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FIDELIO 1: Fully Interregional Dynamic Econometric Long-term Input-Output Model for the EU27

Kurt Kratena, Gerhard Streicher, Umed Temurshoev, Antonio F. Amores, Iñaki Arto, Ignazio Mongelli, Frederik Neuwahl, José M. Rueda-Cantuche, Valeria Andreoni

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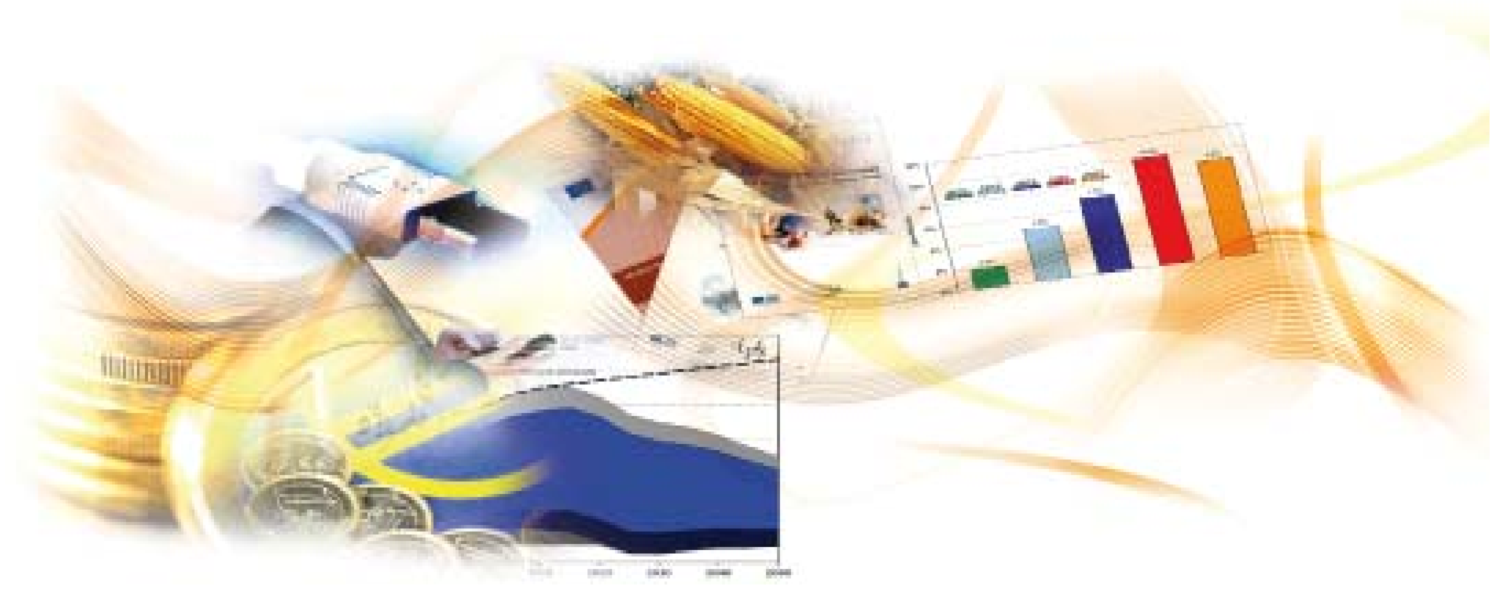

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Preface

Modeling is one per cent inspiration, ninety-nine per cent perspiration. (Slightly modified quotation from Thomas Alva Edison: *Genius is one per cent inspiration, ninety-nine per cent perspiration.*)

The history of FIDELIO starts on February 14 in 2006 at the the European Commission's Joint Research Centre – Institute for Prospective Technological Studies (JRC-IPTS) with an expert workshop on an "Exploratory research project: EU-wide extended input-output analysis tools", where several experts in the field gave presentations on input-output modeling, with a focus on environmental data and analysis. The workshop can ex post be considered as successful, as the "exploratory research project" resulted in several research initiatives at JRC-IPTS, linked to input-output (IO) analysis. One line was the compilation of data for member states, including the derivation of an EU table as well as the construction of time series of supply and use tables. The other line of research was still called "EU-wide extended input-output analysis tools" and mainly consisted of using an extended IO model for the EU for policy simulations. The extensions mainly comprised modeling private consumption and integrating environmental accounts in the IO model.

In 2008 a new step was taken with the organization of an "Econometric IO modeling course" at JRC-IPTS, where the history and methodology of econometric

IO modeling has been laid down in several modules during 2008 and 2009. Special emphasis was given in this course on inter-regional modeling and the implementation of IO models based on supply and use tables in the software package GAMS. The assignments in this course led to first versions of prototype econometric IO models for several EU countries, implemented in GAMS. The next logical step consisted in a research project for a full econometric input-output model for the EU, which is where we stand now. In parallel to this research line, other research projects and activities have continuously provided new and very useful data for this kind of modeling. Especially the output of the World Input-Output Database (WIOD) project has to be mentioned in this context. Another red line of the research leading to FIDELIO has always been the clear distinction of this kind of model from static IO modeling on the one side and from traditional CGE modeling on the other side. The informed reader may find that several features of FIDELIO are very similar to CGE models, whereas in other parts the demand driven and linear 'IO philosophy' still dominates. FIDELIO must be seen as something new that tries to combine aspects of both lines and attempts to give a relevant representation of supply and demand mechanisms of the European economy. It is especially the aspect of dynamic adjustment mechanisms where FIDELIO wants to give a distinct picture of the economy than is laid down in static CGE modeling.

This preface is also a wonderful opportunity to acknowledge the contribution of several people to FIDELIO that are not listed as authors of this technical report. Luis Delgado (JRC-IPTS) has always shown much interest in the econometric IO approach, and in the richness of this type of analysis and therefore his support has made FIDELIO possible. Andreas Loschel was the main force behind organizing the first expert workshop in February 2006 and thereby making the first step towards FIDELIO. Two other members of the JRC-IPTS team that have collaborated at an early stage in important parts are Aurelien Genty and Andreas Ühlein. Michael

Wüger (WIFO) has developed part of the econometric methodology together with the authors of this report and has in certain stages guided the modeling work. Katharina Köberl (WIFO) and Martina Agwi (WIFO) have provided excellent research assistance and helped with the data analysis. Sincere thanks are given to all these people for their help with the construction of FIDELIO.

During the econometric IO modeling course we had somehow established within our group the term DEIO (Dynamic Econometric IO) modeling for what we were doing. In one of the workshops for the project, after a long series of presentations and discussions on technical details, we decided to spend some time with a brainstorming for an appealing acronym. Adding characters to DEIO and playing around and given the fact that there are several friends of the opera in the research group, suddenly the proposal "FIDELIO" with the corresponding interpretation came up. We have no idea what would have resulted from this exercise, if we had had some *aficionados* of ancient history or medieval literature in our group. Wikipedia describes the background of the opera "Fidelio" as "a story of personal sacrifice, heroism and eventual triumph". Although on our way we might more often have seen and experienced the sacrifice and the heroism (especially, when it came to data gaps) than the expectation of eventual triumph, FIDELIO as described below is a working model of the EU 27 economies with relevant features for some of the policy questions of our times.

Glossary

Throughout the book the term *commodity* is used to refer to the COICOP commodity, while the terms *good* and *product* refer to CPA products given in the Supply and Use tables. The following notations for the sets' identifiers and subscripts are adopted.

Sets identifiers

c	private consumption commodity, refers to COICOP category
cd	durable commodity
cf	coefficient in an econometric equation
cn	nondurable commodity
ctn	total of nondurable (QAIDS) commodities
f	final demand category
g	good (product), refers to CPA products
ge	energy good
gm	margin good
gne	non-energy good
gnm	non-margin good
r	region (does not include the rest of the world)
rt	trading region (any region including the rest of the world)

s	sector
sk	labour skill type, indicates high-, medium- or low-skilled labour
st	total sector, represents all intermediate users
t	time
u	user, refers to sectors and final demand categories
utr	trade users, refers only to st and f
v	value added component

Note: Whenever the same sets are used and the necessity of distinguishing between the two arises, numerical subscripts are added to the corresponding identifiers. For example, both r and r_1 refer to the same set of regions, but a sum operator can be defined only over r_1.

Subscripts

1	a variable lagged once
2	a variable lagged twice
bp	basic prices
cif	CIF (cost, insurance, freight; at importers' border) prices
elect	related to commodity Electricity
eu	European Union-related data
fob	FOB (free-on-board; at exporters' border) prices
na.io	National Accounts to input-output data ratio of the same variable
pp	purchasers' prices
privtr	related to commodity Private Transport
qaids	related to the quadratic almost ideal demand system (QAIDS) model
red	stands for 'reduced'

rec.	stands for 'received'
row	rest of the world
tncs	related to the estimation of costs for third-country transport
trf	related to tariffs estimation
xrate	related to exchange rate index
wiod	related to the data of the World Input-Output Database (WIOD) project

Final demand categories

con	private consumption
gov	government (public) consumption
npish	non-profit institutions serving households
inv	investments
invent	changes in inventories
exp	exports

The detailed list of all the variables are given in the Appendix. For variables' notations we used the following general rule: if a variable has at least two dimensions, then its shortcut name is written with uppercase letters only (except for the possibility of having subscripts as defined above); if, on the other hand, a variable has only one dimension, then at least some part of its name is written with lowercase letters. For example, the total number of hours worked in sector s and region r is denoted by HRWK(r,s), while the total number of regional hours worked is denoted by HrWktot(r).

Chapter 1

Macro-overview of FIDELIO

In this chapter we provide a concise macro-overview of FIDELIO. It helps understanding the main mechanisms underlying the model's solution, and as such serves two main purposes. First, it is an adequate material for those who are only interested in FIDELIO's main features and its underlying quantity and price mechanisms. These readers do not have to go into the detailed description of FIDELIO given in Chapters 3 and 4, but are encouraged to read Chapter 2 that presents the economic theories underlying the core blocks of FIDELIO. Second, this chapter makes the process of understanding all the details of FIDELIO easy to those who want to learn (almost) everything about the model. These readers are expected to find helpful the overview of the model flows and prices demonstrated in Figures 1.1 and 1.2, respectively, and are encouraged to go back-and-forth to these charts while learning the material of Chapters 3 and 4.[1]

Figure 1.1 illustrates the main economic *flows* of FIDELIO. Note that flows refer to nominal flows (monetary transactions), and not to real flows (quantities). Real quantities are derived by dividing the flows by the corresponding prices that will

[1]For this reason also the variables' labels are given in these overview charts as they appear in the equations presented in Chapter 4.

be discussed below. A good starting point is the middle top rectangle in Figure 1.1 that represents demand by user u for good g domestically produced in region r and expressed in basic prices, $\mathrm{GD_{bp}(r, g, u)}$. Supply of goods (gross outputs) by sector, $\mathrm{Q(r, s)}$, are derived from these demands using the assumption of *constant market proportions* which implies that the shares of industries' outputs in the production of each good for all simulation years are assumed to be constant at their base-year levels. The implications of this transformation are as follows. First, it implies that FIDELIO is a demand-driven model. This explains the appearance of the term "input-output" or IO in its label because the standard input-output quantity model (Leontief, 1936, 1941) is inherently a demand-driven model.[2] However, as should become evident by the end of this book, FIDELIO is a much more powerful and flexible (hence, realistic) model for policy impact assessment purposes than the standard IO quantity and price models due to the following, among other, reasons:

1. FIDELIO uses various flexible functions (e.g., translog cost functions, QAIDS demand system) that are based on sound economic reasoning/theories,
2. there exist theory-based (direct and indirect) links between prices and quantities, which are entirely separate within the traditional IO framework,
3. while prices in the IO price model are identical for all intermediate and final users, in FIDELIO prices are user-specific due to its proper account of margins, taxes and subsidies, and import shares that are different for each user,
4. final demand categories in FIDELIO are endogenous, while in the IO quantity setting they are set exogenously, and
5. value added components in FIDELIO are endogenous, whereas in the IO price setting they are taken as exogenous.

[2] This is the reason why the IO quantity model is also called as the *demand-pull input-output quantity* model. On the other hand, in the standard IO *price* model, which is independent from the IO quantity model, changes in prices are driven by changes in the value added per output (e.g., wage rates). Therefore, the price model is often referred to as the *cost-push input-output price* model (for details, see e.g., Miller and Blair, 2009).

However, it is important to note that supply-side shocks can be simulated as well, though FIDELIO fits better for the analysis of demand-side shocks.

FIDELIO shows several similarities with computable general equilibrium (CGE) models, partly mentioned above in the five points where it was shown that FIDELIO has a richer potential for policy impact assessment than the static IO model, but also deviates from specifications in CGE models in some important aspects. In FIDELIO the supply side is specified in the dual model, i.e., a cost function that also comprises total factor productivity (TFP). The output firms are supplying is in the dual model with constant returns to scale determined by the demand side. Supply side aspects come into play, as cost factors and prices determine the level of demand. The growth of TFP is the most important long-term supply side force in that sense in FIDELIO. As described in Kratena and Streicher (2009), the differences between econometric IO modeling and CGE modeling have often been exaggerated and can in many cases be reduced to certain features in the macroeconomic closure rules of the models. This view can be upheld, when it comes to the differentiation of FIDELIO from a dynamic CGE model, like the IGEM model for the U.S. economy (Goettle et al., 2007).[3] In CGE it is the changes in prices that bring about equilibrium in all markets. In FIDELIO, however, the equilibrium concept in all markets is based on the *observed* empirical regularities indicating how economies are evolving over time. It is obvious that, in general, the base-year data, used in CGE and other modeling frameworks, are not consistent with the concept of "economic equilibrium" in its strict economic sense. Equilibrium is given in FIDELIO by demand reactions at all levels of users and all types of goods or factor inputs, and by the corresponding supply that is determined under the restrictions given at factor markets. The latter are mainly represented by an exogenous benchmark interest rate and liquidity constraints, as far as the input of capital is concerned, and by the institution of union

[3]The complete description and applications of the IGEM model are available at `http://www.igem.insightworks.com/`.

wage bargaining at industry level, as far as the input of labour is concerned. Savings in the economy (domestic plus external) are not fixed by a fixed current account balance, but are determined in the buffer stock model of consumption, taking into account the wealth position (and therefore also the foreign wealth) of households. Current and lagged household income has an impact on private consumption, as far as lending constraints in the capital market are present and the purchase of consumer durables cannot be fully financed by borrowing.

The public sector in the current version is specified with exogenous expenditure and therefore no explicit restriction on this balance for equilibrium in the economy is integrated into FIDELIO. Anyway, for the construction of a 'baseline' scenario growth rates of transfer payments and public consumption in the different countries that are ex ante in line with their mid-term fiscal stabilization targets (for net lending and public debt as percentage of GDP) are taken into account. This specification could easily be changed into an explicit restriction by endogenizing transfer payments and public consumption, given the target path of net lending as percentage of GDP.

Given the level of production $\mathrm{Q(r,s)}$ and input prices, firms are assumed to minimize their total costs. Input prices are taken as given in this stage because of perfect competition assumption. Further, with a constant returns to scale assumption, minimization of total costs at given level of output becomes equivalent to the minimization of a unit or average cost function, which represents the gross output price by sector, $\mathrm{PQ(r,s)}$. Using the translog cost approach (see Chapter 2.2.2 for details), the derived input demands for five *aggregate* inputs are fist obtained. These are demands for total energy inputs $\mathrm{E(r,s)}$, total domestic non-energy inputs $\mathrm{D(r,s)}$, total imported non-energy inputs $\mathrm{M(r,s)}$ and demands for two primary inputs of labour $\mathrm{L(r,s)}$ and capital $\mathrm{K(r,s)}$. Aggregate labour is further disaggregated into demands for three skill types: high-, medium- and low-skilled labour demands

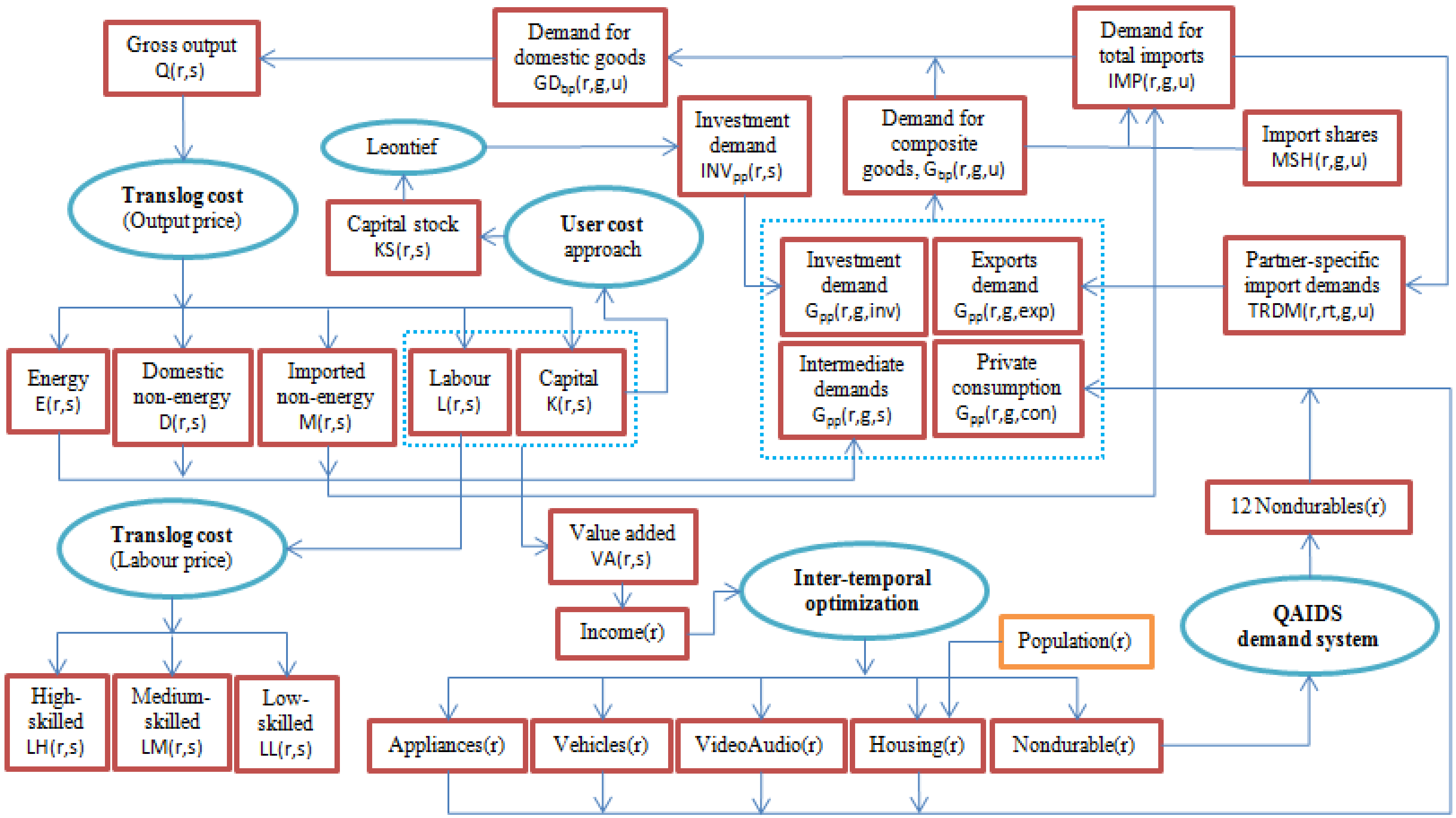

Figure 1.1: Overview of the main economic flows in FIDELIO

The variables included within red rectangles are endogenous variables. The main functional forms and approaches used for the derivation of various parts of the model are mentioned within the blue oval shapes.

denoted, respectively, as $\mathrm{LH}(\mathrm{r},\mathrm{s})$, $\mathrm{LM}(\mathrm{r},\mathrm{s})$ and $\mathrm{LL}(\mathrm{r},\mathrm{s})$. Here also the translog cost approach is employed, where the cost function is the wage earned per hour that determines labour price $\mathrm{PL}(\mathrm{r},\mathrm{s})$ (pricing details are discussed below).

The next step is computing demands for intermediate goods at purchasers' prices, $\mathrm{G_{pp}}(\mathrm{r},\mathrm{g},\mathrm{s})$. This is done by allocating the aggregate intermediate inputs $\mathrm{E}(\mathrm{r},\mathrm{s})$, $\mathrm{D}(\mathrm{r},\mathrm{s})$ and $\mathrm{M}(\mathrm{r},\mathrm{s})$ over all goods g using the corresponding product use structure (proportions) of the base year. Here, again if one expects that, for example, the product structure of energy inputs proportions are going to be different in the future compared to those in the base year, the corresponding information can be exogenously incorporated in the corresponding structure matrix.

The derived input demand (and supply) of labour and capital make up the total value added by sector, i.e., $\mathrm{VA}(\mathrm{r},\mathrm{s}) = \mathrm{L}(\mathrm{r},\mathrm{s}) + \mathrm{K}(\mathrm{r},\mathrm{s})$. At this stage also more components of value added (i.e., wages, social security contributions, production taxes and subsidies, and depreciation) are obtained, which we do not discuss here. Regional public consumption demand and regional non-profit institutions serving households (NPISH) consumption demand, both at purchasers' prices, are currently treated as exogenous. For the construction of a 'baseline' scenario these final demand components are extrapolated according the regional targets for public net lending. In a future version of FIDELIO, NPISH will be included in private consumption and public consumption will be endogenized in a form that the net lending targets are met explicitly. The corresponding totals for regional public consumption demand and regional non-profit institutions serving households (NPISH) are again distributed over all goods g employing the corresponding use structures of the benchmark year, which results in the public and NPISH consumption demands for products at purchasers' prices, i.e., $\mathrm{G_{pp}}(\mathrm{r},\mathrm{g},\mathrm{gov})$ and $\mathrm{G_{pp}}(\mathrm{r},\mathrm{g},\mathrm{npish})$, respectively. (The last are not shown in Figure 1.1 as currently they are exogenous.)

In FIDELIO it is recognized that private consumption is the largest component of aggregate demand, and as such its modeling should be given a very careful consideration. Obtaining private consumption demands at purchasers' prices, $G_{pp}(r, g, con)$, consists of three stages. The first stage is based on a theory of intertemporal optimization of households with buffer stock saving as proposed by Luengo-Prado (2006) and also discussed in Chapter 2.1.1. This theory takes into account that households cannot optimize according to the permanent income hypothesis due to credit market restrictions (liquidity constraints) and down payments for the purchase of durables. From the optimality conditions of the intertemporal problem we derive policy functions of durable and nondurable consumption that turn out to depend on households' wealth, down payment requirement (needed for purchasing durable goods) and the user cost of durables. The last in its turn depends on durables' prices, depreciation rates and the interest rate relevant to households' durables purchases. This theory is used to compute the regional demands for four durable *commodities* Appliances, Vehicles, Video and Audio, and Housing, and one aggregate non-durable commodity. Demand for housing is derived not exactly in the same way as the other three durables' demands, because housing consists of owner occupied houses and houses for rent, where the last is explained by demography (these details are given in Chapter 4.4.1). In the second stage, the derived aggregate nondurable demand is split up into its different components using the Quadratic Almost Ideal Demand System (QAIDS) proposed by Banks, Blundell and Lewbel (1997), which is also discussed in Chapter 2.1.2. Finally, the obtained demands for durable and nondurable commodities consistent with COICOP classification[4] are transformed into private consumption demands for products that are consistent with CPA 2002 classification,[5] $G_{pp}(r, g, con)$, using the corresponding region-specific

[4]COICOP stands for "Classification of Individual Consumption According to Purpose"; see the United Nations Statistics Division's COICOP information at `http://unstats.un.org/unsd/cr/registry/regcst.asp?Cl=5`.

[5]CPA stands for "Classification of Products by Activity", for details see the Eurostat's CPA information at `http://ec.europa.eu/eurostat/ramon/nomenclatures/index.cfm?TargetUrl=LST_NOM_DTL&StrNom=CPA&StrLanguageCode=EN&IntPcKey=&StrLayoutCode=HIERARCHIC`.

bridge matrices between the two systems.

Sectoral capital stocks $\mathrm{KS}(\mathrm{r}, \mathrm{s})$ are obtained from the assumption that their total *user cost* value is equal to the sectoral capital compensation (cash flow). Two options of static and dynamic concepts of user cost of capital can be employed, both of which assume that capital market is in equilibrium in each period (Jorgenson, 1967; Christensen and Jorgenson, 1969). User cost of capital depends on price of investments, interest rate for capital costs of firms' purchases and depreciation rate by industry (see Chapter 4.4.3). Then using Leontief technology, in this case the base-year investment-to-capital stock proportions, the investment demand by sector in purchasers' prices $\mathrm{INV}_{\mathrm{pp}}(\mathrm{r}, \mathrm{s})$ is obtained. These are finally transformed into the investment demands for products at purchasers' prices $\mathrm{G}_{\mathrm{pp}}(\mathrm{r}, \mathrm{g}, \mathrm{inv})$ using the benchmark-year product structure of investments.

Demands for exports in purchasers' prices $\mathrm{G}_{\mathrm{pp}}(\mathrm{r}, \mathrm{g}, \mathrm{u})$ are obtained from the endogenous trade flows between the model regions (which in Figure 1.1 is denoted as $\mathrm{TRDM}(\mathrm{r}, \mathrm{rt}, \mathrm{g}, \mathrm{u})$ and indicates region r's demands for imports from its trade partner rt) plus the exports to the rest of the world. The last component of final demand – demand for inventory – is assumed to be fixed at its base-year use level for all products, and for this reason is not given in the middle-right square of Figure 1.1 that includes four endogenous components of final demand. These are all demands for both domestically produced and imported goods (or composite goods, for short). Taking into account trade and transport margins and taxes less subsidies on products, these purchasers' price demands are translated into the demands for composite goods at basic prices $\mathrm{G}_{\mathrm{bp}}(\mathrm{r}, \mathrm{g}, \mathrm{u})$, the details of which are given in Chapter 4.5.

Multiplication of total import shares $\mathrm{MSH}(\mathrm{r}, \mathrm{g}, \mathrm{f})$ by the corresponding basic price demands for composite goods $\mathrm{G}_{\mathrm{bp}}(\mathrm{r}, \mathrm{g}, \mathrm{f})$ gives final user f's demand in region r for total imported good g valued at CIF prices, $\mathrm{IMP}(\mathrm{r}, \mathrm{g}, \mathrm{f})$. Sectoral demand for intermediate imports of energy goods $\mathrm{IMP}(\mathrm{r}, \mathrm{ge}, \mathrm{s})$ is derived similarly, but that for non-energy good $\mathrm{IMP(r, gne, s)}$ is obtained differently, namely, by multiplication of the total demand for non-energy imported inputs (from the Translog model of factor demands) with the use-structure matrix for imported non-energy intermediates. This use-structure matrix and the total import shares are assumed to be the same as those of the base year for all users, *except* for private consumers where the Armington approach is applied (i.e., the total import shares of consumers $\mathrm{MSH(r, g, con)}$ depend on domestic and import prices). The partner-specific import demands $\mathrm{TRDM(r, rt, g, u)}$ are computed from the total imports demand $\mathrm{IMP(r, g, u)}$ using the base-year trade shares in combination with the Armington approach (for details, see Chapter 4.6).

And finally, deducting imports $\mathrm{IMP(r, g, u)}$ from demands for composite goods at basic prices $\mathrm{G_{bp}(r, g, u)}$ gives demand for domestically produced goods in basic prices, $\mathrm{GD_{bp}(r, g, u)}$, with which we have started the brief explanation of the main economic flows demonstrated in Figure 1.1. This closes the loop of the main flows interactions with the understanding that quite crucial details behind these dependencies are skipped for simplicity purposes and are discussed in the following chapters.

We now turn to the discussion of various prices (but not all prices, similar to the flows discussion above), which naturally affect directly and/or indirectly all the endogenous variables discussed so far. The derivation of all the prices is discussed in Chapter 4.8, while the required theoretical reasonings are presented in the next chapter. The overview of the main prices are illustrated in Figure 1.2, where prices are juxtaposed on the flows chart of Figure 1.1. Wherever possible, prices are positioned close to the transactions which they refer to.

Let us start with the gross output prices $\mathrm{PQ(r, s)}$ that are basic prices determined, through the translog cost approach, from the prices of energy inputs $\mathrm{PE(r, s)}$, of domestic non-energy inputs $\mathrm{PD(r, s)}$, of imported non-energy inputs $\mathrm{PM(r, s)}$, of labour $\mathrm{PL(r, s)}$, and of capital $\mathrm{PK(r, s)}$, and time (in order to take into account

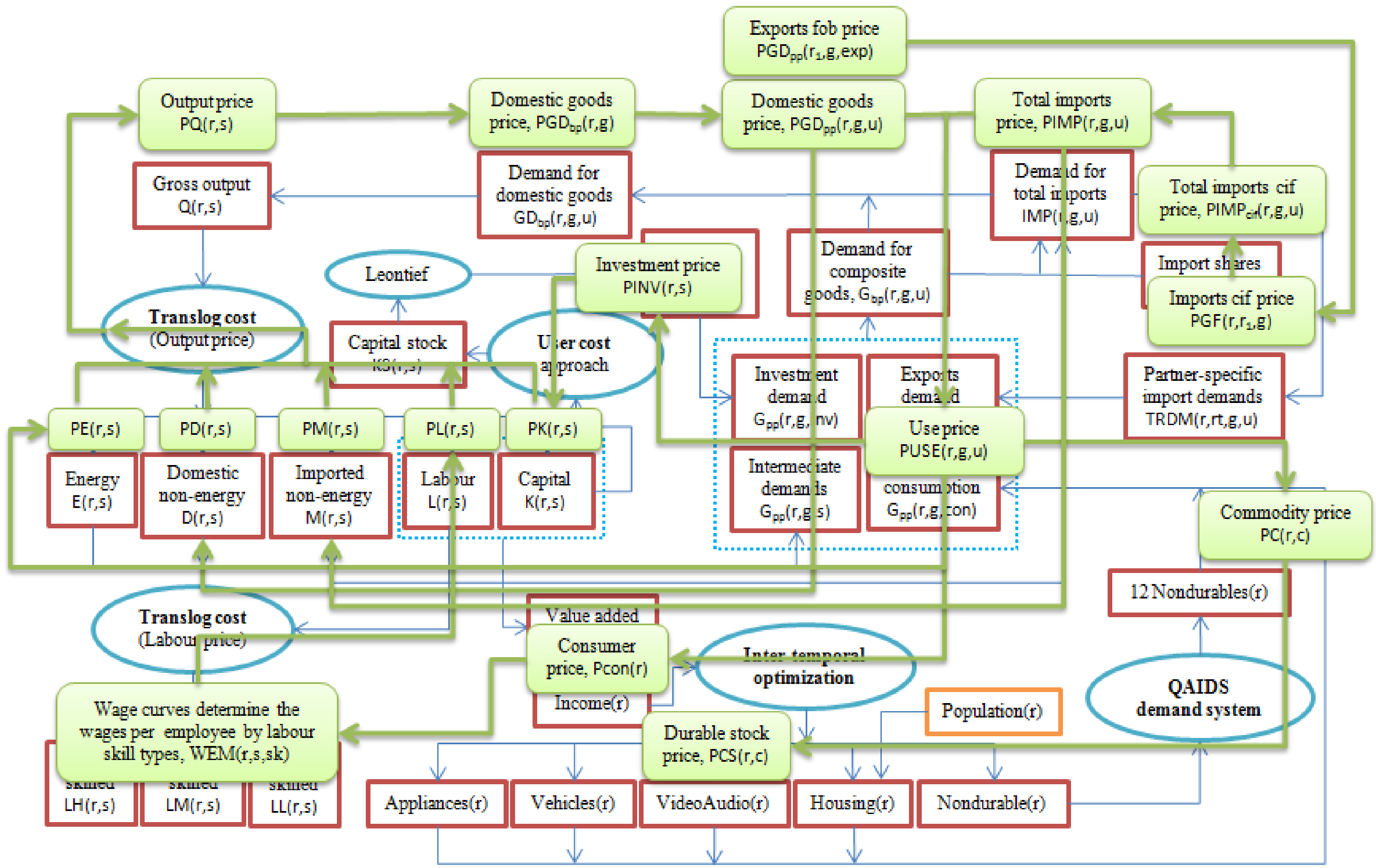

Figure 1.2: Overview of selected prices in FIDELIO

Wherever possible, prices (defined within the green rectangles) are positioned/juxtaposed with the transactions which they refer to.

the effect of technical progress due to TFP growth in the unit cost function and factor-biased technical progress). The last five mentioned prices also enter in the derivation of derived demands for aggregate inputs.

Basic prices of domestic products $\mathrm{PGD_{bp}(r, g)}$ are obtained as weighted averages of the sectoral gross output prices, where the base-year market shares of sectors are used as weights. Note that the last price is the same for all users, similar to the standard IO price model. However, taking into account the fact that in purchasers' prices, demand for products is essentially demand for a composite good, i.e., the good itself, trade and transport margins, and taxes less subsidies on the good, the purchaser prices of domestically produced products $\mathrm{PGD_{pp}(r, g, u)}$ become user-specific. The FOB price of exports in the exporter region r_1 is $\mathrm{PGD_{pp}(r_1, g, exp)}$, which once corrected for the exchange rates and augmented by international transport costs and tariffs gives the CIF prices at the border of the importing region r for goods imported from region r_1, $\mathrm{PGF(r, r_1, g)}$. The corresponding CIF prices for imports from the rest of the world $\mathrm{PGF(r, row, g)}$ are taken exogenous to the model.

Next, the weighted average of the import prices of trading partners (i.e., PGF's) gives the total import CIF price at the border of region r for good g and user u, $\mathrm{PIMP_{cif}(r, g, u)}$, where the endogenously determined trading partner-specific import shares are taken as weights. Further accounting for domestic markups turn $\mathrm{PIMP_{cif}(r, g, u)}$ into the total import prices including domestic margins and taxes less subsidies on products, $\mathrm{PIMP(r, g, u)}$.

Products' use prices for intermediate and final users $\mathrm{PUSE(r, g, u)}$ are the weighted averages of the purchasers' prices of domestic products $\mathrm{PGD_{pp}(r, g, u)}$ and import prices $\mathrm{PIMP(r, g, u)}$ using the import shares $\mathrm{MSH(r, g, u)}$ as the corresponding weights. The aggregate price of energy inputs $\mathrm{PE(r, s)}$ is determined using the base-year product structure of energy inputs and the corresponding sectoral use prices $\mathrm{PUSE(r, g, s)}$. Similarly, combining the purchasers' prices of domestic goods

$\mathrm{PGD_{pp}(r,g,u)}$ (resp. the import prices $\mathrm{PIMP(r,g,u)}$) with the base-year product structure of domestic (resp. imported) non-energy inputs results in the aggregate prices of domestic (resp. imported) non-energy inputs $\mathrm{PD(r,s)}$ (resp. $\mathrm{PM(r,s)}$). By the same principle, the prices of investments $\mathrm{PINV(r,s)}$ are determined from the products' use prices for investments and the base-year product structure of investments.

