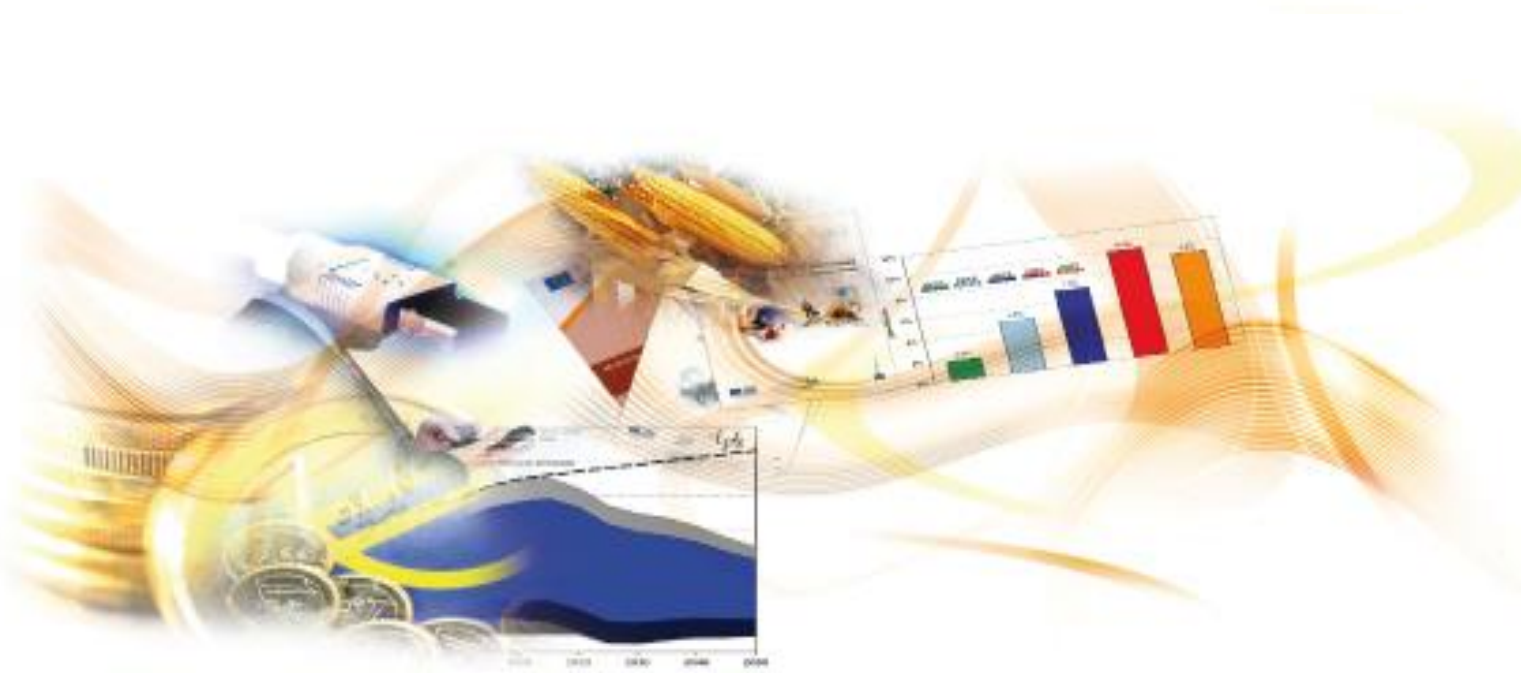


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# A CGE Model with ICT and R&D-driven Endogenous Growth: A Detailed Model Description

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2015

Report EUR 27548 EN

**European Commission**  
Joint Research Centre  
Institute for Prospective Technological Studies

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JRC97908

EUR 27548 EN

ISBN 978-92-79-53360-0 (PDF)

ISSN 1831-9424 (online)

doi:10.2791/60328

Luxembourg: Publications Office of the European Union, 2015

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**Abstract**

We present a multi-country, multi-sector dynamic general equilibrium model with ICT and R&D-driven endogenous growth. The model presented has been developed to study the economic effects of public support to ICT R&D in the European Union. It accommodates alternative policy instruments that could be used in an attempt to stimulate private ICT R&D expenditures, including general production grants, tax credit or subsidies targeted at specific inputs. The model is calibrated to data from four country blocs Germany, France, the Rest of the EU and the Rest of the World.

## **Acknowledgements**

The author acknowledge helpful comments and suggestions from Peter Stephensen (DREAM), Andries Brandsma, Wojciech Szewczyk, Marc Bogdanowicz, Andrea de Panizza, Giuditta De Prato, Ibrahim Rohman (JRC-IPTS), Gianluca Papa, Christophe Doin (DG CNECT), as well as participants of seminars and workshops at the European Commission and at DREAM, Copenhagen. Finally, thorough checking and editing of the text by Patricia Farrer is gratefully acknowledged. The author is solely responsible for the content of the paper.

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# A CGE model with ICT and R&D-driven endogenous growth: A detailed model description

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## Abstract

We present a multi-country, multi-sector dynamic general equilibrium model with ICT and R&D-driven endogenous growth. The model presented has been developed to study the economic effects of public support to ICT R&D in the European Union. It accommodates alternative policy instruments that could be used in an attempt to stimulate private ICT R&D expenditures, including general production grants, tax credit or subsidies targeted at specific inputs. The model is calibrated to data from four country blocs Germany, France, the Rest of the EU and the Rest of the World.

*Key words:* Economic modelling, R&D, ICT, Endogenous growth

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## 1 Introduction

We present a multi-country, multi-sector dynamic Computable General Equilibrium Model (CGE) with ICT and R&D-driven endogenous growth. The

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\* The research is carried out in the context of PREDICT 2, a research project co-financed by the Directorate General for Communication Networks, Content and Technology (CNECT) and JRC-IPTS. One of the objectives of PREDICT 2 is the development of a macroeconomic model allowing for the economic analysis of policy scenarios related to national public support to ICT R&D in the European Union. The views expressed are purely those of the author and may not in any circumstances be regarded as stating an official position of the European Commission.

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model has been developed to study the economic effects of public support to ICT R&D in the European Union. The model has, thus, been designed to address some issues that are central to an applied economic analysis of alternative R&D promoting policies. First, the model accommodates a range of policy instruments that may be used to stimulate R&D spending in the economy. These policy instruments include production grants, tax credits, public procurements or subsidies targeted at specific inputs.

Second, the model addresses some special characteristics of R&D and the dissemination of knowledge that are important for an analysis of policies to promote R&D. The output of the R&D process is a special form of intangible capital – knowledge. Knowledge is non-rival – its use by one person does not preclude its use by another person. Knowledge is not used up in the production process but continues to contribute to the common pool of knowledge. In addition, knowledge in the form of a new idea, design or blueprint may be non-excludable with one firm unable to prevent other firms from using it. When it is non-rival and non-excludable, knowledge takes the form of a public good, which discourages R&D as it prevents firms from recovering their initial R&D expenditures. This market failure results in R&D expenditure levels below the social optimum. However, knowledge may in some cases be embedded in human capital, or firms may attempt at a cost to protect their idea or design through legal procedures such as patents or copyright. Hence, knowledge is sometimes characterised by some degree of rivalry and excludability which gives rise to market power. Under these circumstances, monopoly rent makes it possible for a firm to finance R&D projects, which provides an incentive for innovation. In the model presented here knowledge enters in both a non-excludable and an excludable fashion. R&D output in the model contributes to the common pool of knowledge available to firms across sectors and countries. Hence, a firm’s R&D-produced knowledge serves as a non-rival non-excludable public good benefitting all firms. However, the new knowledge in the model is also sold as blueprints that provide the purchasing firm with the exclusive right to its use. This excludable knowledge in the model entitles the purchasing firm to monopoly rent that covers the initial R&D expenditures.

Third, R&D in the model serves as an engine for growth through multiple channels. R&D-produced knowledge expands the common pool of knowledge, which spills over into increased productivity of R&D producers as these benefit from the production of new knowledge and insights. R&D also leads to the development of new designs or blueprints, expanding the range of available production technologies. This is the case for ICT and for non-ICT production technologies. ICT technology in the model is a multipurpose technology, widely adopted across the economy in combination with skilled labour. Hence, an R&D-induced increase in new ICT technologies affects the production of goods and services across a broad range of sectors in the economy.

Fourth, the model attempts to capture multiple transmission channels through which R&D activity affects the economy. A change in R&D activity affects relative prices and the allocation of labour, capital and goods in the economy. The model therefore allows us to examine a wide set of economic impacts derived from the implementation of a specific policy reform. This is especially useful when the expected effects of policy implementation are complex and materialize through different transmission channels.

The model we propose contains R&D-driven endogenous growth. Hence, it falls within the large body of literature that studies the origin and causes of economic growth. Within this literature different routes have been taken to expand and improve upon the neoclassical growth model in which the presence of long-term growth depends crucially on exogenous technological progress.

One strand of the literature focusses on broadening the notion of capital in the model to include more than buildings and machines. Romer (1986) reinterpreted capital as a combination of physical capital, and blueprints and knowledge, the latter being the outcome of R&D investments. By introducing blueprints and new production techniques into the definition of capital, Romer allowed for the presence of externalities from capital as firms learn from each other. Such externalities contribute to sustained growth. However, large externalities are required in such models to make sustained growth feasible. In addition, this class of models suffers from the definition of capital becoming very broad.

A second strand of the literature redefines capital to include physical capital and human capital (Lucas (1988), Jones and Manuelli (1990) and Rebelo (1991)). Introducing the accumulation of human capital in the same manner as physical capital allows for sustained growth without the presence of exogenous technological progress or externalities from capital. As noted by Rebelo (1998) a problem with this class of models is that growth occurs because workers become more productive over time in a way that does not interact with the invention of new technologies.

A third strand of the literature focusses on technological progress driven by R&D. Romer (1990) proposed a model where profit-oriented innovations from R&D lead to accumulation of non-rival technological knowledge and the development of new patented capital varieties. The development of new capital varieties ensure sustained growth. In the Romer setting, the firm obtains a permanent monopoly in the new capital variety allowing it to cover the fixed cost of R&D. Aghion and Howitt (1992) and Grossman and Helpman (1991) develop models in which the monopoly power is only temporary as patented intermediate goods are replaced by new that can be more effectively used in the production process. The R&D-driven endogenous growth models of Romer (1990), Aghion and Howitt (1992) and Grossman and Helpman (1991) share

the common feature that as the population grows, so does the rate of technological progress and output per person. This is referred to as the "scale effect" in the literature. Variants of the R&D-driven endogenous growth model with no scale effects have been developed. In these models, the outcome of R&D can either be utilised to increase productivity within a product line, or it can be utilised to increase the total number of available products. Hence, larger economies will have to allocate more R&D workers to improve a larger number of products (see Young (1998), Howitt (1999), Dinopoulos and Thompson (1998)).

The R&D-driven endogenous growth model also has its critics. It has been criticised for constituting a "knife-edge case" in the sense that it requires the knowledge production function to exhibit exactly constant returns to past discoveries. Furthermore it has been argued that even small deviations from constant returns dramatically alter the nature of the growth equilibrium. Allowing for slightly increasing returns to past discoveries would lead growth to explode, while imposing slightly decreasing returns to past discoveries would lead growth to halt in the absence of population growth (Jones (1999) and Li (2000,2002)). However, Dalgaard and Kreiner (2003) argue that the R&D-driven endogenous growth model is more general than it appears. They demonstrate that the knowledge production function may exhibit decreasing returns, increasing return or alternate over time between decreasing returns and increasing return. The crucial condition for endogenous growth is that the marginal product of knowledge in producing new ideas converges towards some positive constant in the long run. This implies that a new piece of information never becomes unproductive in producing new ideas even if infinitely many other pieces of information exist.

Another criticism of the R&D-driven endogenous growth model put forward has been that it implies that permanent changes in R&D promoting policies will permanently affect long-term growth rates. Jones (1995a) raise this criticism. In a time series analysis of post-Second World War growth paths for the OECD countries, he finds no evidence of permanent effects on long-term growth rates stemming from permanent changes in government policies. These findings suggest that the stock of knowledge may be subject to depreciation over time due to obsolescence. However, other empirical studies do find that permanent changes in government policy have permanent effects on income and growth (Kocherlatota and Yi (1997) and Kneller, Bleaney and Gemmell (1999)). Jones (1995b) suggests a modification of the Romer (1990) endogenous growth models in which long-term growth rates are invariant to permanent changes in government policies. Subsidies to R&D and to capital accumulation have no long-term effects but instead affect growth only along the transition path to the new steady state. Although the long-term growth rates become a function of exogenous parameters, Jones argues that the model contains semi-endogenous growth in the sense that technological change itself



is endogenous as a result of the pursuit of new technologies by rational profit maximising agents. However, not only semi-endogenous growth models may have the feature that government policy does not affect long-term growth. Dalgaard and Kreiner (2003) show that the R&D-driven endogenous growth model may also be specified in such a way that policy in the long run has no impact on the marginal product of existing knowledge in producing new ideas. Under these specifications, policy changes in the R&D-driven endogenous growth model will only temporarily affect the growth rate as is the case in the semi-endogenous growth model.

A number of studies have applied the Romer (1990) R&D-driven endogenous growth setting in CGE models to examine the effect of trade and R&D-promoting policies on growth and welfare. Diao et al. (1999) examine the effect of R&D-promoting policies using a CGE model calibrated to data from Japan. Ghosh (2007) studies R&D-promoting policies using a CGE model calibrated to the Canadian economy. Křístková (2013) uses a multi-sector CGE model of the Czech economy to study the long-term growth impact of R&D policies, while Bye et al. (2009) examine the effects of R&D-promoting policies using a multi-sector CGE model with strong open economy features<sup>1</sup> calibrated to Norwegian data.

The model presented in this paper also applies the Romer (1990) R&D-driven endogenous growth setting. However, to allow us to examine the effects of ICT R&D promoting policies, the R&D activities related to ICT are distinguished from other R&D activities. Furthermore, growth generated from development of new ICT capital varieties is distinguished from growth generated from other forms of capital. The R&D-driven endogenous growth model has not yet been applied to the analysis of R&D-promoting policies targeted at ICT specifically.

