The Applicability of Mathematics and The Naturalist Die

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Abstract

Philosopher and Christian apologist William Lane Craig has proposed a valid deductive argument for God's existence that is rooted in the applicability of mathematics to the physical universe. This argument was presented by Craig during a debate with philosopher and atheist Alex Rosenberg. During the debate, Rosenberg presented a rebuttal to the soundness of this argument by appealing to chance as an explanation to the applicability of mathematics to the physical universe. Rosenberg argues that some mathematics will apply to the universe when considering all the alternative mathematics that don't apply to the universe. In this paper, we will contend that the naturalist position is unable to produce chance application of mathematics when assuming a mathematical fictionalist position. We shall then defend the soundness of the argument in light of the ontology and effectiveness of mathematics. Pertinently, the problem of God and abstract objects will be addressed. A new modified version of the argument will be proposed to emphasize the unreasonable effectiveness of mathematics. The paper concludes with a biblical motivation for the study of the argument.

1 Introduction

William Lane Craig is a prominent Christian apologist that has debated atheists and scholars over various decades. His most well-known book and website are both titled *Reasonable Faith* (see [6] and [14]). On February of 2013, William Lane Craig had a debate with philosopher and prominent atheist Alex Rosenberg at Purdue University, West Lafayette, Indiana (see [11] and [12]). The debate, titled *Is Faith in God Reasonable?*, consisted of various arguments for or against the reasonability of God's existence. In this paper, we shall scrutinize Craig's deductive argument for God's existence via the applicability of mathematics as presented at the debate. We shall label this argument **AM**. The argument consists of the following premises:

AM:

1. If God does not exist, then the applicability of mathematics would be a happy coincidence.
2. The applicability of mathematics is not a happy coincidence.
3. Therefore, God exists.

This deductive argument is valid by virtue of its Modus Tollens form. The term "happy coincidence" and its contextual usage must be clarified. "Happy coincidence" is a term borrowed from philosopher of mathematics Mary Leng to seemingly indicate an unlikely (small probability) correspondence between mathematical objects and the physical world (see [13]). To be clear, mathematical theory indeed yields predictable results in all of the natural sciences. Put another way, premise 2 is accepted by everyone based on the empirical consistency of applied mathematics. The debate consequently lies in premise 1. Essentially, with premise 1, Craig attempts to point out the inability of naturalism to address the astonishing effectiveness of mathematics. The reason for this, as Craig points out during the debate, is a default anti-platonist (or mathematical fictionalist) position. That is, Craig and Rosenberg both believe mathematical objects are fictions; mathematical fictions do not exist in some metaphysical sense other than as concepts. In this paper we will see that if mathematical objects are just fictions, and there is no God, the applicability of mathematics is unlikely.

Given that the second premise of **AM** is universally accepted, Rosenberg's rebuttal to the **AM** argument consists of attacking the truthfulness of the first premise in order to avoid the guaranteed conclusive third statement. Videlicet, since mathematical fictionalists must avoid coincidences, Rosenberg must show that in a godless universe the applicability of mathematics is not by happy coincidence. To do this, Rosenberg invokes the infinitely many mathematical objects and geometries that do not apply to the real world in order to establish math applications by stating that "any one of an indefinitely large number could perfectly well apply in the universe" (see [12]). To establish an analogy, Rosenberg is saying that when we roll a traditional die, one of the six sides will land. The difference of course is that Rosenberg's naturalist die contains infinitely many sides, each with a different geometry or set of mathematical rules and objects. Craig seems to deny Rosenberg's rebuttal by essentially saying that even if Rosenberg's naturalist die allows for some math to apply, the link between this die and reality is still unexplained.

In this paper, we will take a different approach from Craig to denying Rosenberg's rebuttal by examining the naturalist math die itself. In passing, we will explore the ontology of mathematics and its relationship to naturalism and theism. From this examination on the ontology of mathematics the Christian shall adopt the anti-platonist position due to a concern with the problem of God and abstract objects (see [9]). Particularly, our anti-platonism will be established via mathematical fictionalism. In other words we shall specifically deny the existence of mathematical objects, not the abstract nature of mathematical objects. While the naturalist is not concerned with the problem of God and abstract objects, we shall also see that the naturalist should take a fictionalist position for a different reason. We shall then present a somewhat enhanced version of Craig's argument inspired by his quoting of physicist Eugene Wigner. The paper will conclude with a biblical motivation of the persuasiveness and personal benefit of studying the unreasonable effectiveness of applied mathematics.

