

# Loop Filters with Controllable Doppler Jitter for Standard and High Sensitivity GNSS Receivers

Nadezda Sokolova<sup>\*</sup>, Daniele Borio<sup>\*\*</sup>, Börje Forsell<sup>†</sup>

<sup>\*</sup> SINTEF ICT, Trondheim, Norway. Email: [nadia.sokolova@sintef.no](mailto:nadia.sokolova@sintef.no)

<sup>\*\*</sup> European Commission Joint Research Centre, Ispra, Italy. Email: [borio@jrc.ec.europa.eu](mailto:borio@jrc.ec.europa.eu)

<sup>†</sup> NTNU, IET, Trondheim, Norway. Email: [forssell@iet.ntnu.no](mailto:forssell@iet.ntnu.no)

**Abstract**— Standard tracking loops are usually designed by selecting the loop bandwidth that controls the variance of the phase observable. In this paper, a new approach for the loop filter design is introduced. The proposed methodology is based on the concept of Doppler bandwidth and provides control over the variance of Doppler frequency measurements. Doppler measurements reflect the relative motion between receiver and satellites and can be used to simplify the selection of the loop parameters that can be directly determined as a function of the user dynamics. Two different GNSS carrier tracking loop architectures are considered, namely standard and memory discriminator based tracking loops, and a design example is provided to show the effectiveness of the proposed approach.

**Keywords** — Controlled root formulation, Doppler bandwidth, Doppler measurements, Global Navigation Satellite Systems, GNSS, PLL

## I. INTRODUCTION

Standard tracking loops are essentially designed by setting the loop bandwidth [1]. More specifically, a tracking loop is approximated by a linear device that produces smooth phase estimates from noisy input data samples. The loop bandwidth defines the frequency spread of the loop transfer function and determines the amount of noise that is transferred from the input samples to the final phase estimates. At the same time, the loop bandwidth determines the responsiveness of the loop to changes in the phase of the input samples. Although the loop design is performed with respect to the phase observable, the natural output of a tracking loop is Doppler measurement that is directly provided by the loop filter. In addition to this, Doppler frequency observations are a direct measure of the relative dynamics between receiver and satellites. For these reasons, the linear theory of tracking loops has been recently restated with respect to the Doppler frequency observable [2-3] and the concept of Doppler bandwidth introduced. The Doppler bandwidth defines the frequency spread of the transfer function between input samples and final Doppler estimates and determines the amount of noise present in the final Doppler measurements.

In this paper, the theoretical analysis developed in [2-3] is exploited and an innovative technique for the design of tracking loops is proposed. The approach is based on the Doppler bandwidth and provides control over the variance of the Doppler frequency measurements. More specifically, the controlled root formulation proposed by [4] has been modified in order to achieve the desired Doppler bandwidth

rather than the desired loop bandwidth. This approach allows one to control the variance of the final Doppler estimates given specific levels of noise and dynamics. In addition to this, setting the Doppler bandwidth is usually simpler than determining the loop bandwidth. Since the Doppler shift is the projection of the relative user to satellite velocity on the receiver to satellite line-of-sight, the level of user dynamics can be directly used to determine the Doppler bandwidth. A design example considering the case of a pedestrian user is provided in Section V to show the effectiveness of the proposed approach and its advantages with respect to standard design techniques.

The generality of the proposed approach is shown by considering two different Global Navigation Satellite Systems (GNSS) tracking loop architectures, namely standard and memory discriminator based tracking loops [5]. Memory discriminators allow long non-coherent integration without degrading the phase information and thus are suitable for High Sensitivity (HS) GNSS receivers.

The remainder of this paper is organized as follows. Section II briefly summarizes results on Doppler frequency estimation, providing the notions of Doppler bandwidth and Doppler jitter. The proposed loop design technique is detailed in Section III and its noise performance is discussed in Section IV. In Section V, a case study considering the case of pedestrian dynamics is detailed. Finally, Section VI concludes the paper.