Although the role of ICT has not featured prominently in CGE models with endogenous growth, a vast number of empirical studies have examined the impact of ICT on economic growth. These point to ICT as an important source of economic growth. The contribution from ICT to growth in terms of capital-deepening and total factor productivity (TFP) growth have been examined by empirical studies using various growth accounting methodologies derived from the endogenous growth literature. Oliner et al. (2007) use growth accounting to examine the contribution from ICT to growth by including in their estimates the accumulation a various ICT capital varieties (computer hardware, computer software and communication equipment). This approach is in line with the R&D-driven endogenous growth model strand of the literature in

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<sup>1</sup> The open economy features include strong technology spillover from abroad through the use off all types of production inputs, internationally given interest rates and internationally mobile capital varieties that are exported at prices determined abroad.

which R&D leads to expanding capital varieties. Examining US growth in the period 1973-2006, Oliner et al. found that the growth effects from ICT accelerated sharply during the second half of the 1990s, but became less important after 2000.<sup>2</sup> The growth effect from ICT can be attributed partly to the use of ICT capital in the private non-farm business sector of the economy and partly to growth in TFP in the ICT producing sectors. After 2000, the growth effect from ICT capital deepening and from TFP increases in the ICT producing sectors has decreased from the high rates observed in the late 1990s. Similar results are found by Jorgensen et al. (2008). Examining US growth using a growth accounting framework with ICT capital and a broader sectoral coverage<sup>3</sup>, Jorgensen et al. also find a large contribution from ICT during the late 1990s, which has declined since 2000. The growth accounting framework has also been used to examine the contribution to growth in countries other than the US. Colechia and Schreyer (2002) examine the contribution of ICT capital to economic growth in nine OECD countries. They also find that the growth contribution from ICT rose during the second half of the 1990s in all countries. hence, not only the US benefitted from the surge in ICT capital investment in that period. In the period from 1995-1999 ICT is estimated to have contributed between 0.3 and 0.9 percentage points per year to economic growth in the countries studied. This result indicates that the existence of a large ICT-producing industry is neither a necessary nor a sufficient condition to successfully experience the growth effects of ICT. An estimated contribution from ICT to growth is also found in studies of France (Cette et al. (2002)) and the UK (Oulton (2001)). The findings from the growth accounting literature underline the potentially important effect of ICT R&D on economic growth in the EU.

The R&D-driven endogenous growth model emphasises the spillover of R&D-produced knowledge into increased productivity as a source of growth. A large number of empirical studies have tested for the presence of a spillover of this kind at various degrees of aggregation - total economy, industry or intra-industry level. A number of studies found trade to be an important source of R&D spillover. Coe and Helpman (1995) examine cross-border spillover effects using a sample of 22 OECD countries. They find that R&D not only makes a significant contribution to TFP in the performing countries but also significantly benefits its trade partners. The spillover effects from foreign R&D are found to be as big as the effect from domestic R&D in smaller countries, while

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<sup>2</sup> The total contribution from ICT to growth is estimated at 0.74 percentage points per year for the period 1973-1995, 1.84 percentage points per year for the period 1995-2000 and 1.12 percentage points per year for the period 2000-2006.

<sup>3</sup> Jorgensen et al. includes the flow of services from owner-occupied housing and from consumer durable goods into both output and capital input. Rapid growth in these assets results in estimates of non-ICT capital deepening that are larger than those of Oliner et al.

the effects from own R&D dominate the effects from cross-border spillover in larger G-7 countries. Xu and Wang (1999) examine spillover of R&D in the G7 countries to TFP growth in 21 OECD countries. They find capital goods trade to be a significant channel of R&D spill-over. About half of the return on R&D investment in a G7 country spills over to other OECD countries. Sakurai et al. (1997) examine the effect on TFP of direct R&D and of R&D embedded in traded investment and intermediate goods for 10 OECD countries. They find that the effects from direct R&D and from trade in investment and intermediate goods vary widely across sectors. Especially strong spillover from trade in investment goods was found for the ICT service sector. This indicates that the purchase of R&D-intensive investment goods is an important source of TFP growth.

Other empirical studies have examined the extent to which Foreign Direct Investments (FDI) leads to productivity increases for domestic firms. Xu (2000) uses data on US outward FDI from firms in the manufacturing sector into 40 countries between 1966-1994, and finds a positive relation between FDI and productivity growth. Spillovers are found to be stronger in the richer than in the poorer countries, indicating that the receiving country has to reach a minimum human capital threshold in order to benefit from the technology transfer from FDI. Other studies examine spillovers from FDI using firm level data. Haskel et al. (2002) study inward FDI using a panel of UK manufacturing firms for the period 1973-1992. They find evidence of significant positive FDI spillover effect. The spillover is found to be stronger for US and French FDI, suggesting different spillover potential for different countries. Griffith et al. (2002) also find significant positive spillover effects from inward FDI using data for UK manufacturing firms. Keller and Yeaple (2003) study FDI activity in the US and find the spillovers from FDI to be positive and large. They estimate that FDI in the US accounts for about 14 % of productivity growth in US firms in the period 1987-1996.

Empirical studies, thus, generally find evidence of R&D spillover and identify international trade and FDI as two major channels for this knowledge spillover.

Clearly factors affecting growth are numerous and cannot all be captured in a single model. However, adopting a R&D-driven CGE model allows us, in a tractable manner, to illustrate how R&D and the accumulation of knowledge affects economic growth. Using the CGE model allows us to simulate different scenarios and to examine the extent to which variations in economic structures and public funding policies might lead to variations in productivity and growth.

The paper is organised as follows. Section 2 presents the model in detail. Section 3 describes data and the calibration methodology. Section 4 provides some concluding remarks.

## 2 The model

The model is a multi-country, multi-sector dynamic general equilibrium model with R&D-driven endogenous growth from Romer’s (1990) expanding capital varieties. The model contains four country blocs: Germany, France, the Rest of EU (REU) and the Rest of the World (ROW). Each country bloc is inhabited by a set of representative households which consumes final goods and supplies labour of a given skills level to the domestic production sectors. Each country bloc contains a set of final goods sectors, homogenous capital producers, differentiated capital producers and R&D producers. The national government in each country bloc collects taxes, pays out transfers and provides government consumption free of charge to its resident-households. The EU Member States also pay contributions to the EU and receive EU transfers.

### 2.1 Production of final output

Final output production occurs in 5 sectors: (1) Agriculture, food and beverages, (2) Low-tech manufacturing, (3) High-tech manufacturing, (4) Services, (5) Information and communication technologies (ICT). Each sector produces one type of output with a constant return to scale technology using intermediate goods, labour, and capital as input. Factors of production are perfectly mobile across sectors within a country bloc but immobile internationally. Final output sector firms face perfectly competitive output markets. The final output from each sector are used as intermediate input in all final output sectors and in the production of R&D. In addition final outputs are consumed by domestic and foreign households and national governments. Final outputs are also used for investment in homogenous and differentiated capital.

The final output production is represented by a nested Constant Elasticity of Substitution (CES) - Leontief production function.<sup>4</sup> The upper nest is given by a CES function that captures the substitution between an aggregate of intermediate inputs and value added. The demand for aggregate intermediate input and value added is given by

$$Y_{I,t}(i) = \theta_{F(i)} \left[ \frac{P_{I,t}(i)}{MC_{F,t}(i)} \right]^{-\sigma_{F(i)}} Y_{F,t}(i)$$

$$VA_t(i) = (1 - \theta_{F(i)}) \left[ \frac{P_{VA,t}(i)}{MC_{F,t}(i)} \right]^{-\sigma_{F(i)}} Y_{F,t}(i)$$

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<sup>4</sup> A presentation of the complete nesting structure of the model can be found in the supplementary technical appendix.

where  $Y_F$  is the final output of sector  $i$ ,  $Y_I$  is the aggregate composite of intermediate inputs and  $VA$  is value added. The parameter  $\theta_F$  governs the share of the aggregate intermediate input composite used in production. The cost index  $MC_F$  is the marginal cost of producing a unit of the final output,  $P_I$  is the price of the composite aggregate of intermediate inputs and  $P_{VA}$  is the price of value added. The composite of intermediate inputs are aggregated from the 5 types of intermediate inputs according to a Leontief production function. Final output is produced using both domestic and imported intermediate inputs. Imported intermediate inputs are considered to be imperfect substitutes for the domestically produced intermediate inputs. The final output producer chooses domestic and imported goods according to a CES function (an Armington function).

The Imported intermediate inputs are sources across all countries and aggregated according to a CES function. Import of intermediate input carry transportation costs purchased from a provider of international transport services. We assume that international transportation services enter on a proportional basis with imported inputs. The demand for domestic intermediate inputs, imported intermediate inputs and international transportation services are given by

$$Y_{D,t}(i, j) = z_{I(i,j)} \theta_{A(i,j)} \left[ \frac{P_{D,t}(i, j)}{P_{A,t}(i, j)} \right]^{-\sigma_{A(i,j)}} Y_{I,t}(i)$$

$$Y_{S,t}(i, j, s) = \left( 1 - z_{ST(i,j,s)} \right) z_{I(i,j)} \theta_{M(i,j,s)} \left( 1 - \theta_{A(i,j)} \right) \\ \times \left[ \frac{P_{ST,t}(i, j, s)}{P_{M,t}(i, j)} \right]^{-\sigma_{M(i,j)}} \left[ \frac{P_{M,t}(i, j)}{P_{A,t}(i, j)} \right]^{-\sigma_{A(i,j)}} Y_{I,t}(i)$$

$$Y_{T,t}(i, j, s) = z_{ST(i,j,s)} z_{I(i,j)} \theta_{M(i,j,s)} \left( 1 - \theta_{A(i,j)} \right) \\ \times \left[ \frac{P_{ST,t}(i, j, s)}{P_{M,t}(i, j)} \right]^{-\sigma_{M(i,j)}} \left[ \frac{P_{M,t}(i, j)}{P_{A,t}(i, j)} \right]^{-\sigma_{A(i,j)}} Y_{I,t}(i)$$

where  $Y_D$  is domestic produced intermediate inputs of type  $j$ ,  $Y_S$  is demand for imported intermediate input of type  $j$  sourced from country  $s$ ,  $Y_T$  is demand for international transport services for the imported intermediate good,  $z_I$  is a share parameter for the aggregation of intermediate inputs,  $\theta_A$  is a share parameter governing the proportion of domestic inputs,  $\theta_M$  is a share parameter for the proportion of imported intermediate inputs sourced from country  $s$ , and  $z_{ST}$  is a share parameter governing the proportion of transportation services associated with imported intermediated inputs. The the price of domestic input of type  $j$  is given by  $P_D$ ,  $P_{ST}$  is a composite price index of imported input of type  $j$  from country  $s$  (including cost of transportation),  $P_A$  is a composite

price index of intermediate input of type  $j$  and  $P_M$  is a composite price index of imported intermediate input.

The final output producing firm pays an input tax  $\tau_D$  on domestic inputs which are added to the price charged by the seller.

$$P_{D,t}(i, j) = (1 + \tau_{D,t}(i, j))\tilde{P}_{F,t}(j)$$

The final output producer also pays an input tax  $\tau_S$  on imported inputs which are added to the sellers price. Import tariffs  $\tau_M$ , are levied on imported inputs. In addition exported inputs are given an export subsidy,  $\tau_X$ . Hence, the price for a given imported input of type  $j$  from country  $s$  are given by the sellers price in country  $s$  in domestic currency less the export subsidy and added the import tariff and the input tax. Suppressing the domestic country index the price of imported inputs may be expressed as

$$P_{S,t}(i, j, s) = (1 + \tau_{S,t}(i, j))(1 + \tau_{M,t}(i, j, s))(1 - \tau_{X,t}(j, s))S_t(s)\tilde{P}_{F,t}^s(j)$$

where  $S$  is the exchange rate. International transportation services are also subject to input taxation and import tariffs. We assume that international transport services are paid in the currency used in the ROW country bloc. Hence, the price of international transportation services is given by

$$P_{T,t}(i, j, s) = (1 + \tau_{S,t}(i, j))(1 + \tau_{M,t}(i, j, s))S_t^{ROW}\tilde{P}_{T,t}(j)$$

Value added is made from a nested CES function with homogeneous capital, differentiated capital of two types (ICT and high-tech) and labour of the various skills types. Differentiated capital is nested with high skilled labour to form respectively a ICT composite and a high-tech composite in the production function. This nesting structure is based on the assumption that the operation of advanced capital requires a highly skilled labour force.

The differentiated ICT capital services are rented from capital producers each producing their own variety of differentiated ICT capital. At any given period  $A_{ICT}$  variety of ICT capital exists. We assume Spence-Dixit-Stiglitz (love of variety) preferences. Hence, the composite of ICT capital services is given by

$$K_{ICT,t}(i) = \left(\tilde{A}_{ICT,t-1}(i)\right)^{v_{ICT}(i)} \left(\int_0^{A_{ICT,t}} \left(K_{ICTV,t}(i, a)^{\frac{\sigma_{ICTV}(i)-1}{\sigma_{ICTV}(i)}}\right) da\right)^{\frac{\sigma_{ICTV}(i)}{\sigma_{ICTV}(i)-1}}$$

where  $K_{ICTV}$  is the capital service from differentiated capital of type  $a$ , and  $\tilde{A}_{ICT,t-1}$  is an exogenous productivity parameter. The rental price of ICT capital service of type  $a$  is denoted by  $R_{ICTV}$ . The cost minimization index

for ICT capital is

$$R_{ICT,t}(i) = \frac{1}{\left(\tilde{A}_{ICT,t-1}(i)\right)^{v_{ICT}(i)}} \left( \int_0^{A_{ICT,t}} R_{ICTV,t}(i, a)^{1-\sigma_{ICTV}(i,j)} da \right)^{\frac{1}{1-\sigma_{ICTV}(i)}}$$

The demand for ICT capital variety  $a$  by the final goods producer in sector  $i$  is given by

$$\begin{aligned} K_{ICTV,t}(i, a) &= \frac{(1 - \theta_{VA(i)}) (1 - \theta_{ICT(i)})}{\left(\tilde{A}_{ICT,t-1}(i)\right)^{v_{ICT}(i)(1-\sigma_{ICTV}(i))}} \left[ \frac{R_{ICTV,t}(i, a)}{R_{ICT,t}(i)} \right]^{-\sigma_{ICTV}(i)} \\ &\quad \times \left[ \frac{R_{ICT,t}(i)}{P_{ICT,t}(i)} \right]^{-\sigma_{ICT}(i)} \left[ \frac{P_{ICT,t}(i)}{P_{VA,t}(i)} \right]^{-\sigma_{VA(i)}} VA_t(i) \end{aligned}$$

where  $\theta_{VA}$  is a share parameter governing the share of the ICT composite in value added,  $\theta_{ICT}$  is a share parameter for the proportion of ICT capital services in the ICT composite, and  $P_{ICT}$  is the price index of the ICT composite.

The high-tech composite in the production function is nested with a low-tech composite of homogenous capital, medium skilled labour and low skilled labour. The demand for differentiated high-tech capital services is given by

$$\begin{aligned} K_{HTV,t}(i, e) &= \frac{\theta_{VA(i)} (1 - \theta_{KL(i)}) (1 - \theta_{HT(i)})}{\left(\tilde{A}_{HT,t-1}(i)\right)^{v_{HT}(i)(1-\sigma_{HTV}(i))}} \left[ \frac{R_{HTV,t}(i, e)}{R_{HT,t}(i)} \right]^{-\sigma_{HTV}(i)} \\ &\quad \times \left[ \frac{R_{HT,t}(i)}{P_{HT,t}(i)} \right]^{-\sigma_{HT}(i)} \left[ \frac{P_{HT,t}(i)}{P_{KL,t}(i)} \right]^{-\sigma_{KL(i)}} \\ &\quad \times \left[ \frac{P_{KL,t}(i)}{P_{VA,t}(i)} \right]^{-\sigma_{VA(i)}} VA_t(i) \end{aligned}$$

where  $\theta_{KL}$  and  $\theta_{HT}$  are share parameters,  $P_{KL}$ ,  $P_{HT}$  and  $R_{HT}$  are composite price indices, and  $\tilde{A}_{HT}$  is an exogenous productivity parameter.

