Charles Babbage and Mathematical Aspects of the Miraculous

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1 Introduction

Our story begins in February of 1829 with the death of the Right Honourable and Reverend Francis Henry Edgerton, eighth Earl of Bridgewater. This unconventional man, with eccentricities that included dressing his dogs in custom-made clothing, had a deep interest in natural theology. This passion ran so deep that the Earl of Bridgewater's will made provision for eight thousand pounds to be invested and placed at the disposal of the President of the Royal Society of London. The person or persons selected by the president should be appointed to write, print, and publish one thousand copies of a work:

> On the Power, Wisdom, and Goodness of God as manifested in the Creation; illustrating such work by all reasonable arguments, as for instance the variety and formation of God's creatures in the animal, vegetable, and mineral kingdoms, the effect of digestion, and thereby of conversion; the construction of the hand of man; and an infinite variety of other arguments. [1]

Eight men were chosen by the President of the Royal Society to carry out the wishes of the late Earl of Bridgewater. The Reverend William Whewell, fellow of Trinity College Cambridge, authored the first published Bridgewater treatise entitled *Astronomy and General Physics Considered with Reference to Natural Theology*. In it he made the following comment:

> We may thus, with the greatest propriety, deny to the mechanical philosophers and mathematicians of recent times any authority with regard to their views of the administration of the universe; we have no reason whatever to expect from their speculations any help, when we ascend to the first cause and supreme rule of the universe. But we might perhaps go farther, and assert that they are in some respects less likely than men employed in other pursuits, to make any clear advance toward such a subject of speculation. [7]

Upon reading this, Charles Babbage decided to respond. At the time, the 45 year old was well known throughout Great Britain as a mathematician and inventor. He was Lucasian Professor of Mathematics at Cambridge, a post that Newton once held. Babbage's analytical difference engine, a precursor to our computer, had been funded by an act of Parliament to aid in navigational calculations.

Provoked by Whewell, Babbage replied to him, and the Bridgewater Treatises in general, by publishing a what he referred to as “a fragment” of his own. He titled it *The Ninth Bridgewater Treatise*. Although he was quick to note in the preface that this work was not part of the original set of treatises, he justified his appropriation of the title because he was furthering the intentions of the late Earl of Bridgewater. To make clear his intentions, on its title page he included the quotation from Whewell, and stated in the preface that he wrote his treatise due to the prejudice he encountered in Whewell’s Bridgewater Treatise.

Babbage’s motivation was not merely due to Whewell’s provocation. Later in the preface Babbage stated, “One of the chief defects of the Treatises above referred to appear to me to arise from their not pursuing the argument to a sufficient extent.” He went on to assert that some of his abstract mathematical inquiries, “most removed from any practical application” have led to new perspectives and analogies concerning natural theology. This is illustrated most fully in Chapters VIII through X of *The Ninth Bridgewater Treatise*.

2 Miracles in *The Ninth Bridgewater*

In chapter VIII of *The Ninth Bridgewater*, Babbage examined the nature of miracles. The theological question of how God interacts with his creation had received renewed interest in pre-Victorian England. Much of this was due to a variety of scientific discoveries, particularly in geology. Miracles in particular drew the attention of several writers of natural theology. [3] *The Ninth Bridgewater* can be examined as one of several works during this time concerning the miraculous. According to Babbage

> ... it is more consistent with the attributes of the Deity to look upon miracles not as deviations from the laws assigned by the Almighty for the government of matter and of mind; but as the exact fulfillment of much more extensive laws than those we suppose to exist. [1]

Babbage remarked that such a view of the miraculous assigns greater power and knowledge to God than a God who is constantly intervening or even interfering in creation. He then proceeded to illustrate his views with two extended mathematical examples.

The first of these concerns a mechanical calculator known as a difference engine. Babbage described God as a master programmer of the universe. He is one who programmed the apparent exceptions - the miracles - to follow a uniform natural law. Babbage envisioned an observer of a difference engine who witnesses a sequence of numbers as outputs of the machine. Without fail, every one of these numbers is a perfect square. After millions of observations, one of the numbers is not a perfect square. The pattern of square numbers then returns exactly where it had left off, and continues on for every other observation.