## II. PLL LINEAR FREQUENCY MODEL

A carrier tracking loop, in this case a Phase Lock Loop (PLL), is usually analyzed by considering its linear approximation. More specifically, the non-linear discriminator used for extracting phase information is replaced by a constant gain and the loop is described in terms of linear operations on the phase observable. A linear model with respect to Doppler measurements has been derived in [2-3] and it is briefly summarized in the following.

The extended linear model of a standard PLL is provided in Fig. 1. The standard model, enclosed in the dotted line, has been extended by considering the frequency observable. More specifically, the frequency of the input signal,  $f_d[k]$  is integrated and the phase observable,  $\phi[k]$  is obtained.  $\phi[k]$  is compared against the previous phase loop estimates and the error signal  $\Delta\phi[k]$  is evaluated.  $N_f[k]$  and  $N_d[k]$  model the effects of the noise present in the input signal and propagated at the discriminator output.

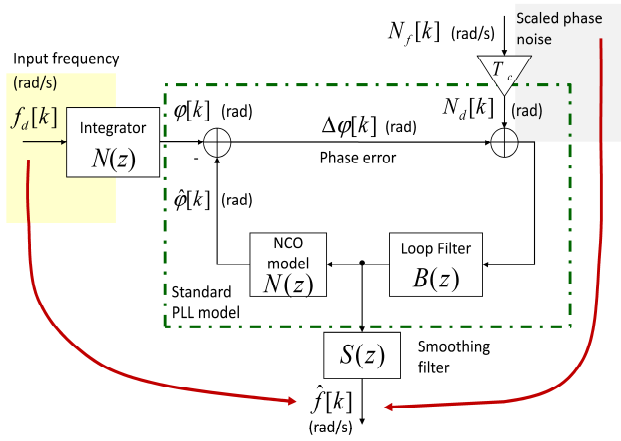


Figure 1. PLL extended linear model. The traditional PLL model is restated with respect to the Doppler observable.

The term  $\Delta\phi[k] + N_d[k]$  is filtered and a new raw frequency estimate,  $f_{raw}[k]$ , is finally obtained.  $f_{raw}[k]$  is integrated to generate a new phase estimate  $\hat{\phi}[k+1]$ .  $f_{raw}[k]$  can be further smoothed to generate the final Doppler observable,  $\hat{f}[k]$ . In Fig. 1,  $B(z)$ , denotes the loop filter transfer function,  $N(z)$  the Numerically Controlled Oscillator (NCO) transfer function and  $S(z)$  characterizes the smoothing filter used to determine the final Doppler estimates.

Using the model in Fig. 1, it is possible to show [2-3] that the final frequency estimate,  $\hat{f}[k]$ , is obtained as a linear combination of the filtered input frequency,  $f_d[k]$ , and filtered normalized phase noise,  $N_f[k]$  [2-3]:

$$\hat{f}(z) = \frac{S(z)B(z)N(z)}{1+B(z)N(z)} \cdot f_d(z) + \frac{T_c S(z)B(z)}{1+B(z)N(z)} \cdot N_f(z) \quad (1)$$

where  $H_f(f)$  and  $H_{nf}(f)$  are the Doppler and frequency noise transfer functions. These two functions characterize the behavior of the loop (in the linear approximation) with respect to the Doppler observable.

In order to quantify the portion of noise transferred from the input signal to the final frequency estimate the concept of Doppler bandwidth can be introduced:

$$B_d = \frac{1}{2} \int_{-0.5/T_c}^{0.5/T_c} |H_{nf}(e^{j2\pi f T_c})|^2 df \quad [Hz] \quad (2)$$

where  $T_c$  is the coherent integration time used by the PLL to determine the phase and frequency observables. The Doppler bandwidth has been derived as a counterpart of the loop noise bandwidth,  $B_n$  and summarizes in a single

parameter the ability of the tracking loop to produce smooth frequency estimates including the effects of all the loop components. The Doppler bandwidth can then be used to determine the jitter (normalized standard deviation) of the final frequency estimates provided by the PLL [2-3]:

$$\sigma_f = \frac{1}{T_c} \sqrt{\frac{B_d}{C/N_0} \left(1 + \frac{1}{2T_c C/N_0}\right)} \left[ \frac{rad}{s} \right] \quad (3)$$

The Doppler bandwidth and Doppler jitter will be used in the next section as basis for the loop filter design algorithm introduced in this paper.