The demand for high skilled labour is given by

$$\begin{aligned}
L_{HS,t}(i) &= \frac{\theta_{VA(i)} (1 - \theta_{KL(i)}) \theta_{HT(i)}}{H_{HT,t}(i)^{1-\sigma_{HT(i)}}} \left[ \frac{W_{HT,t}(i)}{P_{HT,t}(i)} \right]^{-\sigma_{HT(i)}} \left[ \frac{P_{HT,t}(i)}{P_{KL,t}(i)} \right]^{-\sigma_{KL(i)}} \\
&\quad \times \left[ \frac{P_{KL,t}(i)}{P_{VA,t}(i)} \right]^{-\sigma_{VA(i)}} VA_t(i) \\
&\quad + \frac{(1 - \theta_{VA(i)}) \theta_{ICT(i)}}{H_{ICT,t}(i)^{1-\sigma_{ICT(i)}}} \left[ \frac{W_{ICT,t}(i)}{P_{ICT,t}(i)} \right]^{-\sigma_{ICT(i)}} \\
&\quad \times \left[ \frac{P_{ICT,t}(i)}{P_{VA,t}(i)} \right]^{-\sigma_{VA(i)}} VA_t(i)
\end{aligned}$$

where  $W_{ICT}$  and  $W_{HT}$  is the cost of a unit of labour working with ICT or high-tech capital respectively. We allow for skills specific changes in productivity  $H_{ICT}$  and  $H_{HT}$ . The skills specific changes in productivity are assumed to be driven by an exogenous process, reflecting productivity gains due to for example education, training or organizational and managerial improvements.

Homogenous capital is nested with a labour composite of low and medium skilled labour. The demand for homogenous capital services is given by

$$\begin{aligned}
K_{LT,t}(i) &= \theta_{VA(i)} \theta_{KL(i)} (1 - \theta_{LT(i)}) \left[ \frac{R_{LT,t}(i)}{P_{LT,t}(i)} \right]^{-\sigma_{LT(i)}} \left[ \frac{P_{LT,t}(i)}{P_{KL,t}(i)} \right]^{-\sigma_{KL(i)}} \\
&\quad \times \left[ \frac{P_{KL,t}(i)}{P_{VA,t}(i)} \right]^{-\sigma_{VA(i)}} VA_t(i)
\end{aligned}$$

where  $R_{LT}$  is the rental price of homogenous capital,  $P_{LT}$  is the price index of the low-tech composite, and  $\theta_{LT}$  is a share parameter governing the proportion of homogenous capital services in the low-tech composite. The demand for low skilled labour and medium skilled labour is given by

$$\begin{aligned}
L_{LS,t}(i) &= \frac{\theta_{VA(i)} \theta_{KL(i)} \theta_{LT(i)} \theta_{LM(i)}}{H_{LS,t}(i)^{1-\sigma_{LM(i)}}} \left[ \frac{W_{LS,t}(i)}{W_{LM,t}(i)} \right]^{-\sigma_{LM(i)}} \left[ \frac{W_{LM,t}(i)}{P_{LT,t}(i)} \right]^{-\sigma_{LT(i)}} \\
&\quad \times \left[ \frac{P_{LT,t}(i)}{P_{KL,t}(i)} \right]^{-\sigma_{KL(i)}} \left[ \frac{P_{KL,t}(i)}{P_{VA,t}(i)} \right]^{-\sigma_{VA(i)}} VA_t(i)
\end{aligned}$$

$$\begin{aligned}
L_{MS,t}(i) &= \frac{\theta_{VA(i)} \theta_{KL(i)} \theta_{LT(i)} (1 - \theta_{LM(i)})}{H_{MS,t}(i)^{1-\sigma_{LM(i)}}} \left[ \frac{W_{MS,t}(i)}{W_{LM,t}(i)} \right]^{-\sigma_{LM(i)}} \\
&\quad \times \left[ \frac{W_{LM,t}(i)}{P_{LT,t}(i)} \right]^{-\sigma_{LT(i)}} \left[ \frac{P_{LT,t}(i)}{P_{KL,t}(i)} \right]^{-\sigma_{KL(i)}} \left[ \frac{P_{KL,t}(i)}{P_{VA,t}(i)} \right]^{-\sigma_{VA(i)}} VA_t(i)
\end{aligned}$$

where  $\theta_{LM}$  is a share parameter governing the proportion of low-skilled labour



in the low-medium skilled labour composite,  $W_{LS}$  and  $W_{MS}$  is the cost of a unit of low skilled and medium skilled labour respectively, and  $W_{LM}$  is a cost index of the low-medium skilled labour composite. We again allow for skills specific changes in productivity  $H_{LS}$  and  $H_{MS}$ .

The final output producer pays taxes on labour and capital factor inputs. The wage cost paid by the firm to low skilled labour (supplied by household of type  $h$ ) is given by the market wage  $W$  added the factor input tax  $\tau_{FL}$ .

$$W_{LS,t}(i) = (1 + \tau_{FL,t}(i, h))W(h)$$

The rental cost of homogenous capital  $R_{LT}$  is given as the market rental price  $\tilde{R}_{LT}$  added the factor input tax  $\tau_{FK_{LT}}$ .

$$R_{LT,t}(i) = (1 + \tau_{FK_{LT},t}(i))\tilde{R}_{LT}$$

The wage cost for the other types of labour and the capital rental prices for the two types of differentiated capital can be found in a similar fashion. The final output producer pays a gross output tax,  $\tau_O$ . The relation between the firms cost minimization and the price set by the producer is given by.

$$MC_{F,t}(i) = \tilde{P}_{F,t}(j)(1 - \tau_{O,t}(i)) , \text{ for } j = i$$

## 2.2 Production of R&D

The model contains two types of R&D producing firms, producers of ICT R&D and producers of non-ICT R&D. That R&D production is modelled separately from the production of final goods and the investment decision is a simplifying assumption of the model that does not alter the outcome of the policy analysis. Each type of R&D firms produce new ideas and blueprints which are sold as patents and used in the production of new differentiated capital goods. New ideas and blueprints are produced from intermediate inputs and primary factors. Output of each individual R&D producer expands the common pool of technological knowledge available to R&D producers. These technological spillovers constitute a form of positive externality which leads to an increase in the productivity of the R&D active labour force. Consequently, spillover becomes a source of further technological progress increasing the efficiency of producing additional blueprints.

The production function of the R&D sector is represented by a nested CES-Leontief production function. The upper nest captures the substitution between an aggregate of intermediate inputs and value added. The demand for

aggregate intermediate input and value added is given by

$$Y_{I,t}(r) = \theta_{R(r)} \left[ \frac{P_{I,t}(r)}{MC_{XD,t}(r)} \right]^{-\sigma_{R(r)}} \widetilde{XD}_t(r)$$

$$VA_{I,t}(r) = (1 - \theta_{R(r)}) \left[ \frac{P_{VA,t}(r)}{MC_{XD,t}(r)} \right]^{-\sigma_{R(r)}} \widetilde{XD}_t(r)$$

where  $\widetilde{XD}$  is the gross output of R&D of type  $r$ ,  $Y_I$  and  $VA$  are respectively the composite of intermediate inputs and value added. The parameter  $\theta_R$  governs the share of the intermediate input composite used in R&D production. The variable  $MC_{XD}$  denotes marginal costs,  $P_I$  in the price index of the intermediate input composite and  $P_{VA}$  is the price index of value added. The composite of intermediate inputs is aggregated according to a Leontief production function.

The R&D producer uses domestic and imported intermediate inputs. Imported material inputs are imperfect substitutes for the domestic material inputs. Import of intermediate input carry transportation costs purchased from a provider of international transport services. We assume that international transportation services enter on a proportional basis with imported inputs. The demand for domestic intermediate inputs, imported intermediate inputs and transport services is given by

$$Y_{D,t}(r, j) = z_{I(r,j)} \theta_{A(r,j)} \left[ \frac{P_{D,t}(r, j)}{P_{A,t}(r, j)} \right]^{-\sigma_{A(i,j)}} Y_{I,t}(r)$$

$$Y_{S,t}(r, j, s) = z_{I(r,j)} (1 - z_{ST(r,j,s)}) (1 - \theta_{A(r,j)}) \theta_{M(r,j,s)} \times \left[ \frac{P_{ST,t}(r, j, s)}{P_{M,t}(r, j)} \right]^{-\sigma_{M(r,j)}} \left[ \frac{P_{M,t}(r, j)}{P_{A,t}(r, j)} \right]^{-\sigma_{A(i,j)}} Y_{I,t}(r)$$

$$Y_{T,t}(r, j, s) = z_{I(r,j)} z_{ST(r,j,s)} (1 - \theta_{A(r,j)}) \theta_{M(r,j,s)} \left[ \frac{P_{ST,t}(r, j, s)}{P_{M,t}(r, j)} \right]^{-\sigma_{M(r,j)}} \times \left[ \frac{P_{M,t}(r, j)}{P_{A,t}(r, j)} \right]^{-\sigma_{A(i,j)}} Y_{I,t}(r)$$

where  $Y_D$  is domestic produced intermediate inputs of type  $j$ ,  $Y_S$  is demand for imported intermediate input of type  $j$  sourced from country  $s$ ,  $Y_T$  is demand for international transport services arising from the import of intermediate good of type  $j$  sourced from country  $s$ . The share parameter  $z_I$  governs the share of intermediate input of type  $j$ ,  $\theta_A$  is a share parameter governing the

proportion of domestic inputs,  $\theta_M$  is a share parameter for the proportion of imported intermediate inputs sourced from country  $s$ , and  $z_{ST}$  is a share parameter governing the proportion of transportation services associated with imported intermediated inputs of a given type and sourcing country. The price of domestic input of type  $j$  is given by  $P_D$ ,  $P_{ST}$  is a composite price index of imported input of type  $j$  from country  $s$  (including cost of transportation),  $P_A$  is a composite price index of intermediate input of type  $j$  and  $P_M$  is a composite price index of imported intermediate input.

The R&D producer also pays an input tax  $\tau_D$  on the domestic produced input and an input tax  $\tau_S$  on the imported intermediate inputs and transport services. In addition import tariffs  $\tau_M$  are levied on imported intermediate inputs and exported input is given an export subsidy  $\tau_X$ . Suppressing the home country index the price of domestic input, foreign input and international transportation services is given by

$$P_{D,t}(r, j) = (1 + \tau_{D,t}(r, j)) \tilde{P}_{F,t}(j)$$

$$P_{S,t}(r, j, s) = (1 + \tau_{S,t}(r, j))(1 + \tau_{M,t}(r, j, s))(1 - \tau_{X,t}(j, s)) S_{t,s} \tilde{P}_{F,t}^s(j)$$

$$P_{T,t}(r, j, s) = (1 + \tau_{S,t}(r, j))(1 + \tau_{M,t}(r, j, s)) S_t^{ROW} \tilde{P}_{T,t}(j)$$

Value added is produced from a combination of homogenous capital and low-, medium- and high skilled labour.<sup>5</sup> Demand for homogenous capital services is given by

$$K_{LT,t}(r) = \theta_{VA(r)} (1 - \theta_{LT(r)}) \left[ \frac{R_{LT,t}(r)}{P_{LT,t}(r)} \right]^{-\sigma_{LT(r)}} \left[ \frac{P_{LT,t}(r)}{P_{VA,t}(r)} \right]^{-\sigma_{VA(r)}} VA_t(r)$$

where  $\theta_{VA}$  and  $\theta_{LT}$  are share parameters,  $R_{LT}$  is the rental price of homogenous capital and  $P_{LT}$  is a cost index of a low-tech composite aggregated from homogenous capital, low skilled labour and medium skilled labour.

Demand for low, medium and high skilled labour is given by

$$L_{LS,t}(r) = \frac{\theta_{VA(r)} \theta_{LT(r)} \theta_{LM(r)}}{H_{LS,t}(r)^{1-\sigma_{LM(r)}}} \left[ \frac{W_{LS,t}(r)}{W_{LM,t}(r)} \right]^{-\sigma_{LM(r)}} \left[ \frac{W_{LM,t}(r)}{P_{LT,t}(r)} \right]^{-\sigma_{LT(r)}} \\ \times \left[ \frac{P_{LT,t}(r)}{P_{VA,t}(r)} \right]^{-\sigma_{VA(r)}} VA_t(r)$$

<sup>5</sup> Differentiated capital is excluded from the nested production function of the R&D producers to avoid cumulative multipliers of the love-of-variety effects.

$$L_{MS,t}(r) = \frac{\theta_{VA(r)}\theta_{LT(r)}(1 - \theta_{LM(r)})}{H_{MS,t}(r)^{1-\sigma_{LM(r)}}} \left[ \frac{W_{MS,t}(r)}{W_{LM,t}(r)} \right]^{-\sigma_{LM(i)}} \left[ \frac{W_{LM,t}(r)}{P_{LT,t}(r)} \right]^{-\sigma_{LT(r)}} \\ \times \left[ \frac{P_{LT,t}(r)}{P_{VA,t}(r)} \right]^{-\sigma_{VA(r)}} VA_t(r)$$

$$L_{HS,t}(r) = \frac{(1 - \theta_{VA(r)})}{H_{HS,t}(r)^{1-\sigma_{VA(r)}}} \left[ \frac{W_{HS,t}(r)}{P_{VA,t}(r)} \right]^{-\sigma_{VA(r)}} VA_t(r)$$

where  $W_{LS}$ ,  $W_{MS}$  and  $W_{HS}$  is the wage of one unit of low skilled labour, medium skilled labour and high skilled labour respectively. The cost of the composite of low and medium skilled labour is given by  $W_{LM}$ . The parameter  $\theta_{LM}$  governs the proportion of low-skilled labour in the low-medium skilled labour composite. We allow for skills specific changes in productivity for low and high skilled labour ( $H_{LS}$  and  $H_{MS}$ ) driven by an exogenous process. The productivity of high skilled research staff  $H_{HS}$  is given endogenous in the model. We assume that it rises as the common pool of knowledge resulting from R&D expands.

