dimensional elasticity parameters.

Also the boundary conditions reduce to

$$\frac{\partial\psi}{\partial y}=\mp\beta\frac{\partial^2\psi}{\partial y^2}, at\quad y=\pm\eta=\pm\left[1+\varepsilon\sin 2\pi(x-t)\right], \tag{21}$$

$$\theta=\mp\gamma\frac{\partial\theta}{\partial y}, at\quad y=\pm\eta, \tag{22}$$

$$\begin{aligned} &\left[1+\delta\lambda_1\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\right]\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial t\partial x}+E_4\frac{\partial^5}{\partial x^5}+E_5\frac{\partial}{\partial x}\right]\eta= \\ &=\left[1+\delta\lambda_1\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\right]\left[\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-M^2\psi_y-Re\delta\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\right.\right. \\ &\left.\left.-\psi_x\frac{\partial}{\partial y}\right)\psi_y\right]-\frac{1}{K}\psi_y, at\quad y=\pm\eta \end{aligned} \tag{23}$$

It is worth pointing out that the present model can be further developed in future using the following geometry of the channal wall:
1. Sinusoidal wave:

$$\eta(x,t) = d + bX + a\sin\frac{2\pi}{\lambda}(X-ct).$$

2. Triangular wave:

$$\eta(x,t) = d + bX + a\left\{\frac{8}{\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\frac{2\pi(2n-1)}{\lambda}(X-ct)\right\}.$$

3. Square wave:

$$\eta(x,t) = d + bX + a\left\{\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\sin\frac{2\pi(2n-1)}{\lambda}(X-ct)\right\}.$$

2. Solution of the Problem

As the Eqs. (16)–(23) are highly non-linear differential equations,analytical solutions valid for all arbitrary parameters involving in these equations, seems to be impossible to find.We have used perturbation method in this section to find analytical solution. For perturbation solution , we expand flow quantities in a power series of δ as follows

$$\left\{\begin{array}{l} \psi = \psi_0 + \delta\psi_1 + \delta^2\psi_2 + ..., \\ \theta = \theta_0 + \delta\theta_1 + \delta^2\theta_2 + ..., \\ S_{xx} = S_{0xx} + \delta S_{1xx} + \delta^2 S_{2xx} + ..., \\ S_{xy} = S_{0xy} + \delta S_{1xy} + \delta^2 S_{2xy} + ..., \\ S_{yy} = S_{0yy} + \delta S_{1yy} + \delta^2 S_{2yy} + ..., \end{array}\right. \tag{24}$$

If we substitute (24) into (16)–(23) and seperate the terms of different orders in δ, we obtain the following system of partial differential equations for stream function and temperature together with boundary conditions

2.1. Zeroth Order System

$$\frac{\partial^2 S_{0xy}}{\partial y^2} - N^2\psi_{0yy} = 0, \tag{25}$$

$$\frac{\partial^2\theta_0}{\partial y^2} + B_r\psi_{0yy}S_{0xy} = 0, \tag{26}$$

$$S_{0xx} - 2\lambda_1\psi_{0yy}S_{0xy} = 0, \tag{27}$$

$$S_{0xy} - \lambda_1\psi_{0yy}S_{0yy} = \psi_{0yy}, \tag{28}$$

$$S_{0yy} = 0, \tag{29}$$

$$\frac{\partial\psi_0}{\partial y} = \mp\beta\frac{\partial^2\psi_0}{\partial y^2}, at \quad y = \pm\eta, \tag{30}$$

$$\theta_0 = \mp\gamma\frac{\partial\theta_0}{\partial y}, at \quad y = \pm\eta, \tag{31}$$

$$\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial t\partial x}+E_4\frac{\partial^5}{\partial x^5}+E_5\frac{\partial}{\partial x}\right]\eta=\frac{\partial S_{0xy}}{\partial y}-N^2\psi_{0y}, at \quad y=\pm\eta \tag{32}$$

where $N^2=M^2+\dfrac{1}{K}$

2.2. First Order System

$$\begin{aligned}&Re\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\psi_{0yy}=\lambda_1\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2 S_{0xy}}{\partial y^2}-M^2\psi_{0yy}\right)\\&+\frac{\partial^2}{\partial x\partial y}(S_{0xx}-S_{0yy})-N^2\psi_{1yy}+\frac{\partial^2 S_{1xy}}{\partial y^2},\end{aligned} \tag{33}$$