### III. LOOP FILTER DESIGN

The introduction of the Doppler bandwidth parameter allows one to design the PLL loop filter in order to control the variance of the Doppler frequency measurements (3). In order to implement such a design, the controlled root formulation proposed by [5] has been modified in order to achieve a desired Doppler bandwidth.

In the controlled-root formulation, the poles of the loop are constrained to lie on specific geometric loci parameterized by a control parameter that is adjusted in order to meet specific constraints. The shape of the geometric loci and the control parameter determine the decay-rate and damping factor of the loop response. In the original formulation [4], the poles were positioned in order to ensure a stable loop and the design parameter adjusted in order to achieve specific loop bandwidth requirements. According to [4], the system poles are parameterized as follows:

$$\begin{aligned} & \{z_1, z_2; z_3, z_4; z_5, z_6; \dots\} \\ & = \left\{ e^{-\beta(1 \pm \eta_1)T_c}, e^{-\beta\lambda_2(1 \pm \eta_2)T_c}; e^{-\beta\lambda_3(1 \pm \eta_3)T_c}; \dots \right\} \end{aligned} \quad (4)$$

where  $\{z_i\}_{i=1}^L$  are the poles of the transfer function of the loop,  $\beta$  is the control parameter that is adjusted to meet the bandwidth requirements and determines the decay rate of the loop impulse response.  $\{\lambda_i\}$  and  $\{\eta_i\}$  are constants determining the damping characteristics of the loop. In this paper, the control root formulation is extended and the control parameter is determined in order to obtain a specific Doppler bandwidth rather than a desired loop bandwidth. A schematic implementation of the proposed algorithm is shown in Fig. 2.

In the implemented design, the loop poles are fixed according to an initial value of the control parameter,  $\beta$ . The values of these  $L$  poles are used to determine the corresponding integrator gains<sup>1</sup> and finally evaluate the Doppler bandwidth,  $B_d$ . Then, as it is illustrated in Fig. 2, the control parameter,  $\beta$ , is iteratively adjusted until the required Doppler bandwidth is obtained.

<sup>1</sup> The transfer function of the loop filter is assumed to be given by the linear combination of integrators of different orders. The integrator gains define the properties of the loop.

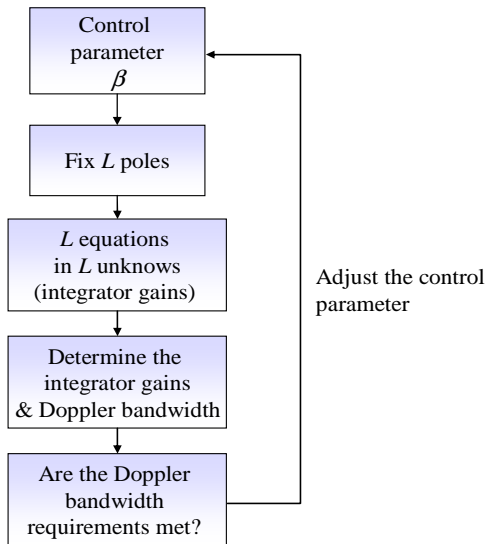


Figure 2. Iterative algorithm used for loop filter design based on Doppler bandwidth.

It is important to emphasize that similarly to the original formulation, in this case as well the stability of the tracking loop is limited by the product of the coherent integration time and the Doppler bandwidth,  $B_d T_c$ . By studying the root location of the system it has been found that the maximum achievable value of  $B_d T_c$  providing a stable loop is 0.27 for a third order loop.

A similar design algorithm can be applied to memory discriminator based PLLs [5]. But due to the use of exponential filtering in the memory discriminator [5], an additional pole is introduced into the system. Therefore, the procedure shown in Fig. 2 has to be further modified to account for the effect of this additional pole. In particular, after initializing the control parameter, first the position of the  $L$  poles can be fixed, whereas the position of the remaining pole can be determined according to the constraints imposed by the structure of the transfer function of this type of the loop. Then the same procedure as illustrated in Fig. 2 can be followed.