The regional use price for each user is the aggregate price of "inputs" for that user, and is obtained as the weighted average of the corresponding use prices with weights representing the product shares of endogenously derived demands for goods in purchasers' prices. If the user is private consumer, then the corresponding regional use price is the consumer price, $\mathrm{Pcon(r)}$. Using the COICOP-CPA bridge matrices of the base year, products' use prices for private consumption $\mathrm{PUSE(r,g,con)}$ are translated into the prices of durable and nondurable consumption commodities $\mathrm{PC(r,c)}$. The prices of stocks of durable commodities $\mathrm{PCS(r,c)}$ are obtained using the concept of user cost of durable goods.

The wages per employee of the high-, medium- and low-skilled labour, denoted respectively as $\mathrm{WEM(r,s,high)}$, $\mathrm{WEM(r,s,med)}$ and $\mathrm{WEM(r,s,low)}$, are determined by wage curves (Blanchflower and Oswald, 1994; see also Chapter 2.3.2), which in FIDELIO relate labour skill type wages to labour productivity, consumer price and skill-specific unemployment rates. These wages together with time components (in order to take into account the effects of technical change in the aggregate wage rate function and skill-biased technical progress) within the translog cost framework determine average wage earned per hour, which in turn defines the price of labour $\mathrm{PL(r,s)}$. Finally, the price of sectoral capital stock $\mathrm{PK(r,s)}$ is obtained from the investments prices $\mathrm{PINV(r,s)}$ using the notion of the user cost of capital.

By now it should, in principle, become clear why the model is called Fully Interregional Dynamic Econometric Long-term Input-Output (FIDELIO) model. It is “fully interregional” because it is a complete inter-regional economic model that takes into account the most important features (for policy analysis purposes) of consumption, production, labour market, trade between regions, and the environment. However, the full-fledged environmental block is (still) not discussed in this book, keeping in mind that some simple environmental impact calculations are straightforward to make once production- and consumption-side impacts are obtained from the model. The quantity and price interactions between regions (currently 27 EU Member States and one rest of the world region) are taken into account by comprehensive modeling of interregional trade flows. It is “dynamic” because, as mentioned above, the consumption block is based on an inter-temporal optimization approach and capital stocks (and investments) derivation is based on dynamic neoclassic theory of optimal capital accumulation. Further, time is explicitly included in the derivations of prices and firms’ demands for intermediate and primary inputs, while various lagged flow and price variables are contained in the consumption demands. These ultimately lead the model to be inherently a dynamic model in the sense of its over-time policy impact assessment capability. The model is “econometric” because the crucial parameters’ values in the functions, characterizing economic agents’ reactions in consumption, production and labour blocks, are estimated from the appropriate time series data employing most relevant (advanced) econometric techniques. The model is “long-term” because the durable and nondurable consumption demands are expressed in the form of long-run equilibrium relationships of an error correction model specification. This allows computing not only short-run effects, but also long-run equilibrium effects and the adjustment speeds of the short-run deviations toward the long-run equilibrium. Finally, the “input-output” part has been already explained in the beginning of this chapter. Here, it additionally needs to be noted that detailed supply and use tables make the core data of FIDELIO, which constitute the building blocks of the commodity-industry approach in input-output analysis.

Chapter 2

Theoretical foundations of FIDELIO

2.1 Consumption block

Since private consumption is by far the largest component of aggregate demand, it is of utmost importance to pay special attention to consumption behavior modeling. The consumption block of FIDELIO consists of two nests. First, households' demands for four types of durable and one aggregate non-durable commodities are derived, and then the aggregate nondurables obtained from the first step is split into its different categories. The theories behind these steps are explained below.

2.1.1 Households' demands for four types of durable and total non-durable commodities

This part of FIDELIO's consumption block is based on a theory of inter-temporal optimization problem of households along the lines of the 'buffer stock model' of consumption. The traditional way of linking a consumption block to an input-output (IO) model is the social accounting matrix (SAM) multiplier model, where the accounts for household income are linked to value added on the one side and to consumption on the other side. This specification is based on the Keynesian theory of consumption, and consumption mainly depends on current disposable income. There is – to our knowledge – only one attempt in the literature to link the IO model to a dynamic model of consumption, based on the permanent income hypothesis, namely Chen et al. (2010). However, the permanent income hypothesis has been challenged by different empirical puzzles that show a certain dependence of household consumption on current household income. This has been motivated by the existence of liquidity constraints and 'buffer stock' savings behavior in order to build up reserves for unexpected events and expenditure. Carroll (1997) has laid down the base of the buffer stock model, starting from the empirical puzzles that the permanent income hypothesis was not able to resolve. In FIDELIO we use a form of the buffer stock model, where households save for the purchase of durables, as described in Luengo-Prado (2006). Consumers maximize the present discounted value of expected utility from consumption of nondurable commodity C_t and from the service flow provided by the stocks of durable commodity K_t, subject to the budget and collateralized constraints. A very crucial and novel feature of this model is consideration of the last constraint imposed on consumers. It includes the so-called *down payment requirement* parameter, $\theta \in [0, 1]$, which represents the fraction of durables that a household is *not* allowed to finance. Hence, the borrowing limit (or maximum loan) of an individual is equal to $(1 - \theta)$ fraction (share) of the stocks of durable commodities. The mentioned constraint then implies that, at any point in time, the household is only required to keep an accumulated durable equity equal to θK_t, i.e., to θ fraction of the stocks of durable commodities.

Without going into all the mathematical details of the mentioned problem, we will just briefly discuss the main results of Luengo-Prado's (2006) paper. Using the same notation, the interest and depreciation factors are denoted, respectively, by $R \equiv 1 + r$ and $\psi \equiv 1 - \delta$, where r and δ are the rates of interest and depreciation, respectively. Then $(R - \psi)/R = (r + \delta)/(1 + r)$ is known as the *user cost of the durable*, or rental equivalent cost of one durable unit. The dependence of user cost on interest and depreciation rates has the following reasoning. The user cost increases if the interest rate r goes up because then the opportunity cost of investing in the durable increases: an euro invested in the durable commodity would have given a return of r if invested in financial assets. It is also evident that the higher depreciation erodes consumers' investments in the durable, which is equivalent to increasing the cost of using such a durable.

The following definitions will be useful. First, *cash-on-hand* is defined as the sum of assets holding, stocks of durables and income, i.e., $X_t \equiv (1 + r)A_{t-1} + (1 - \delta)K_{t-1} + Y_t$. Total wealth of a household, $A_t + K_t$, is comprised of the required down payment, θK_t, and *voluntary equity holding*, $Q_t \equiv A_t + (1 - \theta)K_t$. Using these definitions and the budget constraint $A_t = (1+r)A_{t-1} + Y_t - C_t - (K_t - (1-\delta)K_{t-1})$, it can be readily seen that the difference between cash-on-hand and voluntary equity holding is the sum of nondurable consumption and down payment share of the durable stock consumption, i.e.,

$$X_t - Q_t = C_t + \theta K_t. \tag{2.1}$$

The analysis then proceeds with normalized variables, where all the variables are divided by permanent income, in order to deal with the nonstationarity of income, as proposed by Carroll (1997). As far as the collateralized constraint is concerned, two cases should be distinguished:

(i) Constrained agents: the normalized cash-on-hand, x, is below the unique threshold level, $x^*(\theta) \equiv c + \theta k$, when the agent will be left without any resources once she pays for nondurable consumption and the down payment requirement. Here, thus no voluntary equity is carried over to the next period, i.e., $q = a + (1 - \theta)k = 0$.

(ii) Non-constrained agents: normalized cash-on-hand is higher than $x^*(\theta)$, hence some voluntary equity is accumulated, i.e., $q > 0$.

Luengo-Prado (2006) derives the policy functions of durable and nondurable consumption that turn out to be a function of the difference of cash-on-hand and voluntary equity holding, $x_t - q_t$, down payment parameter, θ, and

$$\Omega \equiv \varphi^{-1/\rho} \left[\frac{r+\delta}{1+r}\right]^{1/\rho}, \tag{2.2}$$

where φ and ρ are consumer's preference parameters with $\rho > 0$ implying risk-averse agent with precautionary motive for saving. Note that the expression in brackets in (2.2) is the user cost of the durable. Let us now discuss the main results of Luengo-Prada (2006) that are relevant for FIDELIO.

First, for nonconstrained agents, i.e., when $x_t > x_t^*(\theta)$, irrespective of the value of down payment parameter, θ, it must be the case that $c_t/k_t = C_t/K_t = \Omega$, where the policy functions take the following forms:

$$c_t = \frac{\Omega}{\Omega + \theta}(x_t - q_t), \tag{2.3}$$

$$k_t = \frac{1}{\Omega + \theta}(x_t - q_t). \tag{2.4}$$

That is, when the collateralized constrained is not binding, once the agent decides on her voluntary equity to be kept on to the next period, she simply spends fixed proportions of the cash-on-hand leftover between the durable and nondurable commodities.

The richness of the model, however, becomes apparent when one considers the second case of constrained agents with $x_t \leq x_t^*(\theta)$. Here everything depends on the value of θ and, crucially, its link to the user cost term. In particular, Proposition 2 in Luengo-Prado (2006) proves that:

(i) if $\theta = 0$, then $c_t = x_t$ and $k_t = (1/\Omega)x^*(\theta)$,

(ii) if $\theta < (R-\psi)/R$, then c_t (resp. k_t) is a convex (resp. concave) function of x_t,

(iii) if $\theta = (R-\psi)/R$, then the policy function are linear: $c_t = [\Omega/(\Omega+\theta)]x_t$ and $k_t = [1/(\Omega+\theta)]x_t$, and

(iv) if $(R-\psi)/R < \theta \leq 1$, then c_t (resp. k_t) is a concave (resp. convex) function of x_t.

The policy functions in all the above considered cases are illustrated in Figure 2.1. It turns out that all this flexibility of the derived policy functions (in terms of their curvature and flatness) makes the model capable of explaining such concepts in macroeconomics as *excess smoothness* and *excess sensitivity* observed in US and other countries' aggregate data. These were puzzling observations because they were not in line with the predictions of the life cycle-permanent income hypothesis that states that consumption is determined by the expected value of lifetime resources or permanent income. Excess smoothness refers to the empirical observations that consumption is excessively smooth, i.e., consumption growth is smoother than permanent income. It has been also found that consumption is excessively sensitive: C_{t+1} reacts to date t or earlier variables other than C_t (for example, income at time t), whereas the standard inter-temporal optimization condition would state that it should not (Hall, 1978, is the pioneering work in this field) because all past and predictable information is already incorporated in current consumption so that no lagged information can provide additional explanatory power in accounting for variations in future consumption. It is important to realize that consumption can simultaneously display excess sensitivity and excess smoothness. This is because excess sensitivity refers to how consumption reacts to past, thus predictable, income

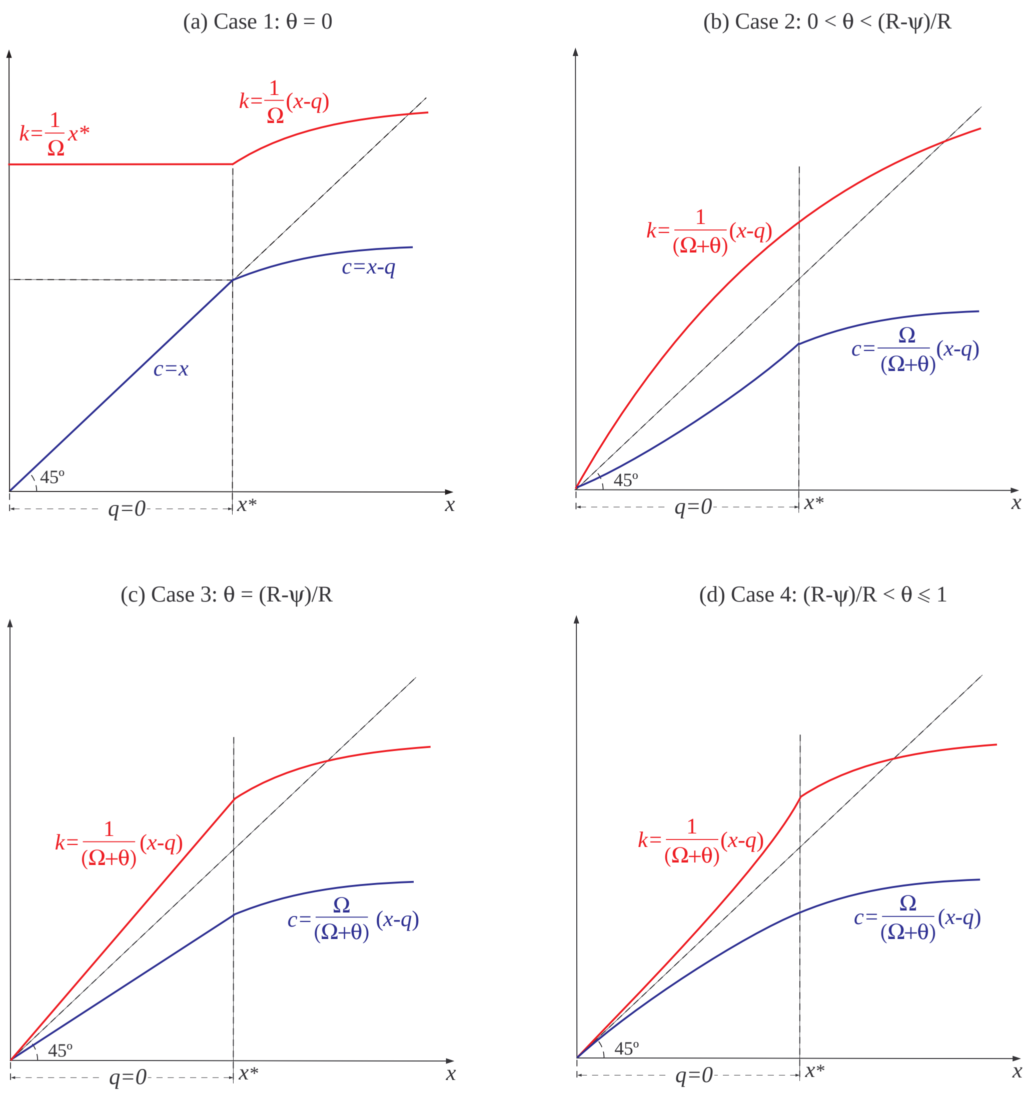

Figure 2.1: Policy functions. Durable and nondurable as a function of cash-on-hand

shocks whereas excess smoothness refers to how consumption reacts to present, thus unpredictable, income shocks.

From Figure 2.1 it follows, for example, that for a constrained household as the down payment requirement increases, the policy function for the nondurable commodity becomes flatter, while the opposite is true for durable consumption (compare the curvature of the policy functions in the range $x_t \leq x_t^*(\theta)$ once moving from subfigure (a) to (b) to (d) and finally to (c)). This implies that for constrained agents, the marginal propensity to consume out of cash-on-hand for the nondurable commodity C_t is higher the lower the down payment parameter θ (which is, in fact, equal to one when $\theta = 0$). This is an indication that with low down payments, there is higher nondurable volatility and lower durable volatility. It then could be proved that in the model nondurable consumption becomes smoother relative to income for higher down payments. One of the reasons for such behavior is that once there is a positive permanent income shock, the agent chooses not to fully adjust her consumption, but rather tends to spread out the cost of accumulating the down payment.

How do we use the above results in FIDELIO? First, it can be shown that the normalization procedure (i.e., dividing all the variables by permanent income) is equivalent to assuming an equilibrium relationship between equity including durable stocks and the long-run path of income. This also implies – due to the building up of 'voluntary equity' out of savings – an equilibrium relationship between non-durable consumption and permanent income. Permanent income in the buffer stock model is usually specified as a difference stationary process with transitory shocks, so that a co-integrating relationship between permanent income and the consumption variables can be assumed. We proceed by this methodology and therefore instead of normalizing the consumption variables by permanent income, set up an error correction model. Second, take logarithm of both sides of the functions in (2.3)-(2.4), assume linearity in the user cost term (i.e., in (2.2) we set $\varphi = \rho = 1$), and write the results in terms of more flexible functions so that the mentioned different curvatures can be taken into account. All these steps give the empirical counterparts of the policy functions for nondurable and durable consumption (co-integrating equations), respectively, as

$$\ln C_t = \tilde{\alpha}_0 + \tilde{\alpha}_1\{\ln p_t(r_t+\delta_t) - \ln[\theta_t + p_t(r_t+\delta_t)]\} + \tilde{\alpha}_2 \ln(X_t - Q_t), \tag{2.5}$$

$$\ln K_t = \tilde{\beta}_0 - \tilde{\beta}_1 \ln[\theta_t + p_t(r_t+\delta_t)] + \tilde{\beta}_2 \ln(X_t - Q_t), \tag{2.6}$$

where two other changes have been made in the durable user cost term: (a) price of the durables p_t was explicitly introduced, and (b) for simplicity the one-period discounting term was omitted. For simplicity, define

$$Z_t \equiv X_t - Q_t, \;\; T_t \equiv \theta_t + p_t(r_t+\delta_t), \;\; \text{and} \;\; N_t \equiv p_t(r_t+\delta_t)/T_t, \tag{2.7}$$

and use tilde for the logarithm sign, e.g., $\widetilde{T}_t \equiv \ln T_t$. Then the counterparts of (2.5) and (2.6) in the form of *autoregressive distributed lag (2,2,2)* models (ADL(2,2,2) models) are respectively

$$\widetilde{C}_{it} = \sum_{j=1}^{2} \alpha_j \widetilde{C}_{i,t-j} + \sum_{j=0}^{2} \alpha_{3+j} \widetilde{N}_{i,t-j} + \sum_{j=0}^{2} \alpha_{6+j} \widetilde{Z}_{i,t-j} + \epsilon_{it}, \tag{2.8}$$

$$\widetilde{K}_{it} = \sum_{j=1}^{2} \beta_j \widetilde{K}_{i,t-j} + \sum_{j=0}^{2} \beta_{3+j} \widetilde{T}_{i,t-j} + \sum_{j=0}^{2} \beta_{6+j} \widetilde{Z}_{i,t-j} + \nu_{it}, \tag{2.9}$$

where the subscript i refers to the model countries for which the required data are available (21 EU countries). The error components in (2.8)-(2.9) can be decomposed in the usual way into the time invariant fixed effects and the error term in time, e.g., $\epsilon_{it} = \epsilon_i + \eta_{it}$. The above equations can be transformed into the error correction model (ECM) specification (see Banerjee et al., 1990). The *long-run equilibrium relationships* are quantified by dropping the time subscripts from (2.8) and (2.9), where the resulting coefficients reflect the corresponding *long-run multipliers*:

$$\widetilde{C}_i = \frac{\alpha_3+\alpha_4+\alpha_5}{1-\alpha_1-\alpha_2}\widetilde{N}_i + \frac{\alpha_6+\alpha_7+\alpha_8}{1-\alpha_1-\alpha_2}\widetilde{Z}_i + \epsilon_i, \tag{2.10}$$

$$\widetilde{K}_i = \frac{\beta_3+\beta_4+\beta_5}{1-\beta_1-\beta_2}\widetilde{T}_i + \frac{\beta_6+\beta_7+\beta_8}{1-\beta_1-\beta_2}\widetilde{Z}_i + \nu_i, \tag{2.11}$$

For example, the second coefficient in (2.10) is the long-run income multiplier for nondurable consumption, that is, it quantifies the long-term equilibrium impact of changes in income (cash-on-hand net of voluntary equity holding) on the household's demand for nondurable commodities.

Finally, if one writes (2.8) and (2.9) in the ECM form, then it immediately becomes evident that the *error-correction parameters* in the corresponding equations equal, respectively,

$$-(1-\alpha_1-\alpha_2) \text{ and } -(1-\beta_1-\beta_2), \tag{2.12}$$

which determine the *speed of adjustment* towards the long-run equilibrium.

The error correction models equivalent to the ADL (2,2,2) models in (2.8) and (2.9) have been estimated using the GMM estimator for dynamic panel data proposed by Blundell and Bond (1998). The estimates of all the coefficients, error-correction parameters (ECP) and the long-run 'durable cost'[1] and income multipliers (M_1 and M_2, respectively) are presented in the table below.

Table 2.1: Parameters for computing the durable and nondurable demands

Nondurables	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	ECP	M_1	M_2
Aggregate	1.277	-0.432	0.063	0.059	-0.099	0.185	-0.534	0.422	-0.155	0,153	0.472
Durables	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	ECP	M_1	M_2
VideoAudio	1.585	-0.660	-0.107	0.104	-0.015	0.095	-0.124	0.076	-0.075	-0.244	0.632
Vehicles	1.771	-0.859	-0.261	0.458	-0.162	0.090	-0.312	0.250	-0.088	0.404	0.321
Appliances	1.619	-0.677	-0.075	0.077	-0.017	0.025	-0.036	0.023	-0.058	-0.259	0.193
Housing	1.840	-1.098	0.677	-0.168	-0.675	-0.710	0.637	0.283	-0.259	-0.643	0.812

Note: ECP = error correction parameter in (2.12), M_1 and M_2 are the long-run durable cost and income multipliers in (2.10) and (2.11).

The results in Table 2.1 show, for example, that the adjustment speed to long-run equilibrium is highest for Housing and lowest for Appliances, which also have, respectively, the biggest and lowest income long-run multipliers, M_2. Note

[1]These have different interpretations for C_t and K_t: for nondurables, it is the elasticity of consumption demand with respect to the share of the durable user cost in the user cost plus down payment requirement, $p_t(r_t+\delta_t)/[\theta_t+p_t(r_t+\delta_t)]$, and for durables, these are the elasticities of consumption demand with respect to the durable user cost plus the down payment requirement.

that the stocks of vehicles, video/audio, appliances and housing variables are in current prices, therefore a long-run multiplier $M_1 < 1$ guarantees a negative own price elasticity. This reasoning does not apply to non-durables, as the corresponding multiplier M_1 measures cross-price elasticity (i.e., reaction of non-durable consumption to durable costs) and therefore any positive value guarantees a *substitution effect*.

2.1.2 Splitting aggregate nondurable commodity into its different categories

Once consumption of total nondurable commodity has been estimated using the approach discussed in Chapter 2.1.1, we need to split up this aggregate demand into its different components.[2] For the purposes of this second step allocation the so-called *Quadratic Almost Ideal Demand System* (QAIDS) proposed by Banks, Blundell and Lewbel (1997) is used. This model is quite popular and widely-used approach in applied microeconomics for estimating demand functions. Therefore, without going into all the details, the main result is that the *expenditure share* equation for the i-th nondurable commodity of a utility-maximizing consumer has the following form:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln \left[\frac{C}{a(\mathbf{p})} \right] + \frac{\lambda_i}{b(\mathbf{p})} \left\{ \ln \left[\frac{C}{a(\mathbf{p})} \right] \right\}^2 , \qquad (2.13)$$

where w_i is the expenditure share of nondurable commodity i, $\mathbf{p} = (p_1, p_2, \ldots, p_n)'$ is the vector of prices of the n nondurable commodities, $a(\mathbf{p})$ is the price index used to deflate nominal aggregate consumption C to arrive at real total expenditure, $b(\mathbf{p})$ is another price index reflecting the cost of bliss (within AIDS model), and

[2]As pointed out by Attanasio and Weber (1995, p. 1144), this strategy is consistent with a two step interpretation of the intertemporal optimization problem: in the first step, the consumer decides how to allocate expenditure across time periods, while in the second step, she allocates the derived expenditure for each time period to its different consumption categories. This second step allocation depends on the prices of consumption categories and the corresponding total expenditure.

the rest are parameters to be estimated. If $\lambda_i = 0$, then (2.13) reduces to the AIDS model of Deaton and Muellbauer (1980). Thus, the AIDS model assumes that demand (or expenditure share) equations are linear in log of real income, but its extension – QAIDS allows for non-linear Engel curves that could be observed for some commodities in practice (e.g., alcohol and clothing).

The (logarithm of the) first price index $\ln a(\mathbf{p})$ has the translog form, while the second price index $b(\mathbf{p})$ is a simple Cobb-Douglas aggregator of commodities' prices, thus are defined, respectively, as follows:

$$\ln a(\mathbf{p}) = \alpha_0 + \sum_i \alpha_i \ln p_i + 0.5 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j, \tag{2.14}$$

$$b(\mathbf{p}) = \prod_i p_i^{\beta_i}. \tag{2.15}$$

The above functional forms are determined by the conditions that have to hold for exact aggregation over all households (for details of aggregation theory, see Muellbauer, 1975, 1976).

To be consistent with the demand theory, the following three restrictions need to be imposed on the parameters of the expenditure shares equations.

- *Additivity*: expenditure shares should add up to one, i.e., $\sum_i w_i = 1$.
- *Homogeneity in prices and total expenditure*: equal increases in prices and income should leave demand unchanged.
- *Slutsky symmetry*: the Hicksian (or compensated) demand (see below for details) response to prices, or equivalently cross-substitution effects, are symmetric, i.e., $\partial h_i/\partial p_j = \partial h_j/\partial p_i$. This implies that the nature of complementarity or substitutability between the goods cannot change whether we work with $\partial h_i/\partial p_j$ or $\partial h_j/\partial p_i$, which is not the case if one instead uses Marshallian demand functions. Slutsky symmetry follows from the assumption that the so-called expenditure function has continuous second partial derivatives (see

e.g., Gravelle and Rees, 2004).

The above restrictions within the QAIDS framework are, respectively, equivalent to:

$$\sum_i \alpha_i = 1, \quad \sum_i \gamma_{ij} = 0, \quad \sum_i \beta_i = 0, \tag{2.16}$$

$$\sum_j \gamma_{ij} = 0, \tag{2.17}$$

$$\gamma_{ij} = \gamma_{ji}. \tag{2.18}$$

One of the main benefits of the estimated parameters of the QAIDS model comes in the elasticity calculations. To calculate the income and price elasticities, first, differentiate (2.13) with respect to (log of) income and prices to obtain:

$$\mu_i \equiv \frac{\partial w_i}{\partial \ln C} = \beta_i + \frac{2\lambda_i}{b(\mathbf{p})} \ln \left[\frac{C}{a(\mathbf{p})} \right], \tag{2.19}$$

$$\mu_{ij} \equiv \frac{\partial w_i}{\partial \ln p_j} = \gamma_{ij} - \mu_i \Big[\alpha_j + \sum_k \gamma_{jk} \ln p_k \Big] - \frac{\lambda_i \beta_j}{b(\mathbf{p})} \left\{ \ln \left[\frac{C}{a(\mathbf{p})} \right] \right\}^2. \tag{2.20}$$

Using (2.19) and (2.20), the income elasticity, e_i, uncompensated price elasticity, e^u_{ij}, and the compensated price elasticity, e^c_{ij}, are easily derived as follows:

$$e_i = \mu_i / w_i + 1, \tag{2.21}$$

$$e^u_{ij} = \mu_{ij} / w_i - \delta_{ij}, \tag{2.22}$$

$$e^c_{ij} = e^u_{ij} + e_i w_j, \tag{2.23}$$

where δ_{ij} is the Kronecker delta.

Banks et al. (1997, p. 529) find that the Engel curves for clothing and alcohol have inverse U-shape. In terms of (2.19) this is equivalent to having $\beta_i > 0$ and $\lambda_i < 0$ for i={clothing, alcohol}. Therefore, in that case the income (or budget) elasticities (2.21) "will be seen greater than unity at low levels of expenditure, eventually becoming less than unity as the total expenditure increases and the term in λ_i becomes more important. Such commodities therefore have the characteristics of *luxuries* at low levels of total expenditure and *necessities* at high levels" (Banks et al., 1997, p. 534, emphasis added).

Recall from consumer theory that there are two demand curves which do *not* respond identically to a price change. These are Marshallian demand (after the economist Alfred Marshall, 1842–1924) and Hicksian demand (after the economist John Richard Hicks, 1904–1989) curves. Marshallian demand quantifies how the quantity of a commodity demanded change in response to the change of the price of that commodity, *holding income and all other prices constant.* As such they combine both the well-known (to economists) *income* and *substitution effects* of a price change. Thus, Marshallian demand curves can be also called "net" demands because they aggregate the two conceptually distinct consumers' behaviorial responses to price changes.

Hicksian demand function, however, shows how the quantity demanded change with a price of the good, *holding consumer utility constant.* But to hold consumer utility constant (or keep the consumer on the same indifference curve) as prices vary, adjustments to the consumer's income are necessary, i.e., the consumer must be compensated. Therefore, Hicksian demand is called "compensated" demand, and for the analogous reason Marshallian demand is called "uncompensated" demand. The *Slutsky equation* (2.23) is used to calculate the set of compensated price elasticities.

The parameters estimated from the QAIDS model, which are used in the consumption block calibration of FIDELIO are given in Table 2.2. Note that the reported estimates are provided for ten nondurables, where Energy includes Heating and Electricity, and Transport includes Private Transport and Public Transport. Notice that all the presented estimates obey the additivity, homogeneity and symmetry restrictions given in (2.16), (2.17) and (2.18), respectively. And because of the last restriction, for simplicity, we skip all the γ_{ij}'s for all $j > i$. Also observe that since the values of λ_i are almost all zero, then the AIDS specification of the linear

Table 2.2: Parameters of the QAIDS model

Nondurable i	Estimates of the paramaters in (2.13), (2.14) and (2.15)												
	α_i	β_i	λ_i	$\gamma_{i,1}$	$\gamma_{i,2}$	$\gamma_{i,3}$	$\gamma_{i,4}$	$\gamma_{i,5}$	$\gamma_{i,6}$	$\gamma_{i,7}$	$\gamma_{i,8}$	$\gamma_{i,9}$	$\gamma_{i,10}$
Food	0.4576	-0.1061	0.0029	0.099									
Alcohol	0.0756	-0.0040	0.0001	0.022	0.016								
Clothing	0.1033	-0.0055	0.0001	-0.025	-0.005	0.024							
Energy	0.0686	-0.0157	0.0004	-0.008	-0.006	-0.008	0.022						
Transport	0.0867	0.0144	-0.0004	-0.062	-0.005	0.006	0.004	0.050					
Communication	-0.0284	0.0208	-0.0006	0.006	0.001	-0.002	0.004	0.000	-0.003				
Recreation	0.0279	0.0202	-0.0006	-0.005	-0.003	0.001	0.002	0.003	-0.001	0.006			
Health	0.0060	0.0282	-0.0008	-0.008	0.005	0.001	-0.005	-0.003	-0.001	0.001	0.011		
Hotels & restaurants	0.1795	-0.0026	0.0001	-0.024	-0.024	0.009	-0.011	0.012	0.000	-0.002	0.000	0.038	
Other nondurables	0.0231	0.0502	-0.0014	0.006	0.000	-0.001	0.006	-0.005	-0.004	-0.001	-0.001	0.001	-0.001
Sum	1.0000	0.0000	0.0000	0.000	-0.022	0.030	0.023	0.057	-0.009	0.003	0.010	0.040	-0.001

Note: Since the parameters γ_{ij} are symmetric, for simplicity the estimates of $\gamma_{ij} = \gamma_{ji}$ are skipped from the table for all $j > i$. Numbers $j = 1, 2, \ldots, 10$ stand for the nondurable commodities according to the order appearing in the first column of this table, e.g., 1 = Food, 2 = Alcohol, and so forth up to 10 = Other nondurables.

Engel curves for all nondurables seems not to contradict the data. In FIDELIO all these parameters are assumed to be the same for all countries.

Energy and Transport now have to be split into Electricity and Heating, and Private Transport and Public Transport, respectively. For this purpose, for energy splitting first the following regression has been run:

$$\text{Electricity share} = \mathrm{c}_1 + \mathrm{c}_2 \ln(\mathrm{P}_{\text{electricity}}/\mathrm{P}_{\text{heating}}) + \mathrm{c}_3 \ln(\text{Energy}/\mathrm{P}_{\text{energy}}), \quad (2.24)$$

where $\mathrm{P}_{\text{electricity}}$ stands for the price of electricity. Note that (2.24) is nothing else as the AIDS model for two nondurables Electricity and Heating, i.e., (2.13) without its last term. It is given only for Electricity because Heating share will be derived as a residual using the adding-up restriction. Similar approach has been used for computing (calibrating) the share of Private Transport in total Transport, while Public Transport is treated as the residual in this category. Table 2.3 shows the estimates of the parameters for heating and public transport shares equations that are used in FIDELIO for splitting Energy and Transport into their corresponding two components.

Table 2.3: AIDS parameters for splitting Energy and Transport

Share of	c_1	c_2	c_3
Electricity in Energy	0.4216	0.0170	0.0052
Private Transport in Transport	0.8123	0.0537	0.1300

The estimates reported in Table 2.3 are assumed to be the same for all countries.

2.2 Production block

The production block of FIDELIO is based on cost minimization approach, where the cost function has been chosen to have a flexible functional form known in the literature as *transendental logarithmic* function, or *translog* function for short. In applied econometrics flexible functional forms are used for the purpose of modeling second-order effects (e.g., elasticities of substitution) that are functions of the second derivatives of cost, utility or production functions. Given the importance of the translog function, we first provide a brief overview of its derivation, and then discuss FIDELIO nests of the production block.

2.2.1 The translog function

The most popular flexible functional form used in the empirical studies of production is the *translog* function, which is interpreted as a second-order approximation of an *unknown* function of interest. We provide a brief discussion of this function, while for further details the reader is referred to Christensen, Jorgenson and Lau (1973), Bernt and Christensen (1973), Christensen, Jorgenson and Lau (1975), Bernt and Wood (1975), Christensen and Greene (1976), and Greene (2003).