$$\frac{\partial^2\theta_1}{\partial y^2}=ReP_r\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\theta_0-B_r[\psi_{0yy}S_{1xy}+\psi_{1yy}S_{0xy}+\psi_{0xy}(S_{0xx}-S_{0yy})], \tag{34}$$

$$S_{1xx}+\lambda_1\left[\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)S_{0xx}-2(S_{0xx}\psi_{0xy}+S_{0xy}\psi_{1yy}+S_{1xy}\psi_{0yy})\right]=2\psi_{0xy}, \tag{35}$$

$$S_{1yy}+\lambda_1\left[\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)S_{0yy}+2S_{0yy}\psi_{0xy}\right]=-2\psi_{0xy}, \tag{36}$$

$$S_{1xy}+\lambda_1\left[\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)S_{0xy}-(S_{0yy}\psi_{1yy}+S_{1yy}\psi_{0yy})\right]=\psi_{1yy}, \tag{37}$$

$$\frac{\partial\psi_1}{\partial y}=\mp\beta\frac{\partial^2\psi_1}{\partial y^2}, at \quad y=\pm\eta, \tag{38}$$

$$\theta_1=\mp\gamma\frac{\partial\theta_1}{\partial y}, at \quad y=\pm\eta, \tag{39}$$

$$\begin{aligned}&\lambda_1\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial t\partial x}+E_4\frac{\partial^5}{\partial x^5}+E_5\frac{\partial}{\partial x}\right]\eta=\\&=\frac{\partial S_{0xy}}{\partial x}+\frac{\partial S_{1xy}}{\partial y}-N^2\psi_{1y}+\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\left[\lambda_1\frac{\partial S_{0xy}}{\partial y}-(\lambda_1M^2+Re)\psi_{0y}\right],\\&\qquad at \quad y=\pm\eta\end{aligned} \tag{40}$$

2.3. Zeroth Order Solution

The solutions of Equs.(25),(26), satisfying conditions(30)-(32) straight forward can be written as

$$\psi_0=L\left\{\frac{\sinh Ny}{N(\cosh N\eta+\beta N\sinh N\eta)}-y\right\}, \tag{41}$$

$$\begin{aligned}\theta_0=&\frac{B_rL^2}{8(\cosh N\eta+\beta N\sinh N\eta)^2}\{(\cosh 2N\eta+2\gamma N\sinh 2N\eta-\cosh 2Ny)+\\&+2N^2(y^2-\eta^2-2\gamma\eta)\}\end{aligned} \tag{42}$$

where $L=\dfrac{8\varepsilon\pi^3}{N^2}\left[\dfrac{E_3}{2\pi}\sin 2\pi(x-t)-\left(E_1+E_2-4\pi^2E_4-\dfrac{E_5}{4\pi^2}\right)\cos 2\pi(x-t)\right]$

The non-dimensional heat transfer coefficient at the wall is given by

$$Z_0=\eta_x\theta_{0y}(\eta)=\frac{B_rL^2\eta_x}{4(\cosh N\eta+\beta N\sinh N\eta)^2}(2N^2\eta-N\sinh 2N\eta) \tag{43}$$

2.4. First Order Solution

Invoking the value of ψ_0 in Equs.(35)-(37), the solution of Equs.(33), (34)satisfying conditions (38)-(40) can be written as

$$\psi_1 = A_3 y + A_4 \sinh Ny + A_5 y \cosh Ny + A_6 y^2 \sinh Ny + A_7 \sinh 2Ny \tag{44}$$

$$\begin{aligned}\theta_1 = & L_{12}(y^2-\eta^2-2\eta\gamma)+L_{13}(y^4-\eta^4-4\eta^3\gamma)+L_{14}(\cosh Ny-\cosh N\eta-N\gamma\sinh N\eta)+\\ &+L_{15}(\cosh 2Ny-\cosh 2N\eta-2N\gamma\sinh 2N\eta)+L_{16}(\cosh 3Ny-\cosh 3N\eta-3N\gamma\sinh 3N\eta)-\\ &-\frac{2}{N}(L_6L_7+4L_4)(y\sinh Ny-\eta\sinh N\eta-\gamma\sinh N\eta-N\gamma\eta\cosh N\eta)+L_{17}(y\sinh 2Ny-\\ &-\eta\sinh 2N\eta-\gamma\sinh 2N\eta-2N\gamma\eta\cosh 2N\eta)+2L_4(y^2\cosh Ny-\eta^2\cosh N\eta-2\eta\gamma cosh N\eta-\\ &-N\gamma\eta^2\sinh N\eta)+\frac{L_{10}}{4N^2}(y^2\cosh 2Ny-\eta^2\cosh 2N\eta-2\eta\gamma\cosh 2N\eta-2N\gamma\eta^2\sinh 2N\eta)\end{aligned} \tag{45}$$