The R&D producer also pays taxes on labour and capital factor inputs. The wage cost paid by the firm to low skilled labour is given by

$$W_{LS,t}(r) = (1 + \tau_{FL,t}(r, h))W(h)$$

The wage costs paid for the other labour types can be found in a similar fashion. The rental cost of homogenous capital  $R_{LT}$  is given by

$$R_{LT,t}(r) = (1 + \tau_{FK_{LT,t}(r)})\tilde{R}_{LT}$$

We assume that it is costly for the R&D producer to adjust the growth rate of R&D output. The cost of adjustment takes the form of foregone R&D output. The gross R&D output  $\tilde{XD}$  is given by

$$\tilde{XD}_t(r) = XD_t(r) + \frac{\phi_{XD(r)}}{2} (XD_t(r) - (1 + gr)XD_{t-1}(r))^2$$

where  $\phi_{XD}$  is a parameter that governs the cost of adjustment, and  $gr$  is the model's long term real growth rate. The R&D producer pays output tax  $\tau_O$  on blueprints produced. Net profit from R&D production is

$$\pi_{XD,t}(r) = P_{XD,t}(r)XD_t(r) (1 - \tau_{O,t}(r)) - MC_{XD,t}(r)\tilde{XD}_t(r)$$

The relation between the R&D producers marginal cost and the output price of blueprints is given by

$$P_{XD,t}(r) = \frac{1}{1 - \tau_{O,t}(r)} \times \left( MC_{XD,t}(r)(1 + \phi_{XD(r)}(XD_t(r) - (1 + gr)XD_{t-1}(r))) - \frac{\phi_{XD(r)}}{1+i_t} MC_{XD,t+1}(r)(XD_{t+1}(r) - (1 + gr)XD_t(r)) \right)$$

We assume that once a new idea or blueprints is produced it is patented and sold to a capital producer who utilize it in the production of a new capital variety. However, the knowledge produced also diffuse into the stock of common knowledge available to all R&D firms. We assume no depreciation of previously R&D produced new knowledge (an assumption that could be modified). Hence, the accumulation of new ideas is given by

$$A_{RD,t+1}(r) = A_{RD,t}(r) + XD_t(r)$$

The stock of new ideas in a given sector potentially effect the productivity of all high skilled research staff domestically and abroad. The overall change in productivity of the research staff in R&D sector  $r$  due to accumulation of knowledge is given by

$$H_{HS,t}(r) = \tilde{A}_{RD,t}(r)^{\nu_{RD}} \prod_{\tilde{r}=1}^{nr} A_{RD,t}(\tilde{r})^{\sigma_{RDd}(r,\tilde{r})} \prod_{\tilde{r}=1}^{nr} \prod_{s=1}^{ns} \omega_{R(r,\tilde{r},s)} A_{RD,t}(\tilde{r},s)^{\sigma_{RDf}(r,\tilde{r},s)}$$

where  $\sigma_{RDd}(r,\tilde{r})$  is the elasticity of the spill-over from new ideas produced domestically by R&D firms of type  $\tilde{r}$  to productivity of research staff in R&D firms of type  $r$ , the elasticity of international spill-over from knowledge of type  $\tilde{r}$  in country  $s$  to productivity of research staff in the domestic R&D firm of type  $r$  is given by  $\sigma_{RDf}(r,\tilde{r})$ . The coefficient  $\omega_{R(r,\tilde{r},s)}$  weights the spill-overs from internal knowledge of type  $\tilde{r}$  in country  $s$  to productivity of domestic R&D firms of type  $r$ . We assume that the weights of the international spill-over coefficient are related to bilateral trade flows.<sup>6</sup> The variable  $\tilde{A}_{RD}$  is an exogenous productivity parameter.

### 2.3 Production of capital varieties

All country blocs host two differentiated capital producing sectors, ICT and other high-tech. Each of the two sectors consists of a number of firms each producing a variety of differential capital. A producer of differentiated capital must first purchase a patented blueprint from a R&D producer. Firms in the ICT capital producing sector purchase blueprints from the ICT R&D producers, while firms in the high-tech capital producing sector purchase blueprints

<sup>6</sup> Alternatively the weights could be moddeld simply as a direct diffusion of ideas.

from the non-ICT R&D producers. Once the patent is bought it excludes other firms from producing identical capital varieties. However, substitutes exist in the form of other capital varieties. Having obtained a patent for a given capital variety the capital producer invests in its capital variety and rents it to the firms in the final output sectors.

Consider a producer of ICT differentiated capital. The firm invest in its ICT capital variety  $a$  and rents it out to final output producers. To invest in its capital variety the ICT capital producer must produce an ICT investment good composite. The ICT investment good composite is produced from intermediate inputs with a nested Leontief-CES production function. In the production of the investment good composite the ICT capital producer uses a mix of domestic and imported intermediate goods according to a CES Armington function. Import of intermediate commodities gives rise to transportation costs. We assume that international transportation services enter on a proportional basis with imported inputs

Demand for domestic intermediate inputs, imported intermediate inputs and international transport services are

$$Y_{D,t}(a, j) = z_{ICT(a,j)} \theta_{A(a,j)} \left[ \frac{P_{D,t}(a, j)}{P_{A,t}(a, j)} \right]^{-\sigma_{A(a,j)}} Y_{ICTV,t}(a)$$

$$Y_{S,t}(a, j, s) = \left(1 - z_{ST(a,j,s)}\right) \theta_{M(a,j,s)} \left(1 - \theta_{A(a,j)}\right) \left[ \frac{P_{ST,t}(a, j, s)}{P_{M,t}(a, j)} \right]^{-\sigma_{M(a,j)}} \\ \times \left[ \frac{P_{M,t}(a, j)}{P_{A,t}(a, j)} \right]^{-\sigma_{A(a,j)}} Y_{ICTV,t}(a, j)$$

$$Y_{T,t}(a, j, s) = z_{ST(a,j,s)} \theta_{M(a,j,s)} \left(1 - \theta_{A(a,j)}\right) \left[ \frac{P_{ST,t}(a, j, s)}{P_{M,t}(a, j)} \right]^{-\sigma_{M(a,j)}} \\ \times \left[ \frac{P_{M,t}(a, j)}{P_{A,t}(a, j)} \right]^{-\sigma_{A(a,j)}} Y_{ICTV,t}(a, j)$$

where  $Y_D$  is the domestic intermediate input of type  $j$ ,  $Y_S$  is demand for imported intermediate input of type  $j$  sourced from country  $s$ ,  $Y_T$  is demand for international transport services for the imported intermediate good and  $Y_{ICTV}$  is the investment goods composite. The share parameters  $z_{ICT}$ ,  $\theta_A$ ,  $\theta_M$  and  $z_{ST}$  govern respectively the share of intermediate input of type  $j$ , the proportion of domestic inputs, sourcing of importing inputs from country  $s$ , and the proportion of international transport services. The price of domestic

inputs is denoted by  $P_D$ ,  $P_{ST}$  is the price of intermediate inputs including transportation,  $P_M$  and  $P_A$  is composite indices.

An input tax  $\tau_D$  is levied on the domestic produced input. The firm also pays an input tax  $\tau_S$  on the imported inputs which are added to the sellers price of the intermediate input. Import tariffs  $\tau_M$  are levied on imported inputs. In addition the exported input is given an export subsidy  $\tau_X$ . International transportation services are also subject to input taxation and import tariffs. Suppressing the domestic country index the price of domestic inputs, imported inputs and international capital services may be expressed as

$$P_{D,t}(a, j) = (1 + \tau_{D,t}(a, j))\tilde{P}_{F,t}(j)$$

$$P_{S,t}(a, j, s) = (1 + \tau_{S,t}(a, j))(1 + \tau_{M,t}(a, j, s))(1 - \tau_{X,t}(j, s))S_t(s)\tilde{P}_{F,t}^s(j)$$

$$P_{T,t}(a, j, s) = (1 + \tau_{S,t}(a, j))(1 + \tau_{M,t}(a, j, s))S_t^{ROW}\tilde{P}_{T,t}(j)$$

The presence of markets power as a consequence of the differentiated characteristics of each ICT capital variety means that firms can charge a rental price of capital above its marginal costs. Define net return of firm  $a$  in period  $t$  as

$$\pi_{ICTV,t}(a) = \tilde{R}_{ICTV,t}(a)\widehat{K}_{ICTV,t}(a)u_{ICTV,t}(a) - MC_{ICT,t}(a)I_{ICTV,t}(a)$$

where  $R_{ICTV}$  is the rental price of ICT capital variety  $a$ ,  $\widehat{K}_{ICTV}$  is the stock of capital variety  $a$ ,  $u_{ICTV}$  is the capital utilization rate and  $I_{ICTV}$  is new investments by the firm. The ICT capital is rented out to firms in all domestic final good sectors. Hence the total rental of ICT capital of type  $a$  is given by

$$\tilde{R}_{ICTV,t}(a)\widehat{K}_{ICTV,t}(a)u_{ICTV,t}(a) = \tilde{R}_{ICTV,t}(a)\sum_{i=1}^{ni}K_{ICTV,t}(i, a)$$

We assume that the capital utilization can be varied from its long term rate  $\bar{u}_{ICTV}$  at the cost of higher capital depreciation. The cost of varying the capital utilization rate is given by

$$\phi_{u_{ICT},t}(a) = \varphi_{u1_{ICT}(a)}(u_{ICTV,t}(a) - \bar{u}_{ICTV}(a)) + \frac{\varphi_{u2_{ICT}(a)}}{2}(u_{ICTV,t}(a) - \bar{u}_{ICTV}(a))^2$$

where  $\varphi_{u1_{ICT}}$  and  $\varphi_{u2_{ICT}}$  are parameters that govern the cost of varying the capital utilization rate.

Accumulation of ICT capital follows

$$\begin{aligned}\widehat{K}_{ICTV,t+1}(a) &= I_{ICTV,t}(a) + \widehat{K}_{ICTV,t}(a)(1 - \delta_{ICT} - \phi_{u_{ICT},t}(a)) \\ &\quad - \frac{\phi_{K_{ICT}(a)}}{2} \frac{(\widehat{K}_{ICTV,t+1}(a) - \widehat{K}_{ICTV,t}(a))^2}{\widehat{K}_{ICTV,t}(a)} \\ &\quad - \frac{\phi_{I_{ICT}(a)}}{2} (\tilde{I}_{ICTV,t}(a) - \tilde{I}_{ICTV,t-1}(a))^2\end{aligned}$$

where  $\delta_{ICT}$  is the depreciation rate and  $\tilde{I}_{ICTV}$  is net investments. The parameters  $\phi_{K_{ICT}}$  and  $\phi_{I_{ICT}}$  govern the cost of adjusting respectively the firms capital stock and the firms net investment levels.

The ICT capital producer sets the capital utilization rate and makes its investment decision such as to maximise future expected profit

$$\sum_{j=0}^{\infty} \left( \left( \frac{1}{1+i} \right)^j \left( \begin{aligned} &\tilde{R}_{ICTV,t+j}(a) \widehat{K}_{ICTV,t+j}(a) u_{ICTV,t+j}(a) \\ &- MC_{ICT,t+j}(a) I_{ICTV,t+j}(a) \end{aligned} \right) \right)$$

subject to the capital accumulation process and demand for its capital variety. The first order conditions are given by

$$\tilde{R}_{ICTV,t}(a) = MC_{ICT,t}(a) \left( \varphi_{u_{1ICT}(a)} + \varphi_{u_{2ICT}(a)} (u_{ICTV,t}(a) - \bar{u}_{ICTV}(a)) \right)$$

$$\begin{aligned}&\tilde{R}_{ICTV,t+1}(a) u_{ICTV,t+1}(a) \\ &= \frac{\sigma_{ICTV}}{\sigma_{ICTV-1}} \left( \begin{aligned} &(1+i_t) MC_{ICT,t}(a) - MC_{ICT,t+1}(a)(1 - \delta_{ICT} - \phi_{u,t+1}(a)) \\ &+ \Psi_{ICTa}(a) + \Psi_{ICTb}(a) - (1 - \delta_{ICT}) \Psi_{ICTc}(a) \end{aligned} \right)\end{aligned}$$

where

$$\begin{aligned}\Psi_{ICTa}(a) &= (1+i_t) MC_{ICT,t}(a) \phi_{K_{ICT}(a)} \left( \frac{\widehat{K}_{ICTV,t+1}(a)}{\widehat{K}_{ICTV,t}(a)} - 1 \right) \\ &\quad - MC_{ICT,t+1}(a) \frac{\phi_{K_{ICT}(a)}}{2} \left( \frac{(\widehat{K}_{ICTV,t+2}(a))^2}{(\widehat{K}_{ICTV,t+1}(a))^2} - 1 \right)\end{aligned}$$

$$\begin{aligned}\Psi_{ICTb}(a) &= (1+i_t) MC_{ICT,t}(a) \phi_{I_{ICT}(a)} (\tilde{I}_{ICTV,t}(a) - \tilde{I}_{ICTV,t-1}(a)) \\ &\quad - MC_{ICT,t+1}(a) \phi_{I_{ICT}(a)} (\tilde{I}_{ICTV,t+1}(a) - \tilde{I}_{ICTV,t}(a))\end{aligned}$$



$$\begin{aligned}\Psi_{ICTc}(a) &= MC_{ICT,t+1}(a)\phi_{ICT(a)}\left(\tilde{I}_{ICTV,t+1}(a) - \tilde{I}_{ICTV,t}(a)\right) \\ &\quad - \frac{1}{1+i_{t+1}}MC_{ICT,t+2}(a)\phi_{ICT(a)}\left(\tilde{I}_{ICTV,t+2}(a) - \tilde{I}_{ICTV,t+1}(a)\right)\end{aligned}$$

The capital utilization rate is set such that marginal revenue equals marginal cost of the chosen utilization rate. The firm invest such as the expected rental price is equal to a constant mark-up over the risk free return on its investment adjusted for losses due to depreciation, expected gains due to changes in next periods price of its investment goods and adjustment costs. We assume that all ICT capital producers share an identical production technology and, hence, all set identical capital rental prices and faces the same demand for their ICT capital variety.

The value of the ICT capital producer of variety  $a$  entering the market at time  $t$  is given by

$$\begin{aligned}V_{ICTV,t}(a) &= \sum_{j=1}^{\infty} \left( \left( \frac{1}{1+i} \right)^j (\pi_{ICTV,t+j}(a)) \right) \\ &\quad - P_{XD,t}(r) + MC_{ICT,t}(a)\widehat{K}_{ICTV,t}(a)\end{aligned}$$

where  $P_{XD}$  is the price of the patent purchased from the ICT R&D producer prior to capital production. Hence, the value of the ICT capital producing firm entering the market equals the value of the newly produced capital to be rented out next period plus expected future income from capital rental less the initial cost of purchasing the patent required for entering the market.

Total investment by the ICT capital producers are given by

$$I_{ICT,t} = A_{ICT,t}I_{ICTV,t}(a) + XD_t(r)\widehat{K}_{ICTV,t+1}(a)$$

Investment in ICT capital totals investment carried out by existing ICT capital producers and investments by producers of new patented ICT capital varieties that enters the market.

We assume that there are free entry to the market for differentiated capital production. Asset market equilibrium requires that the following non-arbitrage condition must hold

$$\pi_{ICTV,t}(a) + V_{ICTV,t}(a) - V_{ICTV,t-1}(a) = iV_{ICTV,t-1}(a)$$

An investor is indifferent between investing in the ICT capital producer and holding a risk free bond. In equilibrium the price of patents for new ICT capital varieties equals the expected discounted future return from producing the capital variety. Firms are entering the market for ICT capital production until the firm's discounted net profit is equal to the entry costs. The price for

new ICT patens is thus given by

$$P_{XD,t}(r) = \sum_{j=0}^{\infty} \left( \left( \frac{1}{1+i} \right)^j \left( \begin{array}{c} \tilde{R}_{ICTV,t+j}(a) \widehat{K}_{ICTV,t+j}(a) u_{ICTV,t+j}(a) \\ -MC_{ICT,t+j}(a) I_{ICTV,t+j}(a) \end{array} \right) \right)$$

Total dividend of the ICT producing sector in any given period is given by

$$\begin{aligned} \pi_{ICT,t}(a) = & A_{ICT,t} \tilde{R}_{ICT,t}(a) \widehat{K}_{ICTV,t}(a) u_{ICTV,t}(a) \\ & - MC_{ICT,t}(a) I_{ICT,t}(a) - P_{XD,t}(r) XD_t(r) \end{aligned}$$

The production technology of the high-tech capital producer is assumed to be similar to that of the ICT capital producer. Hence, investment decisions of the differentiated high-tech capital producers can be found in a similar fashion.

