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Journal of Siberian Federal University. Mathematics & Physics 2011, 4(1), 18–31

УДК 517.946

# Effects of Heat and Mass Transfer on MHD Free Convection Flow Near a Moving Vertical Plate of a Radiating and Chemically Reacting Fluid

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Received 05.09.2010, received in revised form 20.10.2010, accepted 20.111.2010

*The problem of unsteady MHD free convection flow and mass transfer of a viscous, electrically conducting and chemically reacting incompressible fluid in presence of thermal radiation and under the influence of uniform magnetic field applied normal to an infinite vertical plate, which moves with time dependent velocity is studied. The primary purpose of this study was to characterize the effects of thermal radiative heat transfer, magnetic field parameter, chemical reaction rate constant etc on the flow properties. The fluid is also assumed to be gray; emitting absorbing but non scattering medium and the optically thick radiation limit is considered. The solutions of the present problem are obtained in closed form by Laplace transform technique and the expressions for velocity, temperature, concentration, skin friction,rate of heat and mass transfer has been obtained. Some important applications of physical interest for different type motion of the plate are discussed. The results obtained have also been presented numerically through graphs to observe the effects of various parameters and the physical aspects of the problem.*

*Keywords: free convection, mass transfer, thermal radiation, chemically reacting fluid, MHD flow, Laplace transforms.*

## Introduction

The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes and ionized gasses. At the high temperature attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic field and alter heat transfer and friction characteristic. Recently, it is of great interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. The radiation effect on MHD flow and heat transfer have become more important industrially. At high operating temperature,radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and a knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft,missiles, satellites and space vehicles are examples of such engineering areas. Heat transfer by thermal radiation is becoming of greater importance when we are concerned with space applications, higher operating temperatures and also power engineering.

Extensive research work has been obtained on MHD free convection flow near vertical plate or surface with different boundary conditions [1,4]. Several investigations have been carried out on problem of heat transfer by radiation as an important application of space and temperature

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related problems. Grief et al. [5] obtained an exact solution for the problem of laminar convective flow in a vertical heated channel in the optically thin limit. In the optically thin limit, the fluid does not absorb its own emitted radiation which means that there is no self absorption but the fluid does absorb radiation emitted by the boundaries. The forced convective flow in a horizontal channel permeated by uniform vertical magnetic field taking radiation into account considered by Viskanta [6]. Gupta et al. [7] investigated the effect of radiation on the combined free and forced convection of an electrically conducting fluid flowing inside an open ended vertical channel in presence of uniform magnetic field for the case of optically thin limit. Soundalgakar et al. [8] generalized the problem by considering the effect of radiation on the natural convection flow of a gas past a semi infinite plate using the Cogly Vincentine Gilles equilibrium model (Cogly et al. [9]). Later many workers [10–17],under different boundary conditions analyzed the effect of radiation using the Rosseland diffusion approximation for mixed convection of an optically dense viscous incompressible fluid in presence of magnetic field. Das et al. [18] discussed the effect of radiation on MHD free convection flow of a chemically reacting fluid. Das et al. [19] extended the problem to included the effect of mass transfer on MHD free convection flow of a chemically reacting fluid through a porous medium. The effects of the phenomenon of mass transfer on a free convection flow near an infinite vertical porous plate which moves with time dependent velocity have been studied by Toki [20]. Recently Das et al. [21] studied the heat and mass transfer effects on the unsteady MHD free convection flow near a moving vertical plate in porous medium. Das et al. [22] extended the problem by considering the effect of thermal radiation on MHD free convection flow past an infinite vertical plate which moves with time dependent velocity.

In this study we consider the problem of the unsteady free convection flow and mass transfer of an optically thin viscous, electrically conducting and chemically reacting incompressible fluid near an infinite vertical plate which moves with time dependent velocity in presence of transverse uniform magnetic field and thermal radiation. A general exact solution for the partial differential equation governing these flows is obtained with the aid of the Laplace transform technique. Furthermore, this general solution is applied for the most important cases of flow.

# 1. Mathematical Formulation of the Problem

Consider unsteady free convection flow and mass transfer of a viscous incompressible, electrically conducting, radiating and chemically reacting fluid along an infinite non-conducting vertical flate plate in presence of a uniform transverse magnetic field $B_0$ applied in the direction of the flow. Let $x$-axis be along the plate in the upward direction and the $y$-axis normal to it (Fig. 1). Also let us assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. Initially for time $t \leqslant 0$, the plate and the fluid are at the same constant temperature $T_\infty$ in a stationary condition, with the same species concentration $C_\infty$ at all points. Subsequently ($t > 0$), the plate is assumed to be accelerating with a velocity $U_0 f(t)$ in its own plane along the $x$-axis; instantaneously the temperature of the plate and the concentration are raised to $T_w$ and $C_w$ respectively which are hereafter regarded as constant. For free convection flows, here we also assume that all the physical properties of the fluid is to be in the direction of the $x$-axis, so the physical variables are functions of the space co-ordinate $y$ and time $t$ only.

Under the above assumptions, the fully developed flow of a radiating and chemically reacting fluid is governed by the following set of equations,

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho}, \tag{1.1}$$

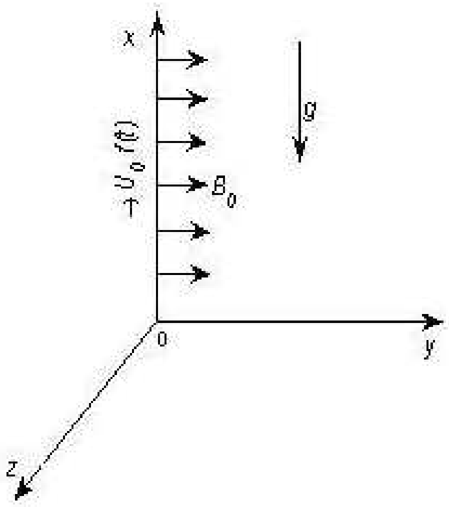


Fig. 1. A schematic of the problem and coordinate system

$$\rho c_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}. \tag{1.2}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} - k_r C. \tag{1.3}$$