Suppose the function is $y = g(x_1, x_2, \ldots, x_n)$, which can be taken as $\ln y = \ln g(x_1, x_2, \ldots, x_n)$. But since $x_i = \exp(\ln x_i)$, we can interpret the function of interest as a function of the logarithms of x_i's, i.e., $\ln y = f(\ln x_1, \ln x_2, \ldots, \ln x_n)$. Next, expand the last function as a second-order Taylor series around the point $\mathbf{x} = (1, 1, \ldots, 1)'$ so that the expansion point is conveniently (and without loss of generality) taken as zero (i.e, $\ln 1 = 0$). This gives

$$\begin{aligned}\ln y =& f(\mathbf{0}) + \sum_i \left[\frac{\partial f(\cdot)}{\partial \ln x_i}\right]_{\ln \mathbf{x}=\mathbf{0}} \ln x_i \\ &+ \frac{1}{2}\sum_i \sum_j \left[\frac{\partial^2 f(\cdot)}{\partial \ln x_i \partial \ln x_j}\right]_{\ln \mathbf{x}=\mathbf{0}} \ln x_i \ln x_j + \varepsilon,\end{aligned} \tag{2.25}$$

where ε is the approximation error. Since the function and its derivatives evaluated at the fixed value $\mathbf{0}$ are constants, these can be seen as the coefficients in a regression setting and thus one can write (2.25) equivalently as

$$\ln y = \beta_0 + \sum_i \beta_i \ln x_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln x_i \ln x_j + \varepsilon. \tag{2.26}$$

Although (2.26) is a linear regression model, in its role of approximating another function it actually captures a significant amount of curvature. If the unknown function is assumed to be continuous and twice continuously differentiable, then by Young's theorem it must be the case that $\gamma_{ij} = \gamma_{ji}$. This assumption of a theory can be tested in the empirical applications of (2.26). Notice also that the other widely-used Cobb-Douglas function (loglinear model) is a special case of the translog function when $\gamma_{ij} = 0$.

2.2.2 Sectoral output prices and derived input demands

Suppose that production is characterized by a production function $Q = f(\mathbf{x})$ and firms are minimizing their costs subject to a fixed level of production. Assuming perfect competition in the input markets, the input prices $\mathbf{p}$ are taken as given by the firms. This produces optimal *input* (or *factor*) *demands* $x_i = x_i(Q, \mathbf{p})$ and the total cost of production is given by the *cost function*

$$C = \sum_i p_i x_i(Q, \mathbf{p}) = C(Q, \mathbf{p}). \tag{2.27}$$

With *constant returns to scale* assumption, the cost function can be shown to take the form $C = Q \cdot c(\mathbf{p})$, where $c(\mathbf{p})$ is the *unit* or *average cost function.* Hence, $\ln C = \ln Q + \ln c(\mathbf{p})$. From microeconomics we know that the optimal (cost-minimizing) input demands x_i are derived using Shepard's lemma as

$$x_i = \frac{\partial C(Q, \mathbf{p})}{\partial p_i} = \frac{Q \cdot \partial c(\mathbf{p})}{\partial p_i}. \tag{2.28}$$

Using (2.28) we obtain the cost-minimizing *cost share* of input i as follow

$$s_i = \frac{\partial \ln C(Q, \mathbf{p})}{\partial \ln p_i} = \frac{p_i}{c(\mathbf{p})} \frac{\partial c(\mathbf{p})}{\partial p_i} = \frac{p_i x_i}{c(\mathbf{p})}. \tag{2.29}$$

In FIDELIO for producing sectoral outputs five types of factor inputs are distinguished: capital (k), labour (l), total energy inputs (e), imported non-energy inputs (m) and domestic non-energy inputs (d). Denote the corresponding output and input prices, respectively, by p_q, p_k, p_l, p_e, p_m and p_d. Adding time, t, to the translog function (2.26) in order to take the effect of technical progress (namely, total factor productivity (TFP) growth in the unit cost function and factor-biased technical progress) into account, the *unit cost*, or equivalently, *output price function* (i.e., $\ln c(\mathbf{p}) \equiv \tilde{p}_q$) can be written as

$$\tilde{p}_q = \beta_0 + \sum_{i \in \{k,l,e,m,d\}} \beta_i \tilde{p}_i + \alpha_1 t + \alpha_2 t^2 + \frac{1}{2} \cdot \sum_{i,j \in \{k,l,e,m,d\}} \gamma_{ij} \tilde{p}_i \tilde{p}_j + \sum_{j \in \{k,l,e,m,d\}} \rho_{tj} t \tilde{p}_j, \quad (2.30)$$

where tilde indicates the logarithm of the variable it refers to, e.g., $\tilde{p}_d \equiv \ln p_d$. In (2.30) TFP effect is represented by the term $\alpha_1 t + \alpha_2 t^2$, while the factor-biased technical progress is captured by $\rho_{tj} t$ for each factor $i = \{k, l, e, m, d\}$. Next imposing the symmetry condition $\gamma_{ij} = \gamma_{ji}$, the cost shares (2.29) take the following form:

$$\begin{aligned}
s_k &= \beta_k + \gamma_{kk} \tilde{p}_k + \gamma_{kl} \tilde{p}_l + \gamma_{ke} \tilde{p}_e + \gamma_{km} \tilde{p}_m + \gamma_{kd} \tilde{p}_d + \rho_{tk} t, \\
s_l &= \beta_l + \gamma_{kl} \tilde{p}_k + \gamma_{ll} \tilde{p}_l + \gamma_{le} \tilde{p}_e + \gamma_{lm} \tilde{p}_m + \gamma_{ld} \tilde{p}_d + \rho_{tl} t, \\
s_e &= \beta_e + \gamma_{ke} \tilde{p}_k + \gamma_{le} \tilde{p}_l + \gamma_{ee} \tilde{p}_e + \gamma_{em} \tilde{p}_m + \gamma_{ed} \tilde{p}_d + \rho_{te} t, \\
s_m &= \beta_m + \gamma_{km} \tilde{p}_k + \gamma_{lm} \tilde{p}_l + \gamma_{em} \tilde{p}_e + \gamma_{mm} \tilde{p}_m + \gamma_{md} \tilde{p}_d + \rho_{tm} t, \\
s_d &= \beta_d + \gamma_{kd} \tilde{p}_k + \gamma_{ld} \tilde{p}_l + \gamma_{ed} \tilde{p}_e + \gamma_{md} \tilde{p}_m + \gamma_{dd} \tilde{p}_d + \rho_{td} t.
\end{aligned} \quad (2.31)$$

The cost shares in (2.31) must sum to 1, which implies that the following extra conditions must be imposed

$$\sum_i \beta_i = 1 \text{ and } \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = \sum_j \rho_{tj} = 0, \quad (2.32)$$

where all the summations are taken over all factors, i.e., $i, j \in \{k, l, e, m, d\}$.

Conditions (2.32) imply that the cost (or output price) function (2.30) is *homogeneous of degree one in input prices* – a well-behaved property of the cost function that is of theoretical necessity; that is, total cost (price) increases proportionally when *all* input prices increase proportionally. When conditions (2.32) are imposed through, without loss of generality, the share of domestic non-energy materials d, the input prices in the price function (2.30) and the cost shares (2.31) will enter as *relative input prices* with respect to p_d. For simplicity define $\tilde{p}_{id} \equiv \ln(p_i/p_d)$, thus the final prices of sectoral outputs (i.e., (2.30) with restrictions (2.32) imposed) are computed from

$$\begin{aligned}\tilde{p}_{qd} =& \beta_0 + \sum_{i\in\{k,l,e,m\}} \beta_i \tilde{p}_{id} + \alpha_1 t + \alpha_2 t^2 + \frac{1}{2}\cdot \sum_{i\in\{k,l,e,m\}} \gamma_{ii}(\tilde{p}_{id})^2 + \sum_{j\in\{l,e,m\}} \gamma_{kj}\tilde{p}_{kd}\tilde{p}_{jd} \\ &+ \sum_{j\in\{e,m\}} \gamma_{lj}\tilde{p}_{ld}\tilde{p}_{jd} + \gamma_{em}\tilde{p}_{ed}\tilde{p}_{md} + \sum_{j\in\{k,l,e,m\}} \rho_{tj} t \tilde{p}_{jd}, \qquad (2.33)\end{aligned}$$

once all the 21 parameters in (2.33) have been estimated. These parameters are estimated from the following system of equations of factor shares for k, l, e and m, where the factor share of d is dropped due to the homogeneity restriction (and is computed as a residual):

$$\begin{aligned} s_k &= \beta_k + \gamma_{kk}\tilde{p}_{kd} + \gamma_{kl}\tilde{p}_{ld} + \gamma_{ke}\tilde{p}_{ed} + \gamma_{km}\tilde{p}_{md} + \rho_{tk} t, \\ s_l &= \beta_l + \gamma_{kl}\tilde{p}_{kd} + \gamma_{ll}\tilde{p}_{ld} + \gamma_{le}\tilde{p}_{ed} + \gamma_{lm}\tilde{p}_{md} + \rho_{tl} t, \\ s_e &= \beta_e + \gamma_{ke}\tilde{p}_{kd} + \gamma_{le}\tilde{p}_{ld} + \gamma_{ee}\tilde{p}_{ed} + \gamma_{em}\tilde{p}_{md} + \rho_{te} t, \qquad (2.34) \\ s_m &= \beta_m + \gamma_{km}\tilde{p}_{kd} + \gamma_{lm}\tilde{p}_{ld} + \gamma_{em}\tilde{p}_{ed} + \gamma_{mm}\tilde{p}_{md} + \rho_{tm} t. \end{aligned}$$

System (2.34) has been estimated with SURE or GMM (depending of the goodness of fit) on pooled data of 12 countries for *each industry* with the inclusion of country-specific fixed effects. For GMM the instruments are the lagged endogenous variables. The parameters' estimates for a few selected sectors of Austria are presented in Table 2.4, where the intercepts and the error terms of the cost shares equations are combined. The last implies that the reported estimates of $\beta_i + \varepsilon_i$ for $i \in \{k, l, e, m\}$ from (2.34) are nothing else as the *base-year observed shares* of, respectively, k, l, e and m because for the base year all the price and time terms are zero (e.g., $\tilde{p}_{ed} = \ln(p_e/p_d) = \ln(1/1) = 0$). The base-year observed share of the domestic non-energy input can be derived as the residual, if needed. For example, for sector sec01 it is equal to $\overline{\beta}_d = 1-\overline{\beta}_k-\overline{\beta}_l-\overline{\beta}_e-\overline{\beta}_m = 1-0.366-0.074-0.069-0.093 = 0.398$.

The important results derived from the parameters' estimates of the unit cost function reported in Table 2.4 are the *elasticities of substitution* between the factors of production and the *price elasticities of demand*. For the translog cost function the Allen partial elasticities of substitution between inputs i and j are defined as follows:

$$\sigma_{ij} = \frac{c(\mathbf{p}) \cdot [\partial^2 c(\mathbf{p})/(\partial p_i \partial p_j)]}{[\partial c(\mathbf{p})/\partial p_i] \cdot [\partial c(\mathbf{p})/\partial p_j]} = \begin{cases} (\gamma_{ij} + s_i s_j)/(s_i s_j) & \text{if } i \neq j, \\ (\gamma_{ij} + s_i^2 - s_i)/s_i^2 & \text{if } i = j, \end{cases} \tag{2.35}$$

and the price elasticity of demand for factor i with respect to input price j, given output quantity and all other input prices, are derived from

$$\varepsilon_{ij} = s_i \sigma_{ij} = \begin{cases} (\gamma_{ij} + s_i s_j)/s_j & \text{if } i \neq j, \\ (\gamma_{ij} + s_i^2 - s_i)/s_i & \text{if } i = j. \end{cases} \tag{2.36}$$

Observe from (2.35) and (2.36) that although $\sigma_{ij} = \sigma_{ji}$ for $i \neq j$, in general, the price elasticities are not symmetric (i.e., $\varepsilon_{ij} \neq \varepsilon_{ji}$) because the corresponding factor shares are different. Note that a positive (resp. negative) value of σ_{ij} or ε_{ij} implies that factors i and j are substitutes (resp. complements), which is important information for policy objectives. But it should be kept in mind that these are partial equilibrium concepts and miss various crucial and complex feedback mechanisms that are captured in models like FIDELIO (for example, all input prices change). Therefore, the most likely effects on factor demand cannot be directly seen from the computed price elasticities.

Using (2.36) and Table 2.4 we can compute the price elasticities of demand. For example, from the last table we see that for the first sector in the base year the share of its capital is $s_k = 0.366$ and $\gamma_{kk} = -0.091$. Plugging these values in

Table 2.4: Estimates of the translog parameters in (2.33) of selected Austrian industries

Sector	$\overline{\beta}_k$	$\overline{\beta}_l$	$\overline{\beta}_e$	$\overline{\beta}_m$	α_1	α_2	γ_{kk}	γ_{ll}	γ_{ee}	γ_{mm}	γ_{kl}	γ_{ke}	γ_{km}	γ_{le}	γ_{lm}	γ_{em}	ρ_{tk}	ρ_{tl}	ρ_{te}	ρ_{tm}
sec01	0.366	0.074	0.069	0.093	0.013	-0.001	-0.091	0.055	0.050	0.007	0.108	0.011	0.009	-0.004	-0.002	0.022	-0.003	-0.006	0.001	0.001
sec02	0.438	0.061	0.026	0.022	0.013	-0.001	-0.140	0.046	0.020	0.003	0.135	0.032	0.042	0.000	0.003	0.010	-0.003	-0.006	0.001	0.001
sec05	0.230	0.094	0.070	0.120	0.013	-0.001	-0.026	0.068	0.050	0.005	0.063	0.007	-0.001	-0.005	-0.005	0.020	-0.003	-0.006	0.001	0.001
sec10	0.377	0.435	0.069	0.020	-0.018	0.000	-0.043	0.157	0.017	0.004	-0.078	0.007	0.035	0.058	0.012	0.005	0.009	-0.009	0.000	0.000
sec11	0.539	0.183	0.077	0.027	-0.018	0.000	-0.148	0.112	0.018	0.005	0.025	0.005	0.047	0.023	0.004	0.005	0.009	-0.009	0.000	0.000
sec12	0	0	0	0	-0.018	0.000	0	0	0	0	0	0	0	0	0	0	0.009	-0.009	0.000	0.000
sec13	0	0	0	0	-0.018	0.000	0	0	0	0	0	0	0	0	0	0	0.009	-0.009	0.000	0.000
sec14	0.249	0.199	0.111	0.08	-0.018	0.000	0.004	0.119	0.023	0.011	0.007	-0.01	0.008	0.018	-0.01	0.001	0.009	-0.009	0.000	0.000
sec75	0.105	0.564	0.028	0.037	0.011	0.000	0.001	0.115	0.005	0.006	-0.005	-0.001	0.002	0.015	0.009	0.001	-0.002	0.001	0.000	0.000
sec80	0.085	0.751	0.033	0.019	0.014	-0.001	-0.138	0.093	0.019	0.008	0.073	0.015	0.016	-0.001	-0.007	0.003	0.004	-0.008	0.000	0.001
sec85	0.113	0.523	0.036	0.100	-0.005	0.001	0.044	0.087	0.034	0.027	-0.024	-0.005	-0.010	-0.021	-0.023	0.009	0.002	-0.004	0.000	0.000
sec90	0.231	0.288	0.036	0.019	0.011	0.000	-0.030	0.094	0.021	0.004	0.031	-0.002	0.010	-0.009	0.005	0.009	0.004	-0.006	0.000	0.001
sec91	0.083	0.467	0.065	0.046	0.011	0.000	0.002	0.068	0.036	0.008	-0.004	-0.003	0.001	-0.028	-0.005	0.014	0.004	-0.006	0.000	0.001
sec92	0.290	0.293	0.030	0.073	0.011	0.000	-0.055	0.094	0.018	0.011	0.037	0.000	-0.003	-0.007	-0.011	0.006	0.004	-0.006	0.000	0.001
sec93	0.341	0.342	0.029	0.045	0.011	0.000	-0.082	0.093	0.017	0.008	0.027	0.000	0.005	-0.008	-0.003	0.006	0.004	-0.006	0.000	0.001
sec95	0	1	0	0	0.015	0.000	0	0	0	0	0	0	0	0	0	0	0.000	-0.002	0.000	0.000

Note: The cost shares errors and the corresponding intercepts are combined, i.e., $\overline{\beta}_i = \beta_i + \varepsilon_i$ for $i \in \{k, l, e, m\}$. Since for the base year all other price and time terms of the factor shares equations are zero, $\overline{\beta}_i$ is the base-year observed share of input i.

(2.36) gives $\varepsilon_{kk} = -0.833$, which is negative as expected. Similarly, together with $\gamma_{kl} = \gamma_{lk} = 0.108$ and $s_l = 0.074$ for sec01, we obtain $\varepsilon_{lk} = 0.370$ and $\varepsilon_{kl} = 1.817$. Hence, capital and labour in sec01 are substitutes, but demand for capital is more sensitive to changes in labour price than the reaction of labour demand to changes in capital price.

2.3 Labour market

2.3.1 Demands for labour skill types

Three labour skills are modelled: high-, medium- and low-skilled. Labour demand for skill types is also modelled by a translog model and can be seen as a *second nest* to the modeling of the factor demands for capital, labour, energy, imported and domestic non-energy inputs described in Chapter 2.2.2. While the unit cost in the first nest of factor demands system was the sectoral *output price* function (2.33), its counterpart in the second nest of the labour market is the *wage rate per hour* which then defines the *labour price* (index).

The unit cost function in the second nest of the labour demands for skill types defines the *wage earned per hour* (W) as

$$\begin{aligned} \ln W =& \beta_0 + \sum_{i\in\{l,h\}} \beta_i \tilde{w}_{im} + \ln w_m + \alpha_1 t + \alpha_2 t^2 + \frac{1}{2}\cdot \sum_{i\in\{l,h\}} \gamma_{ii}(\tilde{w}_{im})^2 \\ &+ \gamma_{lh}\tilde{w}_{lm}\tilde{w}_{hm} + \sum_{j\in\{l,h\}} \rho_{tj} t \tilde{w}_{jm}, \end{aligned} \tag{2.37}$$

where h, m and l refer to, respectively, high-, medium- and low-skilled labour, and the hourly wages of high-skilled and low-skilled labour are defined relative to the hourly wages of medium-skilled labour, i.e., $\tilde{w}_{im} \equiv \ln(w_i/w_m)$ for $i = \{l, h\}$. In is important to note that in FIDELIO all the wages in (2.37) are *industry-specific* (as is true with respect to many variables discussed so far in this chapter), but for the sake of simplicity we do not mention industry dimension explicitly. The TFP terms in this equation capture technical progress in the organization of how labour inputs of different skill levels are combined in the total labour input of firms. Applying Shepard's lemma to the aggregate hourly wage equation (2.37) results in the following *labour types shares* (demand) equations:

$$\begin{aligned} v_l &= \beta_l + \gamma_{ll}\tilde{w}_{lm} + \gamma_{lh}\tilde{w}_{hm} + \rho_{tl}t \\ v_h &= \beta_h + \gamma_{lh}\tilde{w}_{lm} + \gamma_{hh}\tilde{w}_{hm} + \rho_{th}t. \end{aligned} \tag{2.38}$$

Note that homogeneity restrictions of the type (2.32) are already imposed in (2.37) and (2.38), hence the share equation of the medium-skilled labour is removed from the last equation. The deterministic time trend in (2.38) captures the *skill-biased technical progress*. Skill-biased technical change is a shift in the production technology that favors skilled (e.g., more experienced, more educated) labour over unskilled labour by increasing its relative productivity and, therefore, its relative demand. All other things being equal, skill-biased technical change induces a rise in the *skill premium* - the ratio of skilled to unskilled wages.

Table 2.5: Parameters of the translog labour price function (2.37)

Sector	β_0	β_l	β_h	α_1	α_2	γ_{ll}	γ_{hh}	γ_{lh}	ρ_{tl}	ρ_{th}
sec01	-0.1783	-0.0150	0.6280	-0.0024	0.0005	-0.0246	-0.2987	0.7972	-0.0067	0.0082
sec02	-0.1783	-0.0150	0.6280	-0.0024	0.0005	-0.0246	-0.2987	0.7972	-0.0067	0.0082
sec05	-0.1783	-0.0150	0.6280	-0.0024	0.0005	-0.0246	-0.2987	0.7972	-0.0067	0.0082
sec10	-0.1010	0.7019	0.0682	0.0004	-0.0004	0.9558	0.0804	-0.0263	-0.0068	0.0048
sec11	-0.1010	0.7019	0.0682	0.0004	-0.0004	0.9558	0.0804	-0.0263	-0.0068	0.0048
sec91	-0.4968	-0.0319	0.2824	0.0039	0.0000	0.3849	0.3667	0.4380	0.0023	0.0026
sec92	-0.4968	-0.0319	0.2824	0.0039	0.0000	0.3849	0.3667	0.4380	0.0023	0.0026
sec93	-0.4968	-0.0319	0.2824	0.0039	0.0000	0.3849	0.3667	0.4380	0.0023	0.0026

The parameters of equation (2.37) have been estimated using SURE on pooled data over 27 countries for *each* industry with country-specific fixed effects. The results are reported in Table 2.5 for selected sectors. In case the estimated parameters did not yield negative own price elasticities, these elasticities have been set equal to zero and not restricted. The philosophy behind such estimation strategy is that substitution between different segments of the labour market is a much stronger "null hypothesis" than between the first nest production factors (i.e., between capital, labour, energy inputs, domestic and imported non-energy inputs) and should be assumed *not* to exist, if it cannot be proven with the data.

2.3.2 Wage curves

Following a meta analysis of Folmer (2009), the sectoral hourly wages by skill type, i.e., w_i's in (2.37) and (2.38), are derived from the so-called *wage curves* (or functions). The wage curve describes the responsiveness of individual real wages to the changing *local* market conditions. In their pathbreaking work, Blanchflower and Oswald (1994) found that there is a stable relationship in the form of downward-sloping convex curve between local unemployment and the level of wages, which establishes the so-called empirical "law" of economics they characterized as "the wage curve". They also argue that for most purposes, the wage curve relation is well approximated by a simple log-linear function.

Based on the detailed review of some hundred articles, books and working papers, Folmer (2009) gathers and computes 1004 elasticities of pay in disaggregated (industry of labour market segment) macro equations. In FIDELIO three wage elasticities reported in the mentioned study are used. These are related to *labour productivity* (Q/L), *consumer price* (PC) and *unemployment rate by skill level* (UR). That is, the first two explanatory variables are invariant with respect to the labour skill type, while the last is labour skill-specific variable. The log-linear versions of the (long-run) wage equations for labour skill type $i = \{l, m, h\}$ that need to be used in the empirical applications are given by

$$\ln w_i = constant_i + \tau_1 \ln(Q/L) + \tau_2 \ln(PC) + \tau_i \ln(UR_i). \qquad (2.39)$$

Again we note that for simplicity of exposition (and to be consistent with previous section notations) we dropped the industry dimension from w_i and the constant term $constant_i$ in (2.40), and country dimension for all the mentioned variables. The constant term accounts for the unexplained industry component in determination of wages by skill level.

In FIDELIO, however, wage curves determine yearly *wages per employee by skill level*, similar to Folmer (2009) meta analysis, and not hourly wages as given in (2.39). That is, we use instead

$$\ln wem_i = constant_i + \tau_1 \ln(Q/L) + \tau_2 \ln(PC) + \tau_i \ln(UR_i), \qquad (2.40)$$

where wem_i denotes wage per employee of labour skill type $i = \{l, m, h\}$. The reasons for using wem_i instead of w_i in the wage curve are mainly two-fold:

1. The wage elasticities in (2.39) and (2.40) "are similar, given that extra remunerations are proportional to the hourly wage rate and that changes in total number of hours worked are taken into account in the yearly wage equation" (Folmer 2009, p. 54).
2. By dealing with costs by employee for labour demand, we take into account certain short-term fixed characteristics of labour cost and the input of labour cannot adjust immediately. Therefore, changing hours per employee becomes a way of adjusting labour (like the German "Kurzarbeit" during the recent recession) or a policy variable.[3]

[3] This design of the wage rate is therefore also chosen with the perspective of a possible simulation of shortening working time. If we do so in FIDELIO, we would on the one hand (ceteris paribus) have the pure calculation effect of distributing given quantity of labour across more employees and on the other hand also the effect of a correspondingly higher wage costs, so that demand for labour will adjust. This latter effect can be seen in analogy to the "rebound effect" of energy efficiency increases.

Thus, the hourly wage by skill type w_i is then simply derived by dividing the wages per employee wem_i by the so-called working time (exogenous) variable which is nothing else as the total hours worked per employee (and varies by sector and region).

The estimates of the required parameters in (2.40) are the benchmark values of the corresponding long-term (average) elasticities reported in Folmer (2009, pp. 38-39). The information on the reported short-term elasticities values have been ignored because the differences between the two are rather small. Folker makes the following breakdown of the countries:

- The Netherlands
- Anglo-Saxon countries: Australia, Canada, New Zealand, UK, USA
- Nordic countries: Denmark, Finland, Norway, Sweden
- All other countries

The values of τ_1, τ_2 and $\tau_3 = \tau_l = \tau_m = \tau_h$ in (2.40) are assumed to be the same for all labour skill types and the corresponding estimates reported in Folmer (2009) are presented in Table 2.6. From the table we observe that whenever unemployment increases, wages in all regions/coutnries decrease. The corresponding elasticity is lowest for Anlgo Saxon countries (-4.3%) and much higher (in absolute terms) for the remaining countries (from -7.9% to -9.0%). This might be explained by the fact that the Anglo Saxon countries have less generous welfare state than the others. "The lower the real unemployment benefits, the stronger the incentive to look for a new job in case of unemployment and the lower the reservation wage. So an additional rise in unemployment may have lesser impact on search intensity and wages than in more generous welfare states like those of Nordic countries and The Netherlands" (Folmer, 2009, p. 40).

The wage equations by skill and industry in each region (group of regions) comprise an industry component that cannot be explained by differences in the skill

Table 2.6: Elasticities of the wage curves in (2.40)

Regions	τ_1	τ_2	τ_3
AUT/BEL/LUX/NLD	0.832	0.584	-0.080
Anglo Saxon	0.921	0.580	-0.043
Nordic countries	0.921	0.520	-0.090
Other countries	0.921	0.580	-0.079

Source: Folmer (2009, pp. 38-39).

structure and labour productivity of industries. The literature on inter-industry wage differentials in Europe (Du Caju et al., 2010) shows the significant magnitude of this differential and reveals the role of industry specific factors as rent-sharing mechanisms. Though from an academic point of view the unresolved puzzles of inter-industry wage differentials might be seen as disappointing, the integration of this component in an inter-industry model like FIDELIO adds an important degree of differentiation.

It is important to note what the wage curve does *not* represent. First, the wage curve is not the famous *Phillips curve* traditionally taught in macroeconomics courses, named after Alban Phillips (see Phillips, 1958). This is because in wage curve contemporaneous unemployment rate determines the *level* of wages (adjusted for permanent market-specific differentials), while Phillips curve postulates the existence of negative relation (or short-term tradeoff) between the contemporaneous unemployment rate and the *rate of change* of wages. Second, Blanchflower and Oswald (1994) argue that their wage curve is not a labour supply function. This could be a plausible interpretation because as “short run changes in employment and unemployment are approximately mirror images, a finding that wages rise with contemporaneous reductions in unemployment may simply reflect movements along an upward-sloping labor supply function” (Card, 1995, p. 795). This interpretation is rejected by Blanchflower and Oswald empirical results which strongly support the view that it is local unemployment, and not local employment or the size of the local labour force, that effects wages. Since most economists think of labour supply curve in terms of total quantity of labour supplied as a function of wages, these findings seem, indeed, to be inconsistent with the second mentioned interpretation. Rejecting both interpretations, Blanchflower and Oswald (1994) argue that their findings represents something new, which thus they call "the wage curve".

In his excellent review of Blanchflower and Oswald's work, Card (1995) concluded that "the existence of a wage curve relation is an important addition to our knowledge about the modern labor market. ... Many readers will be stimulated by the conclusion that the wage curve is "something new": a surrogate supply function that can be combined with a simple demand curve to yield interesting models of the labor market; a challenge to orthodox theories of supply and demand" (p. 798). It is namely in this context that the wage curve is used in FIDELIO, i.e., the "surrogate supply functions" (2.40) together with the derived labour demands (2.38) determine the "equilibrium" quantities of labour skill types.

Chapter 3

Derivation of the base-year data

In this chapter we discuss the relationships among and the derivations of all the necessary variables, mainly for the base year. All the major data sources, which make the core dataset of FIDELIO, are listed in Chapter 5. In the Appendix we also give the definitions all the variables used in FIDELIO, which further contributes to the easier processing of the material presented in this and the next chapters.

3.1 Basic price data

The base-year *use table* at basic prices is, by definition, derived from the use table at purchasers' prices adjusted for the trade and transport margins and taxes less subsidies, i.e.,

$$\mathrm{USE_{bp}(r,g,u) = USE_{pp}(r,g,u) - MRG(r,g,u) - TXS(r,g,u).} \tag{3.1}$$

The values of *gross outputs* (i.e., gross outputs in nominal terms) by region and sector are obtained from the corresponding make matrix by summing over all goods:[1]

[1]Note that Q does not mean "quantities in physical terms" as, for example, in CGE modelling, but denotes "quantities in nominal terms". We could have used VQ instead of Q to indicate this

$$\mathrm{Q(r,s)} = \sum_{\mathrm{g}} \mathrm{MAKE(r,g,s)}. \tag{3.2}$$

Total value added at basic prices can be derived as a residual, i.e.,

$$\mathrm{VA(r,s)} = \mathrm{Q(r,s)} - \sum_{\mathrm{g}} \mathrm{USE_{pp}(r,g,s)}. \tag{3.3}$$

3.2 Shares and structure matrices

In FIDELIO sectoral outputs are computed from the demands for domestically produced goods based on the assumption of constant market shares. The corresponding *market share matrix* shows the proportions (shares) of industries' outputs in the production of each good, and is defined as

$$\mathrm{MKSH(r,g,s)} = \mathrm{MAKE(r,g,s)} / \sum_{\mathrm{s}} \mathrm{MAKE(r,g,s)}. \tag{3.4}$$

Hence, by definition, for each good and each region the sum of the market shares over all sectors equals unity. Equation (3.4) is defined only for goods that are produced domestically. The entries corresponding to zero domestic outputs of the market share matrix are set to zero. Without further mentioning, such a rule with zero denominators is applied to all the relevant cases (formulas) discussed below.

The *total intermediate use at purchasers' prices* for each region and each user (which refers to sectors and final demand categories) is obtained as

$$\mathrm{S(r,u)} = \sum_{\mathrm{g}} \mathrm{USE_{pp}(r,g,u)}. \tag{3.5}$$

Note from (3.5) that S(r,s) represents base-year *demand for total intermediate inputs* by sector s located in region r.

fact, but the last adapted notation is simpler.

Using (3.2) and (3.5), the *total intermediate inputs shares* are obtained from

$$\mathrm{S\_Q(r,s) = S(r,s)/Q(r,s).} \tag{3.6}$$

FIDELIO's production block distinguishes between energy (E) and non-energy (N) intermediate inputs. The *total energy inputs* include domestic and imported intermediate inputs of five energy goods:

$$\mathrm{E(r,s)} = \sum_{\mathrm{ge}} \mathrm{USE_{pp}(r,ge,s)}, \tag{3.7}$$

where *ge* (energy goods identifier) refers to products "Coal and lignite; peat", "Crude petroleum and natural gas", "Uranium and thorium ores", "Coke, refined petroleum production and nuclear fuels", and "Electrical energy, gas, steam and hot water".

Similarly, *total non-energy inputs* is the aggregate of domestic and imported inputs of all non-energy products, i.e.,

$$\mathrm{N(r,s)} = \sum_{\mathrm{gne}} \mathrm{USE_{pp}(r,gne,s)}. \tag{3.8}$$

The *energy inputs shares in gross outputs* by sectors and regions are then defined as

$$\mathrm{E\_Q(r,s) = E(r,s)/Q(r,s).} \tag{3.9}$$

Consequently, the *shares of non-energy inputs* can be obtained as residuals, i.e.,

$$\mathrm{N\_Q(r,s) = S\_Q(r,s) - E\_Q(r,s).} \tag{3.10}$$

To distinguish between domestic and imported non-energy intermediate inputs, we need the *import shares* matrix which shows the proportion of total imported products from the corresponding basic price uses, i.e.,

$$\mathrm{MSH(r,g,u) = MUSE(r,g,u)/USE_{bp}(r,g,u).} \tag{3.11}$$

The matrices of *imported non-energy inputs* (M) and *domestic non-energy inputs* (D) are derived using the import shares matrix (3.11) as follows:

$$\mathrm{M(r,s)} = \sum_{\mathrm{gne}} \mathrm{USE_{pp}(r,gne,s)} \cdot \mathrm{MSH(r,gne,s)}, \tag{3.12}$$

$$\mathrm{D(r,s)} = \sum_{\mathrm{gne}} \mathrm{USE_{pp}(r,gne,s)} \cdot \big[1 - \mathrm{MSH(r,gne,s)}\big]. \tag{3.13}$$

Note that, by definition, $\mathrm{N(r,s) = D(r,s) + M(r,s)}$. The base-year shares of M and D in gross outputs are, respectively,

$$\mathrm{M\_Q(r,s) = M(r,s)/Q(r,s)}, \tag{3.14}$$

$$\mathrm{D\_Q(r,s) = D(r,s)/Q(r,s)}. \tag{3.15}$$

The *shares of labour compensation*, which includes wages and social security contributions, in gross output are obtained from the base-year value added components matrix, VAC, as

$$\mathrm{L\_Q(r,s)} = \big[\mathrm{VAC(r,wage,s) + VAC(r,soc.sec.cont,s)}\big]/\mathrm{Q(r,s)}. \tag{3.16}$$

The sum of all value added components without labour compensation is defined as *capital compensation* or *cash flow*. That is, the value of capital inputs is the sum of production taxes net of production subsidies, depreciation and operating profits. Given the input-output expenditure-side accounting identity, the *share of capital in gross output* can be derived as the residual:

$$\mathrm{K\_Q(r,s) = 1 - S\_Q(r,s) - L\_Q(r,s)}. \tag{3.17}$$

The *shares of employers' social security contributions in total labour compensation* are

$$\mathrm{SSC\_L(r,s)} = \frac{\mathrm{VAC(r,soc.sec.cont,s)}}{\mathrm{VAC(r,wage,s) + VAC(r,soc.sec.cont,s)}}. \tag{3.18}$$

The *shares of production taxes in gross outputs* is obtained from

$$\mathrm{TXP\_Q(r,s) = VAC(r,prdn.tax,s)/Q(r,s)}. \tag{3.19}$$

Similarly, the base-year *shares of production subsidies* and *of depreciation* in gross outputs are given, respectively, by

$$\mathrm{SBP\_Q(r,s) = VAC(r, prdn.sub, s)/Q(r,s)}, \tag{3.20}$$

$$\mathrm{DPR\_Q(r,s) = VAC(r, depr, s)/Q(r,s)}. \tag{3.21}$$

The *shares of investment goods in gross outputs* that could be used in deriving investment demands are obtained as

$$\mathrm{I\_Q(r,s)} = \sum_{\mathrm{g}} \mathrm{INV_{pp}(r,g,s)/Q(r,s)}. \tag{3.22}$$

The option that is used in FIDELIO for deriving investment demands consists of linking investments to sectoral capital stocks using the *investment-capital stock ratios* from the base-year data, i.e.,

$$\mathrm{I\_K(r,s)} = \sum_{\mathrm{g}} \mathrm{INV_{pp}(r,g,s)/KS(r,s)}, \tag{3.23}$$

where KS(r,s) is the *capital stock* of sector s in region r.

