Marin Mersenne: Minim Monk and Messenger; Monotheism, Mathematics, and Music

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Abstract

Marin Mersenne is one of many names in the history of mathematics known more by a couple of key connections than for their overall life and accomplishments. Anyone familiar with number theory has heard of 'Mersenne primes', which even occasionally appear in broader media when a new (and enormous) one is discovered. Instructors delving into the history of calculus a bit may know of him as the interlocutor who drew Fermat, Descartes, and others out to discuss their methods of tangents (and more). In most treatments, these bare facts are all one would learn about him.

But who was Mersenne, what did he actually do, and why? This paper gives a brief introduction to some important points about his overall body of work, using characteristic examples from his first major work to demonstrate them. We'll especially look into why a monk from an order devoted to being the least of all delved so deeply into (among other things) exploratory mathematics, practical acoustics, and defeating freethinkers, and why that might be of importance today.

1 Introduction

The seventeenth century was a time of ferment in the sciences in Western Europe. For instance, the ancient discipline of mathematics was being dramatically changed. At the beginning of this period coordinate geometry did not exist, and 'calculus' was still largely at the place where Archimedes' investigations had left it; by its end, their combination was ready to have a huge impact. Fields such as chemistry and biology were illuminated by experiments on air pressure and by the microscope.

Particularly stunning was the change in how mathematics was applied to what we today call physics. As an example, Copernicus' new model for the heavens was still of essentially the same type as Ptolemy's, with epicycles required to make it as accurate[1]. In the span of less than one hundred years, Brahe's observations, Kepler's empirical laws, Galileo's experiments and dialogues, and finally Newton's justifications completely changed this situation, allowing the *calculation* of the movements of the heavens.

Near the center of these developments (and many others) for many years was a Parisian monk, Marin Mersenne. Because of the influence of the regular scientific meetings in his simple cell, and of his voluminous correspondence, he has become known to Western intellectual history as

[1]See [20] for a discussion of Brahe's view that each system had its places of more accuracy; see also [19].

the preeminent 'intelligencer' of the time, a clearinghouse for all the latest scientific developments. Beyond this more generally known activity, Mersenne also produced much of his own work – his publications are famously massive works of erudition.

The aims of this article[2] are twofold. We will first give a brief introduction to his life and overall work, as many mathematicians (and others) who know the name know little more about him. Then we will give a more in-depth introduction to his approach and some of the topics he cared most deeply about, by examining representative excerpts from one of his earliest books, *Quaestiones celeberrimae in Genesim*.

As we will see in brief here, a large part of his legacy is his willingness to tackle any subject, or any correspondent, that might help understand even a small of God's Scriptural revelation in the physical world. This is a notable cross-current to the prevailing winds of specialization – and hence Mersenne's attitude may be worthy of more attention even today, especially from people of faith in the academy. We will conclude by returning to this point.

2 Life and Work

Mersenne's dates and a few basic facts can be found in most number theory and math history texts – but usually little else. This section gives a longer synopsis of his life and most important work, which is in any case essential context for our discussion of *Quaestiones* in the next section.

We should note here that most sources of the basics of his life – including online ones such as [17] and [22] – really largely go back to the (published) encomium [4] of his comrade in the order, Hilarion de Coste. The standard biography[3], exploring the role of 'mechanism' at length, is [10]. Of the more recent English-language book-length works, [3] places emphasis on his debt to Catholic scholastic (and other) trends, while [2] focuses more on his mentoring and encouragement of both peers and younger scholars.

2.1 Early Life

Born in 1588 in the province of Maine, Mersenne showed early aptitude for and interest in study. His parents (whose economic status is variously described, but was not close to the top stratum) were able to send him first to a grammar school, and then the newly-formed Jesuit school/'college' of La Flèche. By 1611 he had finished further study in theology in Paris and joined the order of the Minims, being ordained a priest shortly thereafter.

As the name indicates, the Minims[4] considered themselves to be the 'least of these', who tried to emulate the humble life of their founder, Saint Francis of Paola. Among other things, the order lived on the Lenten fast rules year-round. For a sincere religious character like Mersenne's ([10] goes on at some length about this), it seems to have been a good match.

His initial activities for the order included teaching philosophy in central France. Once summoned back to Paris in 1619, the order apparently saw Mersenne's greatest value in his writing and studies, including the commentary on Genesis we will discuss below. Throughout his life they granted him

[2] Mersenne is never short-winded; likewise, this article is an expansion of just part of a talk given at the Conference of the Association of Christians in the Mathematical Sciences in 2019. We are grateful to the referees for suggestions on exposition and content.

[3] Unfortunately, still only available in French.

[4] The Minims still exist in a number of countries; see their website at http://www.ordinedeiminimi.it/index.htm.

fairly permissive rules to this end – not just of using their library, having his own equipment in his cell, or of leaving the premises of the monastery, but even of hosting outside scholars there on various occasions.

Most of his first publications[5], such as *L'Impiété des déistes*, in the mid-1620's were clearly aimed at combating the relatively new perils of not just deism, but even outright atheism. Certainly this was a main goal of *Quaestiones*; more subtly in a similar vein was his defense of the rational basis of scientific investigation against those skeptical of all knowledge in *La Vérité des Sciences* (see [3] at length on this issue). We must recall that at this time any such 'freethinking' variant (including in France, Protestantism) was considered not just theologically dangerous, but also as having potential to upend a broader social order with nothing to replace it. It is not surprising that a sincere and devout Catholic of some talent would focus on this threat[6].