2 Platonism at odds with both theism and naturalism

In order to better understand the **AM** argument, we must study the ontology of mathematics as it seems that both Craig and Rosenberg are anti-platonists in this debate. As we will now argue, both theism and naturalism should adopt an anti-platonist position, albeit each for different reasons. To begin, let's explore the Christian perspective on the ontology of mathematics.

Mathematical platonism certainly entices the Christian mathematician. After all, mathematics seems to fit very nicely as eternal ideas in the mind of God. This has surely been the *Christian Platonist* position of Newton and others in the past (see [4]). However, to view mathematics as concepts or ideas in a mind, albeit divine, does not constitute pure platonism. Rather, said position is a pseudo-platonist position at best since pure or traditional platonism means something ontologically stronger than a concept. Platonism is summarized by the following two statements (see [9]):

1. Abstract objects exist. [Platonism]
2. If Abstract objects exist, then they are independent of God. [Platonist assumption]

Once again, we remind ourselves here that platonist existence does not refer to a mere conceptual existence. If a person imagines a unicorn, the concept of the unicorn exists, but the unicorn itself does not exist. Platonism is stronger than this since platonic existence goes beyond a concept or idea. Thus, assuming pure platonism means assuming that there are objects that exist independently from God. A Christian must consequently be careful to classify his or herself as a mathematical platonist. The care is warranted by the aseity-sovereignty doctrine (see [9]):

(i) God does not depend on anything distinct from Himself for his existing, and (ii) everything distinct from God depends on God's creative activity for its existing.

It is clear that Christians traditionally subscribe to the aseity-sovereignty doctrine. If we are to assume that both traditional platonism and the aseity-sovereignty doctrine are true, we inevitably arrive at the so-called problem of God and abstract objects (see [9]). This problem consists of an inconsistent triad that emanates from subscribing to both traditional Christianity and to traditional mathematical platonism:

Problem of God and Abstract Objects

1. Mathematical objects exist. [Mathematical Platonism]
2. If mathematical objects exist, then they are dependent on God. [Aseity-Sovereignty doctrine]
3. If mathematical objects exist, then they are independent of God. [Platonist assumption]

The first and third premises of this inconsistent triad are due to traditional platonism. Particularly, the third premise of the triad states that mathematical objects are independent of God for their existence. On the other hand, the aseity-sovereignty doctrine of Christianity requires that all existing things depend on God for their existence. Hence, one of the three premises in this inconsistent triad must be eliminated. Christian mathematicians must therefore consider their position carefully. While we shall suppress a proper and philosophical exegesis of the following, these passages are of utmost importance for Christians to consider in light of the inconsistent triad:

> *For from him and through him and to him are all things. To him be the glory forever. Amen.* -Romans 11:36

> *Worthy are you, our Lord and God, to receive glory and honor and power, for you created all things, and by your will they existed and were created.* -Revelation 4:11

In consideration of these passages for support of a strict aseity-sovereignty doctrine interpretation, we realize that the first and third statements of the inconsistent triad, traditional platonism, are where we shall fix the inconsistency. Specifically, we shall sacrifice the very existence of mathematical objects (the third premise of the inconsistent triad). To clarify, we are not denying the existence of abstract concepts. Rather, we are denying an ontologically strong existence of abstract objects in the platonist sense. To reiterate our unicorn analogy, we are not denying the existence of the concept of a unicorn, rather we are denying the existence of a unicorn. It is worth noting that the independence of abstract objects does not threaten the aseity-sovereignty doctrine since it claims "everything distinct from God depends on God's creative activity for its **existing.**" Since we are assuming mathematical objects do not exist, the independence of mathematical objects from God is ill-posed. It is therefore advantageous for the Christian to agree with Craig and assume an anti-platonist position via mathematical fictionalism.

It is true that a naturalist has no reason to even consider the aseity-sovereignty doctrine. Therefore, we make no such claim. All we have asserted is that the Christian is forced to adopt mathematical fictionalism because of the aseity-sovereignty doctrine. We however do claim that the naturalist position also leads to fictionalism, but from a different assumption. Traditional platonism is not a possibility for the naturalist worldview since traditional platonism requires a metaphysical narrative to account for the non-physical and abstract existence of mathematical objects. That is to say, the naturalist would have to speak of and accept a platonic realm despite lacking explanations or observable evidence of such a realm, other than the applicability of mathematics. Nonetheless, some naturalists have used an indispensability argument (see for example [13]) to establish the existence of mathematical objects within naturalism. The indispensability argument is made of three premises and a conclusion as seen below:

1. Naturalism: We should look to science, and in particular to the statements that are considered best confirmed according to our ordinary scientific standards, to discover what we ought to believe.