The non-dimensional heat transfer coefficient at the wall is given by

$$\begin{aligned}&Z_1 = \theta_{0x}(\eta)+\eta_x\theta_{1y}(\eta) =\\ &= \frac{B_r L^2 N\eta_x}{4(\cosh N\eta+\beta N\sinh N\eta)^3}[(\cosh N\eta+\beta N\sinh N\eta)\{(\sinh 2N\eta+2\gamma N\cosh 2N\eta-\\ &-2N(\gamma+\eta)\}-2N\gamma(\sinh N\eta+\beta N\cosh N\eta)(\sinh 2N\eta-2N\eta)]+\\ &+\eta_x[2\eta L_{12}+4\eta^3L_{13}+NL_{14}\sinh N\eta+2NL_{15}\sinh 2N\eta+3NL_{16}\sinh 3N\eta-\\ &-\frac{2}{N}(L_6L_7+4L_4)(\sinh N\eta+N\eta\cosh N\eta)+2L_4(2\eta\gamma\cosh N\eta+N\eta^2\sinh N\eta)+\\ &\quad+\frac{L_{10}}{4N^2}(2\eta\gamma\cosh 2N\eta+2N\eta^2\sinh 2N\eta)+L_{17}(\sinh 2N\eta+2N\eta\cosh 2N\eta)]\end{aligned} \tag{46}$$

where

$$A_1 = \frac{N}{(\cosh N\eta+\beta N\sinh N\eta)^2}\ [(L_t-LL_x)(\cosh N\eta+\beta N\sinh N\eta)+ \\ +LN(\sinh N\eta+\beta N\cosh N\eta)(L\eta_x-\eta_t)],$$

$$A_2 = \frac{LL_xN^2}{\cosh N\eta+\beta N\sinh N\eta},$$

$$A_3 = \frac{1}{N^2K}[\{(1-Re)K+\lambda_1\}(NA_1-A_2)\cosh N\eta+A_2N\eta\{(1-Re\eta)K+\lambda_1\eta\}\sinh N\eta]+ \\ +LL_x-L_t+\frac{L}{(\cosh N\eta+\beta N\sinh N\eta)^3}\{L_x(\cosh N\eta+\beta N\sinh N\eta)- \\ -LN\eta_x(\sinh N\eta+\beta N\cosh N\eta)\},$$

$$A_4 = -\frac{1}{N(\cosh N\eta+\beta N\sinh N\eta)}\left\{A_3+(A_5+A_6N\eta^2)\cosh N\eta+A_2N\eta+2A_6(\eta+\beta)+ \right.\\ \left. +2A_5\beta N+A_6N^2\eta^2\beta\right\},$$

$$A_5 = \frac{1}{4N^4K}\left\{(ReK-\lambda_1)(A_1N-A_2)+N^2K\lambda_1(2NA_1-A_2)\right\}-\frac{\lambda_1LL_x}{\cosh N\eta+\beta N\sinh N\eta},$$

$$A_6 = \frac{A_2}{4N^3K}\left\{ReK+(N^2K-1)\lambda_1\right\},$$

$$A_7 = \frac{\lambda_1 L}{3N(\cosh N\eta+\beta N\sinh N\eta)^3}\{L_x(\cosh N\eta+\beta N\sinh N\eta)-LN\eta_x(\sinh N\eta+\beta N\cosh N\eta)\},$$

$$L_1 = \frac{P_rReBrL}{4(\cosh N\eta+\beta N\sinh N\eta)^3}\{L_t(\cosh N\eta+\beta N\sinh N\eta)-L\eta_t(\sinh N\eta+\beta N\cosh N\eta)\},$$