2.2 Mature Work

Commentators disagree on the continuity between his more polemic early work and his more 'scientific' mature opera. Nonetheless, the topics we will see he waxes eloquent on in *Quaestiones* come to full fruition in his best-known works, of which we here briefly mention the two best known.

Mersenne's longest labor (and generally most famous) book is *Harmonie Universelle* [15], the 'Universal Harmony'. Constantly revised, digested, and even translated[7] up through his death, it brings together nearly everything known at the time about all manner of musical instruments, compositions, and practice, and it is still an important source for musicologists. Acoustics take special pride of place in this as in others of his writings, and he is generally acknowledged as being the first person to write down a formula for a vibrating string – and connecting this to vibrating air columns in organ pipes and wind instruments. Experimental matters come to the fore as well; for example, Mersenne precisely determines the difference in pitch arising from making strings of different materials such as iron, silver, or gold.

Not that this is a music textbook. For instance, his theology is unavoidable throughout. Interestingly, in personal correspondence, the author of the paper [21] on Mersenne's 'monochord' (a set of several strings formed in such a way as to investigate different musical intervals with great precision) points out that its three strings should be taken as a figuring of the Trinity. Likewise, Mersenne conceives of the concept of harmony[8] very broadly, interweaving (desired) harmony in politics, mathematics arising from harmony, as well as the physical attributes of harmony. As he was still forming his views on free fall and Galileo's mechanics at the same time, there is discussion of this as well – motion of a string is still motion.

Mechanics show up in many other of his later works, such as *Cogitata Physicomathematica* [16]. This is emblematic of what [3] and others follow Mersenne in calling 'physico-mathematics', the earlier mentioned slow unification of algebraic mathematics with questions about motion and other physical notions. These works are full of references to his own, repeatable and repeated, experiments. For instance, Mersenne's discussion of hydraulics in [16] comes complete with many figures of apparatus in use, doubtless based on ones like those used in his own cell. It is developed in a long series of definitions, 54 propositions, and various scholia.

[5] See [3] or [10] for fairly detailed discussion of these early works.

[6] See [10] pp. 171ff.

[7] Into Latin, for wider readership, though typically for him with many changes and additions. Mersenne cared about writing in the vernacular but also for international audiences, even in a pastoral sense.

[8] See [3] for discussion of this point.

This is the important shift in focus from an Aristotelian worldview to a 'modern' 'mechanistic' one that Lenoble names his book [10] after; this emphasis seems to postdate *Quaestiones*. Again, this is not the exclusive content – the *Cogitata* has a section on music theory, another on weights and measures, alongside one on ballistics! As Lenoble remarks about several of Mersenne's works, one should think of the *Harmonie* or *Cogitata* as a sort of modern *Summa* – just not one of theology, but of music or physics.

In all of these works there are astonishingly long interludes regarding essentially unrelated pure mathematics. For example, [16] is best known for being the first source of the so-called Mersenne primes of the form 2^p+1 for p a prime, his primary modern mathematical claim to fame[9]. However, Mersenne included pure mathematics at any appropriate opportunity; for instance, the *Harmonie Universelle* has much discussion of the combinatorics of musical composition.

Mersenne's mature work was clearly valued by contemporaries, even if they may not all have read it completely through. Boria has an impressive list of scholars from Cavalieri to Cavendish who were influenced by the *Cogitata*, and the *Harmonie Universelle*'s impact was even greater. We have only touched on it here; the website [17] is another good place to start, and Lenoble's biography [10] is mandatory reading on its impact among nascent scientists, in ways that (according to him) compare favorably to Descartes and Pascal – notably his empiricism.

2.3 Messenger

It is impossible to discuss Mersenne without mentioning his correspondence and publishing activity. His early work explicitly (as opposed to implicitly) aimed at defending Catholic thought does not seem to have had a huge impact on that per se. Rather, it attracted attention more from letter-writers who wished to discuss matters further – and whom Mersenne pursued in the furtherance of his investigations. With hindsight, it is clear that Mersenne's largest contribution to intellectual history[10] as a whole is certainly his efforts at making new discoveries known by the means of his correspondence.

Three well-known, and typical, episodes will suffice to give the reader an indication of the overall contribution. First, Mersenne is one of those responsible for collating the various 'Objections' to Descartes' *Meditations*, as well as Descartes' replies. Mersenne had no objection to printing both that which he agreed with and that not, if in the spirit of open inquiry and publishing defenses of his own views. (His toleration of near-atheist Thomas Hobbes as a friend often raises similar questions, but is consistent with his later practice on this score.) There are long stretches of back-and-forth between them long before Descartes finally published his work.

As another example, in 1637-8 Mersenne was so excited to have Fermat and Descartes share their respective methods for finding maxima and minima (and hence tangents) that he brokered their correspondence. As [13] points out[11], Mersenne had probably already overshared a number of Descartes' geometrical and other works, and in this case made some more missteps over Descartes' easily-bruised personality, including sharing Descartes' letters with some of Mersenne's circle in Paris who were known skeptics of his philosophy and approach. In the event, through these exchanges both Fermat and Descartes were forced to divulge – and, crucially, *clarify* – more of their

[9] It is obligatory to mention here [7], an internet-enabled search for larger ones; mathematical readers will also be interested in different points of view in [5] and [2] regarding whether errors in his computations are typographical in nature, and that Mersenne got the idea from Frenicle de Bessy.