The initial and boundary conditions are

$$\left.\begin{array}{l} u = 0, T = T_\infty, C = C_\infty \quad \forall \quad y \geqslant 0, t \leqslant 0, \\ u = U_0 f(t), T = T_w, C = C_w \quad \text{at} \quad y = 0, t > 0, \\ u \to 0, T \to T_\infty, C \to C_\infty \quad \text{as} \quad y \to \infty, t > 0, \end{array}\right\} \tag{1.4}$$

where $u$ is the velocity in the $x$-direction, $\rho$ the density, $g$ the acceleration due to gravity, $\beta$ the volumetric coefficient of thermal expansion, $\beta^*$ the volumetric coefficient of expansion for concentration, $T$ the temperature of the fluid near the plate, $C$ is the species concentration, $c_p$ the specific heat at constant pressure, $\kappa$ the thermal conductivity, $q_r$ the radiative flux, $\nu$ the kinematic coefficient of viscosity, $\sigma$ the electrical conductivity, $D$ the chemical molecular diffusivity and $k_r$ the chemical reaction rate constant.

In the optically thick limit, the fluid does not absorb its own emitted radiation, that is there is no self absorption but it does absorb radiation emitted by the boundaries. It has been shown by Cogly et al. [9] that in the optically thick limit for a non gray gas near equilibrium that

$$\frac{\partial q_r}{\partial y} = 4\left(T - T_\infty\right) \int_0^\infty K_{\lambda\omega} \left(\frac{de_{b\lambda}}{dT}\right)_\omega d\lambda = 4 I_1 \left(T - T_\infty\right), \tag{1.5}$$

where $K_{\lambda\omega}$ is the absorption coefficient, $e_{b\lambda}$ is the Planck function and the subscript $\omega$ refers to values at the wall.

To reduce the above equations into non-dimensional form, let us introduce the following dimensionless variables and parameters:

$$\left.\begin{array}{l} u' = \dfrac{u}{U_0}, t' = \dfrac{tU_0^2}{\nu}, y' = \dfrac{yU_0}{\nu}, \theta' = \dfrac{T - T_\infty}{T_\omega - T_\infty}, C' = \dfrac{C - C_\infty}{C_\omega - C_\infty}, \\ M = \dfrac{\sigma B_0^2 \nu}{\rho U_0^2}, P_r = \dfrac{\mu c_p}{\kappa}, G_r = \dfrac{\nu g \beta (T_\omega - T_\infty)}{U_0^3}, G_m = \dfrac{\nu g \beta^* (C_\omega - C_\infty)}{U_0^3}, \\ F = \dfrac{4 I_1 \nu^2}{\kappa U_0^2}, K_r = \dfrac{k_r \nu^2}{D U_0^2}, S_c = \dfrac{\nu}{D}, \end{array}\right\} \tag{1.6}$$

where $G_r$ is thermal Grashof number, $G_m$ is the mass Grashof number, $S_c$ is Schmidt number, $M$ is magnetic field parameter, $P_r$ is the Prandtl number, $F$ is the radiation parameter, $K_r$ is the chemical reaction rate parameter.

With the help of (1.6), the governing equations (1.1)–(1.3) reduce to(dropping the primes),

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m C - Mu, \tag{1.7}$$

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - F\theta, \tag{1.8}$$

$$S_c \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} - K_r C. \tag{1.9}$$

The corresponding initial and boundary conditions in non-dimensional form are

$$\left.\begin{array}{l} u = 0, \theta = 0, C = 0, \quad \forall \;\; y \geqslant 0, t \leqslant 0, \\ u = f(t), \theta = 1, C = 1 \;\; \text{at} \;\; y = 0, t > 0, \\ u \to 0, \theta \to 0, C \to 0 \;\; \text{as} \;\; y \to \infty, t > 0. \end{array}\right\} \tag{1.10}$$

The system(1.7)–(1.9) of differential equations, subject to the boundary conditions (1.10), includes the effect of free convection and mass transfer on the flows near a moving isothermal vertical plate.

## 2. Solution of the Problem

In order to obtain the analytical solution of the system of differential equations (1.7)–(1.9) subject to the initial and boundary conditions (1.10), we shall use the Laplace transformation technique.

Thus the general solutions of the present problem for the temperature $\theta$(y,t), the species concentration $C(y,t)$ and the velocity $u(y,t)$ for $t > 0$ are given by

$$\theta(y,t) = \frac{1}{2}\left[ e^{-y\sqrt{F}} erfc\left( \frac{y}{2}\sqrt{\frac{P_r}{t}} - \sqrt{\frac{Ft}{P_r}} \right) + e^{y\sqrt{F}} erfc\left( \frac{y}{2}\sqrt{\frac{P_r}{t}} + \sqrt{\frac{Ft}{P_r}} \right) \right], \tag{2.1}$$

$$C(y,t) = \frac{1}{2}\left[ e^{-y\sqrt{K_r}} erfc\left( \frac{y}{2}\sqrt{\frac{S_c}{t}} - \sqrt{\frac{K_r t}{S_c}} \right) + e^{y\sqrt{K_r}} erfc\left( \frac{y}{2}\sqrt{\frac{S_c}{t}} + \sqrt{\frac{K_r t}{S_c}} \right) \right], \tag{2.2}$$

$$u(y,t) = \Phi(y,t) + A(y,t) + B(y,t), \tag{2.3}$$

where

$$\Phi(y,t) = L^{-1}\left[ f(\bar{s}) e^{-qy} \right], f(\bar{s}) = L\left[ f(t) \right], \tag{2.4}$$

