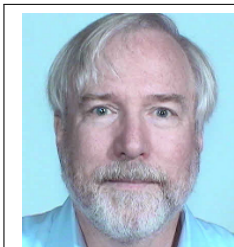# The Search for Hamilton

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Eric Gossett earned his M.S., and Ph.D. in mathematics from the University of Arizona while specializing in combinatorics. He has pioneered the use of Mathematica within the curriculum and published the textbook *Discrete Mathematics with Proof*, Second Edition. He developed Bethel's first generation of web services. He is an avid racquetball player and is the faculty sponsor of the Bethel Anime Club.

1. Introduction

In July and August of 2009 my wife and I went on a four-week trip to Scotland and Ireland. We would be visiting Dublin, so I decided that we should visit the famous bridge where William Rowan Hamilton carved the equations for the quaternions.
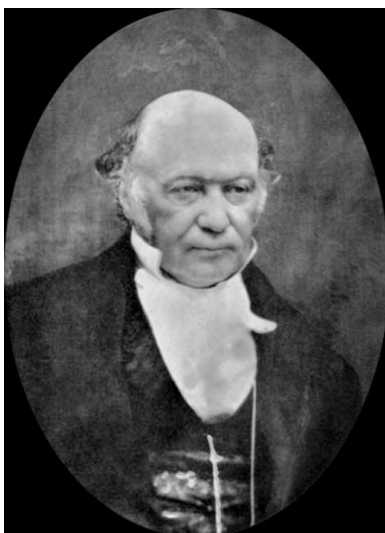
Hamilton had been struggling to find a good way to multiply and divide points in three dimensions but was making no progress. On October 16, 1843 he was walking with his wife along the Royal Canal in the outskirts of Dublin. He had a flash of insight which lead to the now-famous equations that defined the quaternions:

$$i^2 = j^2 = k^2 = ijk = -1.$$

Hamilton was evidently so excited that he carved the equations in the stone of the Broom Street bridge which crossed the canal. (The carving has long since vanished, but a commemorative plaque was added in 1958.

I teach a course in Discrete Mathematics so my interest in Hamilton is actually related to the topic of Hamiltonian cycles in graphs. A Hamiltonion cycle is a walk in the graph that visits every vertex exactly once (except that it starts and ends with the same vertex).

2. The Journey

I had the following useful information from Wikipedia:

> Broom Bridge, also known as Brougham Bridge, is a bridge along Broombridge Road which crosses the Royal Canal in Cabra, Dublin, Ireland: http://en.wikipedia.org/wiki/Broom_Bridge.

When we arrived at our hotel in Dublin, I asked the desk clerks if they knew where the famous bridge was located. They had never heard of the bridge, In fact, they had never heard of a famous Irish mathematician named Hamilton. Our tour guide, bus driver, and city guide were also not very helpful. The bus driver did think that he knew the bridge, but it was in a suburb that was too far to walk to and the tour was not going near that area.

We did have a nice walking map of Dublin which contained the cryptic message:

> Site of No. 36 Sir William Rowan Hamilton World-Famous Mathematician.

The site was part of a row house. We asked several people in the area about the house but none of them had heard of Hamilton. The best I can figure is that Hamilton lived there at one time. The house has been a teacher's club for many years.

Our hotel was next to Trinity College, so we decided to see if the math department there had a display about Hamilton. We started on one end of campus and walked a very long way to get to the math department. It is a small, old building which is surrounded by large, new buildings for departments such as chemistry and pharmacology. It was a weekend and nobody was in the math building. We were unable to find any displays about Hamilton.

We left Dublin in defeat. However, we did find the famous statue of Molly Malone,

saw the Book of Kells,

and attended a Sunday Evensong service at St. Patrick's Cathedral. The service featured a guest choir from America.

We were also able to visit Glendalough which is the site of a famous 6th century Irish monastic community. The tower on the left was for protection from Viking raiders - entry was through the hole about 12 feet from the bottom using a rope ladder which could be retracted.