#### IV. DESIGN VALIDATION: NOISE PERFORMANCE

The fact that the Doppler bandwidth,  $B_d$ , is used as a design parameter in the proposed approach makes it possible to specify a threshold value of the Doppler jitter to be expected at a given level of  $C/N_0$  – for example, the tracking sensitivity limit. By substituting the desired threshold value of the Doppler jitter and the specified value of  $C/N_0$  into the theoretical formula for the Doppler jitter (3), the corresponding value of the Doppler bandwidth can be determined and used for the design of the loop filter. This allows one to design the tracking loop in such a way that the jitter of the obtained Doppler frequency measurements will stay under the desired Doppler jitter until the specified level of  $C/N_0$  is reached. Alternatively, the value of  $B_d$  to be selected in order to match the level of dynamics expect for specific user activities as discussed in the next section.

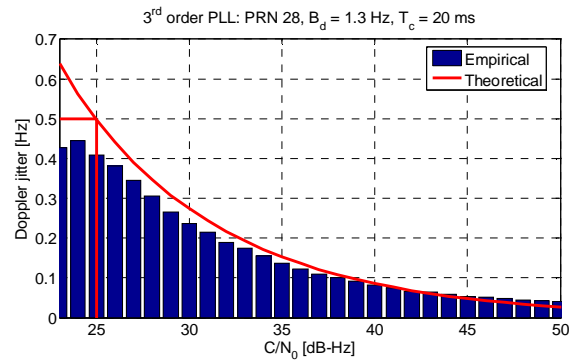


Figure 3. Empirical and theoretical jitter of the raw Doppler measurements as a function of the  $C/N_0$ , obtained using a standard 3rd order PLL designed based on the Doppler bandwidth.

To assess the effectiveness of the proposed design, real GPS L1 C/A data collected in an open sky environment were used. The received signal power was gradually decreased by means of a variable attenuator allowing one to study the obtained results as a function of  $C/N_0$ . The collected data set was processed using two versions of the University of Calgary's GNSS Software Navigation Receiver (GSNRx<sup>TM</sup>) [6], namely standard version and a memory discriminator based architecture.

##### A. Standard PLL

As the data in this test were collected in stationary mode, a fairly low (0.5 Hz) Doppler jitter threshold at a  $C/N_0$  equal to 25 dB-Hz was selected. Substituting the specified Doppler jitter threshold and  $C/N_0$  values into (3), the corresponding value of Doppler bandwidth is found to be equal to 1.3 Hz. The loop filter of a standard PLL was then designed based on the obtained value of  $B_d$  using the procedure described in Fig. 2. In this particular case, a 20 ms coherent integration time was used. The collected data set was then processed using a PLL designed based on the specified parameters.

Fig. 3 compares the expected Doppler jitter values determined using the specified parameters in the theoretical model (3) with the Doppler jitter computed using the empirical data. As it can be seen from Fig. 3, the empirical data closely follow the expected Doppler jitter values and the specified Doppler jitter threshold is not exceeded confirming the validity of the proposed design algorithm.

##### B. Memory discriminator based PLL

Because of the higher noise rejection capability of memory discriminator based tracking loops, a lower Doppler jitter threshold of 0.3 Hz at  $C/N_0$  equal to 25 dB-Hz was selected. Using the same theoretical model of Doppler jitter and substituting the specified values of  $\sigma_f$  and  $C/N_0$ , the corresponding Doppler bandwidth was found to be equal to 0.5 Hz. The loop filter was then designed based on  $B_d = 0.5$  Hz using the approach detailed in Fig. 2. Also in this case a 20 ms coherent integration time was used. Fig. 4 shows the results obtained by processing the collected data set with the memory discriminator based PLL designed using the specified parameters.