$$\begin{aligned} A(y,t) &= \tfrac{G_r}{2a_1}e^{c_1 t}\left[e^{-y\sqrt{b_1}}\left\{erfc\left(\tfrac{y}{2\sqrt{t}}-\sqrt{b_1 t}\right)-erfc\left(\tfrac{y}{2}\sqrt{\tfrac{P_r}{t}}-\sqrt{\tfrac{b_1 t}{P_r}}\right)\right\}+\right.\\ &\left.+e^{y\sqrt{b_1}}\left\{erfc\left(\tfrac{y}{2\sqrt{t}}+\sqrt{b_1 t}\right)-erfc\left(\tfrac{y}{2}\sqrt{\tfrac{P_r}{t}}+\sqrt{\tfrac{b_1 t}{P_r}}\right)\right\}\right]+\\ &+\tfrac{G_r}{2a_1}\left[e^{-y\sqrt{F}}erfc\left(\tfrac{y}{2}\sqrt{\tfrac{P_r}{t}}-\sqrt{\tfrac{Ft}{P_r}}\right)+e^{y\sqrt{F}}erfc\left(\tfrac{y}{2}\sqrt{\tfrac{P_r}{t}}+\sqrt{\tfrac{Ft}{P_r}}\right)\right]-\\ &-\tfrac{G_r}{2a_1}\left[e^{-y\sqrt{M}}erfc\left(\tfrac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+e^{y\sqrt{M}}erfc\left(\tfrac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right], \end{aligned} \tag{2.5}$$

$$\begin{aligned} B(y,t) &= \tfrac{G_m}{2a_2}e^{c_2 t}\left[e^{-y\sqrt{b_2}}\left\{erfc\left(\tfrac{y}{2\sqrt{t}}-\sqrt{b_2 t}\right)-erfc\left(\tfrac{y}{2}\sqrt{\tfrac{S_c}{t}}-\sqrt{\tfrac{b_2 t}{S_c}}\right)\right\}+\right.\\ &\left.+e^{y\sqrt{b_2}}\left\{erfc\left(\tfrac{y}{2\sqrt{t}}+\sqrt{b_2 t}\right)-erfc\left(\tfrac{y}{2}\sqrt{\tfrac{S_c}{t}}+\sqrt{\tfrac{b_2 t}{S_c}}\right)\right\}\right]+\\ &+\tfrac{G_m}{2a_2}\left[e^{-y\sqrt{K_r}}erfc\left(\tfrac{y}{2}\sqrt{\tfrac{S_c}{t}}-\sqrt{\tfrac{K_r t}{S_c}}\right)+e^{y\sqrt{K_r}}erfc\left(\tfrac{y}{2}\sqrt{\tfrac{S_c}{t}}+\sqrt{\tfrac{K_r t}{S_c}}\right)\right]-\\ &-\tfrac{G_m}{2a_2}\left[e^{-y\sqrt{M}}erfc\left(\tfrac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+e^{y\sqrt{M}}erfc\left(\tfrac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right] \end{aligned} \tag{2.6}$$

and $a_1 = M - F$, $b_1 = \dfrac{MP_r - F}{P_r - 1}$, $c_1 = \dfrac{M-F}{P_r-1}$, $a_2 = M - K_r$, $b_2 = \dfrac{MS_c - K_r}{S_c - 1}$, $c_2 = \dfrac{M-K_r}{S_c-1}$.

Since non-dimensional temperature $\theta(y,t)$ and non-dimensional species concentration $C(y,t)$ is clearly described in (2.1) and (2.2), so we shall confine ourselves to non-dimensional velocity $u(y,t)$ for various types of $f(t)$. Here the expressions (2.3)–(2.6) are the general solutions of the present problem which include the effects of heating(cf.term A), diffusion (cf. term B) and the motion of the plate.

## 3. Applications of the General Solution

In this section we now consider some important cases of flow as given below:

**Case (i): motion of the plate with uniform velocity**. Let $f(t) = H(t)$. Heaviside unit function, then

$$\bar{f}(s) = L[f(t)] = \frac{1}{s}. \tag{3.1}$$

In this case we observe that the result (2.1), (2.2) are unaffected and the expression (2.3) for $u(y,t)$ becomes

$$u(y,t) = \frac{1}{2}\left[e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right]+A(y,t)+B(y,t), \tag{3.2}$$

where $A(y,t)$, $B(y,t)$ are given from equation (2.5), (2.6).

**Case (ii): motion of the plate with single acceleration**. Let $f(t) = tH(t)$, then

$$\bar{f}(s) = \frac{1}{s^2}. \tag{3.3}$$

In this case also we observe that the result (2.1), (2.2) remain in the same form but the expression (2.3) for $u(y,t)$ takes the following analytical form :

$$\begin{aligned} u(y,t) &= \frac{1}{2}\left[\left(t-\frac{y}{2\sqrt{M}}\right)e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+\left(t+\frac{y}{2\sqrt{M}}\right)e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right]+\\ &+A(y,t)+B(y,t), \end{aligned} \tag{3.4}$$

where $A(y,t)$, $B(y,t)$ are given from equation (2.5), (2.6).
**Case (iii): motion of the plate with periodic acceleration**. For this case, let $f(t) = H(t)\,cos(\omega t)$, then

$$\bar{f}(s) = \frac{s}{s^2+\omega^2}. \tag{3.5}$$

Then the expression (2.1), (2.2) remains again in the same form but, instead of (2.3), we get the following analytical expression:

$$\begin{aligned} u(y,t) &= \frac{1}{4}e^{i\omega t}\left[e^{y\sqrt{M+i\omega}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M+i\omega)\,t}\right)+e^{-y\sqrt{M+i\omega}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M+i\omega)\,t}\right)\right]+ \\ &+\frac{1}{4}e^{-i\omega t}\left[e^{y\sqrt{M-i\omega}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M-i\omega)\,t}\right)+e^{-y\sqrt{M-i\omega}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M-i\omega)\,t}\right)\right]+ \\ &+A(y,t)+B(y,t), \end{aligned} \tag{3.6}$$

where $A(y,t)$, $B(y,t)$ are given from equation (2.5), (2.6).