#### 2.4 Production of homogenous capital

The model contains one type of low-tech homogenous capital. Firms in the homogenous capital sector invest in homogenous capital goods and rent it to firms producing final outputs and R&D. The homogenous investment good composite  $Y_{LT}$  is produced from intermediate inputs with a nested Leontief-CES production function. The homogenous capital producer uses a mix of domestic and imported intermediate goods according to a CES Armington function. Import of intermediate commodities gives rise to transportation costs. We assume that international transportation services enter on a proportional basis with imported inputs

Demand for domestic intermediate inputs, imported intermediate inputs and international transport services are

$$Y_{D,t}(u, j) = z_{LT(j)} \theta_{A(u,j)} \left[ \frac{P_{D,t}(u, j)}{P_{A,t}(u, j)} \right]^{-\sigma_{A(u,j)}} Y_{LT,t}(u)$$

$$\begin{aligned} Y_{S,t}(u, j, s) = & z_{LT(j)} \left( 1 - z_{ST(u,j,s)} \right) \theta_{M(u,j,s)} \left( 1 - \theta_{A(u,j)} \right) \\ & \times \left[ \frac{P_{ST,t}(u, j, s)}{P_{M,t}(u, j)} \right]^{-\sigma_{M(u,j)}} \left[ \frac{P_{M,t}(u, j)}{P_{A,t}(u, j)} \right]^{-\sigma_{A(u,j)}} Y_{LT,t}(j) \end{aligned}$$

$$Y_{T,t}(u, j, s) = z_{LT}(j)z_{ST}(u, j, s)\theta_{M(u, j, s)} \left(1 - \theta_{A(u, j)}\right) \left[\frac{P_{ST,t}(u, j, s)}{P_{M,t}(u, j)}\right]^{-\sigma_{M(u, j)}} \\ \times \left[\frac{P_{M,t}(u, j)}{P_{A,t}(u, j)}\right]^{-\sigma_{A(u, j)}} Y_{LT,t}(j)$$

where  $Y_D$  is demand for domestic intermediate input of type  $j$ ,  $Y_S$  is demand for imported intermediate input of type  $j$  sourced from country  $s$ ,  $Y_T$  is demand for international transport services for the imported intermediate good.

The homogenous capital producer pays an input tax  $\tau_D$  on the domestic inputs and input tax  $\tau_S$  on the imported input. Import tariffs  $\tau_M$  are also levied on imported inputs. In addition the exported input is given a export subsidy  $\tau_X$ . International transportation services are also subject to input taxation and import tariffs. The price of domestic inputs, imported inputs and international transport service may be expressed as

$$P_{D,t}(u, j) = (1 + \tau_{D,t}(u, j))\tilde{P}_{F,t}(j)$$

$$P_{S,t}(u, j, s) = (1 + \tau_{S,t}(u, j))(1 + \tau_{M,t}(u, j, s))(1 - \tau_{X,t}(j, s))S_t(s)\tilde{P}_{F,t}^s(j)$$

$$P_{T,t}(u, j, s) = (1 + \tau_{S,t}(u, j))(1 + \tau_{M,t}(u, j, s))S_t^{ROW}\tilde{P}_{T,t}(j)$$

where as earlier  $j$  is an index of goods and  $s$  is an index of sourcing countries in the model.

Accumulation of homogenous capital follows

$$\widehat{K}_{LT,t+1} = I_{LT,t} + \widehat{K}_{LT,t}(1 - \delta_{LT} - \phi_{u_{LT,t}}) \\ - \frac{\phi_{K_{LT}}}{2} \frac{(\widehat{K}_{LT,t+1} - \widehat{K}_{LT,t}(1 + g))^2}{\widehat{K}_{LT,t}} \\ - \frac{\phi_{I_{LT}}}{2} (\tilde{I}_{LT,t} - \tilde{I}_{LT,t-1}(1 + g))^2$$

where  $\widehat{K}_{LT}$  is the total stock of homogenous capital and  $u_{LT}$  is the capital utilization rate. The parameters  $\phi_{K_{LT}}$  and  $\phi_{I_{LT}}$  govern the cost of adjusting respectively the firms capital stock and the firms net investment levels. Note that the capital utilization rate can be varied from its long term rate  $\bar{u}_{LT}$  at the cost of higher capital depreciation. The cost of varying the capital utilization rate is given by

$$\phi_{u_{LT,t}} = \varphi_{u_{1LT}(a)} (u_{LT,t} - \bar{u}_{LT}) + \frac{\varphi_{u_{2LT}(a)}}{2} (u_{LT,t} - \bar{u}_{LT})^2$$

Supply of capital services equal the total demand for homogenous capital

services by final goods producers and R&D producers

$$\widehat{K}_{LT}u_{LT} = \sum_{i=1}^{ni} K_{LT}(i) + \sum_{r=1}^{nr} K_{LT}(r)$$

The homogenous capital producer chooses the capital utilisation rate and investment that maximizes the expected profit

$$\pi_{LT,t} = \sum_{s=0}^{\infty} \left( \left( \frac{1}{1+i} \right)^s \left( \tilde{R}_{LT,t+s} \widehat{K}_{LT,t+s} u_{LT,t+s} - MC_{LT,t+s} I_{LT,t+s} \right) \right)$$

subject to the capital accumulation process. The first order conditions are given by

$$\tilde{R}_{LT,t} = MC_{LT,t} \left( \varphi_{u1_{LT}(a)} + \varphi_{u2_{LT}(a)} (u_{LT,t} - \bar{u}_{LT}) \right)$$

$$\begin{aligned} \tilde{R}_{LT,t+1} u_{LT,t+1} + MC_{LT,t+1} (1 - \delta_{LT} - \phi_{u_{LT,t+1}}) = \\ (1 + i_t) MC_{LT,t} + \Psi_{LTa} + \Psi_{LTb} - (1 - \delta_{ICT}) \Psi_{LTc} \end{aligned}$$

where

$$\begin{aligned} \Psi_{LTa} = (1 + i_t) MC_{LT,t} \phi_{K_{LT}} \left( \frac{\widehat{K}_{LT,t+1}}{\widehat{K}_{LT,t}} - (1 + g) \right) \\ - MC_{LT,t+1} \frac{\phi_{K_{LT}}}{2} \left( \frac{(\widehat{K}_{LT,t+2})^2}{(\widehat{K}_{LT,t+1})^2} - (1 + g)^2 \right) \end{aligned}$$

$$\begin{aligned} \Psi_{LTb} = (1 + i_t) MC_{LT,t} \phi_{I_{LT}} \left( \tilde{I}_{LT,t} - \tilde{I}_{LT,t-1} (1 + g) \right) \\ - MC_{LT,t+1} \phi_{I_{LT}} (1 + g) \left( \tilde{I}_{LT,t+1} - \tilde{I}_{LT,t} (1 + g) \right) \end{aligned}$$

$$\begin{aligned} \Psi_{LTc} = MC_{LT,t+1} \phi_{I_{LT}} \left( \tilde{I}_{LT,t+1} - \tilde{I}_{LT,t} (1 + g) \right) \\ - \frac{1}{1 + i_{t+1}} MC_{LT,t+2} \phi_{I_{LT}} (1 + g) \left( \tilde{I}_{LT,t+2} - \tilde{I}_{LT,t+1} (1 + g) \right) \end{aligned}$$

The homogenous capital producer chooses the capital utilization rate that balances marginal revenue with marginal costs. The expected return on investments depends on the risk free interest rate, the depreciation rate, the

expected price of the homogenous capital good in next period and installation costs.

## 2.5 Households

Each country bloc in the model is inhabited by 3 types of representative infinitely lived households. The households supply labour of their given skills type (low, medium or high) to domestic firms, earn income from its holding of financial assets, and consume goods produced by the domestic and foreign final output producers. Labour supply in the model is endogenous with the household at the margin choosing between an extra unit of goods consumption and an extra hour of leisure.

We assume that the representative household supplying low skilled labour are liquidity constrained with no access to credit markets and no holding of financial assets.

### 2.5.1 Non liquidity constrained households

The representative households supplying medium and high skilled labour are assumed to have access to credit markets. These household types are assumed to hold shares in domestic firms, trade in risk free government bonds and hold net deposits at domestic financial intermediates. The representative households derive utility from consumption and leisure. We allow for habit persistence in consumption and leisure to capture a gradual hump-shaped response of real consumption and labour to policy changes.<sup>7</sup> The representative household's utility function is given by

$$U_s(h) = \sum_{t=s}^{\infty} \beta^{t-s} \frac{\left[ (C_t(h) - h_C C_{t-1}(h)) \left( H - \left( \hat{L}_t(h) - h_L \hat{L}_{t-1}(h) \right) \right)^{\eta_{c(h)}} \right]^{1-\phi_{c(h)}}}{1 - \phi_{c(h)}}$$

where  $\beta$  is the rate of time preference,  $C$  is the aggregate bundle of commodities consumed,  $\hat{L}$  is the hours of total labour supply, and  $H$  is the total hours available for leisure or work. The parameter  $\eta$  measure the impact of leisure on welfare of the representative household while the parameter  $\phi$  is the related

<sup>7</sup> Habit persistence are often used in medium scale macroeconomic models to capture the hump shaped response of consumption and labour observed in the data, in which the peak response occurs several periods after the policy implementation, see e.g. Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007) and Ratto et al. (2008).

to the intertemporal rate of substitution.<sup>8</sup> The parameters  $h_c$  and  $h_L$  governs the degree of habit persistence in consumption and leisure respectively.

The aggregated composite of household consumption goods is given by a nested CES function. The household consume domestic commodities  $C_D$  and imported commodities  $C_M$ . Imported commodities are sourced across all countries and brings about international transportation costs. The household's demand for domestic and imported commodities and for international transport services are

$$C_{D,t}(h, j) = \theta_{a(h,j)} \theta_{c(h,j)} \left[ \frac{P_{D,t}(h, j)}{P_{A,t}(h, j)} \right]^{-\sigma_{a(h,j)}} \left[ \frac{P_{A,t}(h, j)}{P_{C,t}} \right]^{-\sigma_{c(h)}} C_t(h)$$

$$\begin{aligned} C_{S,t}(h, j, s) &= \left(1 - z_{ST(h,j,s)}\right) \theta_{m(h,j,s)} \left(1 - \theta_{a(h,j)}\right) \theta_{c(h,j)} \\ &\times \left[ \frac{P_{ST,t}(h, j, s)}{P_{M,t}(h, j)} \right]^{-\sigma_{m(h,j)}} \left[ \frac{P_{M,t}(h, j)}{P_{A,t}(h, j)} \right]^{-\sigma_{a(h,j)}} \\ &\times \left[ \frac{P_{A,t}(h, j)}{P_{C,t}} \right]^{-\sigma_{c(h)}} C_t(h) \end{aligned}$$

$$\begin{aligned} C_{T,t}(h, j, s) &= z_{ST(h,j,s)} \theta_{m(h,j,s)} \left(1 - \theta_{a(h,j)}\right) \theta_{c(h,j)} \left[ \frac{P_{ST,t}(h, j, s)}{P_{M,t}(h, j)} \right]^{-\sigma_{m(h,j)}} \\ &\times \left[ \frac{P_{M,t}(h, j)}{P_{A,t}(h, j)} \right]^{-\sigma_{a(h,j)}} \left[ \frac{P_{A,t}(h, j)}{P_{C,t}} \right]^{-\sigma_{c(h)}} C_t(h) \end{aligned}$$

where  $C_D$  is demand for domestic commodity of type  $j$ ,  $C_S$  is demand for imported commodity of type  $j$  sourced from country  $s$ ,  $C_T$  is demand for international transport services and  $C$  is the bundle of commodities consumed by household of type  $h$ . The parameter  $\theta_c$  governs the share of commodity of type  $j$  in the consumption bundle. The price index for the household's composite consumer bundle is denote by  $P_C$ .

The household pays the consumption tax  $\tau_D$  on domestic commodities and the consumption tax  $\tau_S$  on imported commodities. Import tariffs  $\tau_M$  are levied on imported consumer goods, and the exported consumer goods receive an export subsidy  $\tau_X$ . International transportation services are also subject to taxation and import tariffs. Suppressing the home country index the price of domestic commodities, imported commodities and international transport services are give by

<sup>8</sup> To ensure to ensure that the utility function is concave in the quantities of consumption and leisure we require that  $\phi > 0$ ,  $\eta > 0$ ,  $(1 - \phi)(1 + \eta) < 1$ ,  $\eta(1 - \phi) < 1$ .

$$P_{D,t}(h, j) = (1 + \tau_{D,t}(h, j))\tilde{P}_{F,t}(j)$$

$$P_{S,t}(h, j, s) = (1 + \tau_{S,t}(h, j, s))(1 + \tau_{M,t}(h, j, s))(1 - \tau_{X,t}(j, s))S_t(s)\tilde{P}_{F,t}^s(j)$$

$$P_{T,t}(h, j, s) = (1 + \tau_{S,t}(h, j))(1 + \tau_{M,t}(h, j, s))S_t^{ROW}\tilde{P}_{T,t}(j)$$

The representative household optimizes its utility subject to the budget constraint

$$\begin{aligned} & (1 - \tau_{L,t}(h))W_t(h)\widehat{L}_t(h) \\ & + (1 - \tau_{K,t}(h))(\widehat{\pi}_{HT,t}(h) + \widehat{\pi}_{ICT,t}(h) + \widehat{\pi}_{LT,t}(h) + \widehat{\pi}_{FI,t}(h)) \\ & + \widehat{D}_t(h)(1 + i_{D,t}) + B_{G,t}(h)(1 + i_t) + T_{H,t}(h) \\ = & P_{C,t}(h)C_t(h) + D_{t+1}(h) + B_{G,t+1}(h) \end{aligned}$$

where  $W$  is the wage for one hour of labour of skill  $h$ ,  $\tau_L$  is the income tax rate,  $\widehat{D}$  is the net holding of deposits at the financial intermediaries,  $B_G$  is the net holding of risk-free bonds issued by the national government in the country in which the household resides,  $\widehat{\pi}_{HT}$  and  $\widehat{\pi}_{ICT}$  is dividends from the monopolist capital producers in the high-tech and the ICT sector respectively,  $\widehat{\pi}_{LT}$  is dividends from the low-tech capital producers, and  $\pi_{FI}$  is dividends from the financial intermediates. The household also receives a lump sum transfer,  $T_H$ , from the national government in the country in which it resides.