[10] Even if [8] probably overstates the case over his relatively weak scientific skills.

[11] See Section IV.IV of [13] for every single detail.

proto-calculus than they had intended.

Finally, despite some initial qualms, and despite always treating Galileo's cosmology as a working hypothesis rather than fact[12], Mersenne not only ensured Galileo's publication in Holland, but created a translation-cum-commentary of the *Dialogues* in French for the local market. As various commentators have pointed out, the story of Galileo's reception by the church is more complex than the received wisdom, and Mersenne's bona fides with both church and king in France were well-used.

Mersenne's full edited correspondence runs to seventeen volumes[13]. Because he was persistent in asking questions of every correspondent, its sum has become invaluable for understanding much of scientific philosophy of this era, hence his universal acclamation as the 'intelligencer' for his times.

2.4 The End

As we have seen, over time Mersenne's writing came to focus more and more on the scientific and musical topics that he encountered while refuting his enemies. He also continued to write hundreds of (surviving) letters to all and sundry regarding these same topics. However, he did not travel much outside Paris – one notable longer trip was to Italy, where he hoped to learn more about Torricelli's barometric experiments. In 1648, after nearly thirty years of constant academic activity he fell victim to complications of a lung abscess – missing the chance to confirm the existence of air pressure, which Pascal soon after completed.

Despite a wide-ranging *ouevre*, he never lost his overall focus (despite some commentators' claims to the contrary). As he expressed to one of his first correspondents near his death, "We are in a strange century for the different kinds of libertinism ... neither reasoning nor Scripture can make them yield to the truth ... When videbitur Deus Deorum in Sion[14], we will no longer have wicked people to persecute us, nor impious people to mock God and religion" (quoted in [3], page 203). There does not seem to be any indication that his eulogies were inaccurate as to that he was a pious and sincere man to the end of his days.

3 The Most Famous Questions

We return now to Mersenne's early period, and his monumental early volley against the freethinkers, *Quaestiones celeberrimae in Genesim* [14]. In discussing these 'Most famous/celebrated questions in Genesis,' we will see indications of many of the themes broached in our brief history, exemplified by excerpts we hope will be of interest to the mathematically inclined reader of the twenty-first century.

3.1 Overview

The goals of *Quaestiones*, including two thousand tight columns of text with various adjuncts[15], were not few. The full title page (see[16] Figure 1 on the following page) continues by commending its 'accurate explanation of the text', defeating of 'atheists and deists', 'vindicating the Vulgate from the calumnies of heretics', and 'restoring the music of the Greeks and Hebrews'.

[12] See [6] or [11] for commentary.

[13] See metadata for the entirety at the Bodleian's website, http://emlo-portal.bodleian.ox.ac.uk/collections/?catalogue=marin-mersenne.

[14] When the God of Gods will be seen in Zion.

[15] And then hundreds more columns of *Observationes* usually appended to the *Quaestiones*.

[16] All figures are courtesy of the very generous terms of nonprofit usage of the Gallica online repository, https://gallica.bnf.fr, of the French National Library.


F. MARINI
MERSENNI
ORDINIS MINIMORVM
S FRANCISCI DE PAVLA
QVÆSTIONES CELEBERRIMÆ
IN GENESIM, CVM ACCVRATA
TEXTVS EXPLICATIONE.
Græcorum, & Hebræorum Musica instauratur.
FRANCISCI GEORGII VENETI
CABALISTICA DOGMATA FVSE REFELLVNTVR,
OPVS THEOLOGIS, PHILOSOPHIS, MEDICIS,
LVTETIÆ PARISIORVM.
M. DC. XXIII.

Figure 1: Title page of *Quaestiones*

A brief summary of the contents is hopeless; the index of the 'questions' alone, on everything from angels to architecture, runs to six pages. A modern reader might wonder whether the title page promises too *little*!

Nonetheless, there is a clear structure. *Quaestiones* is assembled as a commentary on the first six chapters of Genesis, and begins with a discussion of the Hebrew word for 'In the beginning' and its connection to 'head'. Even in these opening lines, we see how he uses this to create a different kind of 'Summa'. His first 'problem' is that of whose voice is the first we hear in Scripture, immediately claiming 'without God nothing may be understood'. He swiftly moves to the question of whether God indeed exists, and how to prove this against the atheists – eventually including no fewer than thirty-five rationales for God to exist. (See Figure 2.)

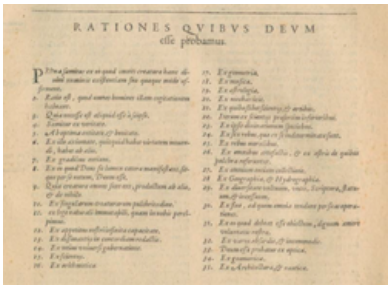

RATIONES QVIBVS DEVM
esse probamus.