It should be noted that our results of case (i) are identical with those of Das[22].

# 4. Skin Friction

Knowing the velocity field, we now study the effect of $t$,$P_r$,$M$,$F$ etc. on the skin friction. In non-dimensional form, it is given by

$$\begin{aligned} \tau &= -\left(\frac{\partial u}{\partial y}\right)_{y=0} = \\ &= -\left(\frac{\partial \Phi}{\partial y}\right)_{y=0}+\frac{G_r}{a_1}e^{c_1 t}\left[\sqrt{b_1}\left\{erf\left(\sqrt{b_1 t}\right)-erf\left(\sqrt{\frac{b_1 t}{P_r}}\right)\right\}+\frac{1}{\sqrt{\pi t}}e^{-b_1 t}-\sqrt{\frac{P_r}{\pi t}}e^{-\frac{b_1 t}{P_r}}\right]- \\ &-\frac{G_r}{a_1}\left[\sqrt{M}erf\left(\sqrt{Mt}\right)+\frac{1}{\sqrt{\pi t}}e^{-Mt}\right]+\frac{G_r}{a_1}\left[\sqrt{F}erf\left(\sqrt{\frac{Ft}{P_r}}\right)+\sqrt{\frac{P_r}{\pi t}}e^{-\frac{Ft}{P_r}}\right]+ \\ &+\frac{G_m}{a_2}e^{c_2 t}\left[\sqrt{b_2}\left\{erf\left(\sqrt{b_2 t}\right)-erf\left(\sqrt{\frac{b_2 t}{S_c}}\right)\right\}+\frac{1}{\sqrt{\pi t}}e^{-b_2 t}-\sqrt{\frac{S_c}{\pi t}}e^{-\frac{b_2 t}{S_c}}\right]- \\ &-\frac{G_m}{a_2}\left[\sqrt{M}erf\left(\sqrt{Mt}\right)+\frac{1}{\sqrt{\pi t}}e^{-Mt}\right]+\frac{G_m}{a_2}\left[\sqrt{K_r}erf\left(\sqrt{\frac{K_r t}{S_c}}\right)+\sqrt{\frac{S_c}{\pi t}}e^{-\frac{K_r t}{S_c}}\right]. \end{aligned} \tag{4.1}$$

When the plate is moving with uniform velocity then,

$$\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = -\left\{\sqrt{M}erf\left(\sqrt{Mt}\right)+\frac{1}{\sqrt{\pi t}}e^{-Mt}\right\}. \tag{4.2}$$

Again when the plate is moving with single acceleration then,

$$\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = -\left\{\sqrt{M}\left(t+\frac{1}{2M}\right)erf\left(\sqrt{Mt}\right)+\sqrt{\frac{t}{\pi}}e^{-Mt}\right\}. \tag{4.3}$$

Lastly when the plate is moving with periodic acceleration then,

$$\begin{aligned} \left(\frac{\partial \Phi}{\partial y}\right)_{y=0} &= -\frac{1}{2\sqrt{t}}\left[e^{i\omega t}\sqrt{(M+i\omega)\,t}erf\sqrt{(M+i\omega)\,t}+\right. \\ &\left.+\,e^{-i\omega t}\sqrt{(M-i\omega)\,t}erf\sqrt{(M-i\omega)\,t}\right]-\sqrt{\frac{t}{\pi}}e^{-Mt}. \end{aligned} \tag{4.4}$$

# 5. Nusselt Number

An important phenomenon in this study is to understand the effect of $t$, $F$ and $P_r$ on the nusselt number. In non-dimensional form, the rate of heat transfer is given by

$$N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \sqrt{F} erf\left(\sqrt{\frac{Ft}{P_r}}\right) + \sqrt{\frac{P_r}{\pi t}} e^{-\frac{Ft}{P_r}}. \tag{5.1}$$

# 6. Sherwood Number

Another important physical quantities of interest is the Sherwood number whose non-dimensional form is

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = \sqrt{K_r} erf\left(\sqrt{\frac{K_r t}{S_c}}\right) + \sqrt{\frac{S_c}{\pi t}} e^{-\frac{K_r t}{S_c}}. \tag{6.1}$$

We study the effects of $t$, $S_c$ and $K_r$ on Sherwood number numerically in the next section for better understanding.

# 7. Numerical Results and Discussion

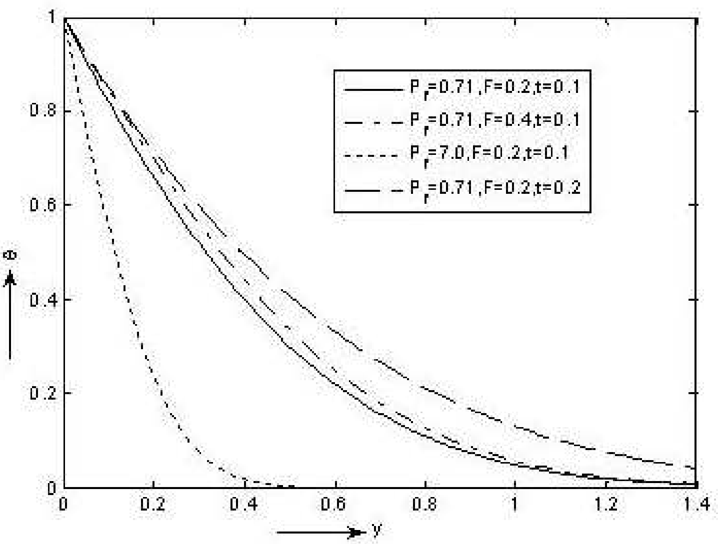


Fig. 2. Effcts $P_r$, $F$and $t$ on concentration distribution

To understand the physical meaning of the problem, we have computed the expression for the velocity $u(y,t)$, the temperature $\theta(y,t)$ and concentration $C(y,t)$ for the case of air $P_r(= 0.70)$ and water $P_r(= 7.0)$ and for different values of magnetic field parameter $M$, the radiation parameter $F$, the chemical reaction rate constant $K_r$, Schmidt number $S_c$ and time $t$. The purpose of the numerical result given here is to assess the effects of different parameters upon the nature of the flow, temperature and concentration etc.