Intertemporal optimization gives the first order conditions

$$U_{C,t}(h) = \beta(1 + i_{D,t+1})\frac{P_{C,t}(h)}{P_{C,t+1}(h)}U_{C,t+1}(h)$$

$$U_{C,t}(h) = \beta(1 + i_{t+1})\frac{P_{C,t}(h)}{P_{C,t+1}(h)}U_{C,t+1}(h)$$

$$(1 - \tau_{L,t}(h))\frac{W_t(h)}{P_{C,t}(h)}U_{C,t}(h) = U_{L,t}(h)$$

where

$$\begin{aligned} U_{C,t}(h) = & (C_t(h) - h_C C_{t-1}(h))^{-\phi_c(h)} \left( H - \left( \widehat{L}_t(h) - h_L \widehat{L}_{t-1}(h) \right) \right)^{(1-\phi_c(h))\eta_c(h)} \\ & - \beta h_C (C_{t+1}(h) - h_C C_t(h))^{-\phi_c(h)} \left( H - \left( \widehat{L}_{t+1}(h) - h_L \widehat{L}_t(h) \right) \right)^{(1-\phi_c(h))\eta_c(h)} \end{aligned}$$

and

$$\begin{aligned}
U_{L,t}(h) = & \\
& \eta_{c(h)} (C_t(h) - h_C C_{t-1}(h))^{1-\phi_{c(h)}} \left( H - \left( \widehat{L}_t(h) - h_L \widehat{L}_{t-1}(h) \right) \right)^{(1-\phi_{c(h)})\eta_{c(h)}-1} \\
& - \eta_{c(h)} \beta h_L (C_{t+1}(h) - h_C C_t(h))^{1-\phi_{c(h)}} \left( H - \left( \widehat{L}_{t+1}(h) - h_L \widehat{L}_t(h) \right) \right)^{(1-\phi_{c(h)})\eta_{c(h)}-1}
\end{aligned}$$

The household balances the marginal utility of current consumption with marginal utility of future consumption. Furthermore, the household supply labour such that the marginal utility of consumption from an additional hour of labour equals the marginal utility from an additional hour of leisure.

### 2.5.2 Liquidity constrained households

In each country bloc a representative household supply low skilled labour to firms producing final goods and R&D. We assume that the household is liquidity constrained with no holding of financial assets. Hence, the household consume all its income each period. Thus the consumption is determined by the budget constraint

$$(1 - \tau_{L,t}(h)) W_t(h) \widehat{L}_t(h) + T_{H,t}(h) = P_{c,t}(h) C_t(h)$$

where  $W$  is the wage for one hour of low skilled labour and  $\tau_L$  is the income tax rate. The household also receives a lump sum transfer,  $T_H$ , from the national government in the country in which it resides.

We assume that the liquidity constrained household possesses an identical utility function to that of the non-liquidity constraint households. The composite of household consumption  $C$  is also defined in a similar fashion to that of the non-liquidity constrained households. Labour supply by the liquidity constraint household, thus, follows a first order condition similar to that of the non-liquidity constraint households.

### 2.6 National governments

The national government in each country collects taxes, pays out subsidies and household transfers and purchases commodities for government consumption which is provided free of charge to the representative households residing in the country. The national governments in the countries which are members of the EU also pay contributions to the EU and receive transfers from the EU. Government consumption is a CES composite of domestic and foreign final goods defined in a similar fashion to that of the private households.



The government budget constraint is given by

$$TR_t - \widehat{B}_{G,t}(1 + i_t) = P_{G,t}G_t + \widehat{T}_{H,t} + T_{EU} - \widehat{B}_{G,t+1}$$

where  $\widehat{B}_G$  is national government issued bonds which is held by domestic households,  $\widehat{T}_H$  is the net transfers to domestic households,  $T_{EU}$  is net transfers to the EU and  $TR$  is the net tax revenue given by

$$TR_t = TR_{L,t} + TR_{K,t} + TR_{O,t} + TR_{C,t} + TR_{G,t} + TR_{I,t} \\ + TR_{F,t} + TR_{R,t} + TR_{FL,t} + TR_{FK,t} + TR_{M,t} - TR_{X,t}$$

The net tax revenue consist of revenue from taxation on the production factors, gross output taxation, consumption taxation, taxes on investment demand, intermediate input taxation, and revenue from import tariffs less export subsidies.

Income taxes are levied on each of the 3 skills types of labour

$$TR_{L,t} = \sum_{h=1}^{nh} \tau_{L,t}(h) W_t \widehat{L}_t(h)$$

Capital income tax is levied on households' dividends income from holding of shares

$$TR_{K,t} = \sum_{h=1}^{nh} \tau_{K,t}(h) (\widehat{\pi}_{HT,t}(h) + \widehat{\pi}_{ICT,t}(h) + \widehat{\pi}_{LT,t}(h) + \widehat{\pi}_{FI,t}(h))$$

An output tax is levied on final outputs and on R&D.

$$TR_{O,t} = \sum_{i=1}^{ni} \tau_{O,t}(i) \widetilde{P}_{F,t}(i) Y_{F,t}(i) + \sum_{r=1}^{nr} \tau_{O,t}(r) P_{XD,t}(r) XD_t(r)$$

Consumption by the household and by the national government is subject to consumption taxation. The revenue is given by

$$TR_{C,t} = \sum_{h=1}^{nh} \sum_{j=1}^{nj} \tau_{D,t}(h, j) \widetilde{P}_{F,t}(h, j) C_{D,t}(h, j) \\ + \sum_{h=1}^{nh} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \left( \tau_{S,t}(h, j, s) (1 + \tau_{M,t}(h, j, s)) (1 - \tau_{X,t}(j, s)) \right) \\ \times S_t(s) \widetilde{P}_{F,t}^s(j) C_{S,t}(h, j, s) \\ + \sum_{h=1}^{nh} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \tau_{S,t}(h, j, s) (1 + \tau_{M,t}(h, j, s)) S_t^{ROW} \widetilde{P}_{T,t}(j) C_{T,t}(h, j, s)$$

and

$$\begin{aligned}
TR_{G,t} &= \sum_{j=1}^{nj} \tau_{D,t}(g, j) \tilde{P}_{F,t}(g, j) G_{D,t}(j) \\
&\quad + \sum_{j=1}^{nj} \sum_{s=1}^{ns} \left( \tau_{S,t}(g, j, s) (1 + \tau_{M,t}(g, j, s)) (1 - \tau_{X,t}(g, s)) \right) \\
&\quad \quad \quad \times S_t(s) \tilde{P}_{F,t}^s(j) G_{S,t}(j, s) \\
&\quad + \sum_{j=1}^{nj} \sum_{s=1}^{ns} \tau_{S,t}(g, j, s) (1 + \tau_{M,t}(g, j, s)) S_t^{ROW} \tilde{P}_{T,t}^s(j) G_{S,t}(j, s)
\end{aligned}$$

Intermediate inputs used in the production of differentiated and homogenous capital is also subject to taxation. Total revenue from taxes on investment is

$$\begin{aligned}
TR_{I,t} &= \sum_{u=1}^{nu} \sum_{j=1}^{nj} \tau_{D,t}(u, j) \tilde{P}_{F,t}(u, j) Y_{D,t}(u, j) \\
&\quad + \sum_{u=1}^{nu} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \left( \tau_{S,t}(u, j, s) (1 + \tau_{M,t}(u, j, s)) (1 - \tau_{X,t}(j, s)) \right) \\
&\quad \quad \quad \times S_t(s) \tilde{P}_{F,t}^s(j) Y_{S,t}(u, j, s) \\
&\quad + \sum_{u=1}^{nu} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \tau_{S,t}(u, j, s) (1 + \tau_{M,t}(u, j, s)) S_t^{ROW} \tilde{P}_{T,t}^s(j) Y_{T,t}(u, j, s) \\
&\quad + A_{ICT,t} \sum_{j=1}^{nj} \tau_{D,t}(a, j) \tilde{P}_{F,t}(a, j) Y_{D,t}(a, j) \\
&\quad + A_{ICT,t} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \left( \tau_{S,t}(a, j, s) (1 + \tau_{M,t}(a, j, s)) (1 - \tau_{X,t}(j, s)) \right) \\
&\quad \quad \quad \times S_t(s) \tilde{P}_{F,t}^s(j) Y_{S,t}(a, j, s) \\
&\quad + A_{ICT,t} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \tau_{S,t}(a, j, s) (1 + \tau_{M,t}(a, j, s)) S_t^{ROW} \tilde{P}_{T,t}^s(j) Y_{T,t}(a, j, s) \\
&\quad + A_{HT,t} \sum_{j=1}^{nj} \tau_{D,t}(b, j) \tilde{P}_{F,t}(b, j) Y_{D,t}(b, j) \\
&\quad + A_{HT,t} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \left( \tau_{S,t}(b, j, s) (1 + \tau_{M,t}(b, j, s)) (1 - \tau_{X,t}(j, s)) \right) \\
&\quad \quad \quad \times S_t(s) \tilde{P}_{F,t}^s(j) Y_{S,t}(b, j, s) \\
&\quad + A_{HT,t} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \tau_{S,t}(b, j, s) (1 + \tau_{M,t}(b, j, s)) S_t^{ROW} \tilde{P}_{T,t}^s(j) Y_{T,t}(b, j, s)
\end{aligned}$$

Final output producers pay tax on intermediate inputs, which gives the total revenue.

$$\begin{aligned}
TR_{F,t} &= \sum_{i=1}^{ni} \sum_{j=1}^{nj} \tau_{d,t}(i, j) \tilde{P}_{F,t}(i, j) Y_{D,t}(i, j) \\
&\quad + \sum_{i=1}^{ni} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \left( \tau_{s,t}(i, j, s) (1 + \tau_{m,t}(i, j, s)) (1 - \tau_{x,t}(j, s)) \right) \\
&\quad \quad \quad \times S_t(s) \tilde{P}_{F,t}^s(j) Y_{S,t}(i, j, s) \\
&\quad + \sum_{i=1}^{ni} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \tau_{S,t}(i, j, s) (1 + \tau_{M,t}(i, j, s)) S_t^{ROW} \tilde{P}_{T,t}(j) Y_{T,t}(i, j, s)
\end{aligned}$$

R&D producers also pays taxes on intermediate inputs.

$$\begin{aligned}
TR_{R,t} &= \sum_{r=1}^{nr} \sum_{j=1}^{nj} \tau_{D,t}(r, j) \tilde{P}_{F,t}(r, j) Y_{D,t}(r, j) \\
&\quad + \sum_{r=1}^{nr} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \left( \tau_{S,t}(r, j, s) (1 + \tau_{M,t}(r, j, s)) (1 - \tau_{X,t}(j, s)) \right) \\
&\quad \quad \quad \times S_t(s) \tilde{P}_{F,t}^s(j) Y_{S,t}(r, j, s) \\
&\quad + \sum_{r=1}^{nr} \sum_{j=1}^{nj} \sum_{s=1}^{ns} \tau_{S,t}(r, j, s) (1 + \tau_{M,t}(r, j, s)) S_t^{ROW} \tilde{P}_{T,t}(j) Y_{T,t}(r, j, s)
\end{aligned}$$

Firms pays production taxes on factor inputs. The revenue from production taxes on labour is given by

$$\begin{aligned}
TR_{FL,t} &= \sum_{i=1}^{ni} \sum_{h=1}^{nh} (1 + \tau_{FL,t}(i, h)) W(h) \\
&\quad + \sum_{r=1}^{nr} \sum_{h=1}^{nh} (1 + \tau_{FL,t}(r, h)) W(h)
\end{aligned}$$

The revenue from production taxes on capital is given by

$$\begin{aligned}
TR_{FK,t} &= \sum_{i=1}^{ni} (1 + \tau_{fk_{LT},t}(i)) \tilde{R}_{LT} \\
&\quad + \sum_{i=1}^{ni} (1 + \tau_{FK_{ICT},t}(i)) \tilde{R}_{ICT} \\
&\quad + \sum_{i=1}^{ni} (1 + \tau_{FK_{HT},t}(i)) \tilde{R}_{HT} \\
&\quad + \sum_{r=1}^{nr} (1 + \tau_{FK_{LT},t}(r)) \tilde{R}_{LT}
\end{aligned}$$

All imported intermediate inputs and commodities are subject to import tar-

iffs. The revenue from import tariffs are

$$TR_{M,t} = \sum_{j=1}^{nj} \sum_{s=1}^{ns} \tau_{M,t}(j, s) (1 - \tau_{X,t}^s(j)) S_t(s) \tilde{P}_{F,t}^s(j, s) M_t(j, s)$$

Final output producers receive export subsidies from the national government

$$TR_{X,t} = \sum_{j=1}^{nj} \sum_{s=1}^{ns} \tau_{X,t}(j, s) \tilde{P}_{F,t}(j) X_t(j, s)$$

Government consumption is assumed to be a fixed proportion of GDP. To rule out explosive levels of government debt we assume that the national government adjust the net transfers to domestic households in response to changes in public debt to GDP ratio.

## 2.7 Financial intermediary

Each country bloc in the model has a financial sector with perfectly competitive financial intermediaries. The financial intermediary receives net deposit from the non-liquidity constrained households and places these in international traded bonds. We assume that the financial intermediary operates at no cost (an assumption that may be relaxed later to allow for financial services funded by an interest rate margin). In a given period the financial intermediary receives net deposit and convert these into internationally traded bonds. Each period the financial intermediary, thus, faces the funding restriction

$$\sum_{h=1}^{nh} D_{t+1}(f, h) = B_{EU,t+1}(f) + S_t^{ROW} B_{ROW,t+1}(f)$$

where  $D$  is the net deposit in financial intermediary  $f$  by household of type  $h$ ,  $B_{EU}$  and  $B_{ROW}$  are net holdings of EU and ROW currency basket denominated bonds respectively, while  $S_t^{ROW}$  is the spot market exchange rate. Note that a positive net deposit by domestic households means that the domestic financial intermediaries holds internationally-traded bonds issued by financial intermediaries in other countries, while a negative net deposit by domestic households means that the domestic financial intermediaries issues internationally-traded bonds held by financial intermediaries in other countries. The financial intermediary takes the deposit rate as given and chooses its bond holdings such that it balances next periods return on household net deposits with the expected return on international bond holding. Hence, each period the financial intermediaries balances

$$\sum_{h=1}^{nh} D_{t+1}(f, h)(1 + i_{D,t+1}) = B_{EU,t+1}(f)(1 + i_{EU,t+1}) \\ + S_{t+1}^{e,ROW} B_{ROW,t+1}(f)(1 + i_{ROW,t+1})$$

where  $i_{EU}$  is the interest rate on bonds denominated in the country's domestic currency and  $i_{ROW}$  is the interest rate on bonds denominated in the ROW currency basket and  $S_{t+1}^{e,ROW}$  is the expected exchange rate for next period. The first order conditions for EU and ROW currency denominated bond holdings is given by

$$(1 + i_{D,t+1}) = (1 + i_{EU,t+1}) \\ (1 + i_{D,t+1}) = \frac{S_{t+1}^{e,ROW}}{S_t^{ROW}}(1 + i_{ROW,t+1})$$

Combining these first order conditions for internationally-traded bonds shows that the uncovered interest parity holds

$$(1 + i_{EU,t+1}) = \frac{S_{t+1}^{e,ROW}}{S_t^{ROW}}(1 + i_{ROW,t+1})$$

## 2.8 The EU

The EU receives net contributions from its member states and revenue from import tariffs. Import tariffs  $\tau_M$  are levied on imported commodities and inputs that are being sourced from non-EU member countries (ROW). Furthermore the EU pays subsidies to firms in the member states. For simplicity we assume that all transfers between EU and agents in the EU member states are carried out with the national governments as mediators. Hence, import tariff are collected by national governments and paid as transfers from the national government to the EU. Likewise subsidies from the EU to firms in a given member state are paid as a transfer to the national government which then in turn pays it to the firm. The net contribution to the EU from member country  $c$  is given by

$$T_{EU,t}(c) = TR_{MS,t}(c) + a_{EU,t}(c)GNI_t(c) - T_{MS,t}(c)$$

where  $TR_{MS}$  is the revenue from tariffs on extra EU imports, the coefficient  $a_{EU}$  determine the country's GNI contribution, and  $T_{MS}$  is a lump sum transfer from the EU to the member country.<sup>9</sup>

Assuming that the EU balances its budget in every period by proportional adjustments of the countries' GNI contributions the EU budget constraint is

<sup>9</sup> The lump sum transfer covers payments related to the Common Agricultural Policy (CAP), structural funds etc.

given by

$$\sum_{c=1}^{nc} T_{EU,t}(c) = 0$$

Note that while the EU budget is assumed to balance in each period, we do not assume that each of the individual member states net contribution to the EU are balanced. Some member states may receive net benefits while others pay net contributions, if this is the case the EU redistributes public funds across member states.