$$L_2 = \cosh 2N\eta+2\gamma N\sinh 2N\eta-2N^2\eta(\eta+2\gamma)$$

$$L_3 = \frac{P_rReB_rL^2N\eta_t}{4(\cosh N\eta+\beta N\sinh N\eta)^2}\{\sinh 2N\eta+2\gamma N\cosh 2N\eta-2N(\eta+\gamma)\},$$

$$L_4 = \frac{P_r Re B_r L^2}{4(\cosh N\eta + \beta N \sinh N\eta)^4} \left\{ L_x(\cosh N\eta + \beta \sinh N\eta) - L\eta_x(\sinh N\eta + \beta N \cosh N\eta) \right\},$$
$$L_5 = \frac{P_r Re B_r L^3 N \eta_x}{4 \cosh N\eta + \beta N \sinh N\eta)^3} \left\{ \sinh 2N\eta + 2\gamma N \cosh 2N\eta - 2N(\eta + \gamma) \right\},$$
$$L_6 = \frac{P_r Re}{N(\cosh N\eta + \beta N \sinh N\eta)^2} \left\{ L_x(\cosh N\eta + \beta N \sinh N\eta) - L\eta_x(\sinh N\eta + \beta N \cosh N\eta) \right\},$$
$$L_7 = \frac{P_r Re B_r L^2 N}{4(\cosh N\eta + \beta N \sinh N\eta)^2},$$
$$L_8 = -\frac{B_r L N}{2(\cosh N\eta + \beta N \sinh N\eta)}(A_4 N^2 + 2NA_5 + 2A_6),$$
$$L_9 = -\frac{B_r L N^2}{2(\cosh N\eta + \beta N \sinh N\eta)}(NA_5 + 4A_6),$$
$$L_{10} = -\frac{B_r L N^3 A_6}{2(\cosh N\eta + \beta N \sinh N\eta)},$$
$$L_{11} = -\frac{2B_r L N^3 A_7}{2(\cosh N\eta + \beta N \sinh N\eta)},$$
$$L_{12} = \frac{1}{2} \left\{ L_3 - L_8 + L_1 L_2 - (L_5 + L_2 L_4)(\cosh N\eta + \beta N \sinh N\eta) \right\},$$
$$L_{13} = \frac{1}{12} \left\{ 2L_x L_7 N - L_1 0 + 2N^2 L_1 - 2L_4 N^2 (\cosh N\eta + \beta N \sinh N\eta) \right\},$$
$$L_{14} = \frac{1}{2N^2}(L_5 + 23L_4 + L_2 L_4 + 7L_6 L_7 - L_1 1),$$
$$L_{15} = \frac{1}{16N^6}[4N^4 \left\{ L_4(\cosh N\eta + \beta N \sinh N\eta) - L_1 + L_8 \right\} + 4N^3(L_x L_7 - L_9) + (4N^2 - 1)L_1 0],$$
$$L_{16} = \frac{1}{18N^2}(2L_1 1 + L_6 L_7 - L_4),$$
$$L_{17} = \frac{1}{4N^3} \left\{ N(L_9 - L_x L_7) - 2L_1 0 \right\}$$

3. Numerical Results and Discussion

This section aims to analyze the behaviours of the streamlines, temperature and heat transfer coefficient graphically for embedded flow parameters in the present problem.

An interesting phenomenon of peristaltic motion in the wave frame is trapping which is basically the formation of an internally circulating bolus of fluid by closed streamlines. This trapped bolus is pushed ahead with the peristaltic wave. The trapping phenomena for different values of M, K, β, E_1, E_2, E_3, E_4 and E_5 are shown in Fig. 2. It is observed from Figs. 2a, 2b that the trapped bolus which are moving as whole decreases in size with the increase in M. The effect of porosity parameter K on the trapping is illustrated in Figs. 2b, 2c and observed that the size of trapped bolus gradually increases with increasing K. Figs. 2b, 2d depict that the size of trapped bolus increases with increasing β. The variation of compliant wall parameters is studied in Figs. 2e–2i. It is observed that the volume of the trapped bolus increases with increasing E_1 and E_2 but the effect is reverse for E_3, E_4 and E_5.