Figure 2: Thirty-five rationales for God's existence

3.2 Examples and Structure

Mersenne's real goal, then, is investigation of many, many questions (and other 'articles' and 'problems' of side commentary) raised by the contents of these verses, in any field of knowledge. All knowledge is part of God's knowledge, to him, and so using what is nominally a commentary to share all sorts of information is a legitimate organizational principle. This is especially so if the information can bring people to a reasonable faith in the God of Scripture and counter poor reasoning as practiced by others (such as the long stretches arguing against witchcraft and deism).

A good example of this *not* connected to scientific investigation is provided by his commentary on Genesis 4:18-19, wherein Cain's descendant Lamech married two women, Adah and Zillah. In Figure 3 we have the (Latin) Vulgate, the Hebrew text, and the (Greek) Septuagint, followed by a commentary. Note the references on the margins, which in general quote quite extensively from Scripture, church Fathers, classical authors, and other primary sources.


VERSICVLVS XVIII. ET XIX.

Figure 3: Commentary on Genesis 4:18-19

This verse also demonstrates how a typical 'Question' arises. In Figure 4 we see Question 54, which answers whether Lamech sinned in having two wives, and under what law polygamy is prohibited. Mersenne quotes in his answer a ninth-century letter from the pope to a Holy Roman Emperor, other examples in Genesis, and Jesus' teaching about (and rejection of) divorce in Matthew 19 (*ab initio non fuit sic*, from the beginning it was not so).


QVÆSTIO LIV.

AN LAMECH PECCAVERIT DVAS VXORES ducendo, & quo iure polygamia prohibita sit.

Figure 4: Question 54

But Mersenne intended *Quaestiones* to be much more. Consider his treatment of Genesis 1:14. The lights God creates in the sky give signs for days and years in this verse. But he does not just discuss God's providence in this passage, but leads into to a very long refutation of all manner of horoscopes—including detailed examples of their construction. Even by today's academic standards, Mersenne is usually punctilious in giving his opponents the best possible discussion of their own views before trying to defeat them.

Likewise, in discussing Genesis 4:20, wherein Lamech's son Jabal is the father of those who live in tents and raise livestock, Mersenne gives a comprehensive discussion of what we would today call the liberal and practical arts, including things like surgery, hydraulics, ethics, and rhetoric – each time with an at-times devotional connection to some episode in Jesus' life or the Apostle's creed[17].

The reader may now, with Lenoble, wonder whether we simply have a 'picturesque disorder' or even 'intellectual incontinence'[18]. Our view is closer to that of Boria; Mersenne may need an editor

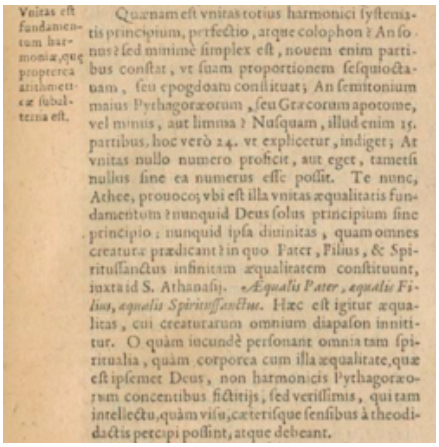
[17]For instance, Caiaphas' comment on one man dying for all the people is connected to geography.
[18]See [10], pages 4 and 25; his language is often more picturesque than that of more recent scholarship.

and enjoys throwing in the kitchen sink, but the structure is not a rambling chaos. In Lenoble's own words, despite Mersenne's 'eclecticism', this is merely a new kind of 'Summa'.

3.3 Music in *Quaestiones*

As noted above, Mersenne's most original investigations were in music theory, particularly in what we would now call the physics of music. (Recall that of the traditional seven liberal arts, music was considered to be applied arithmetic.) Even at the time of *Quaestiones* he has plenty to say about this discipline, and it is appropriate to spend some time with this.

What might be surprising to the modern reader is how universally he considers music to be applicable. Already in the eighteenth rationale for God's existence (see Figure 5) Mersenne is quoting Augustine and Athanasius with respect to music and how it shows humans God.


Vnitas eſt fundamentum harmoniæ, quę propterea arithmeticæ ſubalterna eſt.

Quænam eſt vnitas totius harmonici ſyſtematis principium, perfectio, atque colophon? An ſonus? ſed minimè ſimplex eſt, nouem enim partibus conſtat, vt ſuam proportionem ſeſquioctauam, ſeu epogdoatu conſtituat; An ſemitonium maius Pythagoræorum, ſeu Græcorum apotome, vel minus, aut limma? Nuſquam, illud enim 15. partibus, hoc verò 24. vt explicetur, indiget; At vnitas nullo numero proficit, aut eget, tametſi nullus ſine ea numerus eſſe poſſit. Te nunc, Athee, prouoco; vbi eſt illa vnitas æqualitatis fundamentum? nunquid Deus ſolus principium ſine principio; nunquid ipſa diuinitas, quam omnes creaturæ prædicant? in quo Pater, Filius, & Spiritusſanctus infinitum æqualitatem conſtituunt, iuxta id S. Athanaſij. *Æqualis Pater, æqualis Filius, æqualis Spiritusſanctus.* Hæc eſt igitur æqualitas, cui creaturarum omnium diapaſon innititur. O quàm iucundè perſonant omnia tam ſpiritualia, quàm corporea cum illa æqualitate, quæ eſt ipſemet Deus, non harmonicis Pythagoræorum concentibus ficticijs, ſed veriſſimis, qui tam intellectu, quàm viſu, cæteriſque ſenſibus à theodidactis percipi poſſint, atque debeant.