Fig. 2 depicts the temperature profiles against $y$ (distance from plate). We observe that the temperature for air is greater then that of water, which is due to the fact that thermal conductivity of fluid decreases with increasing $P_r$. We observe that the temperature decreases with increasing $F$ and increases with increasing $t$. The temperature profiles are in good agreement with the results obtained in case of Das [22].

For various values of Schmidt number $S_c$,chemical reaction rate constant $K_r$ and time $t$, the concentration profiles are shown in Fig. 3. The concentration distribution decreases at all points of the flow field with the increase of the chemical reaction rate constant.It is seen from figure that an increase in the Schmidt number leads to an decrease in the concentration but it increases with increasing time. The concentration profiles closely agree with those of Das [22]. Applying numerical values into the expressions of exact solutions for the velocity, we get the

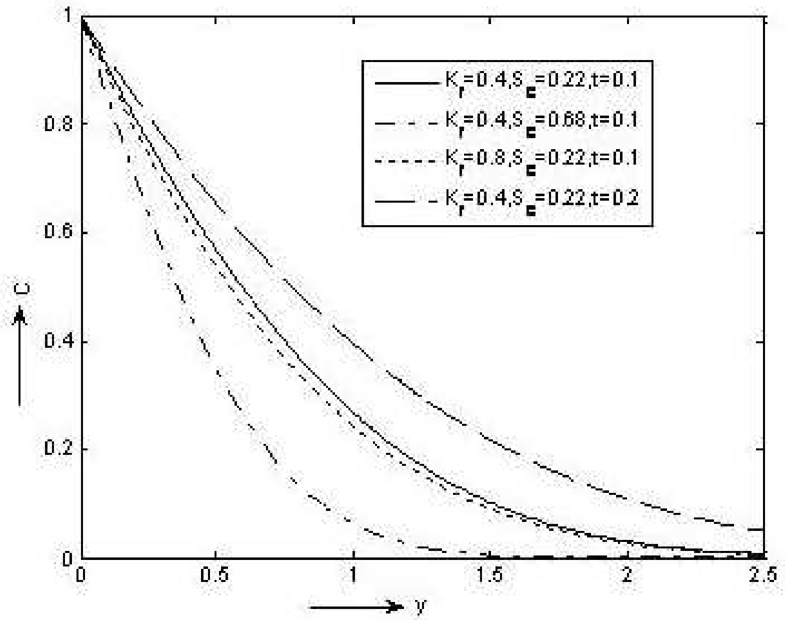


Fig. 3. Effcts $S_c$, $K_r$ and $t$ on temperature profile

velocity profiles of air flows near vertical plate. Figs. 4 and 5 correspond to the plates moving with uniform velocity and with single acceleration respectively. Also Fig. 6 corresponds to the plates which are moved with periodic acceleration. It is seen form these figures that increasing values of $F$, $K_r$ leads to fall in velocity but an increase of $M$ leads to an increase in velocity of air .In case of Das [22], the magnetic parameter $M$ shows reverse effect.

Figs. 7–9 depict skin-friction against time $t$ for different values of $M$, $F$ and $K_r$. It is observed that the skin friction decreases with increasing $F$, $K_r$ and $t$ but effect is reverse for $M$. So wall shear stress decreases with increasing radiation parameter,chemical reaction rate constant and also time. Thus the results are identical with those of Das [22]

Nusselt number is presented in Fig. 10 against time. It decreases with time but increases with increasing the radiation parameter. Also Nusselt number for $P_r = 7$ is higher than that of $P_r = 0.71$.

In Fig. 11, Sherwood number is presented against time t for different values of $S_c$ and $K_r$. We observed that Sherwood number increases with increasing $S_c$ and $K_r$ which is just the reverse of the results obtained in case of Das [22].