The *product use structure* matrix of a region shows for each user the purchasers' price values of the uses of imported and domestic goods per total its (intermediate or final) used goods, hence is defined as

$$\mathrm{USTR(r,g,u) = USE_{pp}(r,g,u)} / \sum_{\mathrm{g}} \mathrm{USE_{pp}(r,g,u)}. \tag{3.24}$$

Similar structure matrices are defined for energy and non-energy inputs separately. Using (3.7), the matrix of *product structure of energy inputs* is obtained from

$$\mathrm{ESTR(r,g,s)} = \begin{cases} \mathrm{USE_{pp}(r,g,s)/E(r,s)} & \text{if } \mathrm{g = ge}, \\ 0 & \text{if } \mathrm{g = gne}. \end{cases} \tag{3.25}$$

Similarly, using total non-energy intermediate inputs (3.8) and the use table at purchasers' prices, the *product structure of non-energy inputs* is derived as

$$\mathrm{NSTR(r,g,s)} = \begin{cases} \mathrm{USE_{pp}(r,g,s)/N(r,s)} & \text{if } \mathrm{g = gne}, \\ 0 & \text{if } \mathrm{g = ge}. \end{cases} \tag{3.26}$$

Using (3.11) and (3.12), the *product structure of imported non-energy inputs* is obtained from

$$\mathrm{MSTR(r,g,s)} = \begin{cases} \left[\mathrm{USE_{pp}(r,g,s) \cdot MSH(r,g,s)}\right]/\mathrm{M(r,s)} \;\text{ if } \mathrm{g = gne}, \\ 0 \;\text{ if } \mathrm{g = ge}. \end{cases} \tag{3.27}$$

Similarly, using the import share matrix (3.11) and the matrix of domestic non-energy intermediate inputs (3.13), the *product structure of domestic non-energy inputs* is derived as follows:

$$\mathrm{DSTR(r,g,s)} = \begin{cases} \left[\mathrm{USE_{pp}(r,g,s) \cdot (1 - MSH(r,g,s))}\right]/\mathrm{D(r,s)} \;\text{ if } \mathrm{g = gne}, \\ 0 \;\text{ if } \mathrm{g = ge}. \end{cases} \tag{3.28}$$

Note that for each region r and each sector s the sum over all goods of the structure matrices ESTR, NSTR, MSTR and DSTR is unity, by construction. This fact is used later in the allocation of aggregate inputs of energy, imported and domestic non-energy materials over all products in order to compute the sectoral demands for intermediate goods at purchasers' prices.

The *product structure of investments* (at purchasers' prices) is defined as

$$\mathrm{ISTR(r,g,s) = INV_{pp}(r,g,s)} / \sum_{\mathrm{g}} \mathrm{INV_{pp}(r,g,s)}. \tag{3.29}$$

For each region and each user, the *product tax net of subsidy rate* is given by

$$\mathrm{TXSR(r,g,u) = TXS(r,g,u)/USE_{pp}(r,g,u)}. \tag{3.30}$$

The *rates of total margins (trade and transport) paid on non-margin goods* are obtained from

$$\begin{aligned} &\mathrm{MGR_{paid}(r,g,u)} \\ &\quad = \begin{cases} \mathrm{MRG(r,g,u)}/\left[\mathrm{USE_{pp}(r,g,u) - TXS(r,g,u)}\right] \;\text{ if } \mathrm{g = gnm}, \\ 0 \;\text{ ifg} = \mathrm{gm}. \end{cases} \end{aligned} \tag{3.31}$$

The *share of total margins received by a margin good* (i.e., share of a margin good in total margins) is computed from

$$\mathrm{MGS_{rec.}(r,g,u)} = \begin{cases} \mathrm{MRG(r,g,u)} / \sum_{\mathrm{gm_1}} \mathrm{MRG(r,gm_1,u)} \ \ \text{if } \mathrm{g = gm}, \\ 0 \ \ \text{if } \mathrm{g = gnm}. \end{cases} \tag{3.32}$$

For simplicity of exposition, let us define for each good and each user, the shares of total margins in the corresponding uses at basic prices plus margins as follows:

$$\mathrm{MG\_MGU(r,g,u)} \equiv \mathrm{MRG(r,g,u)} / \left[\mathrm{USE_{bp}(r,g,u)} + \mathrm{MRG(r,g,u)}\right]. \tag{3.33}$$

Then using the margin shares of margin goods (3.32), the matrix of *price structure of domestically produced goods* can be defined as

$$\begin{aligned} &\mathrm{PSTR(r,g,u,g_1)} \\ &\quad = \begin{cases} 1 \text{ if } \mathrm{g = g_1 = gm}, \\ 1 - \mathrm{MG\_MGU(r,g,u)} \ \ \text{if } \mathrm{g = g_1 = gnm}, \\ \mathrm{MG\_MGU(r,g,u)} \cdot \mathrm{MGS_{rec.}(r,g_1,u)} \text{ if } \mathrm{gn = g \neq g_1 = gm}, \\ 0 \ \ \text{otherwise}. \end{cases} \end{aligned} \tag{3.34}$$

Note that $\sum_{\mathrm{g_1}} \mathrm{PSTR(r,g,u,g_1)} = 1$ for each region r, each good g and each user u. For each margin good $g = gm$, PSTR has only one positive element always equal to unity corresponding to the same margin good. But, for each non-margin good $g = gn$ in $\mathrm{PSTR(r,g,u,g_1)}$ we have the following positive elements: (a) position (gn, gn) indicates the proportion of gn's use at basic prices in its use at basic prices plus margins, and (b) positions $(gn, gm$'s$)$ indicate the proportion of gn's margins in its use at basic prices plus margins that is distributed over all margin goods gm's according to their shares in generating total margins. This price structure matrix along with information on the rates of products' net taxes (3.30) will be used in the model simulations to translate basic prices of domestic goods g_1 into the purchasers' prices of domestically produced goods g.

The following regional shares are based on the data on financial balances of households (FBHH). Taxes on households will be determined from *income tax rate* that is defined as

$$\mathrm{Tx\_Inc(r)} = \frac{\mathrm{FBHH(r, taxes)}}{\mathrm{FBHH(r, wage)} + \mathrm{FBHH(r, oper.surp)} - \mathrm{FBHH(r, soc.con.gov)}}. \tag{3.35}$$

Note that FBHH data are obtained from National Accounts (NA) sources and not from the supply and use tables (SUTs). As such the SUTs-based data need to be re-scaled so as to match those of the NA data. For this purpose, simple scale ratios are used. These are the NA to input-output (IO, or SUT) ratios of wages, social security contributions (Sscn) and operating surplus (Opsp):

$$\mathrm{Wage_{na.io}(r)} = \frac{\mathrm{FBHH(r, wage)}}{\sum_{\mathrm{s}} [\mathrm{VAC(r, wage, s)} + \mathrm{VAC(r, soc.sec.cont, s)}]}, \tag{3.36}$$

$$\mathrm{Sscn_{na.io}(r)} = \frac{\mathrm{FBHH(r, soc.con.gov)}}{\sum_{\mathrm{s}} \mathrm{VAC(r, soc.sec.cont, s)}}, \tag{3.37}$$

$$\mathrm{Opsp_{na.io}(r)} = \frac{\mathrm{FBHH(r, oper.surp)}}{\sum_{\mathrm{s}} \mathrm{VAC(r, oper.surp, s)}}. \tag{3.38}$$

3.3 Trade matrix construction

There are two options for taking into account the costs for third countries to transit *non-service* goods from one region to another. A simple and *ad hoc* option is to use the data for *transit costs* powers[2] that are computed as follows

$$\mathrm{TNCS(r, r_1, g)} = 1 + 0.06 \cdot \frac{\mathrm{Distance(r, r_1)}}{\mathrm{MaxDist}}, \tag{3.39}$$

where MaxDist is the maximum distance between a pair of regions included in the model, except for the rest of the world (RoW). The number 0.06 in (3.39) simply means that the transit costs are assumed not to be larger than 6% of the value

[2]Power of a variable is defined as one plus the rate of change (or interest) of that variable.

of any transaction between countries. Transit costs of all non-service goods to the RoW are set to 1.08 for all regions. For service goods transit costs are zero, hence the corresponding TNCS elements are set to unity.

A more realistic estimates of the international transit costs, that are also used in FIDELIO-1, are derived by Streicher and Stehrer (2012). In contrast to (3.39), for each pair of the exporting and importing countries, these estimates also vary across (non-service) goods. We do not discuss the details of these transit costs derivation here and refer the interested reader to the mentioned study.

Let us denote the WIOD trade matrix by $\mathrm{TRD_{wiod}(r, r_1, g, u)}$, where the first region r is the importing (destination) region, while the second region r_1 is the exporting (source) region. Further, the total EU-imports of each region r given in $\mathrm{MUSE_{eu}(r, g, u)}$ are expressed in national currencies. The initial *trade matrix at CIF prices* expressed in euros is obtained as

$$\mathrm{TRD_{cif}(r, r_1, g, u)} = \begin{cases} 0 \quad \text{if } \mathrm{r = r_1}, \\ \frac{\mathrm{MUSE_{eu}(r,g,u)}}{\mathrm{Xrate(r)}} \cdot \frac{\mathrm{TRD_{wiod}(r,r_1,g,u)}}{\sum_{\mathrm{r_2 \neq r}} \mathrm{TRD_{wiod}(r,r_2,g,u)}} \; \text{ if } \mathrm{r \neq r_1} \end{cases} \tag{3.40}$$

where r is the destination region and r_1 is the source region. In (3.40) the WIOD's trade matrix $\mathrm{TRD_{wiod}}$ is used for the distribution of imports to the trading partners and is not used itself because the product import shares of all sectors in this matrix are identical, while a more realistic picture of the imports is given in the use tables; hence both data are used in (3.40).

The matrix of *trade in FOB prices* is then obtained by taking into account the corresponding transit costs, TNCS:

$$\mathrm{TRD\_fob(r, r_1, g, u) = TRD_{cif}(r, r_1, g, u)/TNCS(r, r_1, g)}. \tag{3.41}$$

However, then one needs to ensure that in the derived trade matrices (3.40) and (3.41): (i) all imports are accounted for (i.e., the sum over all trading partners' shares equals one) and (ii) on the exporting partners' side, the sum of CIF-imports by the trading partners corrected for international trade and transport margins has to equal the export vector valued at FOB (here it is the exports within the EU area, exp_eu). To ensure these two conditions, exports to and imports from the RoW are taken as residuals. The analytical representation of this procedure is given by the following steps:

start loop

$$\mathrm{TRD_{cif}(r, r_1, g, u)} = \frac{\mathrm{MUSE_{eu}(r, g, u)}}{\mathrm{Xrate(r)}} \cdot \frac{\mathrm{TRD_{cif}(r, r_1, g, u)}}{\sum_{\mathrm{r_2}} \mathrm{TRD_{cif}(r, r_2, g, u)}},$$

Re-compute $\mathrm{TRD_{fob}(r, r_1, g, u)}$ using Equation (3.41),

$$\mathrm{TRD_{fob}(r, r_1, g, u)} = \mathrm{TRD_{fob}(r, r_1, g, u)} \cdot \min\left[1,\ \frac{\mathrm{USE_{pp}(r_1, g, exp\_eu)/Xrate(r_1)}}{\sum_{\mathrm{r_2, u_1}} \mathrm{TRD_{fob}(r_2, r_1, g, u_1)}}\right],$$

$$\mathrm{TRD_{cif}(r, r_1, g, u)} = \mathrm{TRD_{fob}(r, r_1, g, u)} \cdot \mathrm{TNCS(r, r_1, g)}$$

end loop

until the trade matrices at CIF prices $\mathrm{TRD_{cif}(r, r_1, g, u)}$ from the first and final steps of a particular loop (described above) converge to each other. Then we obtain again $\mathrm{TRD_{fob}}$ using (3.41). Adding the RoW as additional region to the trade matrices, the imports of an EU-region r from the RoW is computed as the difference between the total imports and the EU-imports of region r:

$$\mathrm{TRD_{cif}(r, row, g, u)} = \frac{\mathrm{MUSE(r, g, u)}}{\mathrm{Xrate(r)}} - \sum_{\mathrm{r_1}} \mathrm{TRD_{cif}(r, r_1, g, u)}. \tag{3.42}$$

Then the new $\mathrm{TRD_{fob}}$ is obtained by plugging (3.42) in (3.41) which now also includes the RoW region. Using this new $\mathrm{TRD_{fob}}$ matrix, total exports of an EU-region r to the RoW is again obtained as a residual ('exp' stands for total exports):

$$\mathrm{TRD_{fob}(row, r, g, exp)} = \frac{\mathrm{USE_{pp}(r, g, exp)}}{\mathrm{Xrate(r)}} - \sum_{\mathrm{r_1, u}} \mathrm{TRD_{fob}(r_1, r, g, u)}. \tag{3.43}$$

From (3.43) we ultimately find the vector of *exports to the RoW in national currencies*, i.e., exports that are not accounted for by trade within the model regions (which will be taken as exogenous in the baseline simulation):

$$\mathrm{EXP}_{\mathrm{row}}(\mathrm{r},\mathrm{g}) = \mathrm{TRD}_{\mathrm{fob}}(\mathrm{row},\mathrm{r},\mathrm{g},\mathrm{exp}) \cdot \mathrm{Xrate}(\mathrm{r}). \tag{3.44}$$

For region r its *trading partners' import shares* in its total imports by each user and each good is defined as

$$\mathrm{TMSH}(\mathrm{r},\mathrm{rt},\mathrm{g},\mathrm{u}) = \frac{\mathrm{TRD}_{\mathrm{cif}}(\mathrm{r},\mathrm{rt},\mathrm{g},\mathrm{u})}{\mathrm{MUSE}(\mathrm{r},\mathrm{g},\mathrm{u})/\mathrm{Xrate}(\mathrm{r})}. \tag{3.45}$$

Note that the first step of the trade matrix adjustment procedure outlined above guarantees that $\sum_{\mathrm{rt}} \mathrm{TMSH}(\mathrm{r},\mathrm{rt},\mathrm{g},\mathrm{u}) = 1$ for all r, g and u. However because of the problem of memory, the number of users needs to be reduced in the trade matrix. Therefore, the *reduced matrix of trading partners' import shares* is defined only for seven trade users (utr), which are all intermediate users as one sector (st) and six final demanders (f). Hence,

$$\mathrm{TMSH}_{\mathrm{red}}(\mathrm{r},\mathrm{rt},\mathrm{g},\mathrm{utr}) = \begin{cases} \mathrm{TMSH}(\mathrm{r},\mathrm{rt},\mathrm{g},\mathrm{f}) & \text{if } \mathrm{utr} = \mathrm{f}, \\ \sum_{\mathrm{s}} \mathrm{TMSH}(\mathrm{r},\mathrm{rt},\mathrm{g},\mathrm{s}) & \text{if } \mathrm{utr} = \mathrm{st}. \end{cases} \tag{3.46}$$

3.4 COICOP-CPA bridge matrices

The base-year and the required lagged data on private consumption of COICOP commodities C(r,c,t) are derived from the national accounts information. For FIDELIO purposes, the 47 components of the individual consumption expenditure of households of the COICOP classification at 3-digit level are aggregated to 17 commodities, from which 12 are nondurables and 5 are durable goods.

As an illustration, Table 3.1 shows the details of these commodities in the example of Austria for the years of 2000 to 2005. In addition to the durables and

Table 3.1: Consumption expenditures of households, Austria (mil. Euros)

Region	COICOP group	2000	2001	2002	2003	2004	2005
	Nondurables						
AUT	Food	14565	15107	15483	15768	16350	16957
AUT	Alcohol	1588	1628	1695	1713	1773	1857
AUT	Clothing	7887	8033	8084	7988	8153	8404
AUT	Heating	2466	2812	2535	2649	2792	3218
AUT	Electricity	1831	1820	1950	2134	2307	2520
AUT	Health	4039	4298	4455	4657	4747	4929
AUT	Private Transport	7683	7756	8212	8448	8830	9438
AUT	Public Transport	2521	2550	2673	2751	2945	3100
AUT	Communication	2816	2927	3046	3219	3407	3446
AUT	Recreation	7526	7832	7849	8059	8335	8631
AUT	Other Nondurables	15253	15377	15527	15646	16559	17798
AUT	Hotel and Restaurants	12630	13382	13770	14694	15070	15323
	Durables						
AUT	Housing	18167	18894	19544	20170	21291	23150
AUT	Appliances	1209	1267	1386	1426	1456	1493
AUT	Vehicles	5253	5106	5000	5300	5461	5352
AUT	Video and Audio	2181	2239	2267	2168	2170	2284
AUT	Other Durables	8033	8167	8045	8077	8426	8655
AUT	Total	115647	119197	121523	124865	130071	136557
AUT	Total Nondurables	80804	83523	85280	87725	91267	95622
AUT	Total Durables	34843	35674	36243	37140	38804	40934
AUT	Energy	4297	4632	4486	4783	5100	5738
AUT	Transport	10204	10306	10885	11198	11775	12538

Source: Eurostat.

nondurables, in the bottom part of Table 3.1 there are two other totals reported: Energy which is the sum of Heating and Electricity, and Transport that includes Private Transport and Public Transport.

In order to translate the COICOP commodities into the CPA products the corresponding *bridge matrices*, denoted by BRG(r,g,c), are required. For any such bridge matrix the following property

$$\sum_{\mathrm{g}} \mathrm{BRG}(\mathrm{r}, \mathrm{g}, \mathrm{c}) = 1 \tag{3.47}$$

holds for all regions r and all commodities c. This condition simply means that

Table 3.2: COICOP-CPA bridge matrix for Spain, 2005

CPA products	Food	Alcohol	Clothing	Housing	Heating	Electricity	Appliances	Health	Vehicles	Private Transport	Public Transport	Communication	Video and Audio	Recreation	Other Nondurables	Other Durables	Hotels, Restaurants
com01	0.16																
com02																0.01	
com05	0.03														0.03		
com10					0.01												
com11																	
com12																	
com13																	
com14	0.00																
com15	0.67	1.00															
com16	0.13																
com17								0.01							0.05	0.11	
com18			0.63														
com19			0.16												0.03	0.05	
⋮																	
com85								0.49							0.10		
com90				0.02													
com91																	
com92								0.22						0.46			
com93			0.08	0.06											0.02		
com95															0.10		
Total	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note: The codes for CPA products correspond to those of the 59 products given in the Eurostat's supply and use tables.

each of the COICOP commodity should be distributed over all the relevant CPA products.

As an example in Table 3.2 we present the (incomplete over the CPA products dimension) bridge matrix of Spain for 2005. The first column of this bridge matrix shows that the COICOP commodity "Food" is distributed according to the given proportions over five CPA products, which are "Products of agriculture, hunting and related services" (code: com01), "Fish and fishing products" (com05), "Other mining and quarrying products" (com14), "Food products and beverages" (com15) and "Tobacco products" (com16). Similarly, commodity "Alcohol" is allocated entirely to "Food products and beverages" (com15) as the corresponding element is unity, i.e., $\mathrm{BRG}(\mathrm{ESP}, \mathrm{com15}, \mathrm{Alcohol}) = 1$. Therefore, the column coefficients of each COICOP commodity of the bridge matrix can be also interpreted as households' *consumption technology* according to which CPA products are used as inputs by individuals to "produce" (a bundle of) commodities for their own needs. This notion was first explored by Lancaster (1966). For recent discussion of this concept see e.g., Mongelli et al. (2010).

3.5 Consumption block residuals

In the consumption block of FIDELIO durable commodities are modelled as stocks, from which then the corresponding flows are obtained. For the explicit distinction of this fact, we denote region r's stock and flow of the durable commodity cd, respectively, as CS(r,cd) and C(r,cd). The estimates of the parameters of the nondurable and durable policy functions in their ADL(2,2,2) specifications (2.8) and (2.9) are included in the consumption block coefficient matrices $\Phi^{\mathrm{r,cd}}_{\mathrm{CS,coefficient}}$ and $\Phi^{\mathrm{r,ctn}}_{\mathrm{C,coefficient}}$, respectively. That is, $(\Phi^{\mathrm{r,ctn}}_{\mathrm{C,1}}, \Phi^{\mathrm{r,ctn}}_{\mathrm{C,2}}, \ldots, \Phi^{\mathrm{r,ctn}}_{\mathrm{C,8}})' = (\alpha_1, \alpha_2, \ldots, \alpha_8)'$ and $(\Phi^{\mathrm{r,cd}}_{\mathrm{CS,1}}, \Phi^{\mathrm{r,cd}}_{\mathrm{CS,2}}, \ldots, \Phi^{\mathrm{r,cd}}_{\mathrm{CS,8}})' = (\beta_1, \beta_2, \ldots, \beta_8)'$. The corresponding values are all reported in Table 2.1. Using durable consumption demand equation (2.9), the corresponding base-year residuals are derived as

$$
\begin{aligned}
\mathrm{RCS(r,cd)} =& \ln \mathrm{CS(r,cd)} \\
& -\big\{ \Phi^{\mathrm{r,cd}}_{\mathrm{CS},1} \cdot \ln \mathrm{CS_1(r,cd)} + \Phi^{\mathrm{r,cd}}_{\mathrm{CS},2} \cdot \ln \mathrm{CS_2(r,cd)} \\
& + \Phi^{\mathrm{r,cd}}_{\mathrm{CS},3} \cdot \ln \left[\underline{\mathrm{Tht}}\mathrm{(r)} + \mathrm{PCS(r,cd)} \right] \\
& + \Phi^{\mathrm{r,cd}}_{\mathrm{CS},4} \cdot \ln \left[\underline{\mathrm{Tht}}_1\mathrm{(r)} + \mathrm{PCS_1(r,cd)} \right] \\
& + \Phi^{\mathrm{r,cd}}_{\mathrm{CS},5} \cdot \ln \left[\underline{\mathrm{Tht}}_2\mathrm{(r)} + \mathrm{PCS_2(r,cd)} \right] \\
& + \Phi^{\mathrm{r,cd}}_{\mathrm{CS},6} \cdot \ln \mathrm{Zz(r)} + \Phi^{\mathrm{r,cd}}_{\mathrm{CS},7} \cdot \ln \mathrm{Zz_1(r)} + \Phi^{\mathrm{r,cd}}_{\mathrm{CS},8} \cdot \ln \mathrm{Zz_2(r)} \big\},
\end{aligned}
\tag{3.48}
$$

where PCS(r,cd) are the observed prices of durable commodities and Zz(r) is the difference between cash-on-hand and voluntary equity holding at time t. The last equals the sum of aggregate nondurable consumption and down payment share of the total stocks of durable commodities as given in (2.1).

Similarly, the calibrated base-year residuals from the nondurable consumption demand (2.8) are obtained as

$$
\begin{aligned}
\mathrm{RCndr(r)} =& \ln \mathrm{Cndr(r)} - \big\{ \Phi^{\mathrm{r,ctn}}_{\mathrm{C},1} \cdot \ln \mathrm{Cndr_1(r)} + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},2} \cdot \ln \mathrm{Cndr_2(r)} \\
& + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},3} \cdot \big(\ln \mathrm{PCStot(r)} - \ln \left[\underline{\mathrm{Tht}}\mathrm{(r)} + \mathrm{PCStot(r)} \right] \big) \\
& + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},4} \cdot \big(\ln \mathrm{PCStot_1(r)} - \ln \left[\underline{\mathrm{Tht}}_1\mathrm{(r)} + \mathrm{PCStot_1(r)} \right] \big) \\
& + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},5} \cdot \big(\ln \mathrm{PCStot_2(r)} - \ln \left[\underline{\mathrm{Tht}}_2\mathrm{(r)} + \mathrm{PCStot_2(r)} \right] \big) \\
& + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},6} \cdot \ln \mathrm{Zz(r)} + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},7} \cdot \ln \mathrm{Zz_1(r)} + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},8} \cdot \ln \mathrm{Zz_2(r)} \big\},
\end{aligned}
\tag{3.49}
$$

where Cndr(r) is the aggregate nondurable commodity in region r and PCStot(r) is the price of total stocks of all durable commodities.

The parameters of the QAIDS expenditure share equation (2.13) used for splitting the aggregate nondurable commodity into its different categories are denoted by $(\Phi^{\mathrm{r,cn}}_{\mathrm{C},1}, \Phi^{\mathrm{r,cn}}_{\mathrm{C},2}, \Phi^{\mathrm{r,cn}}_{\mathrm{C},3})' = (\alpha_{\mathrm{i}}, \beta_{\mathrm{i}}, \lambda_{\mathrm{i}})'$ and $\Gamma^{\mathrm{r}}_{\mathrm{C}}(\mathrm{cn}, \mathrm{cn}_1) = \gamma_{\mathrm{cn},\mathrm{cn}_1}$. The corresponding values are all reported in Table 2.2. The calibrated residuals from the price index $\ln \mathrm{a_{qaids}(r)}$ given in (2.14) are computed as

$$\begin{aligned}\mathrm{Ra_{qaids}(r)} =& \ln \mathrm{a_{qaids}(r)} - \sum_{\mathrm{cn}} \Phi_{\mathrm{C},1}^{\mathrm{r,cn}} \cdot \ln \mathrm{PC(r, cn)} \\ & -0.5 \sum_{\mathrm{cn,cn_1}} \Gamma_{\mathrm{C}}^{\mathrm{r}}(\mathrm{cn, cn_1}) \cdot \ln \mathrm{PC(r, cn)} \cdot \ln \mathrm{PC(r, cn_1)}, \end{aligned} \tag{3.50}$$

where PC(r,c) is the price of commodity c in region r, which are all set to unity for all commodities for the base-year calculations purposes.

The residuals from the expenditure share equation (2.13) are obtained as

$$\begin{aligned}\mathrm{RW_{qaids}(r, cn)} =& \frac{\mathrm{C(r, cn)}}{\sum_{\mathrm{cn_1}} \mathrm{C(r, cn_1)}} - \Phi_{\mathrm{C},1}^{\mathrm{r,cn}} - \sum_{\mathrm{cn_1}} \Gamma_{\mathrm{C}}^{\mathrm{r}}(\mathrm{cn, cn_1}) \cdot \ln \mathrm{PC(r, cn_1)} \\ & -\Phi_{\mathrm{C},2}^{\mathrm{r,cn}} \cdot \ln \frac{\mathrm{Cndr(r)}}{\mathrm{a_{qaids}(r)}} - \frac{\Phi_{\mathrm{C},3}^{\mathrm{r,cn}}}{\mathrm{b_{qaids}(r)}} \cdot \left[\ln \frac{\mathrm{Cndr(r)}}{\mathrm{a_{qaids}(r)}} \right]^2 . \end{aligned} \tag{3.51}$$

For the base year calculations, the two required QAIDS price indices (2.14)-(2.15) used in equations (3.50) and (3.51) are set to unity, i.e., $\mathrm{a_{qaids}(r)} = \mathrm{b_{qaids}(r)} = 1$ for all r.

Next, private Energy consumption must be split into Electricity and Heating. For this purpose we use equation (2.24) and the corresponding estimated parameters are reported in the first row of Table 2.3. For the electricity expenditure share equation (2.24) be consistent with the AIDS specification notation derived from (2.13), these parameters are denoted as $(\Phi_{\mathrm{EE},1}^{\mathrm{r}}, \Phi_{\mathrm{EE},2}^{\mathrm{r}})' = (\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})' = (\mathrm{c_1}, \mathrm{c_3})'$ and $\Gamma_{\mathrm{EE}}^{\mathrm{r}} = \gamma_{\mathrm{elect,heat}} = \mathrm{c_2}$, where elect (or Elect) and heat (or Heat) refer to Electricity and Heating, respectively. The prices of Electricity and Heating are, however, adjusted by the corresponding *efficiencies factors* which are included in CEF(r,c). Hence, the calibrated residuals from the price index $\ln \mathrm{a_{qaids}(r)}$ in (2.14) associated with the AIDS model (2.24), once all the restrictions (2.16)-(2.18) have been imposed, are obtained from

$$\begin{aligned}\mathrm{Ra}_{\mathrm{aids}}^{\mathrm{energy}}(\mathrm{r}) = &\ln \mathrm{a}_{\mathrm{aids}}^{\mathrm{energy}}(\mathrm{r}) - \Phi_{\mathrm{EE},1}^{\mathrm{r}} \cdot \ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{Elect})}{\mathrm{CEF}(\mathrm{r},\mathrm{Elect})} - \Phi_{\mathrm{EE},2}^{\mathrm{r}} \cdot \ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{Heat})}{\mathrm{CEF}(\mathrm{r},\mathrm{Heat})} \\ &- \Gamma_{\mathrm{EE}}^{\mathrm{r}} \cdot \ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{Elect})}{\mathrm{CEF}(\mathrm{r},\mathrm{Elect})} \cdot \ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{Heat})}{\mathrm{CEF}(\mathrm{r},\mathrm{Heat})} \\ &- 0.5 \cdot \Gamma_{\mathrm{EE}}^{\mathrm{r}} \cdot \left[\left(\ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{Elect})}{\mathrm{CEF}(\mathrm{r},\mathrm{Elect})}\right)^2 + \left(\ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{Heat})}{\mathrm{CEF}(\mathrm{r},\mathrm{Heat})}\right)^2\right].\end{aligned} \tag{3.52}$$

Then the residuals from the electricity share equation (2.24) are given by

$$\begin{aligned}\mathrm{RWelect}(\mathrm{r}) = &\frac{\mathrm{C}(\mathrm{r},\mathrm{Elect})}{\mathrm{C}(\mathrm{r},\mathrm{Energy})} - \Phi_{\mathrm{EE},1}^{\mathrm{r}} \\ &- \Gamma_{\mathrm{EE}}^{\mathrm{r}} \cdot \ln \left[\frac{\mathrm{PC}(\mathrm{r},\mathrm{Elect})/\mathrm{CEF}(\mathrm{r},\mathrm{Elect})}{\mathrm{PC}(\mathrm{r},\mathrm{Heating})/\mathrm{CEF}(\mathrm{r},\mathrm{Heating})}\right] \\ &- \Phi_{\mathrm{EE},2}^{\mathrm{r}} \cdot \ln \left[\frac{\mathrm{C}(\mathrm{r},\mathrm{Energy})}{\mathrm{P}_{\mathrm{energy}}(\mathrm{r})}\right],\end{aligned} \tag{3.53}$$

where the (aggregate) price of energy, as follows from the AIDS model, is equivalent to the relevant first price index, i.e.,

$$\mathrm{P}_{\mathrm{energy}}(\mathrm{r}) = \mathrm{a}_{\mathrm{aids}}^{\mathrm{energy}}(\mathrm{r}). \tag{3.54}$$

Similar to the split of Electricity from Energy commodity, one splits Private Transport from total Transport. The corresponding parameters are reported in the second row of Table 2.3 and are denoted as $(\Phi_{\mathrm{TT},1}^{\mathrm{r}}, \Phi_{\mathrm{TT},2}^{\mathrm{r}})' = (\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})' = (\mathrm{c}_1, \mathrm{c}_3)'$ and $\Gamma_{\mathrm{TT}}^{\mathrm{r}} = \gamma_{\mathrm{privtr,pubtr}} = \mathrm{c}_2$, where privtr (or PrivTr) and pubtr (PubTr) refer to, respectively, private transport and public transport. Similar to the base-year residuals from the energy price equation (3.52), the calibrated residuals of the transport price equation are

$$\begin{aligned}\mathrm{Ra}_{\mathrm{aids}}^{\mathrm{tran}}(\mathrm{r}) = &\ln \mathrm{a}_{\mathrm{aids}}^{\mathrm{tran}}(\mathrm{r}) - \Phi_{\mathrm{TT},1}^{\mathrm{r}} \cdot \ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{PrivTr})}{\mathrm{CEF}(\mathrm{r},\mathrm{PrivTr})} - \Phi_{\mathrm{TT},2}^{\mathrm{r}} \cdot \ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{PubTr})}{\mathrm{CEF}(\mathrm{r},\mathrm{PubTr})} \\ &- \Gamma_{\mathrm{TT}}^{\mathrm{r}} \cdot \ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{PrivTr})}{\mathrm{CEF}(\mathrm{r},\mathrm{PrivTr})} \cdot \ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{PubTr})}{\mathrm{CEF}(\mathrm{r},\mathrm{PubTr})} \\ &- 0.5 \cdot \Gamma_{\mathrm{TT}}^{\mathrm{r}} \left[\left(\ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{PrivTr})}{\mathrm{CEF}(\mathrm{r},\mathrm{PrivTr})}\right)^2 + \left(\ln \frac{\mathrm{PC}(\mathrm{r},\mathrm{PubTr})}{\mathrm{CEF}(\mathrm{r},\mathrm{PubTr})}\right)^2\right].\end{aligned} \tag{3.55}$$

Hence, the residuals from the private transport share equation are obtained, similar to (3.53), as follows:

$$\begin{aligned}\mathrm{RWprivtr(r)} =& \frac{\mathrm{C(r, PrivTr)}}{\mathrm{C(r, Transport)}} - \Phi^{\mathrm{r}}_{\mathrm{TT,1}} \\ & -\Gamma^{\mathrm{r}}_{\mathrm{TT}} \cdot \ln\left[\frac{\mathrm{PC(r, PrivTr)}/\underline{\mathrm{CEF}}\mathrm{(r, PrivTr)}}{\mathrm{PC(r, PubTr)}/\underline{\mathrm{CEF}}\mathrm{(r, PubTr)}}\right] \\ & -\Phi^{\mathrm{r}}_{\mathrm{TT,2}} \cdot \ln\left[\frac{\mathrm{C(r, Transport)}}{\mathrm{P_{tran}(r))}}\right],\end{aligned} \tag{3.56}$$

where the (aggregate) price of transport is equal to the corresponding AIDS price index:

$$\mathrm{P_{tran}(r)} = \mathrm{a^{tran}_{aids}(r)}. \tag{3.57}$$

We will (or might) also need the *scale factors for durable commodities* that are used for transforming the durable stocks into the corresponding flow demands, which are defined as

$$\mathrm{ScaleCS(r, cd)} = \frac{\mathrm{C(r, cd)}}{\mathrm{CS(r, cd)} - [1 - \mathrm{DPR(r, cd)}] \cdot \mathrm{CS_1(r, cd)}} \quad \text{if } \mathrm{CS(r, cd)} \neq 0, \tag{3.58}$$

and $\mathrm{ScaleCS(r, cd)} = 1$ if $\mathrm{CS(r, cd)} = 0$, where DPR(r,cd) is the depreciation rate of the durable cd (see Section 3.8 below).