2. Confirmational Holism: The confirmation our theories receive extends to all their statements equally.

3. Indispensability: Statements whose truth would require the existence of mathematical objects are indispensable in formulating our best confirmed theories.

4. Therefore, Mathematical Realism: We ought to believe that there are mathematical objects.

However, Mary Leng rightly argues that confirmational holism is in conflict with naturalism (see [13]). As Leng explains

> It appears, then, that one might reasonably make successful use of a theory while holding back from belief in some of its component parts, either because they are known to be idealizations, and thus to be contributing to theoretical success for reasons other than their truth, or because it has not yet been established that their contribution to the success of our theory will be best accounted for by assuming the existence of their objects, so that it is reasonable to remain agnostic about the objects posited. The fact that we can recognize cases where the practical success of a theoretical hypothesis is not a result of its truth means that we should should hold back from assuming, as Quine's

> confirmational holism does, that a practical decision to adopt a hypothesis as part of our theoretical worldview should always be understood as providing us with a reason to believe that hypothesis.

In other words, there are at least two different reasons for one to not assume confirmational holism. For one, some assumptions in mathematical modeling are idealizations of reality. Secondly, we don't know if the success of a theory is best explained by assuming the existence of the mathematical objects involved in the theory. Whether by negating confirmational holism, or by realizing that naturalism would have to provide a naturalist framework for the platonic realm of mathematical objects, or by the lack of a naturalist link between the platonic real and physical reality (as briefly discussed in section 5 of this paper), we find that naturalism should not co-exist with platonism. Hence we find that both theism and naturalism are at odds with platonism. But this common ground that forces the fictionalist position upon both perspectives turns into an advantage to Craig's argument.

3 The Naturalist Die

In the previous section, I argued that both theism and naturalism have difficulties with a traditional platonist view of mathematical objects. For the theist, the problem of God and abstract objects arises because of the aseity-sovereignty doctrine. For a naturalist, confirmational holism stands in the way of traditional platonism. We now turn our attention to the **AM** argument and Rosenberg's rebuttal. As we have noted earlier, Alex Rosenberg's rebuttal says that "there are" indefinitely many mathematical objects and indefinitely many functions relating these mathematical objects for which only a small number of these have applications to the real world. Thus, since there are so many, it was inevitable that one or a few of these should apply to the real world. As Craig seems to counter, this still fails to account for why mathematical objects are applicable to reality. But there is a greater erroneous subtlety in Rosenberg's answer. To be fair, while not explicitly addressed by Craig, it is very likely Craig was or is aware of this error. The error lies in Rosenberg's words, "there are." Indeed, these words imply that there "exist" alternative mathematical objects or geometries. These abstract objects cannot exist within the naturalist worldview as we have previously mentioned. That is, naturalism does not support a platonist perspective. Consequently, Rosenberg's rebuttal, runs into a fundamental problem: the naturalist die that Alex Rosenberg wishes to roll, does not exist. In other words, the application of these "fictions" to the real world is in fact impossible under a naturalist perspective.

We should clarify why the naturalist die of mathematics does not exist. To do this, we shall further explain the analogy of the die and its associated terms. We shall characterize mathematics that apply to the real world as "linked" to reality. The die with its sides represents all of the possible outcomes of running an experiment (represented by rolling the die). Each side or outcome, represents a geometry that is to be linked or applied to reality. Consequently, rolling the die (running the experiment) is analogous to selecting and linking a geometry or mathematical objects to physical reality. In this analogy, Rosenberg's rebuttal to the **AM** argument is that any of the infinite mathematical sides could land with a roll. Here lies the issue. Simply put, what naturalistic process or experiment (die roll) links a random set of abstract fictions to physical processes? Naturalism, by its own default position cannot answer this question. For why would a naturalistic mindless process randomly select an arbitrary set of abstract fictions (mathematical objects and geometries)? Ergo, Rosenberg's fictionalism defeats his attempt to invoke chance.

It is worth noting that an atheist has a way to avoid this pitfall. The atheist would have to embrace platonism which would require abandoning naturalism. But to do so, the atheist will open his or herself to other arguments for God's existence, different from the one Craig has presented. In essence, the atheist runs into other dangers in retreating from naturalism. We shall address these in section 5 of this paper.

4 Unreasonable Effectiveness of Mathematics

The power of Craig's **AM** argument lies in the lack of a proper naturalistic explanation for the applicability of mathematics to the physical world. As we have noted in the previous section, Rosenberg's rebuttal is flawed in the sense that fictions or concepts cannot be linked to physical processes through any naturalistic mechanism. To clarify Craig's argument to Rosenberg's position, we shall propose a modification to Craig's argument in order to highlight the heart of the issue to the naturalist position. In this modified argument, we shall furthermore emphasize the "unreasonable effectiveness of mathematics." This unreasonable effectiveness, a phrase borrowed from Eugene Wigner's paper (see [8]), is something that Craig uses to defend his argument, though it could be emphasized within the stated argument itself to bolster its persuasive power.