Figure 5: A bit of music and God

If this seems a little far from Anselm's ontological argument (which Mersenne approves of), you are right; as with his science, Mersenne is not looking for a full deductive proof of God's existence, but rather many reasonable arguments that, in total, cannot be confounded[19]. He grounds this reason in the harmony (recall our discussion of [15]) and unity that both God and music provide.

However, Mersenne leaves the majority of his musical thoughts in *Quaestiones* to his commentary on Genesis 4:21, where Cain's descendant Jubal is the father of musicians. Nearly five percent of the *Quaestiones* is a commentary on this verse!

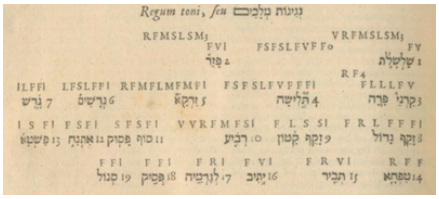


Figure 6: In the mode of the kings

From the title page promising a restoration of Hebrew and Greek music, we might have expected some reconstruction like in Figure 6. The Roman letters correspond to the solfege syllables like Re, Mi, Fa, where Ut, the predecessor to Do, becomes V. Mersenne waxes rhapsodic just before

[19] See [3] on this and its relation to his other writings.

this about how happy is the one who restores the wonderful music of the temple of Jerusalem, and happiest the one who delights in the harmony of the heavens, with which the saints praise God the all-powerful, as in Psalm 83. Perhaps this does belong in a commentary after all!

Yet just before this, Mersenne talks about the way music ought to be composed. Before *that*, there are long extracts describing the various diatonic intervals – perfect fourth, major second, and so forth – with tables. In various articles to the question for this verse he shows correct scansion in various languages, examples of proper four-part music in Latin and French, and a long discussion of diction and what various authors have to say about it. The lyre in Figure 7 is a prefiguring of his detailed accounts of instrument construction in the *Harmonie Universelle*.

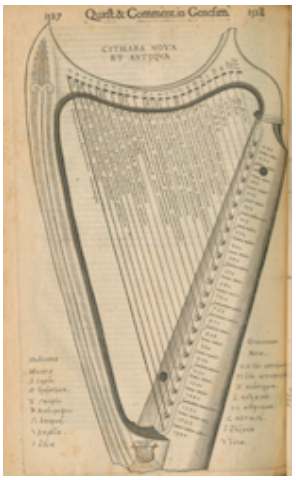


Figure 7: New and ancient lyre

Again, this is not simply the delight of sharing all he knows, though we would argue it is *also* that. The Parisian father is simply keenly interested in every aspect, practical and theoretical, of an earthly instantiation of God's harmony, and uses this verse to shine it through.

3.4 Basic Mathematics in *Quaestiones*

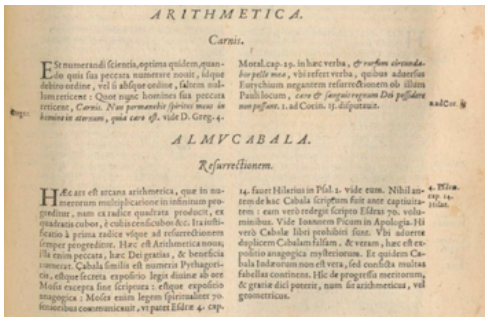
As is well known, it is a very short step from describing the different intervals in harmony to number theory. We should also not forget that Mersenne's Jesuit education at La Flèche would have included a fair amount of Euclidean geometry, courtesy of Christopher Clavius' innovations (see [1]). It is then small wonder that Mersenne was interested in mathematics as an adjunct (and more) to music and harmony, as well as anywhere geometry is important, and that it shows up many times in the *Quaestiones*. By examining the discussions of mathematics in this early work, we may get a sense of how he may have later become interested in deeper mathematical questions.

Many of the places where mathematical content appears in *Quaestiones*, in contradistinction to Mersenne's later work, focuses fairly directly on the theological questions at issue—and sometimes is not very deep. For instance, when attacking cabalistic numerology in the commentary on Genesis 3:20, he does some basic computations to refute it, including comparing the Latin and French versions of the names Maria and Stephen.

Somewhat more interesting is his mention of the number 112400259082719680000 of possible Hebrew 'words' (in a combinatorial sense), which apparently goes back to cryptographer Blaise de Vigenère

in his treatise on ciphers of 1586. In *Quaestiones*, Mersenne simply mentions that the (unnamed) author was unable to explain it, and that rather than being practically infinite, isn't much bigger than 100 quintillion. The later Mersenne we would expect to try to recreate this number, even if to refute its usefulness.

Different, but still not (mathematically) deep, are his discussion of the sciences of mathematics in the earlier-mentioned commentary on Genesis 4:20. In Figure 8 we see arithmetic related to the flesh. Essentially[20], the best knowledge of counting is in knowing how to count one's own sins!



ARITHMETICA.

Carnis.