For the *overall scaling from COICOP commodities to CPA products* of private consumption we need the ratio of their totals for the base year, i.e.,

$$\mathrm{TotalCtoP(r)} = \frac{\sum_{\mathrm{c}} \mathrm{C(r, c)}}{\sum_{\mathrm{g}} \mathrm{USE_{pp}(r, g, con)}}. \tag{3.59}$$

The *additive* and *multiplicative commodity-specific scale factors* for the purpose of transforming the COICOP consumption into CPA consumption are defined, respectively, as

$$\begin{aligned}\mathrm{AddScaleCtoP(r, g)} =& \mathrm{TotalCtoP(r)} \cdot \mathrm{USE_{pp}(r, g, con)} \\ & - \sum_{\mathrm{c}} \mathrm{BRG(r, g, c)} \cdot \mathrm{C(r, c)},\end{aligned} \tag{3.60}$$

$$\mathrm{MulScaleCtoP(r, g)} = \frac{\mathrm{TotalCtoP(r)} \cdot \mathrm{USE_{pp}(r, g, con)}}{\sum_{\mathrm{c}} \mathrm{BRG(r, g, c)} \cdot \mathrm{C(r, c)}}. \tag{3.61}$$

Using the definition of TotalCtoP(r) from equation (3.59) it can be easily seen that the first term on the right-hand side of (3.60), which is also the numerator of (3.61), gives the amount of CPA *product* (or good) g in region r that is derived by applying private consumption composition (i.e., the share of product g in total private consumption) from the use table to the sum of all durable and non-durable commodities $\sum_{\mathrm{c}} \mathrm{C(r,c)}$. The second term in the right-hand side of (3.60), which is also the denominator in (3.61), gives the same quantity but uses the information of each durable and non-durable commodities separately that are translated into CPA consumption products using the corresponding bridge matrix. Given that there is no guarantee that the two estimates are identical, the related difference is captured additively and multiplicatively by the good-specific scale factors in (3.60) and (3.60), respectively. Note that, for example, the sum of (3.60) over all goods is zero, $\sum_{\mathrm{r}} \mathrm{AddScaleCtoP(r,g)} = 0$ (using the property of the bridge matrix given in (3.47)) so that all the product-specific gaps are accounted for in the transformation process, whereas the economy-wide difference of the total commodities and products private consumption to be made with the overall scaling scaling factor (3.59) remains unchanged (see (4.61*) and the follow-up explanation).

3.6 Production block residuals

The estimates of the parameters of the output price equation (2.33) are included in the same order of appearance in the production block coefficient matrix $\Phi^{\mathrm{r,s}}_{\mathrm{P,coefficient}}$. That is, $(\Phi^{\mathrm{r,s}}_{\mathrm{P,1}}, \Phi^{\mathrm{r,s}}_{\mathrm{P,2}}, \ldots, \Phi^{\mathrm{r,s}}_{\mathrm{P,21}})' = (\beta_k, \beta_l, \beta_e, \beta_m, \alpha_1, \alpha_2, \gamma_{kk}, \gamma_{ll}, \gamma_{ee}, \gamma_{mm}, \gamma_{kl}, \gamma_{ke}, \gamma_{km}, \gamma_{le}, \gamma_{lm}, \gamma_{em}, \rho_{tk}, \rho_{tl}, \rho_{te}, \rho_{tm})'$. A few of these parameters are reported in Table 2.4. The factor (or input) prices for capital, labour, energy materials, imported non-energy materials and domestic non-energy materials are defined, respectively, as PK, PL, PE, PM and PD that vary by region and sector. The base year *calibrated* *residuals* of the capital share equation is derived using the corresponding expression (i.e., s_k) from the cost shares system (2.34), where we use the *observed* base-year capital shares in gross outputs K_Q(r,s), i.e.,

$$\begin{aligned}\mathrm{RK}(\mathrm{r,s}) =& \mathrm{K\_Q}(\mathrm{r,s}) - \left[\Phi^{\mathrm{r,s}}_{\mathrm{P},2} + \Phi^{\mathrm{r,s}}_{\mathrm{P},8} \cdot \ln\frac{\mathrm{PK}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},12} \cdot \ln\frac{\mathrm{PL}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})}\right. \\ &\left. + \Phi^{\mathrm{r,s}}_{\mathrm{P},13} \cdot \ln\frac{\mathrm{PE}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},14} \cdot \ln\frac{\mathrm{PM}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},18} \cdot \mathrm{t}\right]. \end{aligned} \tag{3.62}$$

Using the observed labour share compensation (wages and social security contributions) in gross output, L_Q(r,s), the calibrated residuals from the labour share equation s_l in (2.34) are derived as

$$\begin{aligned}\mathrm{RL}(\mathrm{r,s}) =& \mathrm{L\_Q}(\mathrm{r,s}) - \left[\Phi^{\mathrm{r,s}}_{\mathrm{P},3} + \Phi^{\mathrm{r,s}}_{\mathrm{P},12} \cdot \ln\frac{\mathrm{PK}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},9} \cdot \ln\frac{\mathrm{PL}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})}\right. \\ &\left. + \Phi^{\mathrm{r,s}}_{\mathrm{P},15} \cdot \ln\frac{\mathrm{PE}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},16} \cdot \ln\frac{\mathrm{PM}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},19} \cdot \mathrm{t}\right]. \end{aligned} \tag{3.63}$$

Similarly, using the observed base-year shares of energy and imported non-energy inputs in gross outputs E_Q(r,s) and M_Q(r,s) and share equations s_e and s_m in (2.34), the corresponding residuals are derived, respectively, as

$$\begin{aligned}\mathrm{RE}(\mathrm{r,s}) =& \mathrm{E\_Q}(\mathrm{r,s}) - \left[\Phi^{\mathrm{r,s}}_{\mathrm{P},4} + \Phi^{\mathrm{r,s}}_{\mathrm{P},13} \cdot \ln\frac{\mathrm{PK}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},15} \cdot \ln\frac{\mathrm{PL}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})}\right. \\ &\left. + \Phi^{\mathrm{r,s}}_{\mathrm{P},10} \cdot \ln\frac{\mathrm{PE}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},17} \cdot \ln\frac{\mathrm{PM}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},20} \cdot \mathrm{t}\right], \end{aligned} \tag{3.64}$$

$$\begin{aligned}\mathrm{RM}(\mathrm{r,s}) =& \mathrm{M\_Q}(\mathrm{r,s}) - \left[\Phi^{\mathrm{r,s}}_{\mathrm{P},5} + \Phi^{\mathrm{r,s}}_{\mathrm{P},14} \cdot \ln\frac{\mathrm{PK}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},16} \cdot \ln\frac{\mathrm{PL}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})}\right. \\ &\left. + \Phi^{\mathrm{r,s}}_{\mathrm{P},17} \cdot \ln\frac{\mathrm{PE}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},11} \cdot \ln\frac{\mathrm{PM}(\mathrm{r,s})}{\mathrm{PD}(\mathrm{r,s})} + \Phi^{\mathrm{r,s}}_{\mathrm{P},21} \cdot \mathrm{t}\right]. \end{aligned} \tag{3.65}$$

It is important to note that in all the above equations (3.62)-(3.65), the time variable is set to zero, $t = 0$, as these residuals represent base-year residuals. Similarly, since factor prices are defined as indices, the base-year input prices are all set to unity, i.e., $\mathrm{PK}(\mathrm{r,s}) = \mathrm{PL}(\mathrm{r,s}) = \mathrm{PE}(\mathrm{r,s}) = \mathrm{PM}(\mathrm{r,s}) = \mathrm{PD}(\mathrm{r,s}) = 1$ for all regions and all sectors. This would mean that all the price terms in (3.62)-(3.65) are eliminated as well (since $\ln 1 = 0$). However, we chose to present the entire equations for calculating the above residuals in order to make the connection to the cost share equations in (2.34) explicit.

The sectoral output prices, denoted by PQ(r,s), within the model are computed using the price function (2.33). But since we are working with price indices, the base-year output prices are all set to unity, i.e., PQ(r,s) = 1 for all r and s. Defining for simplicity $\widetilde{\mathrm{PF}}(\mathrm{r,s}) \equiv \mathrm{PF}(\mathrm{r,s})/\mathrm{PD}(\mathrm{r,s})$ for factors F=K,L,E,M, the base-year calibrated *residuals of output price* equation is thus computed as follows:

$$
\begin{aligned}
\mathrm{RPQ}(\mathrm{r,s}) = \mathrm{PQ}(\mathrm{r,s}) - \exp\Big\{ & \Phi^{\mathrm{r,s}}_{\mathrm{P},1} + \Phi^{\mathrm{r,s}}_{\mathrm{P},2} \cdot \ln \widetilde{\mathrm{PK}}(\mathrm{r,s}) + \Phi^{\mathrm{r,s}}_{\mathrm{P},3} \cdot \ln \widetilde{\mathrm{PL}}(\mathrm{r,s}) \\
& + \Phi^{\mathrm{r,s}}_{\mathrm{P},4} \cdot \ln \widetilde{\mathrm{PE}}(\mathrm{r,s}) + \Phi^{\mathrm{r,s}}_{\mathrm{P},5} \cdot \ln \widetilde{\mathrm{PM}}(\mathrm{r,s}) + \ln \mathrm{PD}(\mathrm{r,s}) + \Phi^{\mathrm{r,s}}_{\mathrm{P},6} \cdot \mathrm{t} \\
& + 0.5 \cdot \Big[\Phi^{\mathrm{r,s}}_{\mathrm{P},7} \cdot \mathrm{t}^2 + \Phi^{\mathrm{r,s}}_{\mathrm{P},8} \cdot \big(\ln \widetilde{\mathrm{PK}}(\mathrm{r,s}) \big)^2 + \Phi^{\mathrm{r,s}}_{\mathrm{P},9} \cdot \big(\ln \widetilde{\mathrm{PL}}(\mathrm{r,s}) \big)^2 \\
& + \Phi^{\mathrm{r,s}}_{\mathrm{P},10} \cdot \big(\ln \widetilde{\mathrm{PE}}(\mathrm{r,s}) \big)^2 + \Phi^{\mathrm{r,s}}_{\mathrm{P},11} \cdot \big(\ln \widetilde{\mathrm{PM}}(\mathrm{r,s}) \big)^2 \Big] \\
& + \Phi^{\mathrm{r,s}}_{\mathrm{P},12} \cdot \ln \widetilde{\mathrm{PK}}(\mathrm{r,s}) \cdot \ln \widetilde{\mathrm{PL}}(\mathrm{r,s}) + \Phi^{\mathrm{r,s}}_{\mathrm{P},13} \cdot \ln \widetilde{\mathrm{PK}}(\mathrm{r,s}) \cdot \ln \widetilde{\mathrm{PE}}(\mathrm{r,s}) \\
& + \Phi^{\mathrm{r,s}}_{\mathrm{P},14} \cdot \ln \widetilde{\mathrm{PK}}(\mathrm{r,s}) \cdot \ln \widetilde{\mathrm{PM}}(\mathrm{r,s}) + \Phi^{\mathrm{r,s}}_{\mathrm{P},15} \cdot \ln \widetilde{\mathrm{PL}}(\mathrm{r,s}) \cdot \ln \widetilde{\mathrm{PE}}(\mathrm{r,s}) \\
& + \Phi^{\mathrm{r,s}}_{\mathrm{P},16} \cdot \ln \widetilde{\mathrm{PL}}(\mathrm{r,s}) \cdot \ln \widetilde{\mathrm{PM}}(\mathrm{r,s}) + \Phi^{\mathrm{r,s}}_{\mathrm{P},17} \cdot \ln \widetilde{\mathrm{PE}}(\mathrm{r,s}) \cdot \ln \widetilde{\mathrm{PM}}(\mathrm{r,s}) \\
& + \Phi^{\mathrm{r,s}}_{\mathrm{P},18} \cdot \mathrm{t} \cdot \ln \widetilde{\mathrm{PK}}(\mathrm{r,s}) + \Phi^{\mathrm{r,s}}_{\mathrm{P},19} \cdot \mathrm{t} \cdot \ln \widetilde{\mathrm{PL}}(\mathrm{r,s}) \\
& + \Phi^{\mathrm{r,s}}_{\mathrm{P},20} \cdot \mathrm{t} \ln \widetilde{\mathrm{PE}}(\mathrm{r,s}) + \Phi^{\mathrm{r,s}}_{\mathrm{P},21} \cdot \mathrm{t} \cdot \ln \widetilde{\mathrm{PM}}(\mathrm{r,s}) \Big\}.
\end{aligned} \tag{3.66}
$$

Note that for the base year, in (3.66) all logarithmic expressions can be disregarded since they equal zero due to the factor prices being unity. Also the time variable does not play a role for the base year. Hence, the corresponding output price residuals boil down to $\mathrm{RPQ}(\mathrm{r,s}) = 1 - \exp(\Phi^{\mathrm{r,s}}_{\mathrm{P},1})$. Thus, if $\Phi^{\mathrm{r,s}}_{\mathrm{P},1} = 0$, the base-year residuals are zero as well.

3.7 Labour market block residuals

We start with computing the base-year residuals from the wage curves given in (2.40). Let RWEM(r,s,sk) be the base-year residuals of the equations of wages per employee of skill type sk={high, medium (med), low}. The wages per employee by skill type are denoted by WEM(r,s,sk), which for the base year are obtained from the socio-economic accounts of the WIOD database. The estimates of the parameters of the wage curves are contained in the coefficient matrix $\Phi^{\mathrm{r,skill}}_{\mathrm{W,coefficient}}$ that includes the values of τ's reported in Table 2.6. Next, let Pcon(r) be the *consumer price* index, which for the base year is set to unity and otherwise is endogenously derived within the FIDELIO model. Further, let HrWktot(r) be the *total hours worked by employees* of region r which for the base-year calculations also come from the socio-economic accounts of the WIOD database. Finally denoting *unemployment rates by skill type* as UNEMR(r,sk) and the *regional real output* by QReal(r), the residuals of the wage curves (2.40) are obtained as

$$\begin{aligned}\mathrm{RWEM(r,s,sk)}=&\ \ln \mathrm{WEM(r,s,sk)} - \Phi^{\mathrm{r,sk}}_{\mathrm{W,1}} \cdot \ln \left[\mathrm{QReal(r)/HtWktot(r)}\right] \\ &-\Phi^{\mathrm{r,sk}}_{\mathrm{W,2}} \cdot \mathrm{Pcon(r)} - \Phi^{\mathrm{r,sk}}_{\mathrm{W,3}} \cdot \ln \mathrm{UNEMR(r,sk)}. \end{aligned} \tag{3.67}$$

Next, the estimates of the parameters of the labour price (wage) function (2.37) are included in the labour block coefficient matrix $\Phi^{\mathrm{r,s}}_{\mathrm{L,coefficient}}$. That is, $(\Phi^{\mathrm{r,s}}_{\mathrm{L,1}}, \Phi^{\mathrm{r,s}}_{\mathrm{L,2}}, \ldots, \Phi^{\mathrm{r,s}}_{\mathrm{L,10}})' = (\beta_0, \beta_l, \beta_h, \alpha_1, \alpha_2, \gamma_{ll}, \gamma_{hh}, \gamma_{lh}, \rho_{tl}, \rho_{th})'$. The overall labour hourly wage is denoted by $\mathrm{WHR_{tot}(r,s)}$, and, similar to the hourly wages by skill types WHR(r,s,sk) and wages per employee WEM(r,s,sk) is also obtained/computed from the socio-economic accounts of the WIOD database. The base year calibrated residuals of the labour skill type share equation are derived using (2.38), where we use the observed high-, medium- and low-skilled labour shares denoted, respectively, by LH_L(r,s), LM_L(r,s) and LL_L(r,s). Hence, the residuals from the low-and high-skilled labour share equations (2.38) are computed, respectively, as

$$\begin{aligned} \mathrm{RLL(r,s)} =& \mathrm{LL\_L(r,s)} - \left[\Phi^{\mathrm{r,s}}_{\mathrm{L},2} + \Phi^{\mathrm{r,s}}_{\mathrm{L},6} \cdot \ln \frac{\mathrm{WHR(r,s,low)}}{\mathrm{WHR(r,s,med)}} \right. \\ & \left. + \Phi^{\mathrm{r,s}}_{\mathrm{L},8} \cdot \ln \frac{\mathrm{WHR(r,s,high)}}{\mathrm{WHR(r,s,med)}} + \Phi^{\mathrm{r,s}}_{\mathrm{L},9} \cdot \mathrm{t} \right], \end{aligned} \tag{3.68}$$

$$\begin{aligned} \mathrm{RLH(r,s)} =& \mathrm{LH\_L(r,s)} - \left[\Phi^{\mathrm{r,s}}_{\mathrm{L},3} + \Phi^{\mathrm{r,s}}_{\mathrm{L},7} \cdot \ln \frac{\mathrm{WHR(r,s,high)}}{\mathrm{WHR(r,s,med)}} \right. \\ & \left. + \Phi^{\mathrm{r,s}}_{\mathrm{L},8} \cdot \ln \frac{\mathrm{WHR(r,s,low)}}{\mathrm{WHR(r,s,med)}} + \Phi^{\mathrm{r,s}}_{\mathrm{L},10} \cdot \mathrm{t} \right], \end{aligned} \tag{3.69}$$

where $t = 0$. We do not need residuals for the medium-skilled labour shares, since these shares are obtained as residuals.

The unit cost in the labour market is given by the translog function (2.37), which will quantify the average wage per hour for all labour skill types in FIDELIO. Hence, the corresponding base-year residuals of this labour price equation (in its logarithmic form) are obtained from (2.37) as follows

$$\begin{aligned} \mathrm{RWHR(r,s)} = & \ln \mathrm{WHR_{tot}(r,s)} - \Big\{ \Phi^{\mathrm{r,s}}_{\mathrm{L},1} + \Phi^{\mathrm{r,s}}_{\mathrm{L},2} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,low)} \\ & + \Phi^{\mathrm{r,s}}_{\mathrm{L},3} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,high)} + \ln \mathrm{WHR(r,s,med)} + \Phi^{\mathrm{r,s}}_{\mathrm{L},4} \cdot \mathrm{t} \\ & + 0.5 \cdot \left[\Phi^{\mathrm{r,s}}_{\mathrm{L},5} \cdot \mathrm{t}^2 + \Phi^{\mathrm{r,s}}_{\mathrm{L},6} \cdot \left(\ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,low)} \right)^2 + \Phi^{\mathrm{r,s}}_{\mathrm{L},7} \times \right. \\ & \left. \times \left(\ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,high)} \right)^2 \right] + \Phi^{\mathrm{r,s}}_{\mathrm{L},8} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,low)} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,high)} \\ & + \Phi^{\mathrm{r,s}}_{\mathrm{L},9} \cdot \mathrm{t} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,low)} + \Phi^{\mathrm{r,s}}_{\mathrm{L},10} \cdot \mathrm{t} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,high)} \Big\}, \end{aligned} \tag{3.70}$$

where, for simplicity, $\widetilde{\mathrm{WHR}}\mathrm{(r,s,sk)} \equiv \mathrm{WHR(r,s,sk)/WHR(r,s,med)}$ denotes the hourly wage of skill type sk={high, low} relative to that of the medium-skilled labour.

3.8 Other relevant exogenous data

The theoretical model in the consumption block of FIDELIO is based on intertemporal optimization problem of households with durable and non-durable commodities (for details, see Luego-Prado, 2006). A very crucial feature of this model is consideration of a collateralized constraints imposed on consumers. The last includes a *down payment requirement* parameter, $\theta \in [0, 1]$, which represents the fraction of durables that a household is *not* allowed to finance. Hence, the borrowing limit (or maximum loan) of the individual is equal to $(1-\theta)$ fraction (share) of the stocks of durable commodities. The mentioned constraint then implies that, at any point in time, a household is only required to keep an accumulated durable equity equal to θ fraction of the durables stocks. The time series of down payment requirement is denoted by Tht(r,t) and treated as exogenous in the simulations. The corresponding estimates are derived from $(1-\theta)$, which is calculated as the relationship of long term debt to the total stock of durables. Some of the *base year* examples used in the model simulations are: Tht(AUT)=0.709, Tht(DNK)=0.489, Tht(DEU)=0.650, and Tht(LVA)=0.999. In general, these parameters are close to unity, implying the shortage of financing for purchases of durable commodities.

Interest rates (relevant) for capital costs of firms and households' durables purchases are considered as given in the model and are denoted by IntD(r). This rate is taken exogenous in FIDELIO, which is basically determined by the monetary policy. The corresponding values are mainly within the interval [0.03, 0.04] except for a few cases, e.g., for Romania the interest rate for durable purchases is 0.07. These data are the benchmark interest rates in the corresponding bond markets.

Efficiencies of electrical appliances, heating equipment and transport are denoted by CEF(r,c). For example, for the base year of 2005 these efficiencies are assumed (or estimated) to be $\mathrm{CEF(r, Electricity)} = 1.1$, $\mathrm{CEF(r, Heating)} = 1.05$, and

$\mathrm{CEF(r, PrivateTransport)} = \mathrm{CEF(r, PublicTransport)} = 1$ for all regions r. From 2009 onwards it is assumed that all these efficiencies increase by the same rate as between 2007 and 2008.

Depreciation rates of stocks of durable commodities are denoted as DPR(r,cd). Since the category 'Other Durables' has been treated as exogenous and equal to its base-year level, depreciation rates are defined for the other four durable commodities {Housing, Appliances, Vehicles, Video and Audio} and for their aggregate 'Total Durables' (for simplicity, denoted by TotD). It has been assumed that for all r and t, the depreciation rates are equal to 0.083, 0.2, 0.015 and 0.05, respectively, for durables Vehicle, VideoAudio, Housing and Appliances. However, the depreciation rate of total stocks of durables TotD is *endogenous* because it is the weighted average of the four exogenous depreciation rates with weights equal to the shares of the corresponding endogenous stocks of durables in the total of the four durable stocks. It ranges, in general, within the interval [0.02, 0.03].

Aggregate depreciation rates by sector are denoted by DPRS(r,s,t). These rates are based on the capital input data from the EUKLEMS database, which include information on the depreciation rates by asset and the capital stock for each asset by industry. The asset structure of the capital stock by industry has been used for weighting together the depreciation rates by asset in each sector.

In order to obtain the consumption demands for housing additionally the following data is required:

Pop(r) = Population size of region r,
Rent_Pop(r) = Ratio of rented houses stocks in real terms per person in region r,
Phouse(r) = House price in region r,
SprHouse(r) = Spread of housing interest rates from those of consumer durables,
SprAsset(r) = Spread of interest rates on assets from those of consumer durables.

For derivation of aggregate indicators of households' financial status the following data is needed:

PenH(r) = pension funds of households,

CapTranH(r) = capital transfers,

DLiabH(r) = change in other liabilities except debt from year $t-1$ to t,

TransfH(r) = governmental transfers,

DeprH(r) = depreciation accruing to households (for self-employed persons),

OthIncH(r) = other income of households.

For the labour market analyses, an important exogenous policy variable is *working time per employee*, which is denoted by $\mathrm{WKTM}(\mathrm{r}, \mathrm{s})$ for each region r and sector s. In the base year they are derived by dividing the total hours worked in each sector by the number of employees in that sector. Another exogenous variable is *labour supply rate* by skill types $\mathrm{LSR}(\mathrm{r}, \mathrm{sk})$, which is the proportion of labour supply (labour force) in total population.

Chapter 4

FIDELIO equations

In this chapter we present all the details of FIDELIO's equations. For simplicity of exposition, we will skip all the so-called "add" and "fix" terms that are present in the actual (GAMS) code of FIDELIO equations. They, however, play crucial role in the simulation analysis. The role of the "add" terms is simulating the effect of exogenously adding (subtracting) some amount to (from) an endogenous variable. For example, by setting the "add" term $A_X = 100$ in the equation that determines variable X, we would run a simulation of the model economy when X was 100 units higher than it would be implied by the equation for X alone. The role of the "fix" terms is exogenizing (part of) certain endogenous variables. For example, if we have (or activate) the "fix" variable $F_X(sector1) = 10$, then the value of X for sector 1 would be fixed at the level of 10, while those for all other sectors would be determined endogenously. Although the "add" and "fixed" terms are skipped, the symbol (*) is added to the equation number indicating that the corresponding equation has at least one of these two terms.

Throughout this chapter we stick to the following rules. An *underlined* variable indicates that the corresponding variable is exogenously given, hence its value does not change in the simulation analysis. All the variables that have been used/defined in the previous sections are, generally, different from the the variables with the same labels in this chapter. For example, the base-year observed make matrix is *different* from MAKE in this section which is now an endogenous variable. The base-year and simulation variables are the *same* only if the variable is *underlined* (i.e., it is exogenous) *and* it has a superscript 0 indicating base year. For example, the market share matrix MKSH defined in (3.4) is denoted as $\underline{\mathrm{MKSH}}^0$ in the model equations below. Any variable that is *not* underlined is an endogenous variable. Coefficients estimates of various required parameters obtained from econometric estimations are exogenous, but they are not underlined. In general for simplicity, these are all denoted by $\Phi^{\mathrm{region,sector/commodity}}_{\mathrm{block,coefficient}}$. For example, $\Phi^{\mathrm{r,s}}_{\mathrm{P,5}}$ denotes the 5-th coefficient of the output price equation in the production block for region r and sector s. Finally, we do not include time dimension in all the equations for simplicity purposes; however, all the variables without time indication are assumed to be defined for current simulation period t. Lagged variables are not underlined, though they are all taken as given for time t's simulation exercise. Numerical subscripts indicate the exact lagged time of a variable.

4.1 Gross outputs

Time t Make matrix is derived using the assumption of constant market shares. That is, using the base-year market share matrix (3.4), we first obtain

$$\mathrm{MAKE(r,g,s)} = \underline{\mathrm{MKSH}}^0\mathrm{(r,g,s)} \cdot \sum_{\mathrm{u}} \mathrm{GD_{bp}(r,g,u)}, \tag{4.1}$$

where $\mathrm{GD_{bp}}$ is each user's *demand for domestically produced goods* expressed in basic prices. Hence, *sectoral gross outputs* in nominal terms at time t are derived by summing the corresponding make matrix over its product dimension:

$$\mathrm{Q(r,s)} = \sum_{\mathrm{g}} \mathrm{MAKE(r,g,s)}. \qquad (4.2^*)$$

This process of transforming users' demands for products into sectors' supply of products has the following implications. First, it implies that FIDELIO is a demand-driven model. Although supply-side shocks can be simulated as well, FIDELIO fits better for the analysis of demand-side shocks. Second, this step of equating demand and supply of goods makes FIDELIO different from a CGE modelling approach. In CGE it is the changes in prices that bring about equilibrium in the markets for goods. In FIDELIO, however, the equilibrium concept in the goods market is based on the observed empirical regularities indicating how economies are evolving over time. It is obvious that, in general, the base-year data, used in CGE and other modelling frameworks, are not consistent with the concept of "economic equilibrium" in its strict economic sense. As mentioned in Chapter 1, equilibrium in FIDELIO is given by demand reactions at all levels of users and all types of goods or factor inputs, and by the corresponding supply that is determined under the restrictions given at factor markets. It is also important to note that if there is a strong evidence that certain market shares will change in the future, these can be easily incorporated exogenously in the model simulation exercises.

Total *regional real output* is obtained from

$$\mathrm{QReal(r)} = \sum_{\mathrm{s}} \big[\mathrm{Q(r,s)/PQ(r,s)}\big], \qquad (4.3)$$

where $\mathrm{PQ(r,s)}$ are output prices whose derivation is discussed in Chapter 4.8.

4.2 Demand for intermediate and primary inputs

The *intermediate* and *primary inputs demands at purchasers' prices* are derived using the factor (or cost) shares equations that were obtained from the translog cost function approach (for details, see Chapter 2.2.2). To each factor equation we also add the corresponding base-year residuals computed in Chapter 3.6. Hence, the *factor shares* of capital, labour, total energy inputs, and imported non-energy materials are derived, respectively, from

$$\begin{aligned} \mathrm{K\_Q(r,s)} =& \underline{\mathrm{RK}}^0\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P,2}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,8}} \cdot \ln\frac{\mathrm{PK(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,12}} \cdot \ln\frac{\mathrm{PL(r,s)}}{\mathrm{PD(r,s)}} \\ &+\Phi^{\mathrm{r,s}}_{\mathrm{P,13}} \cdot \ln\frac{\mathrm{PE(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,14}} \cdot \ln\frac{\mathrm{PM(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,18}} \cdot \mathrm{t}, \end{aligned} \tag{4.4*}$$

$$\begin{aligned} \mathrm{L\_Q(r,s)} =& \underline{\mathrm{RL}}^0\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P,3}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,12}} \cdot \ln\frac{\mathrm{PK(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,9}} \cdot \ln\frac{\mathrm{PL(r,s)}}{\mathrm{PD(r,s)}} \\ &+\Phi^{\mathrm{r,s}}_{\mathrm{P,15}} \cdot \ln\frac{\mathrm{PE(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,16}} \cdot \ln\frac{\mathrm{PM(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,19}} \cdot \mathrm{t}, \end{aligned} \tag{4.5*}$$

$$\begin{aligned} \mathrm{E\_Q(r,s)} =& \underline{\mathrm{RE}}^0\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P,4}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,13}} \cdot \ln\frac{\mathrm{PK(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,15}} \cdot \ln\frac{\mathrm{PL(r,s)}}{\mathrm{PD(r,s)}} \\ &+\Phi^{\mathrm{r,s}}_{\mathrm{P,10}} \cdot \ln\frac{\mathrm{PE(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,17}} \cdot \ln\frac{\mathrm{PM(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,20}} \cdot \mathrm{t}, \end{aligned} \tag{4.6*}$$

$$\begin{aligned} \mathrm{M\_Q(r,s)} =& \underline{\mathrm{RM}}^0\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P,5}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,14}} \cdot \ln\frac{\mathrm{PK(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,16}} \cdot \ln\frac{\mathrm{PL(r,s)}}{\mathrm{PD(r,s)}} \\ &+\Phi^{\mathrm{r,s}}_{\mathrm{P,17}} \cdot \ln\frac{\mathrm{PE(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,11}} \cdot \ln\frac{\mathrm{PM(r,s)}}{\mathrm{PD(r,s)}} + \Phi^{\mathrm{r,s}}_{\mathrm{P,21}} \cdot \mathrm{t}, \end{aligned} \tag{4.7*}$$

where the base-year calibrated residuals $\underline{\mathrm{RK}}^0$, $\underline{\mathrm{RL}}^0$, $\underline{\mathrm{RE}}^0$ and $\underline{\mathrm{RM}}^0$ computed in (3.62)-(3.65) are added to their corresponding factor shares equations. The shares of total domestic non-energy materials in gross outputs are obtained as residuals, i.e.,

$$\mathrm{D\_Q(r,s)} = 1 - \mathrm{K\_Q(r,s)} - \mathrm{L\_Q(r,s)} - \mathrm{E\_Q(r,s)} - \mathrm{M\_Q(r,s)}. \tag{4.8}$$

Equations (4.6*)-(4.8) determine the shares of total intermediate inputs as well, i.e.,

$$\mathrm{S\_Q(r,s) = E\_Q(r,s) + M\_Q(r,s) + D\_Q(r,s).} \tag{4.9}$$

Multiplication of all the above cost shares with the values of gross outputs obtained from (4.2*) gives the corresponding derived *inputs demands* in nominal terms. Hence, the demands for capital, labour, energy inputs, imported non-energy inputs, and domestic non-energy materials are determined, respectively, as

$$\mathrm{K(r,s) = K\_Q(r,s) \cdot Q(r,s),} \tag{4.10}$$

$$\mathrm{L(r,s) = L\_Q(r,s) \cdot Q(r,s),} \tag{4.11}$$

$$\mathrm{E(r,s) = E\_Q(r,s) \cdot Q(r,s),} \tag{4.12}$$

$$\mathrm{M(r,s) = M\_Q(r,s) \cdot Q(r,s),} \tag{4.13}$$

$$\mathrm{D(r,s) = D\_Q(r,s) \cdot Q(r,s).} \tag{4.14}$$

Using the base-year product structure matrices of energy inputs (ESTR) in (3.25), of imported non-energy inputs (MSTR) in (3.27), of domestic non-energy inputs (DSTR) in (3.28), and the corresponding total inputs demands from (4.12)-(4.14), the *demands for intermediate goods at purchasers' prices* of each sector are derived as follows:

$$\begin{aligned}\mathrm{G_{pp}(r,g,s) =} & \mathrm{\underline{ESTR}^0(r,g,s) \cdot E(r,s) + \underline{MSTR}^0(r,g,s) \cdot M(r,s)} \\ & \mathrm{+ \underline{DSTR}^0(r,g,s) \cdot D(r,s).}\end{aligned} \tag{4.15*}$$

Note that since the sum of the base-year input structure matrices over goods equals unity, in (4.15*) we have simply allocated the *aggregate* intermediate energy and non-energy inputs (4.12)-(4.14) over *all* intermediate products, i.e.,

$$\mathrm{\sum_g G_{pp}(r,g,s) = E(r,s) + M(r,s) + D(r,s) = S(r,s).}$$

The labour demands are distinguished between different skill types, and as explained in Chapter 2.3.1 these are also derived using the translog cost approach. The low- and high-skilled labour shares equations (2.38) with their base-year calibrated residuals obtained from (3.68)-(3.69) are determined, respectively, as

$$\begin{aligned}\mathrm{LL\_L(r,s)} =& \underline{\mathrm{RLL}}^0\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{L},2} + \Phi^{\mathrm{r,s}}_{\mathrm{L},6} \cdot \ln \frac{\mathrm{WHR(r,s,low)}}{\mathrm{WHR(r,s,med)}} \\ &+ \Phi^{\mathrm{r,s}}_{\mathrm{L},8} \cdot \ln \frac{\mathrm{WHR(r,s,high)}}{\mathrm{WHR(r,s,med)}} + \Phi^{\mathrm{r,s}}_{\mathrm{L},9} \cdot \mathrm{t},\end{aligned} \tag{4.16}$$

$$\begin{aligned}\mathrm{LH\_L(r,s)} =& \underline{\mathrm{RLH}}^0\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{L},3} + \Phi^{\mathrm{r,s}}_{\mathrm{L},7} \cdot \ln \frac{\mathrm{WHR(r,s,high)}}{\mathrm{WHR(r,s,med)}} \\ &+ \Phi^{\mathrm{r,s}}_{\mathrm{L},8} \cdot \ln \frac{\mathrm{WHR(r,s,low)}}{\mathrm{WHR(r,s,med)}} + \Phi^{\mathrm{r,s}}_{\mathrm{L},10} \cdot \mathrm{t},\end{aligned} \tag{4.17}$$

where $\mathrm{WHR(r,s,sk)}$ is the wage per hour of labour skill type $\mathrm{sk} = \{$high, medium (med), low$\}$. The shares of the medium skilled labour demands are derived as residuals:

$$\mathrm{LM\_L(r,s)} = 1 - \mathrm{LL\_L(r,s)} - \mathrm{LH\_L(r,s)}. \tag{4.18}$$

Multiplying the above labour skill types shares (4.16)-(4.18) by the total labour demand (4.11) gives the derived labour demand by skill types.