The motivation for emphasizing the unreasonable effectiveness of mathematics is to distinguish unreasonable applications from certain straightforward applications that make much sense and are actually reasonable applications. That is, per Wigner, there are two types of mathematical applications: reasonable and unreasonable; expected and unexpected. Arguably, Craig's argument gains strength in the case of unreasonable applications. For it is the unreasonable applications that highlight the unlikely effectiveness. Therefore, here we shall require a definition that sets the unreasonable applications apart from the reasonable applications. Then we shall construct the modified argument with said definition.

Definition 1. *A physical phenomenon (phenomena) is said to be* ***inherently mathematical*** *if there is a mathematical model that is indispensable to the understanding of its properties and dynamics, indispensable to the intended and unintended accurate predictions of the physical phenomenon and other related phenomena, and does not lack a hidden variable. Furthermore, we assume a phenomenon is inherently mathematical if we assume a current model can be dispensed by a more complete unknown model that satisfies the same above criteria.*

The reader should note that the usage of "*indispensable*" in our definition is not equivalent to the usage of the same word in the previously discussed indispensability argument that is used to establish mathematical realism (platonism). For we have made a case for Christians and naturalists alike to adopt mathematical fictionalism. Thus, in our definition, we use "*indispensable*" to mean absolutely necessary to the scientific theory. In our definition of **inherently mathematical**, it is worth highlighting the usage of *unintended predictions* in the definition. This is is where the mystery of the effectiveness of mathematics lies, according to Wigner. In his paper (see [8]), Wigner notes:

> ... it is important to point out that the mathematical formulation of the physicist's often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena. This shows that the mathematical language has more to commend it than being the only language which we can speak; it shows that it is, in a very real sense, the correct language. Let us consider...elementary quantum

> mechanics. This originated when Max Born noticed that some rules of computation, given by Heisenberg, were formally identical with the rules of computation with matrices, established a long time before by mathematicians. Born, Jordan, and Heisenberg then proposed to replace by matrices the position and momentum variables of the equations of classical mechanics. They applied the rules of matrix mechanics to a few highly idealized problems and the results were quite satisfactory. However, there was, at that time, no rational evidence that their matrix mechanics would prove correct under more realistic conditions. Indeed, they say "if the mechanics as here proposed should already be correct in its essential traits." As a matter of fact, the first application of their mechanics to a realistic problem, that of the hydrogen atom, was given several months later, by Pauli. This application gave results in agreement with experience. This was satisfactory but still understandable because Heisenberg's rules of calculation were abstracted from problems which included the old theory of the hydrogen atom. The miracle occurred only when matrix mechanics, or a mathematically equivalent theory, was applied to problems for which Heisenberg's calculating rules were meaningless. Heisenberg's rules presupposed that the classical equations of motion had solutions with certain periodicity properties; and the equations of motion of the two electrons of the helium atom, or of the even greater number of electrons of heavier atoms, simply do not have these properties, so that Heisenberg's rules cannot be applied to these cases. Nevertheless, the calculation of the lowest energy level of helium, as carried out a few months ago by Kinoshita at Cornell and by Bazley at the Bureau of Standards, agrees with the experimental data within the accuracy of the observations, which is one part in ten million. Surely in this case we "got something out" of the equations that we did not put in.

Here we see the unintended and accurate prediction when Wigner explains that "*we 'got something out' of the equations that we did not put in.*" Another important aspect of our definition of **inherently mathematical** is that the model must not lack a hidden variable in order for it to be indispensable. In quantum mechanics, we are guaranteed that there are no local hidden variables via Bell's theorem (see [3]).

Thus, we can say that the phenomena of quantum mechanics are **inherently mathematical** per our definition. Using this example, we construct our modified applied mathematics argument (which we shall denote **MAM**) as follows:

MAM

1. If God does not exist and mathematical objects do not exist, there are no inherently mathematical phenomena.

2. Mathematical objects do not exist.

3. The physics of quantum mechanics are inherently mathematical.

4. Therefore, God exists.

Mary Leng (see [13]) and others have tried to provide naturalist explanations for the applicability of mathematics to the physical universe. Mary states:

> I argued that a fictionalist account of the role of mathematical hypotheses was possible, which viewed those hypotheses as literally false but useful means of representing how things are taken to be with non-mathematical objects . In particular, if we take the axioms of our favourite version of set theory with urelements to be generative of a make -believe according to which non-mathematical objects can be the members of sets (and therefore can stand in various set-theoretical relations to further sets), we can provide an account of how participating in such a game of make-believe can provide us with a useful means of representing hypotheses concerning non-mathematical objects and their relations.