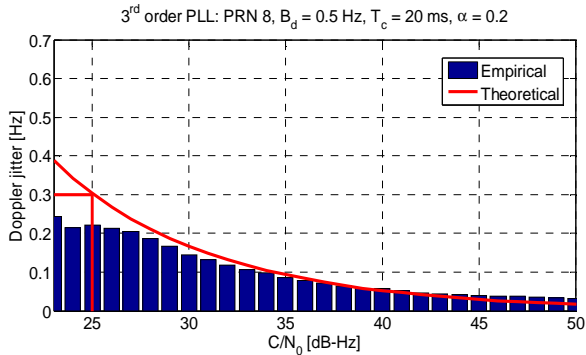


Figure 4. Empirical and theoretical jitter of the raw Doppler measurements as a function of  $C/N_0$ , obtained using a 3rd order memory discriminator based PLL designed based on the Doppler bandwidth.

In Fig.4,  $\alpha$  denotes the forgetting factor parameter of the memory discriminator exponential filter [5]. As it can be seen from Fig. 4, also in this case a good agreement between the expected and empirical Doppler jitter is observed and the specified Doppler jitter threshold is not exceeded.

#### V. A CASE STUDY: PEDESTRIAN MOTION

In the previous section, the Doppler bandwidth was chosen in order to meet specific noise constraints. An alternative approach is to base the filter design on the expected level of dynamics. More specifically, Doppler measurements are a direct measure of the user dynamics and the signal frequency transfer function,  $H_f(f)$ , should be chosen in order to match as much as possible the spectral properties of the input signal frequency  $f_d[k]$ . The spectral characteristics of  $f_d[k]$  can be inferred from velocity measurements characterizing the use motion. This principle has been used in Fig. 5 where velocity measurements have been obtained by using a MEMS accelerometer placed in the user hand. The case where inertial sensors are co-located with GNSS receivers is becoming more and more common in smart-phones and Personal Digital Assistants (PDA) potentially opening to the possibility of the on-line determination of the Doppler bandwidth. The spectrum in Fig. 5 shows several peaks, the first one is the average user velocity, whereas the additional peaks placed at about 1 Hz from the zero frequency reflects the fact that the user is performing a periodic activity corresponding at about a step per second. Since the Doppler shift is a linear projection of the user velocity,  $f_d[k]$  is characterized by a spectral content similar to the one of the velocity measurements.

In Fig. 5, the Doppler bandwidth is chosen in order to preserve the principal frequency components of user dynamics. This is achieved by selecting a Doppler bandwidth equal to 1 Hz and centering the pass-band portion of  $H_f(f)$  on the fundamental step periodicity peak.

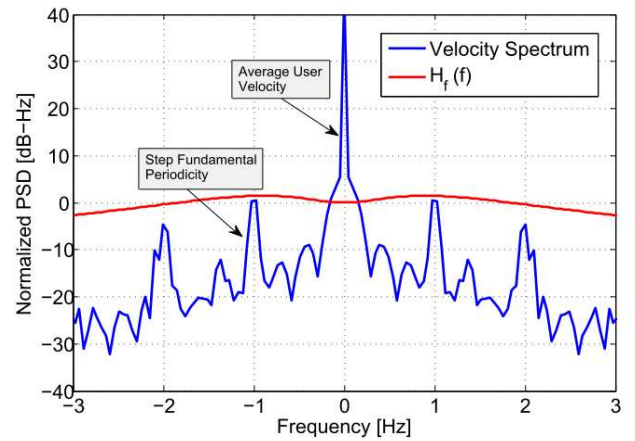


Figure 5. Power Spectral Density (PSD) of velocity measurements obtained by integrating MEMS accelerometers data. The transfer function of the Doppler frequency transfer function has been superimposed to the PSD:  $H_f(f)$  should be chosen in order to match as close as possible, the spectral characteristics of the user dynamics.  $B_d = 1$  and  $T_c = 10$  ms.

#### VI. CONCLUSION

In this paper, the concepts of Doppler bandwidth and Doppler jitter have been exploited to derive a new design approach that allows one to control the variance of Doppler measurements and have a more direct and intuitive relationship with the user dynamics. Tests performed using real GNSS signal show the effectiveness of the proposed approach. Future work includes the generalization of the proposed approach to high order dynamics including user acceleration and Doppler jitter. Multiple constraints on loop and Doppler bandwidth will be also considered.

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