### 2.9 International transport services

A perfectly competitive international transport sector produces transportation services of international traded goods. The transport service is sold to importers of goods across all sectors and countries. The international transport service is produced from intermediate inputs sourced from the service sectors in all countries. The production of international transport services follows a CES production function. The demand for service  $j$  in country  $s$  is given by

$$Y_{TR,t}(j, s) = \theta_{T(j,s)} \left[ \frac{\tilde{P}_{F,t}^S(j, s)/S_t^{ROW}}{P_{TR,t}} \right]^{-\sigma_T} \hat{Y}_{TR,t}$$

where  $Y_{TR}$  is the aggregate supply of international transport services,  $\theta_T$  is a share parameter governing the proportion of inputs sourced from country  $s$ , and  $P_{TR}$  is the corresponding price index

$$P_{TR,t} = \left( \theta_{T(j,s)} \left( \tilde{P}_{F,t}^S(j)/S_t^{ROW} \right)^{1-\sigma_T} \right)^{\frac{1}{1-\sigma_T}}$$

### 2.10 Foreign trade

All final outputs can be traded internationally. Final outputs are sources internationally by firms, households and national governments. Imported final outputs are considered to be imperfect substitutes for domestically produced final outputs. The export price  $P_X$  of commodity  $j$  to country  $c$  including export subsidies can be defined as

$$P_{X,t}(j, c) = (1 - \tau_{X,t}(j, c)) \tilde{P}_{F,t}(j)$$

Suppressing the domestic country index the value of the country's export of good  $j$  to country  $c$  is then given by

$$\begin{aligned}
\sum_{j=1}^{nj} VX_t(j, c) = & \sum_{j=1}^{nj} \left( P_{X,t}(j, c) \sum_{i=1}^{ni} Y_{S,t}(i, j, c) \right) \\
& + \sum_{j=1}^{nj} \left( P_{X,t}(j, c) \sum_{r=1}^{nr} Y_{S,t}(r, j, c) \right) \\
& + \sum_{j=1}^{nj} P_{X,t}(j, c) (A_{ICT,t} Y_{S,t}(a, j, c) + A_{HT,t} Y_{S,t}(e, j, c)) \\
& + \sum_{j=1}^{nj} \left( P_{X,t}(j, c) \sum_{u=1}^{nu} Y_{S,t}(u, j, c) \right) \\
& + \sum_{j=1}^{nj} \left( P_{X,t}(h, c) \sum_{h=1}^{nh} C_{S,t}(h, j, c) \right) \\
& + \sum_{j=1}^{nj} P_{X,t}(j, c) G_{S,t}(j, c)
\end{aligned}$$

The export of services to the international transport sector is given by

$$\sum_{j=1}^{nj} VX_{T,t}(j, c) = \sum_{j=1}^{nj} (P_{X,t}(j, c) Y_{TR,t}(j, c))$$

The value of the country's import of goods and services of type  $j$  from country  $c$  (including transportation services) can be expressed<sup>10</sup> as

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<sup>10</sup> Note that import and export in the definitions used here are both valued at the exporter's customs frontier, referred to as free on board (fob) values. Hence, the value of export and import includes export subsidies but not import tariffs. For further details see for example Eurostat (2001).

$$\begin{aligned}
& \sum_{j=1}^{nj} VM_t(j, c) = \\
& \sum_{j=1}^{nj} \left( S_t(c)P_{X,t}(j, c) \sum_{i=1}^{ni} Y_{S,t}(i, j, c) + S_t^{ROW} \tilde{P}_{T,t}(j) \sum_{i=1}^{ni} Y_{T,t}(i, j) \right) \\
& + \sum_{j=1}^{nj} \left( S_t(c)P_{X,t}(j, c) \sum_{r=1}^{nr} Y_{S,t}(r, j, c) + S_t^{ROW} \tilde{P}_{T,t}(j) \sum_{r=1}^{nr} Y_{T,t}(r, j, c) \right) \\
& + \sum_{j=1}^{nj} S_t(c)P_{X,t}(j, c) \left( A_{ICT,t} Y_{S,t}(a, j, c) + S_t^{ROW} \tilde{P}_{T,t}(j) A_{ICT,t} Y_{T,t}(a, j, c) \right) \\
& \sum_{j=1}^{nj} S_t(c)P_{X,t}(j, c) \left( A_{HT,t} Y_{S,t}(e, j, c) + S_t^{ROW} \tilde{P}_{T,t}(j) A_{HT,t} Y_{T,t}(e, j, c) \right) \\
& + \sum_{j=1}^{nj} \left( S_t(c)P_{X,t}(j, c) \sum_{l=1}^{nl} Y_{S,t}(l, j, c) + S_t^{ROW} \tilde{P}_{T,t}(j) \sum_{l=1}^{nl} Y_{T,t}(l, j, c) \right) \\
& + \sum_{j=1}^{nj} \left( S_t(c)P_{X,t}(j, c) \sum_{h=1}^{nh} C_{S,t}(h, j, c) + S_t^{ROW} \tilde{P}_{T,t}(j) \sum_{h=1}^{nh} C_{T,t}(h, j, c) \right) \\
& + \sum_{j=1}^{nj} \left( S_t(c)P_{X,t}(j, c) G_{S,t}(j, c) + S_t^{ROW} \tilde{P}_{T,t}(j) G_{T,t}(j, c) \right)
\end{aligned}$$

The country's trade balance is given by

$$TB_t = \sum_{j=1}^{nj} \sum_{c=1}^{nc} (VX_t(j, c) + VX_{T,t}(j) - VM_t(j, c))$$

We allow for international capital mobility. Domestic financial intermediaries can lend or borrow through internationally traded risk free bonds. The country's current account is given by

$$CA_t = TB_t - T_{EU,t} + i_{EU,t} \hat{B}_{EU,t} + S_t^{ROW} i_{ROW,t} \hat{B}_{ROW,t}$$

where  $\hat{B}_{EU}$  and  $\hat{B}_{ROW}$  is the country's aggregated holdings of EU and ROW currency denominated internationally traded bonds. This is given as the sum of bond holdings by all domestic financial intermediates

$$\hat{B}_{EU,t} = \sum_{f=1}^{nf} B_{EU,t}(f)$$

The country's accumulated holding of international traded bonds is given by

$$\hat{B}_{EU,t+1} + S_t^{ROW} \hat{B}_{ROW,t+1} = CA_t + \hat{B}_{EU,t} + S_t^{ROW} \hat{B}_{ROW,t}$$



### 2.11 Market clearing

All markets clear in each time period. This requires that (1) demand for each production factor in each country equals its supply, (2) demand for the output from each final output sector in each country equals its supply, (3) The output by R&D producers in each country equals the number of new capital varieties invested, (4) total net holding of internationally traded bonds across all countries equals zero, and (5) total household savings across all countries equals total investment.

## 3 Data and calibration

The model is calibrated to replicate a given initial base year and to generate a specified reference growth path.

### 3.1 Data and parameters

The model is calibrated to a dataset based on the GTAP 8 database. This database covers 129 countries and contains data on value added, material inputs, factor inputs, private consumption, public consumption, investments and international trade for the base year 2007. The GTAP database is modified to a model consistent dataset using weights calculated from the Predict database, national account data and other supplementary datasets.

The 57 sectors in the GTAP 8 database are aggregated to form the model's 5 final output production sectors. The ICT sector is defined here according to definition guidelines from the OECD.<sup>11</sup> The ICT sector, in some cases, represents fractions of the existing GTAP 8 sectors. These GTAP 8 sectors are split into ICT and the non-ICT parts by use of weights calculated from national account data. The aggregation of the GTAP manufacturing sectors into high-tech manufacturing and low-tech manufacturing sectors is done in accordance with guidelines from Eurostat that groups manufacturing sectors by their technological intensity. The remaining final output sectors services and agriculture, food and beverage follows the standard GTAP 8 classification.

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<sup>11</sup> The ICT sector include the following NACE 1.1 sectors:

Manufacturing; Manufacture of office, accounting and computing machinery (30) , Manufacture of radio, television and communication equipment and apparatus (32), Manufacture of medical, precision and optical instruments, watches and clocks (33). Services; Post and telecommunications (64), Computer and related activities (72).

R&D expenditures are calculated for the 5 final output sectors using data from the PREDICT database. The PREDICT database contains data on private sector and public sector R&D expenditures. The R&D expenditure by each sector is subtracted from the final commodity output and allocated to R&D output. R&D expenditures in the ICT sector are allocated to ICT R&D output, the remaining R&D expenditures are allocated to non-ICT R&D output.

Investments in physical capital are divided into the model's three capital types - homogeneous capital, ICT capital and other high-tech capital - using the national accounting classification of gross fixed capital formation by asset types.

The GTAP 8 database splits payments to labour into payments to skilled and unskilled labour. The GTAP-defined payments to skilled labour are taken as payments to high-skilled labour in the model. Payments to unskilled labour are split into payments to low and medium-skilled households using weights taken from the Socio-economic Accounts of the World Input-Output Database (Timmer (2012)). The value of private consumption is split between the three household types using weights based on the skills group's relative income in the base year.

Tax rates on labour and capital income are taken from the OECD tax database. For the tax rate on labour income for low skilled households, we use the average tax wedge for a single person at 67% of average earning with no children. The tax rates for medium and high-skilled households are set as the average tax wedge for a single person at 100% of average earning with no children. The tax rate on dividend income is set to equal the net personal income tax rate on dividend income. The aggregate net lump sum transfer from the national government to households is set in such a way that the government primary budget surplus is consistent with the dataset.

Households supplying low-skilled labour are assumed to be liquidity constrained with no holding of financial assets. The net income from national government lump sum transfers for this household type therefore equals the after-tax labour income, less the value of consumption. The remaining net income from national government lump sum transfers is allocated to households, supplying respectively medium-skilled and high-skilled labour according to their after-tax income from labour and capital.

We assume that the model in the base year is in steady state with a balanced current account. Hence, bilateral trade flows, bond holdings and lump sum transfers in the model are adjusted accordingly.

A number of model parameters are specified exogenously. The elasticities of substitution in the model are set to reflect estimates in the literature (see e.g. Van der Werf (2008), Okagawa and Ban (2008), and Koesler and Schymura (2012)). The elasticity of substitution between intermediate inputs and value

added in the upper nest of the final output production sectors  $\sigma_F$  is based on estimates from Koesler and Schymura (2012), who estimate substitution elasticities using data covering 35 sectors in 40 countries (27 EU countries and 13 other major countries) over a period of 12 years. The substitution elasticities for each of the model's 5 final output producers in each of the model's 4 country blocs are calculated as a weighted average of the estimated elasticities for the relevant sectors. The resulting elasticities are shown in table 1. We have less empirical foundation for the substitution between the various capital-labour composites in the production function. The substitution elasticity between the ICT composite and the KL composite  $\sigma_{VA(i)}$  is set at 1.5. The substitution elasticity between the high-tech composite and the low-tech composite  $\sigma_{KL}$  is also set at 1.5. We assume that the elasticity of substitution between differentiated capital and high-skilled labour,  $\sigma_{ICT}$  and  $\sigma_{HT}$ , is relatively low at 0.15 to reflect the assumption that highly specialised technology requires some inputs from a highly-skilled workforce. The elasticity of substitution between the low-medium labour composite and homogenous capital  $\sigma_{LT}$  is calculated from estimates in Koesler and Schymura (2012). These are shown in table 1. The elasticity of substitution between low-skilled labour and medium-skilled labour  $\sigma_{LM}$  is set at 0.95. The substitution between the different capital varieties,  $\sigma_{HTV}$  and  $\sigma_{ICTV}$ , is not set exogenously but is calibrated to mirror the markup on capital rental in the model given data on investments, capital stocks and return on capital.

**Table 1. Weighted substitution elasticities**

<i>Final output production</i>	$\sigma_F$	$\sigma_{LT}$
Agriculture, food and beverage	0.655	0.286
Low-tech manufacturing	0.258	0.292
High-tech manufacturing	0.773	0.451
Services	0.885	0.367
ICT	0.897	0.433
<i>R&amp;D production</i>	$\sigma_R$	$\sigma_{LT}$
R&D ICT	0.773	0.433
R&D non-ICT	0.897	0.451

*Note: Elasticities are calculated from estimates in Koesler and Schymura (2012)*

The substitution elasticities in R&D production between intermediate inputs and value added  $\sigma_R$  are calculated from Koesler and Schymura (2012), see table 1. We assume that the elasticity of substitution between high skilled labour and the low-tech composite  $\sigma_{VA(r)}$  is set at 1.5. The remaining elas-

elasticities of substitution for R&D producers are set identical to those for final good producers. The substitution elasticities for capital producers are also set identical to those for final good producers.

The elasticity of substitution between commodities in the consumption bundle  $\sigma_C$  is set equal to 1.2. The substitution elasticities for foreign trade is assumed to be relatively high in the model. The elasticity of substitution between domestic and imported commodities  $\sigma_A$  is assumed equal to 4 in production, investment and consumption. The substitution elasticity between imported commodities from the various sourcing countries  $\sigma_M$  are similarly set equal to 4. The only exception being the production of international transport services, where the substitution elasticity between imported services from the different sourcing destinations  $\sigma_T$  is assumed to be 1.25.