Est numerandi scientia, optima quidem, quando quis sua peccata numerare novit, aliquot debito ordine, vel si absque ordine, saltem nullum reticent: Quot nunc homines sua peccata reticent, *Carnis. Non permanebit spiritus meus in hominibus in æternum, quia caro est.* vide D. Greg. 4. Moral. cap. 25. in hæc verba, *& rursum circumdabor pelle mea*, ubi refert verba, quibus adversus Eutychium negantem resurrectionem ob illum Pauli locum, *caro & sanguis regnum Dei possidere non possunt.* 1. ad Corinth. 15. disputavit.

ALMVCABALA.

Resurrectionem.

Hæc ars est arcana arithmetica, quæ in numerorum multiplicatione in infinitum progreditur, nam ex radice quadrata producit, ex quadratis cubos, è cubis cenficubos &c. Ita infinitatio à prima radice vsque ad resurrectionem semper progreditur. Hæc est Arithmetica nova, illa enim peccata, hæc Dei gratias, & beneficia numerat. Cabala similis est numeris Pythagoricis, eliique secreta expositio legis divinæ ab ore Mosis excepta sine scriptura: eliique expositio anagogica: Moses enim legem spiritualiter 70. senioribus communicavit, vt patet Esdræ 4. cap. 14. fuisse Hilarius in Psal. 2. vide eum. Nihil autem de hac Cabala scriptum fuit ante captivitatem: eam verò redegit scripto Esdras 70. voluminibus. Vide Ioannem Picum in Apologia. Hi verò Cabalæ libri prohibiti sunt. Vbi adverte duplicem Cabalam fuisse, & veram, hæc est expositio anagogica mysteriorum. Et quidem Cabala Iudæorum non est vera, sed conficta multas fabellas continens. Hic de progressu meritorum, & gratiæ dici potest, num sit arithmeticus, vel geometricus.

Figure 8: Arithmetic and Al-Muqabalah

Perhaps even more surprisingly, the second paragraph in Figure 8 connects 'Al-muqabalah', or the growing area of algebra[21], to the resurrection. 'This art is the secret arithmetic, which in the multiplication of numbers proceeds to infinity . . . '

Though this is confusing to a modern reader at first, what Mersenne is referring to is the distinction between mere addition and subtraction and the raising to many powers, which is fantastically powerful. For someone educated in Euclidean geometry, discovering cubes, fourth powers, and many higher ones (part of the general sphere of algebra of the time, in addition to solving equations) was doubtless a fascinating new world, even if the vocabulary is as old as Diophantus. He says this is the 'new arithmetic, which is the grace of God, and counts its benefits'. (We would remark that this has parallels to Jesus' calling on us to forgive not just seven times, but seventy times seven, which indeed is a power in both senses.)

3.5 Geometry in *Quaestiones*

So far, we still see Mersenne essentially using rudimentary knowledge about mathematics to make theological points. However, when he turns to geometry, we see that Mersenne is quite capable of understanding the details as well, as proved true in his later work.

In *Quaestiones* this really comes to the fore in his knowledge of contemporary optics. In reason 33 for the existence of God, for thirty columns he uses the 'beauty of optics', bringing in astronomy, geometry, and arithmetic, to support his contention. This includes reiteration of quite detailed computations of various astronomical quantities (see Figure 9 for an example), including the ratio of the diameter of the sun to that of the earth. Mersenne doesn't even neglect to provide a table

[20] I am grateful to my colleague Graeme Bird for assisting and correcting my Latin, though all errors are solely mine.

[21] Al-Khwarizmi's treatise on simple algebra uses two words, of which today we typically only use 'al-jabr' or algebra, but at this time it was reasonable to use the other one to refer to algebra, as Mersenne does here.

of the multiples of sixty (with references to degrees, minutes, etc. of arc) to the *fourteenth* power in order to be sure everyone knows just how accurate[22] an astronomer could be if necessary!

Figure 9: Optics and astronomical calculation

All the while he brings Euclidean propositions, Kepler's observations, ancient authors like Seneca, Plutarch, and Thucydides – and of course the usual orthodox sources – to bear in asserting the relevance and history of this one point. The preponderance of evidence from all respected sources is what will win the fight against the 'impious atheist' whom he addresses.

Along similar lines, Mersenne spends a number of pages refuting the possibility that angelic appearances could have been generated using sophisticated mirrors in some manner. This is an opportunity not lost for describing the full mathematical properties of various sorts of mirrors, including the 'most perfect' parabolic shape (see Figure 10). His goal of defending orthodoxy does not rule out bringing the reader to full scientific understanding as well.