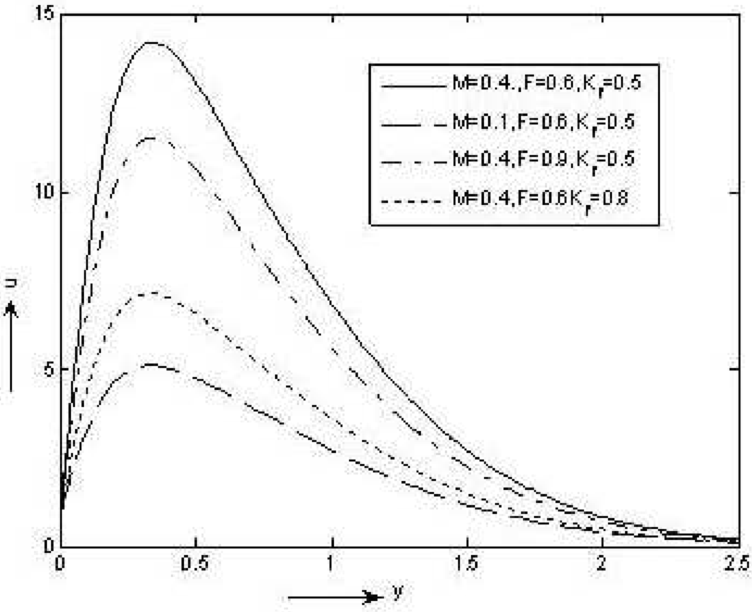


Fig. 4. Velocity profile when the plate moves with uniform velocity

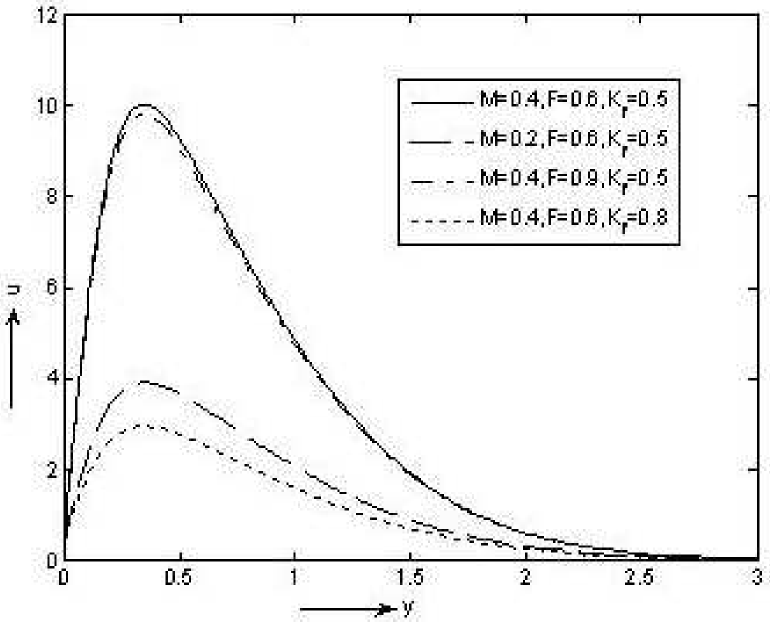


Fig. 5. Velocity profile when the plate moves with single acceleration

# 8. Conclusion

In this study, the effect of thermal radiation and chemical reaction on MHD free convection flow and mass transfer near a moving vertical plate is presented. A general analytical solution

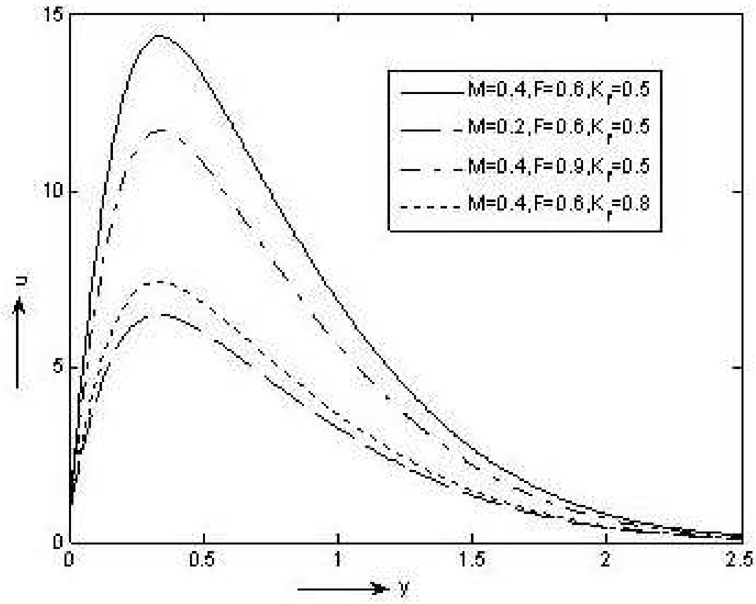


Fig. 6. Velocity profile when the plate moves with periodic acceleration

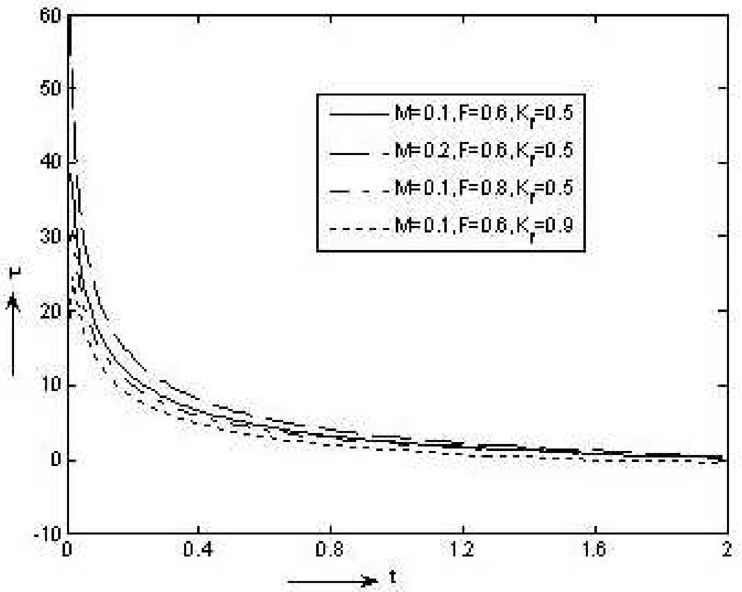


Fig. 7. Skin-friction when the plate moves with uniform velocity

for the problem has been determined without any restriction. Some important applications from the point of view of physical interest was discussed. Also we study a physical example for evaluation of the numerical values of the velocity, temperature and concentration for the case of air($P_r = 0.71$) and water($P_r = 7.0$) and observed that the temperature decreases with increasing $F$ and increases with increasing $t$. The concentration distribution of the flow field decreases at