The effect of various parameters, say K, M, B_r, γ and β on temperature are illustrated in Figs. 3–8.We observed that the temperature increases with increase of γ, B_r and K while it decreases with increasing M and β. Further it can be noted that the temperature at the upper wall is minimum and it increases slowly towardes the middle portion of the channel. Fig. 8 is made to see the variation of the temperature for various values of compliant wall parameters E_1, E_2, E_3, E_4 and E_5. It is observed that the temperature increases with an increase of E_3 and E_4 while it decreases with increasing E_1, E_2 and E_5.

Variations of the heat transfer coefficient at the wall have been presented in Figs. 9–13 for various values of K, M, B_r, γ and β with fixed values of other parameters. One can observe

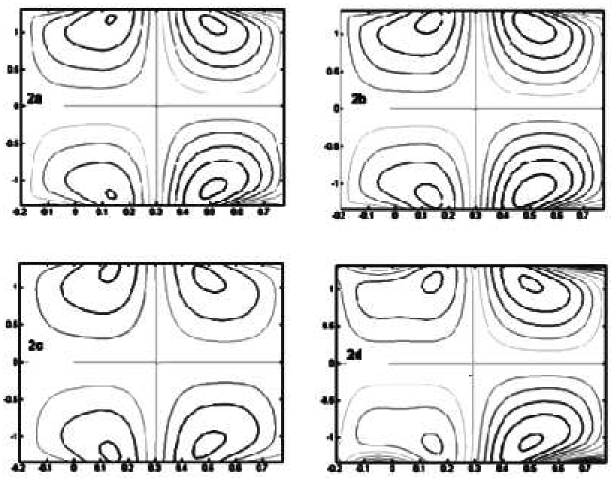

Fig. 2a-2d. Streamlines for different values of M, K, β: (a)M=2, K=0.03, β=0.2, (b)M=1, K=0.03, β=0.2, (c)M=1, K=0.05, β=0.2, (d)M=1, K=0.03, β=0.1

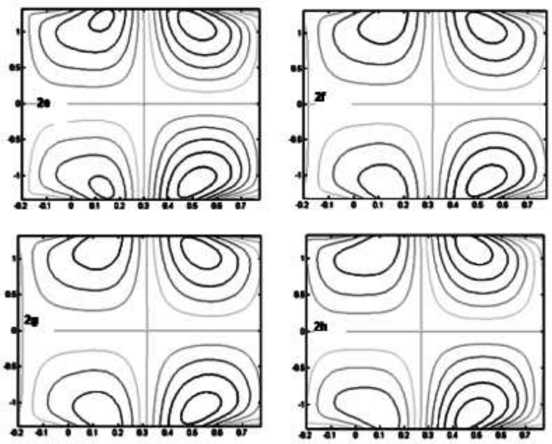

Fig. 2e-2h. Streamlines for different values of E_1, E_2, E_3, E_4, E_5: (e)E_1=0.9, E_2=0.5, E_3=0.2, E_4=0.02, E_5=0.5, (f)E_1=0.7, E_2=0.7, E_3=0.2, E_4=0.02, E_5=0.5, (g)E_1=0.7, E_2=0.5, E_3=0.1, E_4=0.02, E_5=0.5, (h)E_1=0.7, E_2=0.5, E_3=0.2, E_4=0.01, E_5=0.5

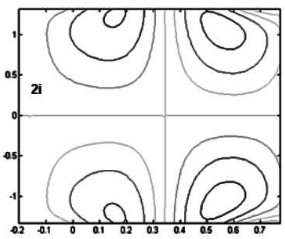

Fig. 2i. Streamlines for different values of E_1, E_2, E_3, E_4, E_5: (i)E_1=0.7, E_2=0.5, E_3=0.2, E_4=0.02, E_5=0.3

that the absolute value of heat transfer coefficient decreases with increase of M and β. However it increases with increasing γ, B_r and K.