Combining equations (4.2*) and (4.9) gives the *total intermediate demand for each sector at purchasers' prices*:

$$\mathrm{S(r,s)} = \mathrm{S\_Q(r,s)} \cdot \mathrm{Q(r,s)}. \tag{4.19*}$$

The previous steps enable us to find *total value added* for each sector from

$$\mathrm{VA(r,s)} = \mathrm{Q(r,s)} - \mathrm{E(r,s)} - \mathrm{M(r,s)} - \mathrm{D(r,s)}. \tag{4.20*}$$

Alternatively, total value added can be derived as the sum of capital and labour demands obtained in (4.10)-(4.11), i.e., $\mathrm{VA(r,s)} = \mathrm{K(r,s)} + \mathrm{L(r,s)}$.

The different components of value added are determined as follows. Wages, social security contributions, production taxes, production subsidies, and depreciation are computed using the corresponding base-year shares given in (3.18)-(3.21), that is,

$$\mathrm{VAC(r, wage, s)} = [1 - \underline{\mathrm{SSC\_L}}^0\mathrm{(r, s)}] \cdot \mathrm{L\_Q(r, s)} \cdot \mathrm{Q(r, s)}, \tag{4.21}$$

$$\mathrm{VAC(r, soc.sec.cont, s)} = \underline{\mathrm{SSC\_L}}^0\mathrm{(r, s)} \cdot \mathrm{L\_Q(r, s)} \cdot \mathrm{Q(r, s)}, \tag{4.22}$$

$$\mathrm{VAC(r, prdn.tax, s)} = \underline{\mathrm{TXP\_Q}}^0\mathrm{(r, s)} \cdot \mathrm{Q(r, s)}, \tag{4.23}$$

$$\mathrm{VAC(r, prdn.sub, s)} = \underline{\mathrm{SBP\_Q}}^0\mathrm{(r, s)} \cdot \mathrm{Q(r, s)}, \tag{4.24}$$

$$\mathrm{VAC(r, depr, s)} = \underline{\mathrm{DPR\_Q}}^0\mathrm{(r, s)} \cdot \mathrm{Q(r, s)}. \tag{4.25}$$

The remaining part of the total value added then gives its last component – operating surplus:

$$\begin{aligned}\mathrm{VAC(r,oper.surp, s)} &= \mathrm{VA(r, s)} - \mathrm{VAC(r, wage, s)} - \mathrm{VAC(r, soc.sec.cont, s)} \\ &\quad - \mathrm{VAC(r, prdn.tax, s)} - \mathrm{VAC(r, prdn.sub, s)} - \mathrm{VAC(r, depr, s)}.\end{aligned} \tag{4.26}$$

Note that the *total labour demand* (4.11) can be also derived by summing wages (2.38) and social security contributions (4.22), that is,

$$\mathrm{L(r, s)} = \mathrm{VAC(r, wage, s)} + \mathrm{VAC(r, soc.sec.cont, s)} = \mathrm{L\_Q(r, s)} \cdot \mathrm{Q(r, s)}.$$

The *regional labour demand* (compensation) is thus given by

$$\mathrm{LReg(r)} = \sum_{\mathrm{s}} [\mathrm{VAC(r, wage, s)} + \mathrm{VAC(r, soc.sec.cont, s)}]. \tag{4.27*}$$

4.3 Labour market equations

Multiplication of total labour demand (4.11) by labour shares by skill types (4.16)-(4.18) gives the labour demand by skill type. Dividing the last by the *hourly wage by* *labour skill type* (or *price of labour skill type*) WHR(r, s, sk) gives the *total number of hours worked by skill type sk* in sector *s* and region *r*:

$$\mathrm{HRWK(r,s,sk)} = \left[\mathrm{L(r,s)} \cdot \mathrm{SK\_L(r,s,sk)}\right] / \mathrm{WHR(r,s,sk)}. \tag{4.28*}$$

where the labour skill type shares are compactly written as SK_L(r, s, low) = LL_L(r, s), SK_L(r, s, med) = LM_L(r, s) and SK_L(r, s, high) = LH_L(r, s).

The *total number of regional hours worked* in region *r* is then given by

$$\mathrm{HrWktot(r)} = \sum_{\mathrm{s}} \sum_{\mathrm{sk}} \mathrm{HRWK(r,s,sk)}. \tag{4.29}$$

Using hours worked by skill type (4.28*) and the exogenous variable of working time per employee WKTM(r, s), we then derive *total number of employees by skill type* in each sector and each region as follows:

$$\mathrm{EMP(r,s,sk)} = \mathrm{HRWK(r,s,sk)} / \underline{\mathrm{WKTM}}\mathrm{(r,s)}. \tag{4.30*}$$

Therefore, *total employment for all skill levels* is

$$\mathrm{EMP_{tot}(r,s)} = \sum_{\mathrm{sk}} \mathrm{EMP(r,s,sk)}. \tag{4.31*}$$

Using the exogenously given labour supply rates for each skill level and population size, *labour supply by skill type* is exogenously given as follows:

$$\underline{\mathrm{LSUP}}\mathrm{(r,sk)} = \underline{\mathrm{Pop}}\mathrm{(r)} \cdot \underline{\mathrm{LSR}}\mathrm{(r,sk)}. \tag{4.32*}$$

Finally, employing (4.31*) and (4.32*), *unemployment rate by labour skill type* is computed as

$$\mathrm{UNEMR(r,sk)} = \left[\underline{\mathrm{LSUP}}\mathrm{(r,sk)} - \sum_{\mathrm{s}} \mathrm{EMP(r,s,sk)}\right] / \underline{\mathrm{LSUP}}\mathrm{(r,sk)}. \tag{4.33*}$$

4.4 Demand for final goods at purchasers' prices

4.4.1 Stocks and flows of durable commodities

In Chapter 2.1 it has been explained that an inter-temporal optimization model is used in order to compute households' demands for durable and total non-durable *commodities.* Durables are modeled as capital stocks: first the stocks of durable consumption commodities are computed and then the corresponding demands are derived using the well-known capital accumulation (or stock-flow) equation also referred to as the perpetual inventory method. For four durable commodities, cd = {Appliances, Vehicles, Video and Audio, Housing}, the optimal consumption function (2.6) in its autoregressive distributed lag (2,2,2) form as given in (2.9) is:

$$\begin{aligned}\mathrm{CS(r,cd)} = \exp\Big\{&\underline{\mathrm{RCS}}^0\mathrm{(r,cd)}\\ &+\Phi^{\mathrm{r,cd}}_{\mathrm{CS,1}}\cdot\ln\mathrm{CS_1(r,cd)}+\Phi^{\mathrm{r,cd}}_{\mathrm{CS,2}}\cdot\ln\mathrm{CS_2(r,cd)}\\ &+\Phi^{\mathrm{r,cd}}_{\mathrm{CS,3}}\cdot\ln\left[\underline{\mathrm{Tht}}\mathrm{(r)}+\mathrm{PCS(r,cd)}\right]\\ &+\Phi^{\mathrm{r,cd}}_{\mathrm{CS,4}}\cdot\ln\left[\underline{\mathrm{Tht}}_1\mathrm{(r)}+\mathrm{PCS_1(r,cd)}\right]\\ &+\Phi^{\mathrm{r,cd}}_{\mathrm{CS,5}}\cdot\ln\left[\underline{\mathrm{Tht}}_2\mathrm{(r)}+\mathrm{PCS_2(r,cd)}\right]\\ &+\Phi^{\mathrm{r,cd}}_{\mathrm{CS,6}}\cdot\ln\mathrm{Zz(r)}+\Phi^{\mathrm{r,cd}}_{\mathrm{CS,7}}\cdot\ln\mathrm{Zz_1(r)}+\Phi^{\mathrm{r,cd}}_{\mathrm{CS,8}}\cdot\ln\mathrm{Zz_2(r)}\Big\}\end{aligned} \tag{4.34*}$$

where $\underline{\mathrm{RCS}}^0$(r,cd) are the base-year residuals derived from (3.48), and Zz(r) is the difference between cash-on-hand and voluntary equity holding at time t which equals the sum of aggregate nondurable consumption and down payment share of the total stocks of durable commodities (2.1), i.e.,

$$\mathrm{Zz(r)} = \mathrm{Cndr(r)} + \underline{\mathrm{Tht}}\mathrm{(r)}\cdot\mathrm{CStot(r)}. \tag{4.35}$$

In (4.35) CStot(r) denotes the *total stocks of durable consumer commodities* in region r which is the sum of consumer demands for four durable stocks, i.e.,

$$\begin{aligned}\mathrm{CStot}(\mathrm{r}) = {} & \mathrm{CS}(\mathrm{r}, \mathrm{Housing}) + \mathrm{CS}(\mathrm{r}, \mathrm{Appliances}) \\ & + \mathrm{CS}(\mathrm{r}, \mathrm{Vehicles}) + \mathrm{CS}(\mathrm{r}, \mathrm{VideoAudio}).\end{aligned} \tag{4.36}$$

Consumption demands for durable commodities cd = {Appliances, Vehicles, Video and Audio} are then computed from the corresponding stocks of the durables using stock-flow equation as follows:

$$\mathrm{C}(\mathrm{r}, \mathrm{cd}) = \underline{\mathrm{ScaleCS}}^0(\mathrm{r}, \mathrm{cd}) \cdot \{\mathrm{CS}(\mathrm{r}, \mathrm{cd}) - [1 - \mathrm{DPR}(\mathrm{r}, \mathrm{cd})] \cdot \mathrm{CS}_1(\mathrm{r}, \mathrm{cd})\} \tag{4.37*}$$

where the scale factor $\underline{\mathrm{ScaleKC}}^0$(r,cd) was defined in (3.58) which reflects the base-year multiplicative difference of the flow and stocks of durable commodities. If the base-year stocks of durable commodities are estimated using the stock accumulation equation then these scale factors will be unity by definition.

Durable commodity 'Other Durables' is taken exogenously at its base-year level. The derivation of *demand for housing* is computed somewhat differently from that of other durable commodities. This is due to the fact that housing comprises two components: *owner occupied houses* and *houses for rent*. The part of the owner occupied houses is included in the equation of durable stocks in (4.34*) (and thus enters the inter-temporal allocation process of consumers), while houses for rent are explained by demography. The *stock of rented houses* in real terms is derived from

$$\underline{\mathrm{Khous.rent}}(\mathrm{r}) = \underline{\mathrm{Rent\_Pop}}(\mathrm{r}) \cdot \underline{\mathrm{Pop}}(\mathrm{r}). \tag{4.38}$$

Therefore, the *total stock of houses* is equal to

$$\mathrm{Khous.tot}(\mathrm{r}) = \frac{\mathrm{CS}(\mathrm{r}, \mathrm{Housing})}{\underline{\mathrm{Phouse}}(\mathrm{r})} + \underline{\mathrm{Khous.rent}}(\mathrm{r}). \tag{4.39}$$

The *total stock of houses in nominal terms* is then obtained as

$$\mathrm{CShouse}(\mathrm{r}) = \mathrm{Khous.tot}(\mathrm{r}) \cdot \underline{\mathrm{Phouse}}(\mathrm{r}). \tag{4.40}$$

Finally, housing expenditure is linked to the total stock of houses (in real terms) by a *user cost* term that includes house prices, depreciation rate and (implicit) rate of return as follows:

$$\mathrm{C(r, Housing)} = \left\{\underline{\mathrm{SprHouse}}(\mathrm{r}) \cdot \underline{\mathrm{IntD}}(\mathrm{r}) + \underline{\mathrm{DPR}}(\mathrm{r, Housing})\right\} \cdot \mathrm{CShouse(r)}. \quad (4.41^*)$$

The sum of the demands for durable commodities (4.37*) and (4.41*) gives the *total regional private consumption of durables*, i.e.,

$$\begin{aligned}\mathrm{Cdur(r)} = {} & \mathrm{C(r, Appliances)} + \mathrm{C(r, Vehicles)} \\ & + \mathrm{C(r, VideoAudio)} + \mathrm{C(r, Housing)}.\end{aligned} \quad (4.42)$$

4.4.2 Demand for non-durable commodities

The optimal consumption function of the *aggregate non-durable commodity* is given by (2.5), while its empirical counterpart in autoregressive distributed lag (2,2,2) form (2.8) is used in the simulations as follows:

$$\begin{aligned}\mathrm{Cndr(r)} = {} & \exp\left\{\underline{\mathrm{RCndr}}^0(\mathrm{r}) + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},1} \cdot \ln \mathrm{Cndr}_1(\mathrm{r}) + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},2} \cdot \ln \mathrm{Cndr}_2(\mathrm{r})\right. \\ & + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},3} \cdot \big(\ln \mathrm{PCStot(r)} - \ln\left[\underline{\mathrm{Tht}}(\mathrm{r}) + \mathrm{PCStot(r)}\right]\big) \\ & + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},4} \cdot \big(\ln \mathrm{PCStot}_1(\mathrm{r}) - \ln\left[\underline{\mathrm{Tht}}_1(\mathrm{r}) + \mathrm{PCStot}_1(\mathrm{r})\right]\big) \\ & + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},5} \cdot \big(\ln \mathrm{PCStot}_2(\mathrm{r}) - \ln\left[\underline{\mathrm{Tht}}_2(\mathrm{r}) + \mathrm{PCStot}_2(\mathrm{r})\right]\big) \\ & \left. + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},6} \cdot \ln \mathrm{Zz(r)} + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},7} \cdot \ln \mathrm{Zz}_1(\mathrm{r}) + \Phi^{\mathrm{r,ctn}}_{\mathrm{C},8} \cdot \ln \mathrm{Zz}_2(\mathrm{r})\right\}\end{aligned} \quad (4.43^*)$$

where PCStot(r) is the price of total stocks of all durable commodities and $\underline{\mathrm{RCndr}}^0(\mathrm{r})$ is the corresponding base-year residual obtained from (3.49).

Next we need to split up the total non-durable consumption Cndr(r) into its different categories. As explained in detail in Chapter 2.1.2, for this purpose the QAIDS demand model. Therefore, we first compute the required price indices a($\mathbf{p}$)

and b(**p**) given, respectively, in (2.14) and (2.15) as:

$$\begin{aligned} \mathrm{a_{qaids}(r)} =& \exp\Big\{\underline{\mathrm{Ra}}^0_{\mathrm{qaids}}(\mathrm{r}) + \sum_{\mathrm{cn}} \Phi^{\mathrm{r,cn}}_{\mathrm{C,1}} \cdot \ln \mathrm{PC(r,cn)} \\ &+0.5 \sum_{\mathrm{cn,cn_1}} \Gamma^{\mathrm{r}}(\mathrm{cn,cn_1}) \cdot \ln \mathrm{PC(r,cn)} \cdot \ln \mathrm{PC(r,cn_1)}\Big\}, \end{aligned} \tag{4.44}$$

$$\mathrm{b_{qaids}(r)} = \prod_{\mathrm{cn}} \mathrm{PC(r,cn)}^{\Phi^{\mathrm{r,cn}}_{\mathrm{C,2}}}, \tag{4.45}$$

where the base-year residuals $\underline{\mathrm{Ra}}^0_{\mathrm{qaids}}(\mathrm{r})$ are computed from (3.50). In practice, one can also simply approximate the first Translog price by the Stone price index given in (4.114).

The implied first-step QAIDS *expenditure shares* of each non-durable commodity given in (2.13) are then obtained as

$$\begin{aligned} \mathrm{W0_{qaids}(r,cn)} =& \underline{\mathrm{RW}}^0_{\mathrm{qaids}}(\mathrm{r,cn}) + \Phi^{\mathrm{r,cn}}_{\mathrm{C,1}} + \sum_{\mathrm{cn_1}} \Gamma^{\mathrm{r}}(\mathrm{cn,cn_1}) \cdot \ln \mathrm{PC(r,cn_1)} \\ &+\Phi^{\mathrm{r,cn}}_{\mathrm{C,2}} \cdot \ln \frac{\mathrm{Cndr(r)}}{\mathrm{a_{qaids}(r)}} + \frac{\Phi^{\mathrm{r,cn}}_{\mathrm{C,3}}}{\mathrm{b_{qaids}(r)}} \cdot \left[\ln \frac{\mathrm{Cndr(r)}}{\mathrm{a_{qaids}(r)}}\right]^2 \end{aligned} \tag{4.46}$$

where $\underline{\mathrm{RW}}^0_{\mathrm{qaids}}(\mathrm{r,cn})$ are again the calibrated residuals of the base-year expenditure shares given in (3.51). However, since $\mathrm{W0_{qaids}(r,cn)}$'s in (4.46) are not always guaranteed to be proper shares (i.e., for every region the corresponding shares should be in the interval [0,1] and sum up to unity), the final QAIDS expenditure shares are obtained as:

$$\mathrm{W_{qaids}(r,cn)} = \mathrm{W0_{qaids}(r,cn)} / \sum_{\mathrm{cn_1}} \mathrm{W0_{qaids}(r,cn_1)}. \tag{4.47}$$

Using (4.43*) and (4.47), the *consumption demands for non-durable commodities* cn = {Food, Alcohol, Clothing, Energy, Transport, Communication, Recreation, Health, Hotel and Restaurants, Other NonDurables} are then obtained as

$$\mathrm{C(r,cn)} = \mathrm{W_{qaids}(r,cn)} \cdot \mathrm{Cndr(r)}. \tag{4.48*}$$

Finally, we further disaggregate private consumption of Energy and Transport into their respective two components. First, let us split Energy consumption into

Electricity and Heating. Using the AIDS electricity share equation (2.24) and the corresponding residuals from (3.53), we have

$$\begin{aligned}\mathrm{Welect\_energy(r)} = {} & \underline{\mathrm{RWelect}}^{0}(\mathrm{r}) + \Phi^{\mathrm{r}}_{\mathrm{EE,1}} \\ & + \Gamma^{\mathrm{r}}_{\mathrm{EE}} \cdot \ln\left[\frac{\mathrm{PC(r,Elect)}/\underline{\mathrm{CEF}}\mathrm{(r,Elect)}}{\mathrm{PC(r,Heating)}/\underline{\mathrm{CEF}}\mathrm{(r,Heating)}}\right] \\ & + \Phi^{\mathrm{r}}_{\mathrm{EE,2}} \cdot \ln\left[\frac{\mathrm{C(r,Energy)}}{\mathrm{Penergy(r)}}\right]\end{aligned} \tag{4.49}$$

where the (aggregate) price of energy is the price index $\mathrm{a}^{\mathrm{energy}}_{\mathrm{aids}}(\mathrm{r})$ in the AIDS system and is determined similar to (4.44), i.e.,

$$\begin{aligned}\mathrm{Penergy(r)} = \exp\Big\{ & \underline{\mathrm{Ra}}^{\mathrm{energy}}_{\mathrm{aids}}(\mathrm{r}) + \Phi^{\mathrm{r}}_{\mathrm{EE,1}} \cdot \ln\frac{\mathrm{PC(r,Elect)}}{\mathrm{CEF(r,Elect)}} + \Phi^{\mathrm{r}}_{\mathrm{EE,2}} \cdot \ln\frac{\mathrm{PC(r,Heat)}}{\mathrm{CEF(r,Heat)}} \\ & + \Gamma^{\mathrm{r}}_{\mathrm{EE}} \cdot \ln\frac{\mathrm{PC(r,Elect)}}{\mathrm{CEF(r,Elect)}} \cdot \ln\frac{\mathrm{PC(r,Heat)}}{\mathrm{CEF(r,Heat)}} \\ & + 0.5 \cdot \Gamma^{\mathrm{r}}_{\mathrm{EE}} \cdot \left[\left(\ln\frac{\mathrm{PC(r,Elect)}}{\mathrm{CEF(r,Elect)}}\right)^2 + \left(\ln\frac{\mathrm{PC(r,Heat)}}{\mathrm{CEF(r,Heat)}}\right)^2\right]\Big\},\end{aligned} \tag{4.50}$$

where $\underline{\mathrm{Ra}}^{\mathrm{energy}}_{\mathrm{aids}}(\mathrm{r})$ are the corresponding base-year residuals of the energy price index equation computed in (3.52). Observe that in (4.49) and (4.50) all the prices of energy types are adjusted for the corresponding *efficiencies factors* of electrical appliances or heating equipments given in CEF matrices. In practice, one can also simply approximate the Translog price (4.50) by the corresponding Stone price index given in (4.116).

The *consumer demands for electricity* and *heating* are then derived, respectively, as:

$$\mathrm{C(r,Elect)} = \mathrm{Welect\_energy(r)} \cdot \mathrm{C(r,Energy)}, \tag{4.51}$$

$$\mathrm{C(r,Heating)} = \mathrm{C(r,Energy)} - \mathrm{C(r,Elect)}. \tag{4.52}$$

In this case usually an adjustment for obtaining proper shares similar to (4.47) is not required as there are only two energy commodities, but could be done only if the obtained share Welect_energy(r) crosses [0,1] border.

Similar to the split of Electricity from Energy commodity, we split Private Transport from total Transport. Using the base-year residuals from the private transport shares equation (3.56), we have

$$\begin{aligned}\mathrm{Wprivtr\_tran(r)} =& \ \underline{\mathrm{RWprivtr}}^0(\mathrm{r}) + \Phi^{\mathrm{r}}_{\mathrm{TT},1} \\ & + \Gamma^{\mathrm{r}}_{\mathrm{TT}} \cdot \ln\left[\frac{\mathrm{PC(r,PrivTr)}/\underline{\mathrm{CEF}}(\mathrm{r,PrivTr})}{\mathrm{PC(r,PubTr)}/\underline{\mathrm{CEF}}(\mathrm{r,PubTr})}\right] \\ & + \Phi^{\mathrm{r}}_{\mathrm{TT},2} \cdot \ln\left[\frac{\mathrm{C(r,Transport)}}{\mathrm{Ptran(r)}}\right],\end{aligned} \tag{4.53}$$

where the AIDS price index $\mathrm{a}^{\mathrm{tran}}_{\mathrm{aids}}(\mathrm{r})$ defines the aggregate transport price as

$$\begin{aligned}\mathrm{Ptran(r)} = \exp\Big\{ & \underline{\mathrm{Ra}}^{\mathrm{tran}}_{\mathrm{aids}}(\mathrm{r}) + \Phi^{\mathrm{r}}_{\mathrm{TT},1} \cdot \ln\frac{\mathrm{PC(r,PrivTr)}}{\mathrm{CEF(r,PrivTr)}} + \Phi^{\mathrm{r}}_{\mathrm{TT},2} \cdot \ln\frac{\mathrm{PC(r,PubTr)}}{\mathrm{CEF(r,PubTr)}} \\ & + \Gamma^{\mathrm{r}}_{\mathrm{TT}} \cdot \ln\frac{\mathrm{PC(r,PrivTr)}}{\mathrm{CEF(r,PrivTr)}} \cdot \ln\frac{\mathrm{PC(r,PubTr)}}{\mathrm{CEF(r,PubTr)}} \\ & + 0.5 \cdot \Gamma^{\mathrm{r}}_{\mathrm{TT}} \cdot \left[\left(\ln\frac{\mathrm{PC(r,PrivTr)}}{\mathrm{CEF(r,PrivTr)}}\right)^2 + \left(\ln\frac{\mathrm{PC(r,PubTr)}}{\mathrm{CEF(r,PubTr)}}\right)^2\right]\Big\},\end{aligned} \tag{4.54}$$

with the base-year residuals $\underline{\mathrm{Ra}}^{\mathrm{tran}}_{\mathrm{aids}}(\mathrm{r})$ are obtained from (3.55). As with the aggregate prices of all non-durables and energy, one can approximate the transport Translog price (4.54) by the corresponding Stone price index given in (4.117).

Finally, the *demands for private transport* and *public transport* are obtained, respectively, from

$$\mathrm{C(r,PrivTr)} = \mathrm{Wprivtr\_tran(r)} \cdot \mathrm{C(r,Transport)}, \tag{4.55}$$

$$\mathrm{C(r,PubTr)} = \mathrm{C(r,Transport)} - \mathrm{C(r,PrivTr)}. \tag{4.56}$$

4.4.3 Sectoral demands for investments

Sectoral capital stocks, denoted by KS, are derived from the assumption that their total user cost value is equal to the sectoral capital compensation (or cash flow), i.e., $\mathrm{KS(r,s)} \cdot \mathrm{UCKS(r,s)} = \mathrm{K(r,s)}$, where UCKS(r,s) is the *user cost of capital by* *sector.* Since cash flow data can be negative, such sectoral base-year capital stocks are nullified by adding the appropriate add-factors AddKS(r,s) to the obtained (negative) capital stocks. Consequently, these values are taken as given in the simulation exercises. Hence, the equation that determines the *stocks of capital by sector* is

$$\mathrm{KS(r,s)} = \mathrm{K(r,s)/UCKS(r,s)} + \underline{\mathrm{AddKS}}^0\mathrm{(r,s)}. \tag{4.57*}$$

There are *static* and *dynamic* concepts of capital user cost, which assume that capital market is in equilibrium in each period (for further details, see e.g., Jorgenson, 1967; Christensen and Jorgenson, 1969). The *static user cost of capital* is computed from

$$\mathrm{UCKS(r,s)} = \mathrm{PINV(r,s)} \cdot \left[\underline{\mathrm{IntD}}\mathrm{(r)} + \underline{\mathrm{DPRS}}\mathrm{(r,s)}\right], \tag{4.58}$$

where $\underline{\mathrm{IntD}}$(r) and $\underline{\mathrm{DPRS}}$(r,s) are, respectively, the exogenously given interest rate for capital costs of firms (and households' durables purchases) and aggregate depreciation rate by industry for year t, while PINV(r,s) is the *price index for investments by sector* whose base-year values are all set to unity.

Alternatively, one can use the *dynamic* concept of capital user cost which is based on Euler equation for capital market equilibrium and is given by

$$\mathrm{UCKS(r,s)} = \mathrm{PINV(r,s)} \cdot \left[\underline{\mathrm{IntD}}\mathrm{(r)} + \underline{\mathrm{DPRS}}\mathrm{(r,s)}\right] - \Delta\mathrm{PINV(r,s)}, \tag{4.59}$$

where the term $\Delta\mathrm{PINV(r,s)} \equiv \mathrm{PINV(r,s,t)} - \mathrm{PINV(r,s,t-1)}$ takes explicitly into account changes in the investments prices from period $t-1$ to period t. Apparently, it is desirable to use the dynamic version of the user cost of capital (4.59), but if it turns out that the changes in investment prices are significantly volatile then a switch to the static version of the user cost given in (4.58) could be more appropriate.

The *investments demand by sector in purchasers' prices* is then derived using the base-year proportion of investments in the capital stocks computed in (3.23), i.e.,

$$\mathrm{INV_{pp}(r,s)} = \underline{\mathrm{I\_K}}^0\mathrm{(r,s)} \cdot \mathrm{KS(r,s)}. \tag{4.60*}$$

4.4.4 Demands for final products at purchasers' prices

Region r's *private consumption demand* for good g at purchasers' prices is derived from the corresponding COICOP commodities demands computed in Chapters 4.4.1 and 4.4.2 as follows:

$$\mathrm{G_{pp}(r,g,con)} = \underline{\mathrm{AddScaleCtoP}}^0\mathrm{(r,g)} + \frac{\sum_\mathrm{c} \underline{\mathrm{BRG}}^0\mathrm{(r,g,c)} \cdot \mathrm{C(r,c)}}{\underline{\mathrm{TotalCtoP}}^0\mathrm{(r)}}. \tag{4.61*}$$

The second term on the right-hand side equation of (4.61*) is the demand for final good g that has been adjusted using the base-year *overall* scale factor of commodities-to-products transformation $\underline{\mathrm{TotalCtoP}}^0\mathrm{(r)}$ given in (3.59). This step of the adjustment ensures that the overall discrepancy between *economy-wide* total private consumption in CPA and total COICOP commodities of the base year is taken into account in the simulation exercises as well. Then, to this term we add the product-specific gaps of COICOP-to-CPA consumption transformation of the base year $\underline{\mathrm{AddScaleCtoP}}^0\mathrm{(r)}$ from (3.60) which completes the proper transformation process.

The *investment demands* for products at purchasers' prices are obtained as follows

$$\mathrm{G_{pp}(r,g,inv)} = \sum_\mathrm{s} \mathrm{INV_{pp}(r,s)} \cdot \underline{\mathrm{ISTR}}^0\mathrm{(r,g,s)}, \tag{4.62*}$$

where $\mathrm{INV_{pp}(r,s)}$ is investment demand given in (4.60*) and $\underline{\mathrm{ISTR}}^0\mathrm{(r,g,s)}$ is the base-year product structure of investments (3.29).

Regional public consumption and *regional NPISH consumption* are given exogenously as $\underline{\mathrm{Cpub}}_\mathrm{pp}\mathrm{(r)}$ and $\underline{\mathrm{Cnpish}}_\mathrm{pp}\mathrm{(r)}$, respectively. This allows for the possibility of construction of a baseline scenario where these final demand components are extrapolated according the regional targets for public net lending. Using these regional totals and the corresponding product use structure of the base year, the *public consumption* and *NPISH consumption demands for products at purchasers' prices* are obtained, respectively, as

$$G_{pp}(r, g, gov) = \underline{USTR}^0_{pp}(r, g, gov) \cdot \underline{Cpub}_{pp}(r), \tag{4.63}$$

$$G_{pp}(r, g, npish) = \underline{USTR}^0_{pp}(r, g, npish) \cdot \underline{Cnpish}_{pp}(r). \tag{4.64}$$

Demands for inventory are assumed to be fixed at their base-year use levels for all products, i.e., $G_{pp}(r, g, invent) = \underline{USE}_{pp}(r, g, invent)$.

And finally, *demands for exports in purchasers' prices* are obtained from the endogenous trade flows with other model regions plus the (exogenous) exports to the rest of the world, i.e.,

$$\begin{aligned} G_{pp}(r, g, exp) &= \underline{Xrate}(r) \cdot \sum_{r_1, utr} \frac{TRDM(r_1, r, g, utr)}{\underline{TNCS}(r_1, r, g) \cdot \underline{TRF}(r_1, r, g)} \\ &+ \begin{cases} \underline{EXP}_{row}(r, g) & \text{for the baseline scenario} \\ \underline{EXP}_{row}(r, g) \cdot \left[\frac{PGD_{pp}(r,g,exp)/\underline{PG}_{row}(g)}{PGD^0_{pp}(r,g,exp)/\underline{PG}^0_{row}(g)}\right]^{-\underline{TREL}(r,g,exp)} & \text{otherwise} \end{cases} \end{aligned} \tag{4.65*}$$

where $TRDM(r_1, r, g, utr)$ is the endogenous trade matrix (representing user *utr*'s demands in region r_1 for imports of good g produced in region r, see Chapter 4.6), $PGD_{pp}(r, g, exp)$ is the FOB price of exports in the exporting region r, and $\underline{PG}_{row}(g)$ is the FOB price of exports of the rest of world, and TREL's are the trade elasticities on the relative FOB-prices of exports. That is, the exports to the rest of the world $\underline{EXP}_{row}(r, g)$ computed in (3.44) are assumed to be exogenous in the base run of the model, while in simulations these values are modified according to the relative FOB-prices: if FOB-prices rise relative to those of the baseline scenario, then exports to the rest of the world decreases.

4.5 Demands for goods at basic prices

To transform products demands at purchasers' prices to demands at basic prices we need to account for total (trade and transport) margins, and net taxes on products.

This transformation involves three steps. First, using the base-year margin *rates* paid on non-margin goods from (3.31), we determine the values of *margins paid on non-margin products* as follows:

$$\mathrm{MRG_{paid}(r,g,u) = G_{pp}(r,g,u) \cdot [1 - \underline{TXSR}(r,g,u)] \cdot \underline{MGR}^0_{paid}(r,g,u)}, \tag{4.66}$$

where the product tax net of subsidy rates are set to those of the base-year derived in (3.30) for the baseline scenario, while could be changed for other simulation runs.

In the second step, the *total margins received by margin goods* are computed using the base-year received margins shares from (3.32) as follows:

$$\mathrm{MRG_{rec.}(r,g,u) = \underline{MGS}^0_{rec.}(r,g,u) \cdot \sum_{g_1} MRG_{paid}(r,g_1,u)}. \tag{4.67}$$

Finally, deducting the net taxes on products and paid margins (4.66) from and adding received margins (4.67) to the demands at purchasers' prices gives the *demands for goods at basic prices*:

$$\begin{aligned}\mathrm{G_{bp}(r,g,u) =} & \mathrm{G_{pp}(r,g,u) \cdot [1 - \underline{TXSR}(r,g,u)]} \\ & \mathrm{- MGR_{paid}(r,g,u) + MRG_{rec.}(r,g,u)}.\end{aligned} \tag{4.68}$$

From all these steps it thus follows that margins and net taxes on products are calculated as "mark-downs" on the demands in purchasers' prices.

4.6 Demands for imported and domestic goods

In deriving imports demands, private consumption (CP) is treated somewhat differently than the other users. Namely, the import shares of private consumption are obtained using the Armington approach, where these shares are a function of domestic and import prices. For all other users, we use the base-year import shares matrix derived in (3.11). Thus, the *total import shares* of each region r are determined as follows:

$$\mathrm{MSH(r,g,f)} = \underline{\mathrm{MSH}}^0\mathrm{(r,g,f)} \cdot \left[\frac{\mathrm{PUSE(r,g,f)}}{\mathrm{PIMP(r,g,f)}}\right]^{\underline{\mathrm{ARM}}\mathrm{(r,g)}-1} \quad \text{for f} = \mathrm{CP}, \tag{4.69*}$$

$$\mathrm{MSH(r,g,u)} = \underline{\mathrm{MSH}}^0\mathrm{(r,g,u)} \quad \text{for all other users.} \tag{4.70}$$

The *demands for total imported goods* by final demanders in region r valued at *CIF prices* are obtained by multiplying the corresponding total import shares (4.69*)-(4.70) by the corresponding demands for products at basic prices (4.68):

$$\mathrm{IMP(r,g,f)} = \mathrm{MSH(r,g,f)} \cdot \mathrm{G_{bp}(r,g,f)}. \tag{4.71*}$$

However, since we have total *non-energy intermediate* imports (at purchasers' prices) $\mathrm{M(r,s)}$ from the production side given in (4.13), the derivation of *demands for intermediate imports at CIF prices* (equivalent to basic prices) are different for energy and non-energy goods. These are computed as follows:

$$\mathrm{IMP(r,g,s)} = \begin{cases} \underline{\mathrm{MSH}}^0\mathrm{(r,g,s)} \cdot \mathrm{G_{bp}(r,g,s)} & \text{if g} = \text{ge}, \\ \underline{\mathrm{MSTR}}^0\mathrm{(r,g,s)} \cdot \mathrm{M(r,s)} \cdot \frac{\mathrm{G_{bp}(r,g,s)}}{\mathrm{G_{pp}(r,g,s)}} & \text{if g} = \text{gne}, \end{cases} \tag{4.72*}$$

where $\underline{\mathrm{MSTR}}^0\mathrm{(r,g,s)}$ is the base-year product structure of imported non-energy inputs obtained from (3.27). Note that in the case of demands for imported non-energy inputs, $\mathrm{IMP(r,gne,s)}$, the first part in (4.72*), i.e., $\underline{\mathrm{MSTR}}^0\mathrm{(r,g,s)} \cdot \mathrm{M(r,s)}$, gives the corresponding values at purchasers' prices because the intermediate input structure is defined at purchasers' prices. The last are then multiplied by $\mathrm{G_{bp}(r,gne,s)/G_{pp}(r,gne,s)}$ in order to be transformed into CIF valuation level.[1] Such transformation is not needed for energy goods, because the import structure is already defined in basic prices, see (3.11).