We wandered around Ireland for several days, then crossed into Northern Ireland. The border really surprised me. The American-Canadian border has armed guards on the American side to keep out attacking hoards of Canadians and to stop foreign terrorists from sneaking into the country. On the Canadian side are armed guards who are desperately trying to keep out American culture. The border between the Republic of Ireland and Northern Ireland has no guards (not even the remains of guard houses), no lines on the road, not even a sign saying "Welcome to the UK" or "now leaving the Republic of Ireland". The only way to tell you have crossed the border is that signs in Ireland use kilometers and kilometers per hour and list places using Gaelic followed by English. In Northern Ireland they use miles and miles per hour and only list places in English.

The image is of a wall mural in Derry/Londonderry Northern Ireland. (The Catholics call the town Derry, the protestants call it Londonderry, and tourists who are unsure about who they are talking with refer to it as "your fair city."

We eventually crossed into Scotland and wandered around there for several days. The image is of Brig O'Doon (the bridge over the river Doon). The bridge is famous for the Broadway musical and also because of the Robert Burns poem "Tam O' Shanter." Tam and his horse escape a group of angry witches by crossing the river via the bridge. Remember the Broom street bridge? We were having a good time, but at the back of my mind I was still disappointed that I missed it.

We eventually ended in Edinburgh, said goodbye to one tour group, then a day later joined a second tour group. After wandering around to more places in Scotland, Northern Ireland, and Ireland, we ended up back in Dublin. Here is yet another bridge: a new bridge across the river Lethe shaped like a celtic harp (the national symbol).

In Dublin we connected with another city guide. This time the city guide and bus driver were confident that they knew how to get to the Broom bridge, The next morning was a free time, so we walked 2 minutes from our hotel and got on a commuter train heading to the suburbs. One train change later we exited onto the Broombridge Station—a bare platform surrounded by weeds and some liter.

Up ahead was a graffiti-covered bridge over the railroad tracks. We walked up the ramp and asked a pedestrian where the Broom bridge was. He said "this is it." I told him about the Hamilton plaque and he said he thought this was the bridge some people were interested in.


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Enter the following coordinates into Google Maps to see a satellite
image of the bridge and station: 53.373023,-6.299992.
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We walked across the bridge and noticed the canal running parallel to the train tracks. There was a walking path next to the canal, so we exited the road and went under the bridge, And there it was: the long-lost plaque.

There was no graffiti on this side of the bridge, but there was some white paint that had been thrown onto the wall and plaque.

The following heavily photo-manipulated image shows the plaque. The contents read:

> Here as he walked by on the 16th of October 1843 Sir William Rowan Hamilton in a flash of genius discovered the fundamental formula for quaternion multiplication
>
> $$i^2 = j^2 = k^2 = ijk = -1$$
>
> and cut it on a stone of this bridge.
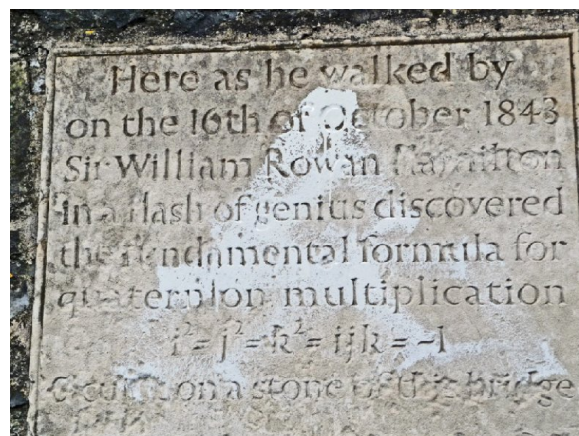
This bridge, so dearly loved by mathematicians, is clearly not on the Dublin Tourist Board's list of places to visit. The site is pretty much off the radar of both the city of Dublin and the nation of Ireland. However, there is one group that is attempting to keep the memory alive: the Mathematics Department at the National University of Ireland at Maynooth.

> In 1990, we (the Department of Mathematics at NUI Maynooth) initiated the annual commemoration on the anniversary of the discovery, in the form of a walk from Dunsink observatory to Broom Bridge. Since then, a growing number of people have been participating. In 1993, the sesquicentenary, the first New Yorker appeared. He had flown across just for the day. http://www.maths.may.ie/hamiltonwalk

3. Epilogue

There is an even happier ending to the story. Two months after our visit, a restored plaque was installed during the 2009 annual walk.

All images except the portrait of Hamilton and the image from the Book of Kells were taken by Eric Gossett and can be used with permission. The exceptions are already in the public domain and can be found on Wikipedia.