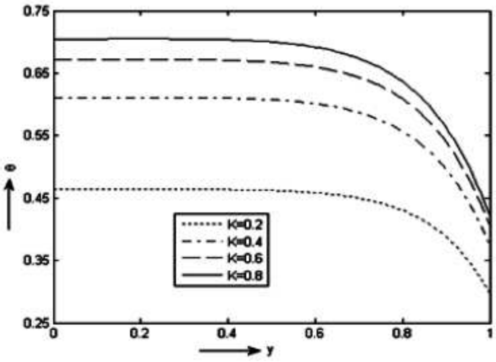

Fig. 3. Variation of temperature with y for different values of K

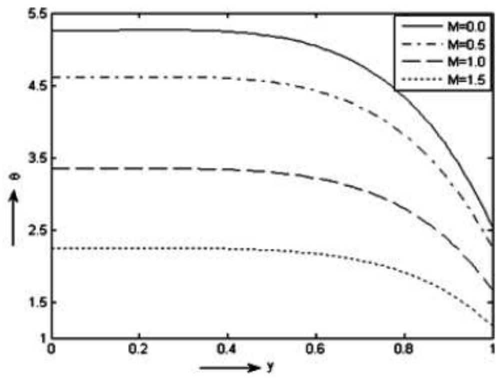

Fig. 4. Variation of temperature with y for different values of M

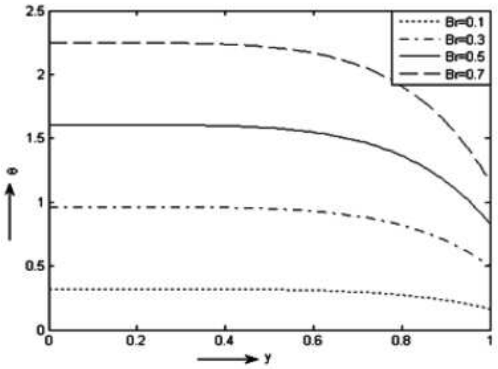

Fig. 5. Variation of temperature with y for different values of B_r

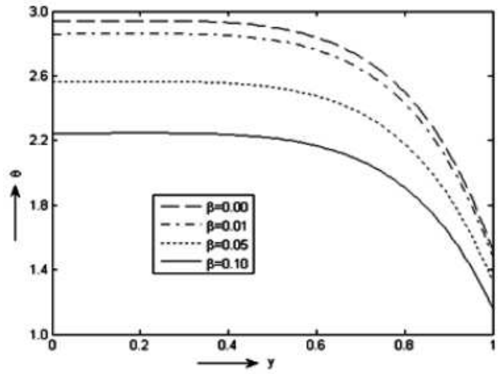

Fig. 6. Variation of temperature with y for different values of β

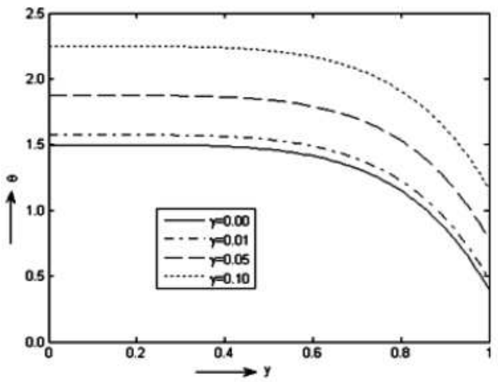

Fig. 7. Variation of temperature with y for different values of γ

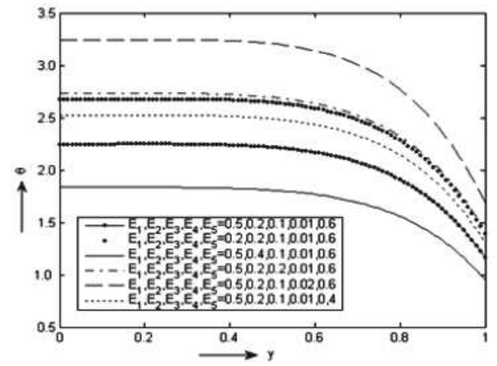

Fig. 8. Variation of temperature with y for different values of E_1, E_2, E_3, E_4 and E_5