[1]Such transformation *could* be a problem when the good under consideration refers to trade and transport margins: demand for margin goods (*gm*) in purchasers' prices can be zero, but demand for *gm* in basic prices is not zero and can be imported as well. In such cases the ratio $\mathrm{G_{bp}(r,gm,s)/G_{pp}(r,gm,s)}$ will not be defined and as such this transformation is somewhat ad hoc. This is one issue that will be solved in a different way in the future version of FIDELIO.

The next step distinguishes between the source regions of these imports. To do so we first obtain the matrix of *shares of trade partners in total imports* in two steps. The initial estimates are based on the base-year source-distinguishing import shares matrix $\underline{\mathrm{TMSH}}^0_{\mathrm{red}}$ from (3.46) as follows:

$$\mathrm{TMSH}_1(\mathrm{r,rt,g,utr}) = \underline{\mathrm{TMSH}}^0_{\mathrm{red}}(\mathrm{r,rt,g,utr}) \cdot \mathrm{PGF}(\mathrm{r,rt,g})^{-\underline{\mathrm{TREL}}(\mathrm{r,g,utr})}, \tag{4.73}$$

where PGF(r,rt,g) is the purchaser (foreign) price of the good produced in trading region *rt* when purchased in region *r*. Recall that due to memory problem the number of trade users (*utr*) has been reduced to seven: total intermediate sector (*st*) and six final demand categories (*f*). Also recall that the set of trading region *rt* in comparison to region set *r* includes additional element for the rest of the world. Since the shares in (4.73) do not generally sum up to one, the next adjustment gives the final *partner-specific import shares* of country *r*'s imports by trading users as follows:

$$\mathrm{TMSH}(\mathrm{r,rt,g,utr}) = \mathrm{TMSH}_1(\mathrm{r,rt,g,utr}) / \sum_{\mathrm{utr}} \mathrm{TMSH}_1(\mathrm{r,rt,g,utr}). \tag{4.74}$$

Using total imports demands (4.71*), the derivation of region *r*'s *demands for imports from its trade partners rt* (could be also termed as a *trade matrix*), all expressed in common currency - euros, is straightforward, i.e.,

$$\mathrm{TRDM}(\mathrm{r,rt,g,utr}) = \begin{cases} \left[\mathrm{TMSH}(\mathrm{r,rt,g,st}) \cdot \sum_{\mathrm{s}} \mathrm{IMP}(\mathrm{r,g,s})\right] / \underline{\mathrm{Xrate}}(\mathrm{r}), \\ \left[\mathrm{TMSH}(\mathrm{r,rt,g,f}) \cdot \mathrm{IMP}(\mathrm{r,g,f})\right] / \underline{\mathrm{Xrate}}(\mathrm{r}). \end{cases} \tag{4.75}$$

Finally, deducting imports (4.71*) from demands for goods at basic prices (4.68) gives *demands for domestically produced goods in basic prices*:

$$\mathrm{GD}_{\mathrm{bp}}(\mathrm{r,g,u}) = \mathrm{G}_{\mathrm{bp}}(\mathrm{r,g,u}) - \mathrm{IMP}(\mathrm{r,g,u}). \tag{4.76*}$$

4.7 Regional indicators

Interest rate on assets of households in region r is exogenous to the model and is equal to the spread of interest rates on assets from those of the consumer durables times the interest rates for households' durable purchases, i.e.,

$$\underline{\mathrm{IntAssetH}}(\mathrm{r}) = \underline{\mathrm{SprAsset}}(\mathrm{r}) \cdot \underline{\mathrm{IntD}}(\mathrm{r}). \tag{4.77}$$

Households' *gross savings*, *net lending*, *net assets*, *debt*, *liabilities*, and *assets* are determined, respectively, as

$$\mathrm{SaveH}(\mathrm{r}) = \mathrm{DInc}(\mathrm{r}) - \sum_{\mathrm{g}} \mathrm{G}_{\mathrm{pp}}(\mathrm{r},\mathrm{g},\mathrm{con}) + \underline{\mathrm{PenH}}(\mathrm{r}), \tag{4.78}$$

$$\mathrm{NLendH}(\mathrm{r}) = \mathrm{SavH}(\mathrm{r}) + \underline{\mathrm{CapTranH}}(\mathrm{r}), \tag{4.79}$$

$$\mathrm{NAssetH}(\mathrm{r}) = \mathrm{NAssetH}_1(\mathrm{r}) + \mathrm{NLendH}(\mathrm{r}), \tag{4.80}$$

$$\mathrm{DebtH}(\mathrm{r}) = (1 - \underline{\mathrm{Tht}}(\mathrm{r})) \cdot \mathrm{K}(\mathrm{r},\mathrm{TotD}), \tag{4.81}$$

$$\mathrm{LiabH}(\mathrm{r}) = \mathrm{LiabH}_1(\mathrm{r}) + (\mathrm{DebtH}(\mathrm{r}) - \mathrm{DebtH}_1(\mathrm{r})) + \underline{\mathrm{DLiabH}}(\mathrm{r}), \tag{4.82}$$

$$\mathrm{AssetH}(\mathrm{r}) = \mathrm{NAssetH}(\mathrm{r}) + \mathrm{LiabH}(\mathrm{r}). \tag{4.83}$$

Disposable income of households, denoted as DInc(r), consists of total wages, profit income, transfers and other income, excluding taxes and depreciation accruing to households. Using the base-year regional shares (3.35)-(3.38) and the endogenous value-added components (4.21)-(4.22) and (4.26) we derive the corresponding disposable income components as follows:

$$\mathrm{WageH}(\mathrm{r}) = \underline{\mathrm{Wage}^0_{\mathrm{na.io}}}(\mathrm{r}) \cdot \sum_{\mathrm{s}} [\mathrm{VAC}(\mathrm{r},\mathrm{wage},\mathrm{s}) + \mathrm{VAC}(\mathrm{r},\mathrm{soc.sec.cont},\mathrm{s})], \tag{4.84*}$$

$$\mathrm{SscnH}(\mathrm{r}) = \underline{\mathrm{Sscn}^0_{\mathrm{na.io}}}(\mathrm{r}) \cdot \sum_{\mathrm{s}} \mathrm{VAC}(\mathrm{r},\mathrm{soc.sec.cont},\mathrm{s}), \tag{4.85*}$$

$$\mathrm{OpspH(r)} = \underline{\mathrm{Opsp}}^{0}_{\mathrm{na.io}}(\mathrm{r}) \cdot \sum_{\mathrm{s}} \mathrm{VAC(r, oper.surp, s)}, \tag{4.86*}$$

$$\mathrm{TaxesH(r)} = \underline{\mathrm{Tx\_Inc}}^{0}(\mathrm{r}) \cdot \big[\mathrm{Wage(r)} + \mathrm{Opsp(r)} - \mathrm{Sscn(r)}\big], \tag{4.87*}$$

$$\mathrm{ProfIncH(r)} = \underline{\mathrm{IntAssetH}}(\mathrm{r}) \cdot \mathrm{AssetH}_1(\mathrm{r}). \tag{4.88*}$$

Hence, households' disposable income is obtained from

$$\begin{aligned} \mathrm{DInc(r)} =& \mathrm{WageH(r)} - \mathrm{SscnH(r)} + \mathrm{OpspH(r)} + \mathrm{ProfIncH(r)} - \mathrm{TaxesH(r)} \\ & + \underline{\mathrm{OthIncH}}(\mathrm{r}) + \underline{\mathrm{TransfH}}(\mathrm{r}) - \underline{\mathrm{DeprH}}(\mathrm{r}). \end{aligned} \tag{4.89*}$$

4.8 Prices

FIDELIO distinguishes between prices at a very detailed level: for any product it allows for different users paying different prices due to the following factors:

- users facing different trade and transport margins,
- users facing different product taxes and subsidies (value added tax, for example, is paid only by private and government consumption as well as NPISH),
- users exhibiting different import shares.

All prices ultimately derive from *output prices* PQ(r,s), which are *basic prices* determined in the translog production block using the price function (2.33) as discussed in Chapter 2.2.2. Also all prices are normalized to unit value for the base year. Using the base-year calibrated residuals RPQ(r,s) computed in (3.66), and for simplicity defining the relative prices as $\widetilde{\mathrm{PF}}(\mathrm{r,s}) \equiv \mathrm{PF(r,s)/PD(r,s)}$ for factors F={K,L,E,M}, the output price is determined as follows:

$$
\begin{aligned}
\mathrm{PQ(r,s)} = {} & \underline{\mathrm{RPQ}}^0\mathrm{(r,s)} + \exp\Big\{\Phi^{\mathrm{r,s}}_{\mathrm{P},1} + \Phi^{\mathrm{r,s}}_{\mathrm{P},2}\cdot\ln\widetilde{\mathrm{PK}}\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P},3}\cdot\ln\widetilde{\mathrm{PL}}\mathrm{(r,s)} \\
& +\Phi^{\mathrm{r,s}}_{\mathrm{P},4}\cdot\ln\widetilde{\mathrm{PE}}\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P},5}\cdot\ln\widetilde{\mathrm{PM}}\mathrm{(r,s)} + \ln\mathrm{PD(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P},6}\cdot\mathrm{t} \\
& +0.5\cdot\Big[\Phi^{\mathrm{r,s}}_{\mathrm{P},7}\cdot\mathrm{t}^2 + \Phi^{\mathrm{r,s}}_{\mathrm{P},8}\cdot\big(\ln\widetilde{\mathrm{PK}}\mathrm{(r,s)}\big)^2 + \Phi^{\mathrm{r,s}}_{\mathrm{P},9}\cdot\big(\ln\widetilde{\mathrm{PL}}\mathrm{(r,s)}\big)^2 \\
& +\Phi^{\mathrm{r,s}}_{\mathrm{P},10}\cdot\big(\ln\widetilde{\mathrm{PE}}\mathrm{(r,s)}\big)^2 + \Phi^{\mathrm{r,s}}_{\mathrm{P},11}\cdot\big(\ln\widetilde{\mathrm{PM}}\mathrm{(r,s)}\big)^2\Big] \\
& +\Phi^{\mathrm{r,s}}_{\mathrm{P},12}\cdot\ln\widetilde{\mathrm{PK}}\mathrm{(r,s)}\cdot\ln\widetilde{\mathrm{PL}}\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P},13}\cdot\ln\widetilde{\mathrm{PK}}\mathrm{(r,s)}\cdot\ln\widetilde{\mathrm{PE}}\mathrm{(r,s)} \qquad (4.90^*) \\
& +\Phi^{\mathrm{r,s}}_{\mathrm{P},14}\cdot\ln\widetilde{\mathrm{PK}}\mathrm{(r,s)}\cdot\ln\widetilde{\mathrm{PM}}\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P},15}\cdot\ln\widetilde{\mathrm{PL}}\mathrm{(r,s)}\cdot\ln\widetilde{\mathrm{PE}}\mathrm{(r,s)} \\
& +\Phi^{\mathrm{r,s}}_{\mathrm{P},16}\cdot\ln\widetilde{\mathrm{PL}}\mathrm{(r,s)}\cdot\ln\widetilde{\mathrm{PM}}\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P},17}\cdot\ln\widetilde{\mathrm{PE}}\mathrm{(r,s)}\cdot\ln\widetilde{\mathrm{PM}}\mathrm{(r,s)} \\
& +\Phi^{\mathrm{r,s}}_{\mathrm{P},18}\cdot\mathrm{t}\cdot\ln\widetilde{\mathrm{PK}}\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P},19}\cdot\mathrm{t}\cdot\ln\widetilde{\mathrm{PL}}\mathrm{(r,s)} \\
& +\Phi^{\mathrm{r,s}}_{\mathrm{P},20}\cdot\mathrm{t}\ln\widetilde{\mathrm{PE}}\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{P},21}\cdot\mathrm{t}\cdot\ln\widetilde{\mathrm{PM}}\mathrm{(r,s)}\Big\}.
\end{aligned}
$$

Note that in the above equation (similar to all other equations that include variable t) time variable series is chosen such that $t = 0$ for the base year.

Starting from the output prices (4.90*), the majority of other prices are derived as appropriate weighted averages. *Basic prices of domestic products* are obtained as weighted average of the sectoral output prices (4.90*), where the base-year market shares of sectors (3.4) are used as weights:

$$\mathrm{PGD_{bp}(r,g)} = \sum_{\mathrm{s}}\underline{\mathrm{MKSH}}^0\mathrm{(r,g,s)}\cdot\mathrm{PQ(r,s)}. \qquad (4.91^*)$$

Note from (4.91*) that the basic prices of domestic good g are the same for all users.

Purchaser prices of domestic products take into account the fact that in purchasers' prices, demand for products is essentially demand for a *composite good*: the good itself, trade and transport margins, and taxes less subsidies on the good. Thus, purchaser prices of domestic goods are derived as follows:

$$\mathrm{PGD_{pp}(r,g,u)} = \frac{1-\underline{\mathrm{TXSR}}^0\mathrm{(r,g,u)}}{1-\underline{\mathrm{TXSR}}\mathrm{(r,g,u)}}\sum_{\mathrm{g_1}}\underline{\mathrm{PSTR}}^0\mathrm{(r,g,u,g_1)}\cdot\mathrm{PGD_{bp}(r,g_1)}, \qquad (4.92)$$

where $\underline{\mathrm{PSTR}}^0(\mathrm{r,g,u,g_1})$ is the base-year price structure matrix obtained in (3.34), and $\underline{\mathrm{TXSR}}^0(\mathrm{r,g,u})$ and $\underline{\mathrm{TXSR}}(\mathrm{r,g,u})$ are, respectively, the matrices of products' net taxes rates for the base year (3.30) and for the simulation exercises (which could be different from its base-year counterpart). So from (4.92) it follows that different prices enter the derivation of $\mathrm{PGD_{pp}(r,g,u)}$: the weighted average of the basic price of the good itself and the prices of the trade and transport services (i.e., $\mathrm{PGD_{bp}}$'s of products 50-52 and 60-63).[2] Product taxes less subsidies constitute then a markup on this composite price. Note that $\mathrm{PGD_{pp}(r,g,u)}$ for some good g is different for different users due to different margins and net taxes.

Next the *price of the good produced in trading region r_1 when purchased in region r*, $\mathrm{PGF(r,r_1,g)}$, is calculated. It is the *FOB price of exports* in the exporter region r_1, $\mathrm{PGD_{pp}(r_1,g,exp)}$, corrected for the exchange rate, and augmented by international transport costs and tariffs. Hence, this arrives at *CIF prices at the border of the importing region r for goods imported from region r_1* as

$$\mathrm{PGF(r,r_1,g)} = \mathrm{PGD_{pp}(r_1,g,exp)} \cdot \underline{\mathrm{I}_{\mathrm{xrate}}}(\mathrm{r,r_1}) \cdot \underline{\mathrm{I}_{\mathrm{trf}}}(\mathrm{r,r_1,g}) \cdot \underline{\mathrm{I}_{\mathrm{tncs}}}(\mathrm{r,r_1,g}), \quad (4.93^*)$$

where the *indexes of exchange rates*, *tariffs* and *costs for third-country transport* (=CIF-FOB) all between regions r and r_1 are defined (relative to the base year) as

$$\underline{\mathrm{I}_{\mathrm{xrate}}}(\mathrm{r,r_1}) = \frac{\underline{\mathrm{Xrate}}(\mathrm{r})/\underline{\mathrm{Xrate}}(\mathrm{r_1})}{\underline{\mathrm{Xrate}}^0(\mathrm{r})/\underline{\mathrm{Xrate}}^0(\mathrm{r_1})}, \quad (4.94)$$

$$\underline{\mathrm{I}_{\mathrm{trf}}}(\mathrm{r,r_1,g}) = \underline{\mathrm{TRF}}(\mathrm{r,r_1,g})/\underline{\mathrm{TRF}}^0(\mathrm{r,r_1,g}), \quad (4.95)$$

$$\underline{\mathrm{I}_{\mathrm{tncs}}}(\mathrm{r,r_1,g}) = \underline{\mathrm{TNCS}}(\mathrm{r,r_1,g})/\underline{\mathrm{TNCS}}^0(\mathrm{r,r_1,g}). \quad (4.96)$$

For the sake of further clarification, if we would have included time dimension explicitly in the above indices, then the index of tariffs, for example, for year t would have been written as

$$\underline{\mathrm{I}_{\mathrm{trf}}}(\mathrm{r,r_1,g,t}) = \underline{\mathrm{TRF}}(\mathrm{r,r_1,g,t})/\underline{\mathrm{TRF}}(\mathrm{r,r_1,g,t=0}),$$

[2] The assumption that in real terms the shares of trade and transport services in the purchaser price $\mathrm{PGD_{pp}(r,g,u)}$ are identical to the base year, however, could be exogenously changed.

where $t = 0$ represents the base year. Therefore for the base year, the CIF prices of imports from region r_1, $\mathrm{PGF}(\mathrm{r}, \mathrm{r}_1, \mathrm{g})$, are equal to the region FOB prices of exports $\mathrm{PGD}_{\mathrm{pp}}(\mathrm{r}_1, \mathrm{g}, \mathrm{exp})$.

The corresponding *CIF prices for imports from the rest of the world* are taken exogenous to the model:

$$\underline{\mathrm{PGF}}(\mathrm{r}, \mathrm{row}, \mathrm{g}) = \underline{\mathrm{PG}_{\mathrm{row}}}(\mathrm{g}) \cdot \underline{\mathrm{I}_{\mathrm{xrate}}}(\mathrm{r}, \mathrm{row}) \cdot \underline{\mathrm{I}_{\mathrm{trf}}}(\mathrm{r}, \mathrm{row}, \mathrm{g}) \cdot \underline{\mathrm{I}_{\mathrm{tncs}}}(\mathrm{r}, \mathrm{row}, \mathrm{g}), \tag{4.97*}$$

where $\underline{\mathrm{PG}_{\mathrm{row}}}(\mathrm{g})$ is the FOB price of exports of the RoW (i.e., prices in the RoW). Recall that the last price $\underline{\mathrm{PG}_{\mathrm{row}}}(\mathrm{g})$ and the purchaser price of exports of region r, $\mathrm{PGD}_{\mathrm{pp}}(\mathrm{r}, \mathrm{g}, \mathrm{exp})$, determined demands for region r's exports to the rest of the world as indicated by (4.65*).

The *total import CIF price at the border of region r for good g and user utr* are computed as the weighted average of the import prices of trading partners PGF's from (4.93*) and (4.97*) with the shares of trading partners endogenously determined in (4.73)-(4.74) as weights, i.e.,

$$\mathrm{PIMP}_{\mathrm{cif}}(\mathrm{r}, \mathrm{g}, \mathrm{utr}) = \sum_{\mathrm{rt}} \mathrm{TMSH}(\mathrm{r}, \mathrm{rt}, \mathrm{g}, \mathrm{utr}) \cdot \mathrm{PGF}(\mathrm{r}, \mathrm{rt}, \mathrm{g}), \tag{4.98*}$$

with $\mathrm{PIMP}_{\mathrm{cif}}(\mathrm{r}, \mathrm{g}, \mathrm{s}) = \mathrm{PIMP}_{\mathrm{cif}}(\mathrm{r}, \mathrm{g}, \mathrm{st})$ for all sectors s. Then the *total import prices including (domestic margins and) taxes less subsidies on products* are derived as follows:

$$\mathrm{PIMP}(\mathrm{r}, \mathrm{g}, \mathrm{u}) = \mathrm{PM}_{\mathrm{cif}}(\mathrm{r}, \mathrm{g}, \mathrm{u}) \cdot \frac{1 - \underline{\mathrm{TXSR}}^0(\mathrm{r}, \mathrm{g}, \mathrm{u})}{1 - \underline{\mathrm{TXSR}}(\mathrm{r}, \mathrm{g}, \mathrm{u})}. \tag{4.99*}$$

The next step computes products' *use prices* for all users $\mathrm{PUSE}(\mathrm{r}, \mathrm{g}, \mathrm{u})$, which are the weighted averages of the purchasers' prices of domestic products $\mathrm{PGD}_{\mathrm{pp}}(\mathrm{r}, \mathrm{g}, \mathrm{u})$ and import prices $\mathrm{PIMP}(\mathrm{r}, \mathrm{g}, \mathrm{u})$ using the import shares $\mathrm{MSH}(\mathrm{r}, \mathrm{g}, \mathrm{u})$ from (4.69*)-(4.70) as weights:

$$\begin{aligned}\mathrm{PUSE}(\mathrm{r}, \mathrm{g}, \mathrm{u}) = & \mathrm{MSH}(\mathrm{r}, \mathrm{g}, \mathrm{u}) \cdot \mathrm{PIMP}(\mathrm{r}, \mathrm{g}, \mathrm{u}) \\ & + [1 - \mathrm{MSH}(\mathrm{r}, \mathrm{g}, \mathrm{u})] \cdot \mathrm{PGD}_{\mathrm{pp}}(\mathrm{r}, \mathrm{g}, \mathrm{u}).\end{aligned} \tag{4.100*}$$

Since (theoretically) it could be the case that $\mathrm{MSH}(\mathrm{r},\mathrm{g},\mathrm{u}) > 1$ for private consumption as follows from (4.69*), we set the corresponding element to unity in computing the use price above.

The *regional use price* $\mathrm{PUSE}_{\mathrm{tot}}(\mathrm{r},\mathrm{u})$ is the aggregate price of "inputs" for each user (user cost of each user), and is the weighted average of the use prices with weights representing the structure of users' demands for goods in purchasers' prices:

$$\mathrm{PUSE}_{\mathrm{tot}}(\mathrm{r},\mathrm{u}) = \sum_{\mathrm{g}} \frac{\mathrm{G}_{\mathrm{pp}}(\mathrm{r},\mathrm{g},\mathrm{u})}{\sum_{\mathrm{g}_1} \mathrm{G}_{\mathrm{pp}}(\mathrm{r},\mathrm{g}_1,\mathrm{u})} \cdot \mathrm{PUSE}(\mathrm{r},\mathrm{g},\mathrm{u}). \tag{4.101}$$

If user in (4.101) is private consumption, then the corresponding price is the *consumer price*, i.e.,

$$\mathrm{Pcon}(\mathrm{r}) = \mathrm{PUSE}_{\mathrm{tot}}(\mathrm{r},\mathrm{con}). \tag{4.102}$$

If user is a sector, then $\mathrm{PUSE}_{\mathrm{tot}}(\mathrm{r},\mathrm{s})$ is the *price of sector s's total intermediate inputs* PS(r,s).

The *aggregate price of energy inputs* PE(r,s) is determined using the base-year product structure of energy inputs ESTR obtained from (3.25) as follows:

$$\mathrm{PE}(\mathrm{r},\mathrm{s}) = \sum_{\mathrm{g}} \underline{\mathrm{ESTR}}^0(\mathrm{r},\mathrm{g},\mathrm{s}) \cdot \mathrm{PUSE}(\mathrm{r},\mathrm{g},\mathrm{s}). \tag{4.103*}$$

Similarly, using the import prices (4.99*), purchaser prices of domestic goods (4.92), and the base-year product structures of imported and domestic non-energy inputs (3.27)-(3.28), we derive the *aggregate prices of imported non-energy* and *domestic non-energy inputs*, respectively, as

$$\mathrm{PM}(\mathrm{r},\mathrm{s}) = \sum_{\mathrm{g}} \underline{\mathrm{MSTR}}^0(\mathrm{r},\mathrm{g},\mathrm{s}) \cdot \mathrm{PIMP}(\mathrm{r},\mathrm{g},\mathrm{s}), \tag{4.104*}$$

$$\mathrm{PD}(\mathrm{r},\mathrm{s}) = \sum_{\mathrm{g}} \underline{\mathrm{DSTR}}^0(\mathrm{r},\mathrm{g},\mathrm{s}) \cdot \mathrm{PGD}_{\mathrm{pp}}(\mathrm{r},\mathrm{g},\mathrm{s}). \tag{4.105*}$$

Next we need to compute the *prices of labour skill types* (wage per employee) based on the wage curves given in (2.40) and discussed in Chapter 2.3.2. Recall that the wage curves in FIDELIO relate labour skill type wages per employee to labour productivity, consumer price and unemployment rate. Hence, using the corresponding base-year residuals computed in (3.67), the wages per employee of the three labour types are derived as follows:[3]

$$\begin{aligned}\mathrm{WEM(r,s,sk)} = \exp\Big\{&\underline{\mathrm{RWEM}}^0\mathrm{(r,s,sk)} + \Phi^{\mathrm{r,sk}}_{\mathrm{W,1}} \cdot \ln\big[\mathrm{QReal(r)/HrWktot(r)}\big] \\ &+\Phi^{\mathrm{r,sk}}_{\mathrm{W,2}} \cdot \ln \mathrm{Pcon(r)} + \Phi^{\mathrm{r,sk}}_{\mathrm{W,3}} \cdot \ln \mathrm{UNEMR(r,sk)}\Big\}. \end{aligned} \tag{4.106*}$$

Now using the exogenous policy variable of working time per employee WKTM(r,s), the wages per employee in (4.106*) are easily translated into the *wages per hour by skill type* as follows:

$$\mathrm{WHR(r,s,sk)} = \mathrm{WEM(r,s,sk)}/\underline{\mathrm{WKTM}}\mathrm{(r,s)}. \tag{4.107*}$$

The *average price of labour* (i.e., average wage per hour for all labour skill types) is then obtained from the unit cost function of the translog cost approach as given in (2.37):

$$\begin{aligned}\mathrm{WHR_{tot}(r,s)} = \exp\Big\{&\underline{\mathrm{RWHR}}^0\mathrm{(r,s)} + \Phi^{\mathrm{r,s}}_{\mathrm{L,1}} + \Phi^{\mathrm{r,s}}_{\mathrm{L,2}} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,low)} \\ &+\Phi^{\mathrm{r,s}}_{\mathrm{L,3}} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,high)} + \ln \mathrm{WHR(r,s,med)} + \Phi^{\mathrm{r,s}}_{\mathrm{L,4}} \cdot \mathrm{t} \\ &+0.5 \cdot \Big[\Phi^{\mathrm{r,s}}_{\mathrm{L,5}} \cdot \mathrm{t}^2 + \Phi^{\mathrm{r,s}}_{\mathrm{L,6}} \cdot \Big(\ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,low)}\Big)^2 + \Phi^{\mathrm{r,s}}_{\mathrm{L,7}} \times \\ &\times \Big(\ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,high)}\Big)^2\Big] + \Phi^{\mathrm{r,s}}_{\mathrm{L,8}} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,low)} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,high)} \\ &+\Phi^{\mathrm{r,s}}_{\mathrm{L,9}} \cdot \mathrm{t} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,low)} + \Phi^{\mathrm{r,s}}_{\mathrm{L,10}} \cdot \mathrm{t} \cdot \ln \widetilde{\mathrm{WHR}}\mathrm{(r,s,high)}\Big\}, \end{aligned} \tag{4.108*}$$

where, for simplicity, $\widetilde{\mathrm{WHR}}\mathrm{(r,s,sk)} \equiv \mathrm{WHR(r,s,sk)/WHR(r,s,med)}$ denotes the hourly wage of skill type sk={high, low} relative to that of the medium-skilled labour, and the base-year residuals $\underline{\mathrm{RWHR}}^0\mathrm{(r,s)}$ are computed in (3.70).

[3]In the wage curve (4.106*) one could instead of the economy-wide productivity variable QReal(r)/HrWktot(r) use the productivity factor $[\mathrm{Q(r,s)/P(r,s)}]/\mathrm{HRWK(r,s)}$ that varies across sectors.

Recall that prices are defined as indices. Thus, the average price of labour at time t relative to that of the base year then gives the *price of labour* which is used in the production block, i.e.,

$$\mathrm{PL(r,s) = WHR_{tot}(r,s)/\underline{WHR}^0_{tot}(r,s).} \tag{4.109*}$$

Similarly, the *price of sectoral capital stock* is obtained as an index of user cost of capital – using its static concept (4.58) or the dynamic one (4.59) – as follows:

$$\mathrm{PK(r,s) = UCKS(r,s)/\underline{UCKS}^0(r,s).} \tag{4.110}$$

The *prices of investments* are determined from the products' use prices for investments and the base-year product structure of investments from (3.29) as:

$$\mathrm{PINV(r,s) = \sum_g \underline{ISTR}^0(r,g,s) \cdot PUSE(r,g,inv).} \tag{4.111*}$$

The *prices of the consumption commodities* (i.e., COICOP commodities) are calculated on the base of CPA-products' use prices for private consumption from (4.100*) and the base-year bridge matrix:

$$\mathrm{PC(r,c) = \sum_g \underline{BRG}^0(r,g,c) \cdot PUSE(r,g,con).} \tag{4.112*}$$

Since non-durable commodities allocation is done within the QAIDS demand system (4.46), the *aggregate price of non-durable commodities* is equivalent to the corresponding price index, i.e.,

$$\mathrm{PCndr(r) = a_{qaids}(r)} \tag{4.113}$$

where $\mathrm{a_{qaids}(r)}$ is computed in (4.44).[4]

[4] It is often the case that in practice instead of the correct Translog price (4.44), its approximate *Stone price index* – a weighted sum of the relevant prices – is used. In such a case, instead of (4.113) the price of aggregate non-durable becomes

$$\mathrm{PCndr(r) = \sum_{cn} \frac{C(r,cn)}{Cndr(r)} PC(r,cn),} \tag{4.114}$$

which is also used for $\mathrm{a_{qaids}(r)}$ in the entire QAIDS system.

Using (4.112*), consumption of durables (4.37*) and (4.41*), and total durable consumption (4.42), the *aggregate price of all durables* is obtained from

$$\mathrm{PCdur(r)} = \sum_{\mathrm{cd}} \frac{\mathrm{C(r,cd)}}{\mathrm{Cdur(r)}} \cdot \mathrm{PC(r,cd)}. \tag{4.115*}$$

Given that energy and transport have been allocated into their corresponding components using the AIDS models in (4.49) and (4.53), respectively, the *aggregate prices of energy* and *transport* are defined within these models as well. These are already given, respectively, in (4.50) and (4.54).[5]

The *price of all commodities* from the QAIDS block is computed as

$$\mathrm{PCtot(r)} = \frac{\mathrm{Cndr(r,c)}}{\mathrm{Ctot(r)}} \cdot \mathrm{PCndr(r)} + \frac{\mathrm{Cdur(r,c)}}{\mathrm{Ctot(r)}} \cdot \mathrm{PCdur(r)}, \tag{4.118*}$$

where $\mathrm{Ctot(r)} = \mathrm{Cndr(r)} + \mathrm{Cdur(r)}$.

The *prices of stocks of durable commodities* are obtained using the user cost concept similar to that for sectoral capital stocks as given in (4.58), i.e.,

$$\mathrm{PCS(r,cd)} = \mathrm{PC(r,cd)} \cdot \left[\underline{\mathrm{IntD}}(\mathrm{r}) + \underline{\mathrm{DPR}}(\mathrm{r,cd})\right], \tag{4.119}$$

where $\underline{\mathrm{DPR}}(\mathrm{r,cd})$ are the exogenously given depreciation rates for durable commodities. Using (4.119) and the demands for stocks of durables from (4.34*) and (4.36), the *price of total stock of all durable commodities* is derived as

$$\mathrm{PCStot(r)} = \sum_{\mathrm{cd}} \frac{\mathrm{CS(r,cd)}}{\mathrm{CStot(r)}} \cdot \mathrm{PCS(r,cd)}. \tag{4.120}$$

[5]Similar to the aggregate price on non-durables (see fn. 4 above), here one can also use the corresponding Stone price indices. Thus, instead of (4.50) and (4.54), the prices of energy and transport can be also derived, respectively, from

$$\mathrm{Penergy(r)} = \sum_{\mathrm{c}\in\{\mathrm{Elect,Heating}\}} \frac{\mathrm{C(r,c)}}{\mathrm{C(r,Energy)}} \cdot \frac{\mathrm{PC(r,c)}}{\mathrm{CEF(r,c)}}, \tag{4.116}$$

$$\mathrm{Ptran(r)} = \sum_{\mathrm{c}\in\{\mathrm{PrivTr,PubTr}\}} \frac{\mathrm{C(r,c)}}{\mathrm{C(r,Transport)}} \cdot \frac{\mathrm{PC(r,c)}}{\mathrm{CEF(r,c)}}. \tag{4.117}$$

Finally, the *average depreciation rate of the stocks of all durable commodities* is obtained using the exogenous depreciation rates of durable stocks and the endogenous demands for the stocks of durables as follows:

$$\mathrm{DPRtot(r)} = \sum_{\mathrm{cd}} \frac{\mathrm{CS(r, cd)}}{\mathrm{CStot(r)}} \cdot \underline{\mathrm{DPR}}\mathrm{(r, cd)}. \tag{4.121}$$

Chapter 5

Data sources

One of the primary data source of FIDELIO is the so-called TIMESUT database. This database contains the TIME Series of Supply and Use Tables of each of the EU member states for the period 1995-2007 in NACE Rev1.1/CPA 2002 (59 industries/59 commodities, for the detailed breakdown, see Tables B.1 and B.2 in the Appendix). In this dataset for each year the following tables are available: Supply Tables at basic prices with transformation into purchaser's prices, Use Tables at purchasers' prices, Use Tables at basic prices, Trade and Transport Margin Tables and Taxes less Subsidies Tables.[1]

The entire TIMESUT database is, however, not publicly available as far as it has been compiled under an Administrative Arrangement between Eurostat and European Commission's Joint Research Centre with confidential information provided by many member states. Part of this information (at least, 2000-2007 annual Supply Tables at basic prices with transformation into purchasers' prices and Use Tables at purchasers' prices, and five-year Input-Output Tables differentiating be-

[1]Financial Intermediation Services Indirectly Measured (FISIM) were originally recorded as an extra industry in the Use tables up to 2004 for many countries. In TIMESUT, FISIM has been redistributed among industries to harmonize with the 2005-onwards FISIM treatment approach.

tween domestic and imported uses) submitted by member states in order to fulfill the ESA95 Transmission Programme (EC, 1996; EC, 2007) are available at Eurostat website.[2] However, note that these available tables contain blank confidential cells for some countries. The confidential information submitted by member states was reviewed and adjusted for consistency. Whenever this information is missing, it was estimated by means of the best methods considering the available information (Rueda-Cantuche et al., 2013).