This scenario is analogous to the way that a natural law would be inferred from numerous observations. In this case, the observer might say, “This machine always produces square numbers.” The one supposed exception to the sequence of square numbers is likened by Babbage as a miracle. This anomaly could have resulted from several causes, but only two were considered. The first explanation was that a secret lever, hidden from view, was activated by the machine’s creator to produce the intended effect. Another explanation of the aberration is that the supposed exception

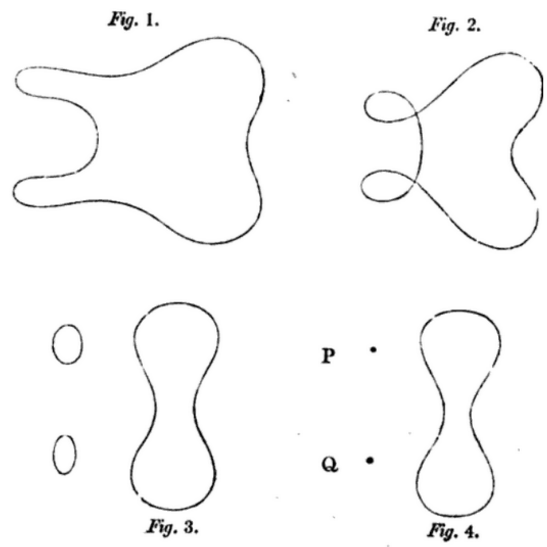

Figure 1: Figure from page 101 of *The Ninth Bridgewater*

of the rule had been programmed into the machine.

Which of these two cases shows the greater power of workmanship for one who built the difference engine? For Babbage, the second case demonstrates that a greater mind and power was at work. In a similar way, miracles, which are supposed exceptions to natural law, are really part of a more complex pattern working.

Babbage's second illustration shared the same perspective of the miraculous as his difference engine thought experiment. However, the second illustration utilized an entirely different area of mathematics. At first glance, the various curves on page 101 of *The Ninth Bridgewater*, which are shown in Figure 1, appear different from one another. The first two are connected, the third contains two portions that are separate from a larger closed curve, and the fourth contains two singular points.

For the fourth curve in particular, sight alone would lead to the conclusion that points P and Q are exceptions to the infinitely many observations that comprise the connected portion. However, as Babbage explained in a note, these singular points as well as points on the curve can be described by the same equation. Moreover, all four the figures on page 101 can be generated from the following equation, where $a, b, c, d,$ and e are all constants that can be varied:

$$(y^2 - 2)^2 = -ax^4 + bx^3 + cx^2 + dx + e. \tag{1}$$

Thus in Figure 1, the points P and Q satisfy the same equation as every point on the curve. The apparent deviations from the closed curve are really manifestations of a higher law, imperceptible to sight, but detectable by mathematics.

Babbage did not give the specific constants used to produce each of the curves in Figure 1. Experimentation, using a graphing utility such as Desmos, achieves a reasonably close approximation. An interesting exercise in curve sketching reveals how the roots of the quartic on the right side of Equation (1) affect the behavior of the curve.

We begin with a quartic polynomial possessing a negative leading coefficient. The other coefficients are given values so that there are only two real roots of the quartic, r_1 and r_2, and the three critical points of the graph of the polynomial are in the interval (r_1, r_2). The left side of Figure 2 shows one example of this. We then vary only the constant term and observe that the shapes similar to those displayed in Figure 1 begin to emerge as the roots of the quartic change. On the right side of Figure 2 there are four real roots, one of which is repeated, a critical point, and between the other two roots. On the left side of Figure 3 there are four distinct real roots. Finally, on the right side of Figure 3 there are four real roots, one of which is repeated, and is not between the other two roots. For these particular values of the coefficient the two singular points appear.

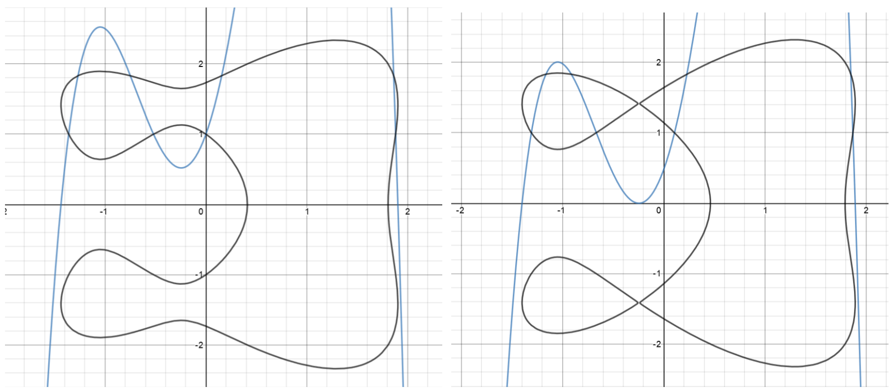

Figure 2: Graphs of $y = -3x^4 + 8.5x^2 + 4x + e$ in blue and $(y^2 - 2)^2 = -3x^4 + 8.5x^2 + 4x + e$ in black for $e = 1$ and 0.48