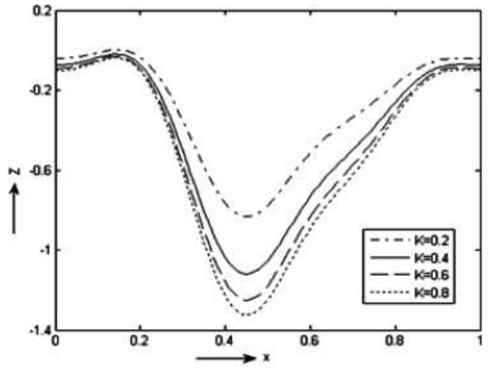

Fig. 9. Effect of K on heat transfer coefficient

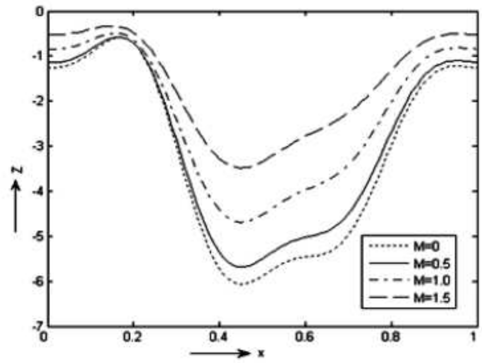

Fig. 10. Effect of M on heat transfer coefficient

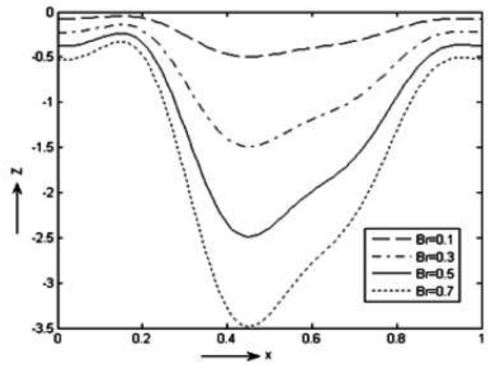

Fig. 11. Effect of B_r on heat transfer coefficient

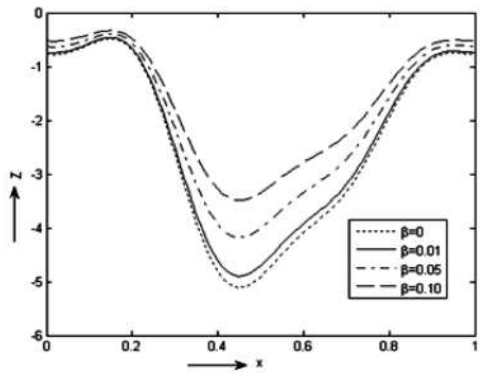

Fig. 12. Effect of β on heat transfer coefficient

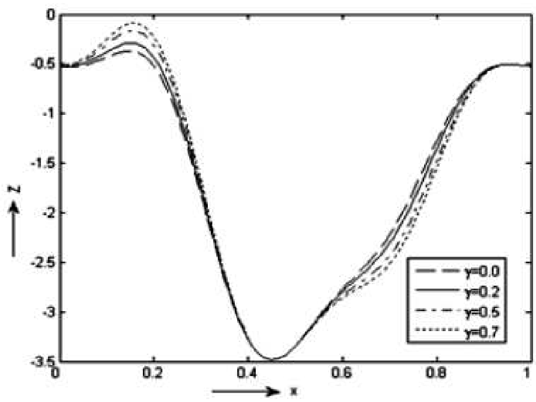

Fig. 13. Effect of γ on heat transfer coefficient

Conclusions

In this work, the combined effects of slip conditions and heat transfer on MHD peristaltic flow of a Maxwell fluid in a porous channel with influence of wall properties are studied. The closed

form analytical solutions of the problem are obtained using perturbation method. The results are discussed through graphs and concluded the following observations :
(i)The volume of the trapped bolus decreases by increasing both M, E_1 and E_2. Moreover the effect is reverse for K, β, E_3, E_4 and E_5.
(ii)The temperature field decreases with increase in both M and β while with increase in γ, B_r and K the temperature field increases.
(iii)The absolute value of heat transfer coefficient decreases with increasing M and β but it increases with the increasing γ, B_r and K in the vicinity of the upper wall.
(iv)The analytical resultes obtained in this work are more generalised form of Hayat et al. [25] and can be taken as a limiting case by taking $\beta \to 0$ and $\gamma \to 0$.

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Совместное влияние условий проскальзывания и свойств стенки на волнообразное МГД-течение максвелловской жидкости с учетом теплопереноса

Калидас Дас

Было изучено влияние условий проскальзывания и теплопереноса на волнообразное магнитогидродинамическое (МГД) течение максвелловской жидкости в пористом плоском канале с упругими стенками. Математическая формулировка задачи основана на модифицированном уравнении Дарси. Аналитическое решение было получено для функции тока и температуры в предположении малых волновых чисел. Полученные результаты представлены в численной и графической форме.

Ключевые слова: волнообразование, максвелловская жидкость, модифицированный закон Дарси, число Бринкмана, число Кнудсена, коэффициент теплопроводности.