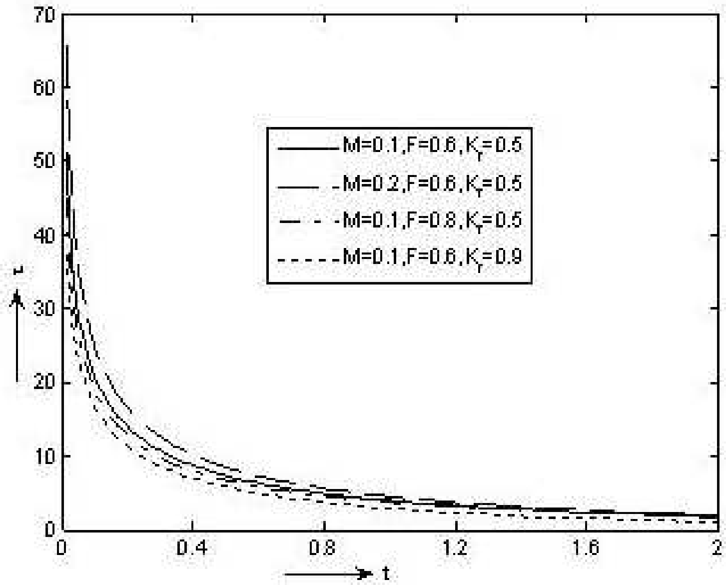


Fig. 8. Skin-friction when the plate moves with single acceleration

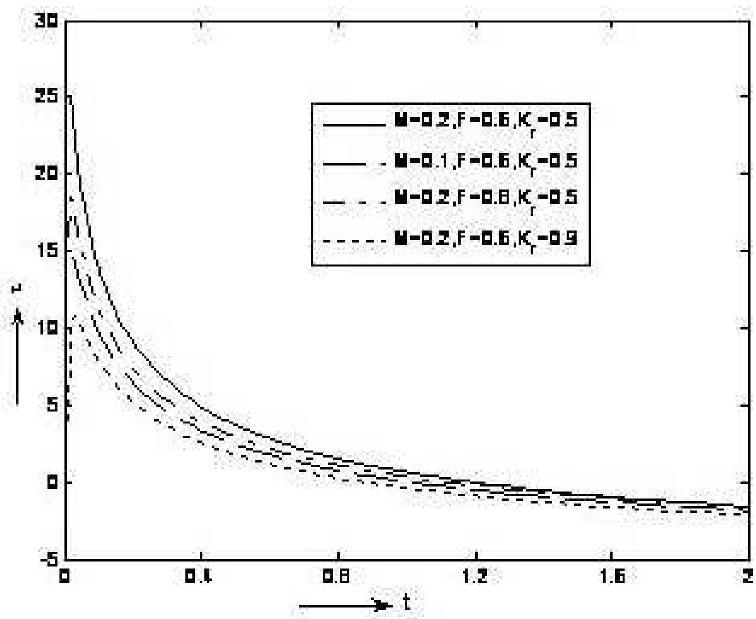


Fig. 9. Skin-friction when the plate moves with periodic acceleration

all points as the Schmidt number $S_c$ and chemical reaction rate constant $K_r$ increases.Also the velocity decreases with increasing $F$ and $K_r$ while $M$ shows the reverse effect. The skin friction coefficient decreases with increasing $F$, $K_r$ and $t$ but the effect is reverse for $M$. The rate of heat transfer for water is more than that of air and it increases with increasing the radiation

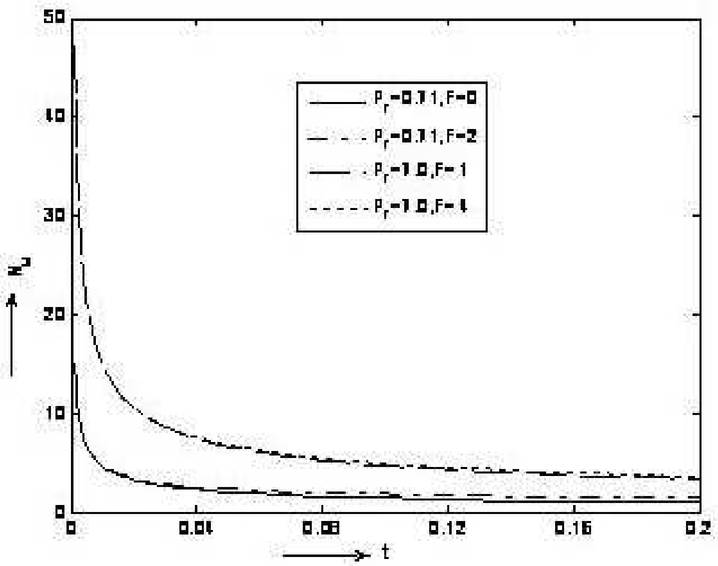


Fig. 10. Effects of $P_r$ and $F$ on Nusselt number

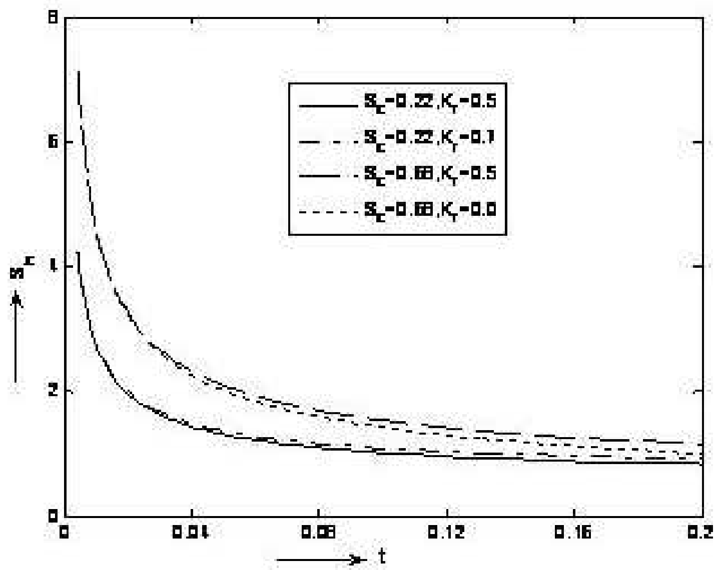


Fig. 11. Effects of $S_c$ and $K_r$ on Sherwood number

parameter. The rate of mass transfer increases with increasing $S_c$ and $K_r$. To our knowledge, this work gives in close form the actual analytical solution of the MHD free convection flow and mass transfer of an optically thin and chemically reacting fluid which has wide application in power engineering and also in the study of vertical air flows into the atmosphere.