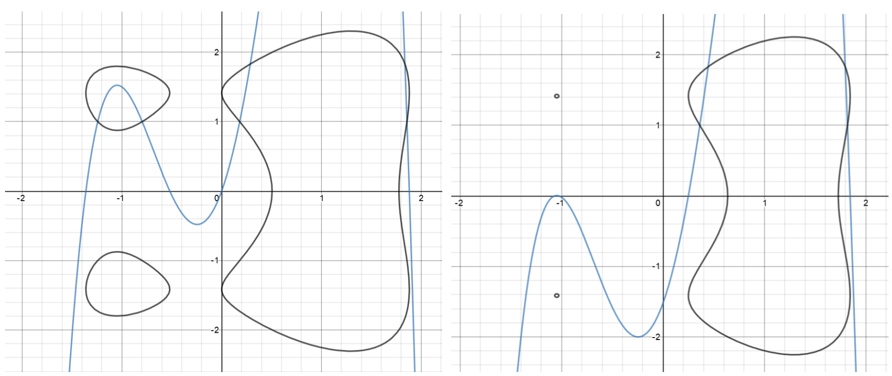

Figure 3: Graphs of $y = -3x^4 + 8.5x^2 + 4x + e$ in blue and $(y^2 - 2)^2 = -3x^4 + 8.5x^2 + 4x + e$ in black for $e = 0$ and -1.52

Babbage had Whewell's quotation in mind as he concluded chapter IX of *The Ninth Bridgewater Treatise*. According to Babbage, this example opened "views of the grandeur of creation perhaps more extensive than any which the sciences of observation or of physics have yet supplied." [1] Not everyone agreed. For instance, the Roman Catholic priest D.W. Cahill commented on the flaws of Babbage's analogy. The miraculous had been reduced to a more extended natural machinery. As Cahill wrote to the bishops of England, "Was there ever published such a monstrous conceit as to reduce miracles to a formula in algebra - to a curve of four dimensions!" [2]

Cahill's comment does spark the interesting question of why Babbage chose this particular curve for his example. Other curves of lower degree will exhibit similar behavior. For instance, the graph of $y^2 = -x^3 + x + c$ changes its number of components and even has a singular point depending on the value of c.

3 Babbage and Hume

Babbage's examination of the miraculous needed to confront a deeper issue that had arisen nearly a century earlier. In 1748 the Scottish philosopher David Hume published *An Enquiry Concerning Human Understanding*. In section X, known as "Of Miracles," Hume contended that miracles, which he considered violation of the laws of nature, were so unlikely as to not exist. There are several elements to "Of Miracles," and the one that drew Babbage attention relates to testimony of miraculous events.

In Hume's words

> The plain consequence is (and it is a general maxim worthy of our attention), that no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous than the fact which it endeavors to establish. [4]

The use of the word "miraculous" concerning testimony has obscured some of Hume's meaning, but reference to another key passage clarifies this usage. Elsewhere in section X Hume remarked

> When any one tells me, that he saw a dead man restored to life, I immediately consider with myself, whether it be more probable, that this person should either deceive or be deceived, or that the fact, which he relates, should really have happened. I weigh the one miracle against the other; and according to the superiority, which I discover, I pronounce my decision, and always reject the greater miracle. [4]

To establish that a miracle has occurred, following Hume, "the falsehood of testimony of the miracle must be *more miraculous* than the miracle." By meeting Hume on his own terms, Babbage recast Hume's criterion to read, "The falsehood of testimony of the miracle must be *more improbable* than the miracle."

This restatement was not original to Babbage. A similar argument can be traced as far back as George Campbell's *A Dissertation on Miracles* in 1762. What was novel in Babbage's approach was to use mathematical formalism to further restate Hume's criterion. A miracle has occurred if

$$P(\text{ Falsehood of Testimony of Miracle }) < P(\text{ Miracle }). \tag{2}$$

Babbage realized that Hume had simply stated an equivalent version of this criterion, and then asserted that it could never be met. Hume had made no effort to weigh the probabilities in the above inequality. Chapter X and Appendix E of *The Ninth Bridgewater* concern the calculation of these probabilities. We will follow and expand on Babbage's line of argument.

We begin with the calculation of the probability of a miracle. In order to assign a numerical value to this probability, Babbage uses Laplace's rule of succession. Laplace derived this result in order to confront the so-called sunrise problem [5]. Given that the sun has risen a specified number of times, and there is no prior knowledge on what it will do tomorrow, what is the probability that the sun rises again? More generally, suppose that there are N observations, which can either be classified as a success or a failure, and that s successes were observed. The probability that the $N + 1$ observation is a success is given by the following formula

$$P(N+1 \text{ observation is a success } | s \text{ successes }) = \frac{s+1}{N+2}. \tag{3}$$

In order to use Equation 3 regarding something as miraculous as a dead person being restored to life, Babbage made several rough estimates and provided us with the details. He first supposed that the world is 6000 years old, and that there have consistently been 30 years per generation. Thus there have been 200 generations of people since the beginning of the world. He further estimates that at every point in history the earth's human population has numbered 1 billion, which was the estimated world population in the mid-nineteenth century. This gave Babbage the estimate that 200 billion people have died, which is used as the value for N in Equation 3. For the sake of argument, Babbage supposed that following the death of every one of these 200 billion, none of them were restored to life. With no successes, the value of $s = 0$. By Laplace's rule of succession, the probability that the next person who dies will be restored to life is

$$\frac{s+1}{N+2} = \frac{0+1}{200{,}000{,}000{,}000+2} = \frac{1}{200{,}000{,}000{,}002} \approx 5 \cdot 10^{-12}.$$

This is an astonishingly small probability, but it is not zero.