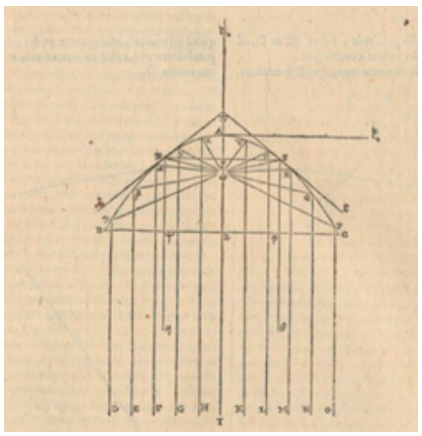


Figure 10: Parabolic reflection

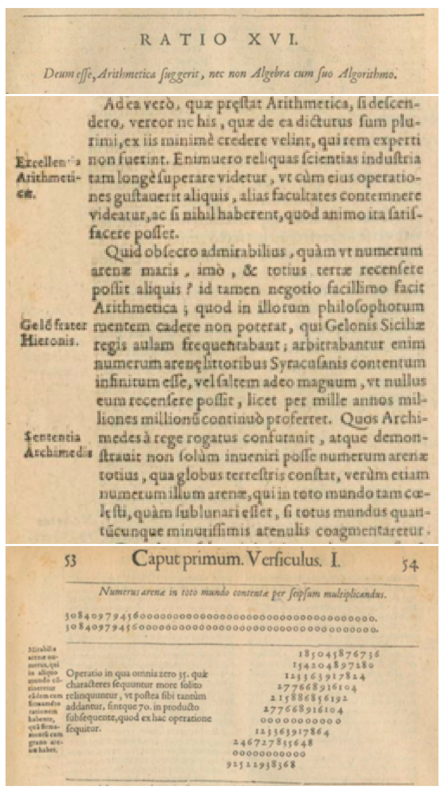
[22] In column 187 he even points out that a year has 525,960 minutes since it is slightly longer than 365 days.

3.6 Mathematics and God's Existence

We have just seen that geometry in the service of optics, to Mersenne, help in making God's existence reasonable. He is far more explicit about this in reasons 16 and 17 (see above Figure 2), where he enlists arithmetic and geometry directly to that cause.

It would be an entire study to go through these rationales in detail. Reason 16 may be roughly translated, 'Arithmetic suggests [that] God exists, as does algebra with its algorithms.' Mersenne starts immediately with the 'excellence of unity', from which the 'infinite storehouse' of arithmetic is generated, waxing quite eloquent about the many varieties of numbers and how God's unity is shown through this. He then continues to explain the excellence of arithmetic in describing the world by giving large parts of Archimedes' argument in *The Sandreckoner* for the number of grains of sand in the universe, particularly the various assumptions needed and then the computation of the cubes of various large numbers.

How does this relate? Because it is so easy for the human mind to compute all this, among other reasons. The explicit calculation of the square of a number of 46 digits (for all this, see Figure 11) needed to compute these numbers in decimal notation[23] helps in his argument: 'You, atheist, behold the prodigious intellect of man' which can calculate the grains of sand.



RATIO XVI.

Deum eſſe, Arithmetica ſuggerit, nec non Algebra cum ſuo Algorithmo.

Excellentia Arithmeticæ.

Ad ea verò, quæ præſtat Arithmetica, ſi deſcendero, vereor ne his, quæ de ea dicturus ſum plurimi, ex iis minimè credere velint, qui rem experti non fuerint. Enimuero reliquas ſcientias induſtria tam longè ſuperare videtur, vt cùm eius operationes guſtauerit aliquis, alias facultates contemnere videatur, ac ſi nihil haberent, quod animo ita ſatiſfacere poſſet.

Gelô frater Hieronis.

Quid obſecro admirabilius, quàm vt numerum arenæ maris, imò, & totius terræ recenſere poſſit aliquis? id tamen negotio facillimo facit Arithmetica; quod in illorum philoſophorum mentem cadere non poterat, qui Gelonis Siciliæ regis aulam frequentabant; arbitrabantur enim numerum arenę littoribus Syracuſanis contentum infinitum eſſe, vel ſaltem adeo magnum, vt nullus eum recenſere poſſit, licet per mille annos milliones millionū continuò proferret. Quos Archimedes à rege rogatus confutauit, atque demonſtrauit non ſolùm inueniri poſſe numerum arenæ totius, qua globus terreſtris conſtat, verùm etiam numerum illum arenæ, qui in toto mundo tam cœleſti, quàm ſublunari eſſet, ſi totus mundus quantūcunque minutiſſimis arenulis coagmentaretur.

Sententia Archimedis

53 Caput primum. Verſiculus. I. 54

Numerus arenæ in toto mundo contentæ per ſeipſum multiplicandus.

3084097945600000000000000000000000000000000000.
3084097945600000000000000000000000000000000000.

Operatio in qua omnia zero 35. quæ characteres ſequuntur more ſolito relinquuntur, vt poſtea ſibi tantùm addantur, ſintque 70. in producto ſubſequente, quod ex hac operatione ſequitur.

185045876736
154204897280
123363917824
277668916104
215886856192
277668916104
00000000000
123363917864
246727835648
00000000000
92512293836 8

Figure 11: Reason 16

Similarly, the amazing power of algebra to solve problems involving high powers or related rates is lauded; for instance, Mersenne quotes and then solves an ancient Greek problem later appearing in many English-language puzzle books about a 'brazen lion'. He ends by calling the atheist to rethink the error of his ways with Plato's quote of God as geometrizing.

[23] Recall the Greeks did not use decimals in the Hindu-Arabic numeral sense.

Reason 17 continues with the same argument, now explicitly from geometry. As one of many mathematical examples, Euclid's Proposition XII.18 is provided, which shows that the volume of spheres are in proportion to the cubes of their diameters. Indeed, he looks at the circles' areas as well, as we see in Figure 12. Why the focus on circles? As he says at the beginning of the section, it is because God is the 'indivisible center, whose irradiation extends to the periphery of all things,' which perfectly understands all angles of creation. Later he says he wishes with all these arguments to recall from their error the detested atheists.