## References

[1] K.Das, D.C.Sanyal, Hydromagnetic unsteady free convection flow past a hot vertical plate, *The Math. Ed.*, **30**(1966), no. 4, 227–236.

[2] V.M. Soundalgekar, U.N.Das, R.K.Deka, Free convection effects on MHD flow past an infinite vertical oscillating plate with constant heat flux, *Indian Jour. of Maths.*, **39**(1997), no. 3, 195–202.

[3] P.K. Sahoo, N. Dutta, S.Biswal, Magnetohydrodynamic unsteady free convection flow past an infinite vertical plate with constant suction and heat sink, *Indian J. Pure and Appl. Maths.*, **34**(2003), no.1, 145–155.

[4] K.Das, Hydromagnetic unsteady free convection flow past an impulsively started vertical plate, *J of Mech. Con. and Math. Sci.*, **1**(2006), no.1, 27–31.

[5] R.Greif, I.S.Habib, J.C.Lin, Laminar convection of a radiating gas in a vertical channel, *Journal of Fluid Dynamics*, **46**(1971), 513–520.

[6] R.Viskanta, Effects of transverse magnetic field on heat transfer to an electrically conducting and thermal radiating fluid flowing a parallel channel, *ZAMP*, **14**(1963), 353–361.

[7] P.S.Gupta, A.S.Gupta, Radiation effect on hydrodynamic convection in a vertical channel, *Int. J. Heat Mass transfer*, **17**(1974), 1437–1442.

[8] V.M.Soundalgekar, H.S.Takhar, Radiative convective flow past a semi infinite vertical plate, *Modeling Measure and Cont.*, **51**(1992), 31–40.

[9] A.C.Cogly, W.C.Vincentine, S.E.Gilles, Differential approximation for radiative transfer in an non-grey gas near equilibrium, *AIAA Journal*, **6**(1968), 551–555.

[10] H.S.Takhar, R.S.R.Gorla, V.M.Soundalgekar, Radiation effects on MHD free convection flow of a radiating gas past a semi infinite vertical plate, *Int. J. Numerical Methods Heat Fluid Flow*, **6**(1996), 77–88.

[11] M.A.Hossain, H.S.Takhar, Radiative effect on mixed convection along a vertical plate with uniform surface temperature, *Int. J. Heat Mass transfer*, **31**(1996), 243–248.

[12] M.A.Hossain, M.A.Alim, D.A.S.Rees, Effects of radiation on free convection from a porous vertical plate, *Int. J. Heat Mass Transfer*, **42**(1999), 181–191.

[13] A.Y.Ghaly, E.M.E.Elbarbary, Radiation effect of MHD free convection flow of a gas at a stretching surface with a uniform free stream, *Jour.of Appl. Math.*, **2**(2002), no. 2, 93–103.

[14] R.Muthcuumaraswamy, G.Senthil Kumar, Heat and mass transfer effects on moving vertical plate in the presence of thermal radiation, *Theoret. Appl. Mach.*, **31**(2004), no. 1, 35–46.

[15] M.K.Majumder, R.K.Deka, MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation, *Rom. Jurn. Phys.*, **52**(2007), no. 5–7, 565–573.

[16] S.Shateyi, P.Sibanda, S.S.Motsa, Magnetohydrodyanamic flow past a vertical plate with radiative heat transfer, *Jour. Heat transfer, ASME*, **129**(2007), 1708–1713.

[17] K.Das, R.Das, MHD free convection flow near a moving vertical plate in presence of thermal radiation :An analytical soluton, *Moldavian J of Phy. Sci.*, **8**(2009), no. 3–4, 358–365.

[18] K.Das, S.Jana, MHD free convection flow of a radiating and chemically reacting fluid, *Int. J. Engg. and Inter. Disci. Maths.*, **1**(2009), no. 1, 23–33.

[19] K.Das, R.Das, Effects of radiation and transpiration on MHD free convection and mass transfer flow through a porous medium of a chemically reacting fluid, *Jour. of Ind. Acad. Math*, Accepted.

[20] C.J.Toki, Free convection and mass transfer flow near a moving vertical porous plate: An analytical soluton, *Jour. Appl. Mach., ASME*, **75**(2008), 0110141–0110148.

[21] K.Das, S.Jana, Heat and mass transfer effects on the unsteady MHD free convection flow near a moving vertical plate in porous medium, *Bull. Soc. Math. Banja. Luka.*, **17**(2010), no. 1, 15–32.

[22] K.Das, Exect solution of MHD free convection flow and mass transfer near a moving vertical plate in presence of thermal radiation, *African J. Math. Phys.*, **8**(2010), no. 1, 29–41.

## Влияние тепло- и массопереноса на свободно конвективное МГД–течение вблизи вертикального движущегося слоя излучающей и химически реагирующей жидкости

**Калидас Дас**

*Изучается задача о нестационарном свободно конвективном МГД–течении и массопереносе вязкой, электропроводной и химически реагирующей несжимаемой жидкости при наличии теплового излучения и под влиянием однородного магнитного поля, приложенного перпендикулярно к бесконечному вертикальному слою, который движется с переменной скоростью. Основной целью данного исследования было рассмотреть влияние теплового излучения, магнитного поля и постоянной химической реакции на параметры течения. Предполагается, что жидкость является серой, излучающей, поглощающей, но не рассеивающей средой. Решение представленной задачи получено в замкнутой форме с использованием преобразования Лапласа. Приводятся выражения для скорости, температуры, концентрации, поверхностного трения и скоростей изменения тепло и массопереноса. Обсуждаются некоторые интересные физические явления, возникающие при различных типах движения слоя. Полученные результаты также представлены в виде графиков, которые показывают влияние различных параметров и физические аспекты задачи.*

*Ключевые слова: свободная конвекция, массоперенос, тепловое излучение, химически реагирующая жидкость, МГД–течение, преобразование Лапласа.*