We now consider the probability of the falsehood of testimony concerning a miracle. Babbage supposed mutually independent witnesses who are reliable 99% of the time. Given two such witnesses, due to the crucial assumption of independence, the probability that they both agree on a falsehood is $(1/100)\cdot(1/100) = 1/10,000$. Each additional independent witness will further reduce the overall probability of mistaken or false testimony. With n such witnesses

$$P(\text{ Falsehood of Testimony }) = (1/100)^n. \tag{4}$$

To return to Hume's criterion, in order to establish by testimony that something as seemingly impossible as a dead person being restored to life had occurred, we would need to combine Equation 4 with our probability of a miracle $2 \cdot 10^{-11}$ in the inequality 2. A miracle has occurred when

$$(1/100)^n < 5 \cdot 10^{-12}.$$

The question then becomes how large must n be in order to satisfy the inequality.

With only $n = 6$ witnesses that match Babbage's description, the probability that they all give false testimony (either intentionally or unintentionally) concerning the restoration of a dead person to life is $(1/100)^6 = 10^{-12}$. This probability is less than the probability of a dead person being restored to life that was calculated above to be approximately $5 \cdot 10^{-12}$. By Hume's own criterion, such a miracle could be established by testimony.

In appendix E of *The Ninth Bridgewater Treatise* by use of several mathematical proofs, each making subtle distinctions in the nature of the testimony of a purported miracle, Babbage considered other situations. For example, what if the witnesses are less reliable than 99% of the time? Variations on the original question were summarized by Babbage (with emphasis in original):

> ... if independent witnesses can be found, who speak truth more frequently than falsehood, it is ALWAYS possible to assign a number of independent witnesses, the improbability of the falsehood of whose concurring testimony shall be greater than the improbability of the miracle itself. [1]

4 Conclusion

Although many of us, including myself, would disagree with Babbage's definition of a miracle, *The Ninth Bridgewater Treatise* provides a fascinating historical example of the integration of faith and mathematics. Babbage's refutation of Hume as well as his dispute with Whewell demonstrated a firm conviction and willingness to engage with his opponent on his opponent's terms, modelling a type of civility in disagreement.

There are several places in the undergraduate mathematics curriculum where the ideas from *The Ninth Bridgewater* could be discussed. The specific calculations performed by Babbage could be demonstrated as in-class examples of probability to illustrate concepts such as independence. An upper-division seminar course could evaluate Babbage's argument and discuss the role of mathematics in Christian apologetics.

A series of questions below would help start a seminar discussion concerning some of the issues underlying Babbage's examination of the miraculous.

- What was some more of the historical context underlying the discussion of miracles? See, for instance, [3] and [6].
- What are some flaws in how Babbage defined a miracle?
- What is your definition of a miracle? How does your definition compare or contrast with Babbage's interpretation? How does your definition compare or contrast with Hume's interpretation?
- Was Babbage's interpretation of a miracle in a naturalistic setting consistent with a Biblical understanding of the miraculous?
- How effective was Babbage's use of mathematical analogies to illustrate his conception of a miracle? What are the advantages and limitations to using mathematical analogies to illustrate theological truths?
- How effective do you think Babbage's argument concerning the miraculous would be to an unbeliever?
- What are some other arguments, besides Babbage's, that refute Hume's "Of Miracles"?
- What is an appropriate response today to those who would challenge the existence of miracles and our faith in them?

References

[1] Charles Babbage. *The Ninth Bridgewater Treatise, Second Edition.* John Murray, London, 1838.

[2] *The Bengal Catholic Herald, V. IX.* P.S. D'Rozario and Co., Calcutta, 1845.

[3] Walter Cannon. "The Problem of Miracles in the 1830's." *Victorian Studies*, vol. 4, no. 1, 1960, pp. 5–32.

[4] David Hume. “On Miracles” *Enquiry Concerning Human Understanding.* 1748

[5] Pierre Simon Laplace. *Essai philosophique sur les probabilités.* 1814.

[6] Joan L. Richards. “The Probable and the Possible in Early Victorian England” In B. Lightman (Ed.) *Victorian Science in Context*, University of Chicago Press, 2008.

[7] William Whewell. *Astronomy and General Physics Considered with Reference to Natural Theology.* William Pickering, London, 1834.