Figure 12: Euclid Proposition XII.18

Yet Mersenne is clearly going far beyond any relatively simplistic analogies, which at least for much of these sections is all he really needs to make his arguments against his targets. He addresses the angle between a circle and its tangent; he complains about some researchers' values for π; he examines some 'excellent' propositions from book 2 of Euclid, the Pythagorean Theorem, various aspects of the parallelogram, and the so-called Conchoid of Nicomedes.

To what purpose? We argue it is both his pure delight in these results (one which he wants the reader to share) *and* his desire to 'offer the atheists medicine'. Geometry offers such a treasury of interesting and useful results, but it also allowed (to Mersenne) God to build all things in number, measure, type, etc. We can see that Mersenne, in indulging his own 'youthful ardor' (so Lenoble of *Quaestiones*) looks forward to his later compilations while also meeting his immediate goal.

3.7 Reception?

In the discussion of his musical proof of God he says, "I provoke you now, Atheist," in asking where else such unity may be found but in the Trinity. This mission was accomplished, as a number of authors, notably Robert Fludd, proceeded to defend themselves and attack Mersenne. This in turn drew the interest of more orthodox commentators, who helped Mersenne obtain many of his contacts and led to his huge legacy of missives. However, it is interesting that none of the his major biographers discuss *Quaestiones*' influence on orthodox developments, and it is not clear there were many. Instead, along with his other early books, it served to introduce the topics he cared about most; at least some authors suggest his change in publication to more strictly scientific work was occasioned by his lack of success in convincing the heretics to change.

4 The Legacy

It took many years for Mersenne to be seen as a figure worthy of study in his own right. Perhaps partly this is because he "combine[d] sophistication and naivety as perhaps only a very kindly and cloistered savant could do"[24]. Certainly his understanding of what Fermat was doing with whole numbers, or Galileo's work, was not as insightful as that of some of his colleagues[25].

Crucially, however, he understood their *importance* – in the case of Fermat's number theory, he was one of the only ones to do so – and he was responsible for their ideas being promulgated in Northwestern Europe. In general his understanding of music, mathematics, and other fields was not simply that of a rank amateur. His was a quest to understand these ideas more fully, and add to the world's compendium of knowledge of how God organized the world. It is not just that Mersenne facilitated the transmission of knowledge, but that he actively sought out correspondents from the great organist Jehan Titelouze and the renowned philosopher Rene Descartes to many far more obscure figures in order to achieve this quest.

In doing so, Mersenne would tackle any subject, or any correspondent, that might help understand even a small part of creation – and bring it to others. A major thesis of [2] is that he was the only one who would deign to report much of this knowledge to many less famous, now-unknown intellectuals of the time (as well as future luminaries when they were unknown). As we mentioned earlier, his commitment to organizing and passing on these insights to a wide audience bore fruit in his later work being much valued for bringing together the known state of many fields (notably music).

Mersenne's true legacy is thus not any of his specific efforts, or his role in intellectual history or music theory, though these are well known to experts in those fields. He was also not precisely a polymathic 'Renaissance man,' though his interests and investigations did range very widely. Nor do his theological arguments from music or mathematics directly contribute to modern debates over the existence or nature of the Deity.

Instead, the example of Mersenne may be thought of as advocating a return to one's first love – both intellectually and spiritually. Above all, he wanted to defend the possibility of knowledge and revelation over against various adversaries (not just freethinkers). And as Dear points out at length in the last chapter of his book, all the various 'mixed mathematical sciences', including acoustics and hence music, were the best place to do this in the seventeenth century.

So Mersenne returned again and again to all of these topics in his work, while not necessarily seeing his experiments in air pressure or long combinatorial digressions[26] as being somehow unrelated to his main purpose. He investigated what he was interested in, then brought it back as best he could to how it showed God's authority, purpose, or handiwork. And he would talk to anyone who could help him in it, great or small, without worrying (too) much about fame or priority, or even whether he was bothering them with his long letters.

It is true that sometimes he also didn't worry too much about coherence for other readers; yet this, in the end, was also a positive for Mersenne. Consider Descartes, who took years to publish

[24] See [12], in the context of keyboard tuning, though it is more generally true.

[25] See e.g. [11] on French scientists preferring Galileo's Italian originals to Mersenne's French translations/condensations.

[26] Or infinitesimal angles of tangents to circles or showing how to properly set four-part music or ...

his deductive superstructure, always worried about whether someone might scoop him (as the Fermat episode demonstrates). Their contemporary John Amos Comenius (Jan Komensky) never finished his 'Pansophic', fully organized compendium of human knowledge[27], despite many pleas from others. But Mersenne got people talking.

In this age of hyper specialization and showing only the polished final product of our work (at least in mathematics), Mersenne affords an alternate view of how to pursue and disseminate intellectual inquiry, even (or particularly) when motivated at root by theological concerns. Perhaps we would do well to consider emulating him a bit more, rather than being a Fermat or a Descartes, waiting until all is perfect to strike the iron.

Not that we should all follow Mersenne's lead, either. But he encourages us to step back a bit from our own legacy-seeking to consider the disciplines on their own merits, wherever they lead. For a father from the order of 'minimum' importance, that is a legacy of maximum value.

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