If there were phenomena for which this explanation would suffice, it could be for so-called "reasonable" applications. But not for unreasonable applications. Applications that are reasonable are therefore the result of selecting axioms that emulate reality. Mary Leng would likely then argue that mathematics is dispensable by arguing that it is possible to reformulate scientific theories without relying on abstract mathematical objects. But quantum mathematics is a known exception to this. Leng (see [13]) has tried to argue otherwise by invoking Balaguer's nominalization attempt (see [1]) to essentially argue that the mathematics of quantum mechanics are dispensable. But the success of this nominalization is easily and substantially contested (see [5]). Thus, in the case of quantum mechanics, the *inherently mathematical* status of its physical phenomena is currently irremovable. Furthermore, there is no heuristic derivation of the mathematics of quantum mechanics. The wave function is not emulating anything in reality. Thus, the effectiveness of mathematics in quantum mechanics cannot be explained by making assumptions that emulate reality.

One might contest that the failure of the nominalization of mathematical applications is a failure for fictionalism since typically, realists (platonists) invoke said failure to promote their view of mathematical platonism. But we must understand that the failure only points to realism if there is no God. The failure to dispense of mathematics in quantum mechanics indeed convinces us, if anything, that quantum mechanics is per our definition, inherently mathematical.

5 Does Platonism allow an atheistic view?

Given that atheistic anti-platonism cannot provide an explanation for the unreasonable effectiveness of mathematical application, the atheist might consider platonism as a viable and godless explanation of the applicability of mathematics. That is, the atheist might consider negating our second premise in **MAM**. But as Leng, Craig (see [7]), and others have noted, if anything, both share in having difficulties with godless explanations for the applicability of mathematics. For while platonism could say that a realist view of mathematical objects is compatible with the truthful predictions one can mathematically make, it still fails to supply a reason for why there is a link between the abstract mathematical objects in the platonic realm and the non-mathematical objects in the physical universe. As Ballaguer notes (see [2]),

> The idea here is that in order to believe that the physical world has the nature that empirical science assigns to it, I have to believe that there are causally inert mathematical objects, existing outside of spacetime."

Furthermore, since naturalism cannot hold hands with platonism per Leng and our previous discussion, the atheist platonist might need to abandon naturalism all together. This of course will provide all sorts of new problems for the atheist. For one, all arguments for the historicity of the resurrection of Christ are typically dismissed simply on naturalistic grounds. For while the New Testament is far more superior to the ancient classical texts in terms of its historical reliability, the historicity of the New Testament is typically dismissed because naturalism does not accept miracles. But without naturalism, an atheist must intellectually struggle with the validity of the historical attestation of the resurrection of Christ.

6 The applicability of mathematics in the Bible

The argument for God's existence based on the applicability of mathematics has a familiar biblical tone. In particular, one could argue that the depth of Romans 1:20 is discovered as we learn more about the mathematical nature of the universe. In Romans 1:20, Paul writes

> For his invisible attributes, namely, his eternal power and divine nature, have been clearly perceived, ever since the creation of the world, in the things that have been made. So they are without excuse.

First we notice that Paul equates God's eternal power and divine nature with his invisible attributes. It is clear that to natural eyes these attributes are invisible. But Paul holds that while these attributes are not visible, they are clearly perceived in creation. I do not intend to say that the original intent of Paul in this passage was to refer specifically to the inherently mathematical phenomena of the universe. Rather, that the perception of God's attributes only becomes "clearer" as we study the intricacies of God's creation. So while a naturalist might argue that science has cast a shadow on this passage, the Christian understands that the perception of the invisible only becomes clearer as we learn more of the mathematical backbone of the universe. As Christian mathematicians, we must continue the legacy of clearly perceiving his invisible attributes by studying the applicability of mathematics to the "things that have been made," as we respond with joyous wonder.

7 Conclusion and Acknowledgement

The goal of this paper has been to persuade the reader of the futility of Rosenberg's rebuttal to Craig's argument (the paradox of the naturalist die), strengthen the argument of the applicability of mathematics by considering unreasonable applications within inherently mathematical phenomena, and motivate the continued study of mathematics applied to the things made in order to gain a "clearer" perception of God's attributes. So we find that our motivation for appreciating the unreasonable applicability of mathematics is twofold: to apologetically persuade those that are not believers and to glorify God as we stand in awe of his mathematically created universe.

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