The inter-temporal elasticity of substitution in household preferences  $\phi_c$  is set at 1.5. The parameter governing household preferences for leisure  $\eta_c$  is also set at 1.5. The parameters governing habit persistence in consumption and labour,  $h_C$  and  $h_L$ , are set to 0.2 and 0.4 respectively. The capital utilization rate for differentiated capital and homogenous capital are assumed to be 0.8. Cost of adjusting capital utilization for all capital types is set as follows. The parameter  $\varphi_{u2}$  is set to 2 for all capital types, while the parameter  $\varphi_{u1}$  is given by the first order condition for capital utilization in the model. The parameters governing cost of adjustment to capital and investment is set to 2 for all capital types. The parameter governing the cost of adjusting R&D output is similarly set to 2.

The remaining parameters in the model are calibrated such that the model replicates the dataset for the reference year and the specified reference growth path.

### *3.2 The reference growth path*

The model is calibrated to generate a reference growth path. We assume a reference growth path where production, GDP, consumption and investments across all countries growth at the model's reference growth rate. The reference growth rate is set to 1.64 pct. p.a. This corresponds to the annual average real growth rate for the 28 EU Member States for the period 1995-2013. It is possible to specify an alternative reference growth path in which specified reference growth rates varies over time and across countries. The annual inflation rate is set to 1.81 pct. p.a. which corresponds to the annual inflation in the implicit GDP deflator for the 28 EU Member States for the period 1995-2013. In its main scenario Eurostat projects the annual growth in the population aged 15-64 in the 28 EU member states from 2015 to 2050 to -0.31 pct. p.a. We set

the annual growth in the labour force accordingly.

The annual nominal interest rate for a risk free bond is set to equal 4 pct. The households' discount rate  $\beta$  is set such that the model generates its long run growth rate.

In the model economic growth is driven partly endogenously through R&D and partly by exogenously specified growth components. The R&D-driven endogenous growth in a country is generated from domestic knowledge accumulation and from cross-border spillover of foreign knowledge. The growth from domestic knowledge accumulation is governed by R&D productivity elasticities with respect to the domestic R&D knowledge stock  $\sigma_{RDd(r,\bar{r})}$ . These are set at levels comparable to estimates of TFP elasticities with respect to domestic R&D stocks reported in Coe et al. (2008). The growth due to cross border spillover of knowledge is governed by the R&D productivity elasticities with respect to the foreign R&D knowledge stocks  $\sigma_{RDf(r,\bar{r},s)}$ . These elasticity parameters are also set to levels comparable with estimates of TFP elasticities with respect to foreign R&D stocks reported in Coe et al. (2008). Given parameters for cross-border spillover, the initial stocks of knowledge are calibrated such that the growth generated from R&D in each country equal the country's reference growth rates. The exogenous growth components is then set such that the model generates its reference growth path. This is done to ensure that the model converge to a balanced growth path in which all endogenous variables in all countries growth at the same long term growth rate and the transversality conditions for bond holding are satisfied.

#### 4 Concluding remarks

This report describes the first version of a CGE model with ICT and R&D-driven endogenous growth. The model is specified in such a way that it captures the multiple channels through which R&D activity in the ICT sector affect the economy. The model is therefore well suited to the analysis of R&D production policies targeted at the ICT sector.

The merits of the model will be explored in future work through a series of policy scenarios covering different amounts of spending, policy instruments and sources of funding. The sensitivity of results to the setting of central model parameters will also be analysed. The model will in subsequent work be extended to include a detailed description of EU Member States and to accommodate a broader spectrum of policy scenarios.

## A Appendix

### A.1 The differentiated capital producer's price setting

To simplify the notation we abstract for variable rates of capital utilization and adjustment costs. The differentiated ICT capital producer set at time  $t$  the rental price of capital  $R_{ICTV,t+1}$  that maximizes expected profit

$$\pi_t(a) = \sum_{j=0}^{\infty} \left( \left( \frac{1}{1+i_t} \right)^j \left( \begin{array}{c} R_{ICTV,t}(a) \widehat{K}_{ICTV,t+j}(a) - \\ MC_{ICT,t+j}(a) I_{ICTV,t+j}(a) - P_{XD,t0}(a, r) \end{array} \right) \right)$$

subject to capital accumulation

$$\widehat{K}_{ICTV,t+1}(a) = I_{ICTV,t}(a) - \widehat{K}_{ICTV,t}(a)(1 - \delta_{ICT})$$

and demand for its ICT capital variety

$$\widehat{K}_{ICTV,t}(a) = \left[ \frac{R_{ICTV,t}(a)}{R_{ICT,t}} \right]^{-\sigma_{ICTV}} K_{ICT,t}$$

where  $P_{XD,t0}$  is the price of the patent purchased from the R&D sector prior to capital production. Substitute for  $I_{ICTV,t}$  using the capital accumulation equation and for  $R_{ICTV,t+1}$  using the inverse demand function.

$$R_{ICTV,t}(a) = \left[ \frac{K_{ICT,t}}{\widehat{K}_{ICTV,t}(a)} \right]^{1/\sigma_{ICTV}} R_{ICT,t}$$

gives

$$\pi_t(a) = \sum_{j=0}^{\infty} \left( \left( \frac{1}{1+i_t} \right)^j \left( \begin{array}{c} \left[ \frac{K_{ICT,t}}{\widehat{K}_{ICTV,t}(a)} \right]^{1/\sigma_{ICTV}} R_{ICT,t} \widehat{K}_{ICTV,t+j}(a) \\ - MC_{ICT,t+j}(a) \\ \times \left( \widehat{K}_{ICTV,t+1}(a) - \widehat{K}_{ICTV,t}(a)(1 - \delta_{ICT}) \right) \\ - P_{XD,t0}(a, r) \end{array} \right) \right)$$

Differentiating with respect to  $\widehat{K}_{ICTV,t+1}$  gives the first order condition

$$\begin{aligned} & -MC_{ICT,t}(a) \\ & + (1 - 1/\sigma_{ICTV}) \frac{1}{1+i_t} \widehat{K}_{ICTV,t+1}(a)^{-1/\sigma_{ICT}} K_{ICT,t+1}^{1/\sigma_{ICTV}} R_{ICT,t+1} \\ & + \frac{1}{1+i_t} MC_{ICT,t+1}(a)(1 - \delta_{ICT}) \\ & = 0 \end{aligned}$$

Substitute

$$\begin{aligned} & -MC_{ICT,t}(a) + (1 - 1/\sigma_{ICTV})\frac{1}{1+i_t}R_{ICTV,t+1}(a) \\ & + \frac{1}{1+i_t}MC_{ICT,t+1}(a)(1 - \delta_{ICTV}) \\ & = 0 \end{aligned}$$

Rearrange

$$R_{ICTV,t+1}(a) = \frac{\sigma_{ICTV}}{\sigma_{ICTV} - 1} ((1 + i_t) MC_{ICT,t}(a) - MC_{ICT,t+1}(a)(1 - \delta_{ICT}))$$

## Acknowledgements

The author acknowledge helpful comments and suggestions from Peter Stephensen (DREAM), Andries Brandsma, Wojciech Szewczyk, Marc Bogdanowicz, Andrea de Panizza, Giuditta De Prato, Ibrahim Rohman (JRC-IPTS), Gianluca Papa, Christophe Doin (DG CNECT), as well as participants of seminars and workshops at the European Commission and at DREAM, Copenhagen. Finally, thorough checking and editing of the text by Patricia Farrer is gratefully acknowledged. The author is solely responsible for the content of the paper.

## References

- [1] Aghion, P. and Howitt, P. (1992) A model of growth through creative destruction, *Econometrica*, 60, 323-351
- [2] Bye, B., Fæhn, T. and Heggedal, T-R. (2009) Welfare and growth impacts of innovation in a small, open economy, an applied general equilibrium analysis. *Economic Modelling*, 26, 6, 1075-1088.
- [3] Cette, G., Mairesse, J. and Kocoglu, Y. (2002) Diffusion of ICTs and Growth of the French Economy over the Long-term, 1980-2000, *International Productivity Monitor*, 4, 27-38.
- [4] Christiano, L., Eichenbaum, M. and Evans, C. (2005) Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113, 1-45.
- [5] Coe, D. and Helpman, E. (1995) International R&D spillovers, *European Economic Review*, 39, 859-887.

- [6] Coe, D., Helpman, E. and Hoffmaister, A. W. (2008) *International R&D Spillovers and Institutions*. IMF Working Paper 104.
- [7] Colecchia, A. and Schreyer, P. (2002) ICT Investment and Economic Growth in the 1990s: Is the United States a Unique Case? *Review of Economic Dynamics*, 5, 408-442.
- [8] Dalgaard, C. J. and Kreiner C. T. (2003) Endogenous Growth: A Knife Edge or the Razor's Edge?, *Scandinavian Journal of Economics*, 105(1), 73-85.
- [9] Diao, X., Roe, T. and Yeldan, E. (1999) Strategic policies and growth: an applied model of R&D-driven endogenous growth, *Journal of Development Economics*, 60, 343-380.
- [10] Dinopoulos, E. and Thompson, P. (1998) Schumpeterian Growth Without Scale Effects. *Journal of Economic Growth*, 3, 313-335.
- [11] Eurostat (2001) *Handbook on price and volume measures in national accounts*. Luxembourg: Office for Official Publications of the European Communities, 2001
- [12] Ghosh, M. (2007) R&D policies and endogenous growth: A dynamic general equilibrium analysis of the case for Canada, *Review of Development Economics*, 11(1), 187-203.
- [13] Griffith, R., Redding, S. and Simpson, H. (2002) *Productivity Convergence and Foreign Ownership at the Establishment Level*, The Institute for Fiscal Studies Working Paper, 02/22
- [14] Grossman, G. and Helpman, E. (1991) Quality ladders in the theory of growth, *Review of Economic Studies*, 58, 43-61.
- [15] Haskel, J. E. , Pereira, S. C. and Slaughter, M. J. (2002) *Does Inward Foreign Direct Investment Boost the Productivity of Domestic Firms?* NBER Working Paper No. 8724.
- [16] Howitt, P. (1999) Steady Endogenous Growth with Population and R&D Inputs Growing, *Journal of Political Economy*, 107(4), 715-730.
- [17] Jones, C. I. (1995a) Time Series Tests of Endogenous Growth Models, *Quarterly Journal of Economics*, 110(2), 495-525.
- [18] Jones, C. I. (1995b) R&D-Based Models of Economic Growth, *Journal of Political Economy*, 103(4), 759-784.
- [19] Jones, C. I. (1999) Growth: With or Without Scale Effects?, *American Economic Review, Papers and Proceedings*, 89, 139-144.
- [20] Jones, L. and Manuelli, R. (1990) A convex model of equilibrium growth: Theory and policy implications, *Journal of Political Economy*, 98, 1008-1038.
- [21] Jorgenson, D. W., Ho, M. S. and Stiroh, K. J (2008) A Retrospective Look at the US Productivity Growth Resurgence, *Journal of Economic Perspectives*, 22(1), 3-24.



- [22] Keller, W and Yeaple, S. R. (2003) *Multinational enterprises, international trade, and productivity growth: Firm level evidence from the United States*. NBER Working Paper 9504.
- [23] Kneller, R., Bleaney, M. and Gemmell, N. (1999) Fiscal Policy and Growth: Evidence from OECD Countries, *Journal of Public Economy*, 74, 171-190.
- [24] Kocherlakota, N. R. and Yi, K.-M. (1997) Is there Endogenous Long-run Growth? Evidence from the United States and the United Kingdom, *Journal of Money, Credit and Banking*, 29, 235-262.
- [25] Koesler, S. and Schymura, M. (2012) *Substitution Elasticities in a CES Production Framework: An Empirical Analysis on the Basis of Non-Linear Least Squares Estimations*. ZEW Discussion Papers, 12-007.
- [26] Křístková, Z. (2013) Analysis of Private R&D Effects in a CGE Model with Capital Varieties: The case of Czech Republic, *Czech Journal of Economics and Finance*, 63, 3, 262-287.
- [27] Li, C.-W. (2000) Endogenous vs. Semi-endogenous Growth in a Two-R&D-sector Model, *Economic Journal*, 110, C109-C122.
- [28] Li, C.-W. (2002) Growth and scale effects: the role of knowledge spillovers, *Economic Letters*, 74, 177-185.
- [29] Lucas, R. E. (1988) On the mechanics of economic development, *Journal of Monetary Economics*, 22, 3-42.
- [30] Okagawa, A. and Ban, K. (2008) Estimation of substitution elasticities for CGE models. Discussion Papers in Economics and Business, no.08-16.
- [31] Oliner, S. D., Sichel, D. E. and Stiroh, K. J. (2007) Explaining a Productive Decade, *Brookings Papers on Economic Activity*, 1, 81-151.
- [32] Oulton, N. (2001) *ICT and Productivity Growth in the UK*, Bank of England Working Paper.
- [33] Ratto, M., Roeger, W. and Veld, Jan in't (2008) *QUEST III: An Estimated Open-Economy DSGE Model of the Euro Area with Fiscal and Monetary Policy*. European Commission, Economic Papers 335.
- [34] Rebelo, S. (1991) Long run policy analysis and long run growth, *Journal of Political Economy*, 99, 500-521.
- [35] Rebelo, S. (1998), *The Role of Knowledge and Capital in Economic Growth*, Research paper 149, World Institute for Development Economics research.
- [36] Romer, P. M. (1986) Increasing returns and long run growth, *Journal of Political Economy*, 94, 1002-1037.
- [37] Romer, P. M. (1990) Endogenous Technological Change, *Journal of Political Economy*, 98(5) pt. 2, S71-S102.

- [38] Sakurai, N., Papaconstantinou, G. and Ioannidis, E. (1997) Impact of R&D and Technology Diffusion on Productivity Growth: Empirical Evidence for 10 OECD Countries, *Economic Systems Research*, 9(1), 81-109.
- [39] Smets, F. and Wouters, R. (2007) Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach, *American Economic Review*, 97, 586-606.
- [40] Timmer, M. P. (ed) (2012) The world Input-Output Database (WIOD): Contents, Sources and Methods. WIOD Working Paper. no.10.
- [41] Van der Werf, E. (2008) Production functions for climate policy modelling: An empirical analysis. *Energy Economics*, vol.33, 2964-2979.
- [42] Xu, B. (2000) Multinational enterprises, technology diffusion, and host country productivity growth, *Journal of Development Economics*, 62, 477-493.
- [43] Xu, B. and Wang, J. (1999) Capital Goods Trade and R&D Spillovers in the OECD, *The Canadian Journal of Economics*, 32(5), 1258-1274.
- [44] Young, A. (1998) Growth without Scale Effects, *Journal of Political Economy*, 160(1), 41-63.

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**EUR 27548 EN – Joint Research Centre – Institute for Prospective Technological Studies**

Title: A CGE model with ICT and R&D-driven endogenous growth: A detailed model description

Author: Martin Aarøe Christensen

Luxembourg: Publications Office of the European Union

2015 – 48 pp. – 21.0 x 29.7 cm

EUR – Scientific and Technical Research series – ISSN 1831-9424 (online)

ISBN 978-92-79-53360-0 (PDF)

doi:10.2791/60328

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doi:10.2791/60328

ISBN 978-92-79-53360-0