It should be noted that in FIDELIO, SUTs (and, in fact, the whole FIDELIO model) are included in national currencies. Exchange rates also come from the corresponding Eurostat dataset. Individual countries are coded according to their ISO 3166-1 alpha-3 codes (ISO, 2006),[3], while the aggregated "Rest of the World" region is coded as 'ROW'. For trade matrix construction, total imports taken from Eurostat import matrices have to be distributed accross all trading partners covered in the model. For this purpose, the inter-regional use tables of the EU-funded project World Input-Output Database (WIOD) have been used (Timmer, 2012). WIOD database comprises 35 industries and 59 commodities. The product split is made according to the TIMESUT database.[4]

The main data source for modeling the production block, which requires price and share information on labour input, capital input, energy intermediate input, domestic non-energy and imported non-energy intermediate inputs, was Socio-Economic Accounts (SEA) files of the WIOD database (December 2011 release).[5] In general, the relevant full dataset has been constructed for the EU 23 (which includes all EU 27 countries without Bulgaria, Cyprus, Estonia and Malta), covering the period

[2] See `http://epp.eurostat.ec.europa.eu/portal/page/portal/esa95_supply_use_input_tables/data/workbooks`.

[3] These codes can be found at `http://unstats.un.org/unsd/tradekb/Knowledgebase/Country-Code`.

[4] For further details about WIOD, see `http://www.wiod.org/`.

[5] The socio-economic accounts in WIOD mainly come from the EUKLEMS project. For further details about EUKLEMS, see `http://www.euklems.net/`.

1995 to 2009. For details, see Kratena and Wüger (2012). This dataset is available in the industry classification of WIOD (35 industries).

The capital stock and investment by industry for the base year have been constructed using the appropriate data from the SEA accounts in the WIOD database. The WIOD's capital stock to output ratios at the 35 industry level are expanded to the NACE Rev1.1 2-digit classification of FIDELIO and used for the derivation of the base-year and one-lagged capital stock variable KS(r,s). Similarly, investment by investing industry (NACE Rev1.1 2-digit) vectors for each country are obtained (by using fixed investment/output ratios). These vectors represent one margin of the investment matrices $\mathrm{INV_{pp}(r, g, s)}$, which for each country r have the dimension of investing industry (NACE Rev1.1 2-digit) $\times$ investment commodity (CPA 2002, 2-digit). These matrices have been adjusted by a RAS procedure towards the commodity vector of investment in SUTs.[6] However, detailed investment matrix was available only for Austria. Hence, the investment structure for Austria was used as the starting point in the RAS procedure, which on the commodity side is restricted by the investment vector from SUTs and on the sector side by WIOD (or EUROSTAT) information on sectoral investments.

The price of capital is derived as an index of user costs of capital. The time series of dynamic user costs, given in (4.59), shows a high variance in some EU countries due to statistical outliers in the inflation rate of investment prices, PINV. Therefore, the full estimation for the EU 23 has been carried out using the static user cost concept, (4.58). The aggregate depreciation rate by industry for the EU 23 countries has been calculated based on the capital input files from the WIOD/EUKLEMS database. These files contain the depreciation rate by asset and the capital stock for each asset by industry. The asset structure of the capital stock by industry has been used for weighting together the depreciation rates by asset in each industry.

[6]For details about the RAS procedure, see e.g., Temurshoev et al. (2013).

For calculating the capital stock in t and $t-1$, depreciation rates for those EU 27 countries that were not available from the WIOD/EUKLEMS capital input files, have been approximated by those from similar countries.

The price of labour input is calculated from the hours worked by industry in the SEA files of the WIOD database and the values for labour compensation from the same source.

The SEA accounts data for total intermediate inputs have been complemented by physical and monetary energy data, constructed from the WIOD energy accounts and the information in the International Energy Agency (IEA) Energy Prices and Taxes.[7] In the first step, the WIOD physical energy data by energy carrier from 1995 to 2009 have been combined with prices (per physical unit) by energy carrier from the OECD energy prices and taxes. Price information for certain energy carriers (district heating, biomass and fuelwood) had to be taken from other national data sources. From that nominal expenditure for energy and an aggregate energy price by industry can be calculated. In the second step, the energy inputs have been subtracted from total intermediate inputs (from the SEA accounts in the WIOD database) in order to derive total non-energy intermediate inputs. Next, the last had to be split up into domestic and imported non-energy intermediates. This has been done based on the International SUT from the WIOD database. The full matrix of the International SUT can be divided into the part of the diagonal matrices, corresponding to the domestic use matrix and the other matrices, by column corresponding to the import use matrix. Prices for domestic and imported have then been calculated by combining the deflator for total intermediate inputs (from the SEA accounts in the WIOD database) with a deflation procedure for imported intermediates, derived from the information in the WIOD database. This procedure encompasses the following steps:

[7] Further details of the IEA statistics can be obtained from `http://www.iea.org/stats/`.

1. converting the gross output deflator (in national currency) from the SEA accounts into national commodity deflators by using market share matrices,
2. converting the commodity deflator into a previous years prices (pyp) series,
3. converting the international SUT from US dollars into national currency and then applying the commodity pyp of each sending country and the exchange rate between the sending and the receiving country in order to generate international SUT at pyp from the perspective of the receiving country.

Then the import matrix from international SUT in national currency and current prices is divided by the import matrix in pyp from step 3, which gives imports in pyp by commodity and user for each country. The sum over all commodities gives the import price (pyp) for each user, which is - in a final step - converted into an import deflator. The price for domestic intermediates is then calculated as a residual by assuming a Divisia price index for the aggregate intermediates price and using total intermediates, imported and domestic intermediates at current prices for the weights in the Divisia price index.

Data for COICOP consumption categories[8], disposable income and financial wealth of households are available for EU 27 from 1995 – 2010 (Eurostat) with some minor data gaps which have been filled by interpolation techniques. Data for the durable stock have been constructed by assuming lifetimes for the 4 durable categories (audio/video goods, passenger cars, household appliances, and owner occupied dwellings with respective depreciation rates of 0.2, 0.083, 0.05 and 0.015) and applying the perpetual inventory method (i.e., stock-flow equation) from a starting value of the durable stock in 1995 on. For the estimation of this starting value other statistical sources that contain information about physical measures of household durables (ODYSSEE database, EUROCONSTRUCT) and information from Euro-

[8] COICOP stands for "Classification of Individual Consumption According to Purpose"; see the United Nations Statistics Division's COICOP information at `http://unstats.un.org/unsd/cr/registry/regcst.asp?Cl=5`.

stat on unit prices in consumer and investment price measurement have been used.[9] The obtained capital stock data are fully consistent wit the COICOP expenditure data (in current prices), applying the depreciation rate according to the lifetimes. Prices for aggregates of durables and non-durables have been derived by applying the Divisia price index.

Eurostat financial and sectoral accounts are the primary source of disposable household income and components (wages and operating surplus, government payments, property income), stock of household debt and stock of net financial assets. Interest rate on assets is the calculated by relating property income of households from sectoral accounts to net financial assets. Interest rate for user costs of durable stock is the bond market rate or household credit prime rate. Borrowing limits $(1-\theta)$ for each country are calculated as the relationship of long term debt to the total stock of durables.

In general, the COICOP data (expenditure, prices) have been taken from Eurostat, only the splitting up of energy into electricity and heating had to rely on additional statistical sources. Electricity expenditure has been estimated by combining IEA energy balance data (for the household sector) with IEA energy prices. The category heating has been treated as the residual in the category Energy. Starting from a bridge matrix between the classification of SUTs (CPA 2002, 2-digit) and COICOP for Austria in purchaser prices, bridge matrices for the other countries have been constructed by application of RAS. Energy efficiency indices for heating and electrical appliances are taken from the ODYSSEE database, while TREMOVE database[10] was the primary source of the energy efficiency index for vehicles.

Finally, the SEA accounts of the WIOD database (release December 2011) have been used to obtain the following labour market data set:

[9] Further information about the ODYSSEE and EUROCONSTRUCT databases can be found, respectively, at `http://www.odyssee-indicators.org/` and `http://www.euroconstruct.org/`.

[10] For more information visit the website `http://www.tremove.org/`.

- labour compensation of employees by skill type (high, medium and low),
- hours worked of employees by skill type, and
- number of high-, medium- and low-skilled employees.

That, in turn, led to a calculation of hourly wages for the three types of skill. These data have been complemented by unemployment rates for the same three skill types from EUROSTAT.

Appendix A

List of FIDELIO variables

Below we provide the complete list all the variables of FIDELIO's equations that show up in Chapters 3 and 4. For simplicity, basic prices and purchasers' prices in the description of the variables are denoted as bp and pp, respectively.

Notation	Description of the variable
AddKS(r,s)	Add factor to capital stock equation that avoids negative stocks, if existed in the base year
$\mathrm{a}_{\mathrm{aids}}^{\mathrm{energy}}$(r)	Energy price in the AIDS system (used for energy split) for region r
$\mathrm{a}_{\mathrm{aids}}^{\mathrm{tran}}$(r)	Transport price in the AIDS system (for transport split) for region r
$\mathrm{a}_{\mathrm{qaids}}$(r)	First price index in QAIDS demand system for region r
AssetH(r)	Assets of households in region r
$\mathrm{b}_{\mathrm{qaids}}$(r)	Second price index in QAIDS demand system for region r
BRG(r,g,c)	Bridge matrix between the COICOP commodities c and the CPA products g
CapTranH(r)	Capital transfers of households in region r
Cdur(r)	Sum of durable commodities demanded by consumers in region r
CEF(r,c)	Efficiency factor of commodity c in region r

Notation	Description of the variable
Cndr(r)	Aggregate non-durable commodity demanded by consumers in region r
$\mathrm{Cnpish_{pp}(r)}$	Regional NPISH consumption at pp
$\mathrm{Cpub_{pp}(r)}$	Regional public consumption at pp
CS(r,cd)	Stocks of durable consumption commodity cd demanded in region r
CShouse(r)	Total stock of rented and owner occupied houses (in nominal terms)
CStot(r)	Total stocks of durable consumer commodities in region r
Ctot(r)	Total consumption of durable and non-durable commodities in region r
D(r,s)	Total domestic non-energy inputs at pp of sector s in region r
D_Q(r,s)	Share of domestic non-energy inputs at pp in gross output at bp of sector s in region r
DebtH(r)	Debt of households in region r
DeprH(r)	Depreciation accruing to households in region r
DInc(r)	Disposable income of households in region r
DLiabH(r)	Change in other households' liabilities except debt in region r
DPR(r,cd)	Depreciation rate of consumer durable *cd* in region r
DPR_Q(r,s)	Share of depreciation in gross output of sector s in region r
DPRS(r,s)	Depreciation rate of capital in sector s and region r
DPRtot(r)	Average depreciation rate of all consumer durables in region r
DSTR(r,g,u)	Share of domestic non-energy good g in total domestic non-energy inputs used by sector s in region r
E(r,s)	Total (domestic and imported) energy inputs at pp of sector s in r
E_Q(r,s)	Share of total energy inputs at pp in gross output at bp of sector s in r
EMP(r,s,sk)	Employment by skill type *sk* in sector s and region r
$\mathrm{EMP_{tot}(r, s)}$	Total number of employees in sector s and region r
ESTR(r,g,s)	Share of energy good g (domestic and imported) in total energy inputs used by sector s in region r
$\mathrm{EXP_{row}(r, g)}$	Exports of good g to the rest of the world (in national currency of the exporting region)
$\mathrm{EXP_{row}(r, g)}$	Region r's exports of good g to the rest of the world

Notation	**Description of the variable**
FBHH(r,k)	Financial balance k (e.g., wages, social security contributions) of households in region r
$\mathrm{G_{bp}(r, g, u)}$	Demand for composite (domestic and imported) good g at bp by user u in region r
$\mathrm{GD_{bp}(r, g, u)}$	Demand for domestically produced good g at bp by user u in region r
$\mathrm{G_{pp}(r, g, u)}$	Demand for composite (domestic and imported) good g at pp by user u in region r
HRWK(r,s)	Number of hours worked in sector s and region r
HrWktot(r)	Total number of hours worked in region r
I_K(r,s)	Ratio of investment goods at pp to capital stock at bp of sector s in r
I_Q(r,s)	Share of investment goods at pp in gross output at bp of sector s in r
IMP(r,g,u)	Demand for total imports at CIF prices of good g by user u in region r
IntAssetH(r)	Interest rate on assets of households in region r
IntD(r)	Interest rate relevant for capital costs of firms and households' purchases of durable commodities
$\mathrm{INV_{pp}(r, s)}$	Demand for investments at pp by sector s in region r
ISTR(r,g,s)	Share of investment good g in total investment goods used by sector s in region r
$\mathrm{I_{tncs}(r, r_1, g)}$	Index of transit costs of good g delivered from region r to region r_1 (defined with respect to (w.r.t.) the base year)
$\mathrm{I_{trf}(r, r_1, g)}$	Index of tariffs on good g between regions r and r_1 (w.r.t. the base year)
$\mathrm{I_{xrate}(r, r_1)}$	Index of exchange rate between regions r and r_1 (w.r.t. the base year)
K(r,s)	Capital inputs (cash flow) of sector s in region r
K_Q(r,s)	Share of capital input in gross output (both at bp) of sector s in r
Khous.rent(r)	Stock of rented houses in region r (in real terms)
Khous.tot(r)	Total stock of rented and owner occupied houses (in real terms)
KS(r,s)	Capital stock of sector s in region r
L(r,s)	Labour inputs of sector s in region r
L_Q(r,s)	Share of labour input in gross output (both at bp) of sector s in r

Notation	**Description of the variable**
LH_L(r,s)	High-skilled labour share in total labour inputs of sector s in region r
LiabH(r)	Liabilities of households in region r
LL_L(r,s)	Low-skilled labour share in total labour inputs of sector s in region r
LM_L(r,s)	Medium-skilled labour share in total labour inputs of sector s in r
LReg(r)	Total labour demand in region r
LReg(r)	Total labour demand in region r
LSR(r,sk)	Proportion of labour supply of skill type sk in total population
LSUP(r,sk)	Labour supply of skill type sk in region r
M(r,s)	Total imported non-energy inputs at pp of sector s in region r
M_Q(r,s)	Share of imported non-energy inputs at pp in gross output at bp of sector s in region r
MAKE(r,g,s)	Total supply of good g by sector s in r (i.e., make matrix element)
$\mathrm{MGR_{paid}(r,g,u)}$	Total margins rate (w.r.t. total use at pp minus net taxes) paid on non-margin good g by user u in region r
$\mathrm{MGS_{rec.}(r,g,u)}$	Share of margin good g in total margins by user u and region r
MKSH(r,g,s)	Share of output of sector s in the production of good g in region r (i.e., market share matrix entry)
MRG(r,g,u)	Trade and transport margins of good g paid by user u in region r
MSH(r,g,u)	Share of imports in total use (both at bp) of good g used by user u in r
MSTR(r,g,s)	Share of imported non-energy good g in total imported non-energy inputs used by sector s in region r
MUSE(r,g,u)	Imported use at bp of good g by user u in region r
$\mathrm{MUSE_{eu}(r,g,u)}$	EU-imports of good g used by user u in region r (in national currency)
N(r,s)	Total (domestic and imported) non-energy inputs at pp of sector s in region r [=D(r,s)+M(r,s)]
N_Q(r,s)	Share of total non-energy inputs at pp in gross output at bp of sector s in region r
NAssetH(r)	Net assets of households in region r
NLendH(r)	Net lending of households in region r

Notation	**Description of the variable**
NSTR(r,g,s)	Share of non-energy good g (domestic and imported) in total non-energy inputs used by s in region r
OpspH(r)	Operating surplus of households in region r
$\mathrm{Opsp}_{\mathrm{na.io}}(\mathrm{r})$	Ratio of households' operating surplus in national accounts to those in input-output tables
OthIncH(r)	Other income of households in region r
PC(r,c)	Price of consumption commodity c in region r
PCdur(r)	Aggregate price of all durable commodities in region r
PCndr(r)	Aggregate price of all non-durable commodities in region r
Pcon(r)	Consumer price in region r
PCS(r,cd)	Price of stocks of durable commodity cd in region r
PCStot(r)	Price of stock of all durable commodities in region r
PCtot(r)	Aggregate price of all commodities in region r
PD(r,s)	Price of domestic non-energy inputs of sector s in region r
PE(r,s)	Price of energy inputs of sector s in region r
Penergy(r)	Aggregate price of energy (electricity and heating) in region r
PenH(r)	Pension funds of households in region r
$\mathrm{PGD}_{\mathrm{bp}}(\mathrm{r}, \mathrm{g})$	Basic price of domestic good g in region r (note that this price is not user-specific)
$\mathrm{PGD}_{\mathrm{pp}}(\mathrm{r}, \mathrm{g}, \mathrm{exp})$	FOB price of exports of good g in the exporting region r
$\mathrm{PGD}_{\mathrm{pp}}(\mathrm{r}, \mathrm{g}, \mathrm{u})$	Purchaser price of domestic good g paid by user u in region r
$\mathrm{PGF}(\mathrm{r}, \mathrm{r}_1, \mathrm{g})$	Purchaser (foreign) price of imported good g produced in region r_1 when purchased in region r; equivalently, CIF price at the border of importing region r for good g imported from region r_1
$\mathrm{PG}_{\mathrm{row}}(\mathrm{r})$	FOB price of exports of the rest of the world
Phouse(r)	House price in region r
PIMP(r,g,u)	Total import price of good g including domestic margins and net taxes on products
$\mathrm{PIMP}_{\mathrm{cif}}(\mathrm{r}, \mathrm{g}, \mathrm{utr})$	Total imports CIF price at the border of importing region r for good g and user utr

Notation	Description of the variable
PINV(r,s)	Price of investments of sector s in region r
PK(r,s)	Price of capital stock of sector s in region r
PL(r,s)	Price of labour of sector s in region r
PM(r,s)	Price of imported non-energy inputs of sector s in region r
Pop(r)	Population size in region r
PQ(r,s)	Output price of sector s in region r
ProfIncH(r)	Profit income of households in region r
$\mathrm{PSTR}(\mathrm{r}, \mathrm{g}, \mathrm{u}, \mathrm{g}_1)$	Price structure of domestically produced goods (used in translating bp of g_1 into the pp of g)
Ptran(r)	Aggregate price of (private and public) transport in region r
PUSE(r,g,u)	Use price of composite (domestic and imported) good g paid by user u in region r
PUSEtot(r,con)	Consumer price in region r, Pcon(r)
PUSEtot(r,u)	Aggregate use price of all "inputs" (user cost) of user u in region r
Q(r,s)	The value of gross output at bp of sector s in region r
QReal(r)	Total real gross output of region r
$\mathrm{Ra}_{\mathrm{aids}}^{\mathrm{energy}}(\mathrm{r})$	Base-year residuals from the AIDS energy price index equation
$\mathrm{Ra}_{\mathrm{qaids}}(\mathrm{r})$	Base-year residuals from the first QAIDS price index equation
$\mathrm{Ra}_{\mathrm{aids}}^{\mathrm{tran}}(\mathrm{r})$	Base-year residuals from the AIDS energy price index equation
RCndr(r)	Base-year residuals from the non-durable consumption demand equation
RCS(r,cd)	Base-year residuals from the stocks of durable consumption demand equation
RE(r,s)	Base-year residuals from the energy share equation
Rent_Pop(r)	Ratio of the stock of rented houses in real terms per person in region r
RK(r,s)	Base-year residuals from the capital share equation
RL(r,s)	Base-year residuals from the labour share equation
RLH(r,s)	Base-year residuals from the high-skilled labour share equation
RLL(r,s)	Base-year residuals from the low-skilled labour share equation
RM(r,s)	Base-year residuals from the imported non-energy share equation

Notation	**Description of the variable**
RPQ(r,s)	Base-year residuals from the output price equation
RWelect(r)	Base-year residuals from the AIDS electricity share equations
RWEM(r,s,sk)	Base-year residuals from the wage curve equations (see also WEM)
RWHR(r,s)	Base-year residuals from the average hourly wage (i.e., labour price) equation
RWprivtr(r)	Base-year residuals from the AIDS private transport share equations
RWqaids(r,cn)	Base-year residuals from the QAIDS non-durables expenditure shares equations
S(r,u)	Total (domestic an imported) intermediate use at pp of user u in r
S_Q(r,s)	Share of total intermediate inputs at pp in gross output at bp of sector s in region r
SaveH(r)	Gross savings of households in region r
SBP_Q(r,s)	Share of production subsidies in gross output of sector s in region r
SK_L(r,s,high)	Same as LH_L(r,s)
SK_L(r,s,low)	Same as LL_L(r,s)
SK_L(r,s,med)	Same as LM_L(r,s)
SprAsset(r)	Spread of interest rates on households' assets from those of consumer durables
SprHouse(r)	Spread of housing interest rates from those of consumer durables
SSC_L(r,s)	Share of employers' social security contributions in total labour compensation
SscnH(r)	Social security contributions of households in region r
$\mathrm{Sscn}_{\mathrm{na.io}}(\mathrm{r})$	Ratio of households' social security contributions in national accounts to those in input-output tables
t	Time variable
TaxesH(r)	Taxes payed by households in region r
Tht(r)	Down payment requirement parameter of region r
$\mathrm{TMSH}(\mathrm{r}, \mathrm{r}_1, \mathrm{g}, \mathrm{u})$	Share of good g imported from r_1 to r and used by u in the total imports of region r (in Euros)

Notation	Description of the variable
$\mathrm{TMSH_{red}(r, r_1, g, utr)}$	Reduced partners' import shares: same as $\mathrm{TMSH(r, r_1, g, u)}$ but with only one intermediate sector
$\mathrm{TNCS(r, r_1, g)}$	Transit costs (power) for third countries to transit non-service good g from r_1 to r (i.e., cif-fob gap)
TransfH(r)	Governmental transfers to households in region r
$\mathrm{TRD_{cif}(r, r_1, g, u)}$	Trade matrix at cif prices: trade of good g from r_1 to r used by u
$\mathrm{TRD_{fob}(r, r_1, g, u)}$	Trade matrix at fob prices: trade of good g from r_1 to r used by u
$\mathrm{TRDM(r, r_1, g, utr)}$	Use utr's demands in region r for imports of good g from region r_1 (trade matrix, in euros)
$\mathrm{TRD_{wiod}(r, r_1, g, u)}$	WIOD trade matrix: trade of good g from r_1 to r by user u
$\mathrm{TRF(r, r_1, g)}$	Tariff on good g imported from region r_1 to region r
Tx_Inc(r)	Income tax rate applied to households in region r
TXP_Q(r,s)	Share of production taxes in gross output of sector s in region r
TXS(r,g,u)	Taxes less subsidies on good g paid by user u in region r
TXSR(r,g,u)	Product tax net of subsidy rate (w.r.t. total use at pp) of good g used by user u in region r
UCKS(r,s)	User cost of capital of sector s in region r
UNEMR(r,sk)	Unemployment rate of labour skill type sk in region r
$\mathrm{USE_{bp}(r, g, u)}$	Total (domestic and imported) use at bp of good g by user u in r
$\mathrm{USE_{pp}(r, g, u)}$	Total (domestic and imported) use at pp of good g by user u in r
USTR(r,g,u)	Share of good g (domestic and imported) in total goods used by user u in region r
VA(r,s)	Total value added at bp of sector s in region r
VAC(r,v,s)	Component v of total value added of sector s in region r
WageH(r)	Wages of households in region r
$\mathrm{Wage_{na.io}(r)}$	Ratio of households' wages in national accounts to those in input-output tables
Welect_energy(r)	AIDS expenditure share of electricity in energy for region r
WEM(r,s,sk)	Wage per employee of labour skill type sk in sector s and region r

Notation	Description of the variable
WHR(r,s,sk)	Hourly wage of labour skill type *sk* in sector *s* and region *r*
WHRtot(r,s)	Average hourly wage of all labour skill types in sector *s* and region *r*
WKTM(r,s)	Working time per employee in sector *s* and region *r*
Wprivtr_tran(r)	AIDS expenditure share of private transport in total transport for *r*
$\mathrm{W_{qaids}(r, cn)}$	QAIDS expenditure share of non-durable commodity *cn* in region *r*
Xrate(r)	Exchange rate: national currency of region *r* per Euro
Zz(r)	Cash-on-hand minus voluntary equity holdings of households in *r*

Appendix B

Sector and product classifications

Industry classification used in FIDELIO corresponds to the statistical classification of economic activities in the European Community, NACE Rev1.1 (EC, 2002a) and is presented in Table B.1.[1] The term NACE is derived from the French *Nomenclature statistique des activités économiques dans la Communautéuropéenne*. It should be noted that this classification is compatible with the United Nations International Standard Industrial Classification of All Economic Activities, ISIC Rev3.1.

Products classification used in FIDELIO corresponds to the Classification of Product by Activities, CPA (EC, 2002b) and is given in Table B.2.[2] This classification, in turn, is compatible with the United Nations Central Product Classification, CPC Rev1.1.

[1] More detailed breakdown of NACE Rev1.1. can be found from `http://ec.europa.eu/eurostat/ramon/nomenclatures/index.cfm?TargetUrl=ACT_OTH_DFLT_LAYOUT&StrNom=NACE_1_1&StrLanguageCode=EN`.

[2] Further details of CPA 2002 can be obtained from `http://ec.europa.eu/eurostat/ramon/nomenclatures/index.cfm?TargetUrl=LST_NOM_DTL&StrNom=CPA&StrLanguageCode=EN&IntPcKey=&StrLayoutCode=HIERARCHIC`.

Table B.1: Statistical classification of economic activities in the European Community, NACE Rev1.1 (EC, 2002a)

No.	FIDELIO code	NACE Rev1.1	Description
1	sec01	A01	Agriculture, hunting and related service activities
2	sec02	A02	Forestry, logging and related service activities
3	sec05	B05	Fishing, operating of fish hatcheries and fish farms; service activities incidental to fishing
4	sec10	CA10	Mining of coal and lignite; extraction of peat
5	sec11	CA11	Extraction of crude petroleum and natural gas; service activities incidental to oil and gas extraction excluding surveying
6	sec12	CA12	Mining of uranium and thorium ores
7	sec13	CB13	Mining of metal ores
8	sec14	CB14	Other mining and quarrying
9	sec15	DA15	Manufacture of food products and beverages
10	sec16	DA16	Manufacture of tobacco products
11	sec17	DB17	Manufacture of textiles
12	sec18	DB18	Manufacture of wearing apparel; dressing and dyeing of fur
13	sec19	DC19	Tanning and dressing of leather; manufacture of luggage, handbags, saddlery, harness and footwear
14	sec20	DD20	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
15	sec21	DE21	Manufacture of pulp, paper and paper products
16	sec22	DE22	Publishing, printing and reproduction of recorded media
17	sec23	DF23	Manufacture of coke, refined petroleum products and nuclear fuels
18	sec24	DG24	Manufacture of chemicals and chemical products
19	sec25	DH25	Manufacture of rubber and plastic products
20	sec26	DI26	Manufacture of other non-metallic mineral products
21	sec27	DJ27	Manufacture of basic metals
22	sec28	DJ28	Manufacture of fabricated metal products, except machinery and equipment
23	sec29	DK29	Manufacture of machinery and equipment n.e.c.
24	sec30	DL30	Manufacture of office machinery and computers
25	sec31	DL31	Manufacture of electrical machinery and apparatus n.e.c.
26	sec32	DL32	Manufacture of radio, television and communication equipment and apparatus
27	sec33	DL33	Manufacture of medical, precision and optical instruments, watches and clocks

No.	FIDELIO code	NACE Rev1.1	Description
28	sec34	DM34	Manufacture of motor vehicles, trailers and semi-trailers
29	sec35	DM35	Manufacture of other transport equipment
30	sec36	DN36	Manufacture of furniture; manufacturing n.e.c.
31	sec37	DN37	Recycling
32	sec40	E40	Electricity, gas, steam and hot water supply
33	sec41	E41	Collection, purification and distribution of water
34	sec45	F45	Construction
35	sec50	G50	Sale, maintenance and repair of motor vehicles and motorcycles; retail sale services of automotive fuel
36	sec51	G51	Wholesale trade and commission trade, except of motor vehicles and motorcycles
37	sec52	G52	Retail trade, except of motor vehicles and motorcycles; repair of personal and household goods
38	sec55	H55	Hotels and restaurants
39	sec60	I60	Land transport; transport via pipelines
40	sec61	I61	Water transport
41	sec62	I62	Air transport
42	sec63	I63	Supporting and auxiliary transport activities; activities of travel agencies
43	sec64	I64	Post and telecommunications
44	sec65	J65	Financial intermediation, except insurance and pension funding
45	sec66	J66	Insurance and pension funding, except compulsory social security
46	sec67	J67	Activities auxiliary to financial intermediation
47	sec70	K70	Real estate activities
48	sec71	K71	Renting of machinery and equipment without operator and of personal and household goods
49	sec72	K72	Computer and related activities
50	sec73	K73	Research and development
51	sec74	K74	Other business activities
52	sec75	L75	Public administration and defence; compulsory social security
53	sec80	M80	Education
54	sec85	N85	Health and social work
55	sec90	O90	Sewage and refuse disposal, sanitation and similar activities
56	sec91	O91	Activities of membership organisation n.e.c.
57	sec92	O92	Recreational, cultural and sporting activities
58	sec93	O93	Other service activities
59	sec95	P95	Private households with employed persons

Table B.2: Classification of Product by Activities, CPA (EC, 2002b)

No.	FIDELIO code	CPA Rev.1	Description
1	com01	CPA_A01	Products of agriculture, hunting and related services
2	com02	CPA_A02	Products of forestry, logging and related services
3	com05	CPA_B05	Fish and other fishing products; services incidental of fishing
4	com10	CPA_CA10	Coal and lignite; peat
5	com11	CPA_CA11	Crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying
6	com12	CPA_CA12	Uranium and thorium ores
7	com13	CPA_CB13	Metal ores
8	com14	CPA_CB14	Other mining and quarrying products
9	com15	CPA_DA15	Food products and beverages
10	com16	CPA_DA16	Tobacco products
11	com17	CPA_DB17	Textiles
12	com18	CPA_DB18	Wearing apparel; furs
13	com19	CPA_DC19	Leather and leather products
14	com20	CPA_DD20	Wood and products of wood and cork (except furniture); articles of straw and plaiting materials
15	com21	CPA_DE21	Pulp, paper and paper products
16	com22	CPA_DE22	Printed matter and recorded media
17	com23	CPA_DF23	Coke, refined petroleum products and nuclear fuels
18	com24	CPA_DG24	Chemicals, chemical products and man-made fibres
19	com25	CPA_DH25	Rubber and plastic products
20	com26	CPA_DI26	Other non-metallic mineral products
21	com27	CPA_DJ27	Basic metals
22	com28	CPA_DJ28	Fabricated metal products, except machinery and equipment
23	com29	CPA_DK29	Machinery and equipment n.e.c.
24	com30	CPA_DL30	Office machinery and computers
25	com31	CPA_DL31	Electrical machinery and apparatus n.e.c.
26	com32	CPA_DL32	Radio, television and communication equipment and apparatus
27	com33	CPA_DL33	Medical, precision and optical instruments, watches and clocks
28	com34	CPA_DM34	Motor vehicles, trailers and semi-trailers
29	com35	CPA_DM35	Other transport equipment
30	com36	CPA_DN36	Furniture; other manufactured goods n.e.c.

No.	FIDELIO code	CPA Rev.1	Description
31	com37	CPA_DN37	Secondary raw materials
32	com40	CPA_E40	Electrical energy, gas, steam and hot water
33	com41	CPA_E41	Collected and purified water, distribution services of water
34	com45	CPA_F45	Construction work
35	com50	CPA_G50	Trade, maintenance and repair services of motor vehicles and motorcycles; retail sale of automotive fuel
36	com51	CPA_G51	Wholesale trade and commission trade services, except of motor vehicles and motorcycles
37	com52	CPA_G52	Retail trade services, except of motor vehicles and motorcycles; repair services of personal and household goods
38	com55	CPA_H55	Hotel and restaurant services
39	com60	CPA_I60	Land transport; transport via pipeline services
40	com61	CPA_I61	Water transport services
41	com62	CPA_I62	Air transport services
42	com63	CPA_I63	Supporting and auxiliary transport services; travel agency services
43	com64	CPA_I64	Post and telecommunication services
44	com65	CPA_J65	Financial intermediation services, except insurance and pension funding services
45	com66	CPA_J66	Insurance and pension funding services, except compulsory social security services
46	com67	CPA_J67	Services auxiliary to financial intermediation
47	com70	CPA_K70	Real estate services
48	com71	CPA_K71	Renting services of machinery and equipment without operator and of personal and household goods
49	com72	CPA_K72	Computer and related services
50	com73	CPA_K73	Research and development services
51	com74	CPA_K74	Other business services
52	com75	CPA_L75	Public administration and defence services; compulsory social security services
53	com80	CPA_M80	Education services
54	com85	CPA_N85	Health and social work services
55	com90	CPA_O90	Sewage and refuse disposal services, sanitation and similar services
56	com91	CPA_O91	Membership organisation services n.e.c.
57	com92	CPA_O92	Recreational, cultural and sporting services
58	com93	CPA_O93	Other services
59	com95	CPA_P95	Private households with employed persons

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Abstract

In this report we present complete information about the Fully Interregional Dynamic Econometric Long-term Input-Output Model for the EU27 (FIDELIO 1). First, the macro overview of the model is discussed, which presents the main mechanisms of interactions between various blocks of FIDELIO. The second chapter explains the main economic theories underlying FIDELIO consumption, production and labour market blocks. Here, further econometric approaches for estimation of the parameters of all behavioural equations and their results are presented. Then, derivation of all the necessary base-year data (e.g., various commodity use structure and price structure matrices, trade matrix, base-year residuals, etc.) are discussed in detail. All FIDELIO equations are presented (with discussions) in Chapter 4. Finally, a full description of the data sources is given in the last chapter. It will become clear from this description document that FIDELIO is appropriate for the impact assessment purposes of diverse (economic and/or environmental) policy questions of our times.

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