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The Role of Contagion in Financial Crises: an Uncertainty Test on Interbank Patterns

S. Zedda^{1,2}, G. Cannas¹, C. Galliani¹, R. De Lisa²

1. European Commission, Joint Research Centre, Scientific Support to Financial Analysis, Institute for the Protection and Security of the Citizens

2. University of Cagliari

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European Commission
Joint Research Centre
Institute for the Protection and Security of the Citizen

Contact information

Stefano Zedda
E-mail: stefano.zedda@jrc.ec.europa.eu – szedda@unica.it
Phone: +39 0332 78 5103
Fax: +39 0332 78 5733

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Abstract

The main lesson learned from the recent financial crisis is the crucial role of interconnectedness between banks as a factor that can push the effects of bank defaults to extreme levels.

One bank in distress can compromise the ability to repay obligations of its creditor banks, thereby inducing a more general crisis that spreads from the banking system towards the real economy. Several empirical and theoretical studies have focused on the role of the interbank market in causing contagion in financial crises.

In this regard, one frequent problem encountered in dealing with contagion risk in the banking system is that only data on interbank credits and debts aggregated at bank level are publicly available, whereas the whole matrix of interbank linkages would be needed in order to estimate systemic risk correctly.

One common solution is to assume that banks maximise the dispersion of their interbank credits and debts, so that the interbank matrix can be approximated by its maximum entropy.

This paper tests the influence of this hypothesis on simulations by verifying if variations in the structure of the interbank matrix lead to significant changes in the magnitude of contagion.

In order to do this, an algorithm was developed that generates interbank matrices with higher concentration. Then a Monte Carlo simulation was run by making use of the SYMBOL model (**SY**stemic **M**odel of **B**anking **O**riginated **L**osses) jointly developed by the JRC, DG MARKT, and experts of banking regulation (see De Lisa *et al.*, 2010). We then compared results obtained using the maximum entropy approximated matrix with those obtained from more concentrated matrices.

Numerical experiments, performed on samples of banks from four European countries, highlight that concentration in interbank loans does affect results but that, when considering the probability distribution of losses, even significant changes in the interbank matrix do not deeply affect results.

Keywords: Financial contagion, interbank lending, systemic crisis, systemic risk.

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1. Introduction

Interbank markets are important for the proper functioning of modern financial systems. They therefore need to be considered in any banking model aiming at estimating the probability of a systemic banking crisis. One of the effects of interbank connections is that one initial bank failure could have domino effects on the whole system: interbank markets can be a major carrier of contagion among banks, as problems affecting one bank may spread to others.

Contagion results from two risks: first, the risk that at least one component of the system could default (probability of a bank defaulting) and, second, the risk that this shock could propagate through the system (potential impact of the default). As the former can stem from a variety of unexpected situations, and is driven mainly by assets' riskiness and solvency, this research focuses on the latter. In particular, the goal of this paper is to assess how a hypothesis on the structure of the interbank market (i.e. the matrix of credit and debts among banks) affects the magnitude of a systemic banking crisis.

One common problem in dealing with interbank market structures is that only partial data are available, as balance sheets report only aggregated interbank assets and liabilities. Maximum entropy approximation offers a way to proxy interbank bilateral exposures, assuming that banks maximise the dispersion of their interbank credits and debts. But what is the cost of such an approximation?

This paper assesses the influence of the maximum entropy hypothesis by verifying if variations in the matrix structure lead to significantly different results in systemic excess losses, i.e. losses that exceed capital requirements. The model generates losses in the banking systems of four countries (Belgium, Ireland, Italy and Portugal) via Monte Carlo simulations.

This is achieved by making use of the SYMBOL model (**SY**stemic **M**odel of **B**anking **O**riginated **L**osses) jointly developed by the JRC, DG MARKT, and experts of banking regulation (see De Lisa *et al.* (2011)), that estimates aggregate losses, country by country, on the basis of individual banks' asset default probabilities, calculated by means of the Basel FIRB (Foundation Internal Ratings Based) formula.

Interbank exposures are initially modelled using a matrix that maximises the dispersion of banks' bilateral exposures. Contagion results obtained from this scenario are then compared with those achieved with a more concentrated interbank matrix, in order to evaluate if contagion is influenced by hypotheses on interbank exposures.

Results show that relaxing the hypothesis of a maximum entropy interbank matrix does affect systemic excess losses. This holds true in Belgium, Ireland and Portugal, whereas in Italy results are more stable. By contrast, probability distributions are rather robust to variations in the interbank matrix in all four countries.

Furthermore, a distinction is drawn between financial crises in which contagion plays a prominent role and cases in which contagion is not so relevant. When contagion effects are small, systemic excess losses seem to be underestimated by the maximum entropy hypothesis in countries with large interbank exposures. Conversely, with large contagion effects excess losses are overestimated.

This paper is structured as follows: Section 2 gives an overview of the literature on interbank market contagion; Section 3 explains the maximum entropy matrix approximation, the algorithm to adjust the interbank exposures matrix and the scenario generation procedure; Section 4 presents data used to perform the numerical analysis; Section 5 shows results; and conclusions are drawn in Section 6.

2. Literature review

It is well-known that if a failing bank does not repay its obligations in the interbank market, this could compromise the solvency of its creditor banks and lead to a domino effect in the banking system. Hence, contagion occurs when the financial distress of a single bank affects one bank's ability to pay debts to other financial institutions. Therefore, interlinkages between banks could eventually have an impact on the whole financial system and, beyond that, on the state of the entire economy.

Moreover, the pattern of the interbank linkages could affect the way a crisis propagates through the system. Theoretical studies often apply network theory to the banking system and, in particular, focus on the completeness and connectedness of the interbank matrix. According to Allen and Gale (2000), three main forms of interbank network can be distinguished: (i) the 'complete interbank structure' where banks are linked to all other banks, (ii) the 'incomplete interbank structure' where banks are just linked to neighbouring banks (i.e. banks specialise in particular areas of business or have closer connections with banks that operate in the same geographical or political unit) and (iii) the 'disconnected (incomplete) market structure' where there are different disconnected regions of banks (i.e. banks A and B trade with each other, but not with banks C and D that, in turn, hold deposits in each other). Allen and Gale (2000) argue that contagion effects are less likely to occur in a complete interbank structure, since the relationships with a large number of banks act as a buffer on the impact of a single bank in financial distress. A fourth form of interbank linkage is known as the 'money centre' (Freixas, Parigi and Rochet, 2000) where banks are not linked together but only a central money institution is connected to each financial institution. In this case, it is possible that the failure of a single bank will not trigger the failure of the money centre, but if the money centre itself goes bankrupt this can have a domino effect on the whole interbank market. In addition, a 'multiple money centre' structure occurs when the interbank market consists of a number of banking groups, each led by a money centre, where interbank claims are traded solely between banks in the same group.

Many empirical contributions have focused on the role played by the interbank market in spreading financial contagion. A summary can be found in Upper (2011).

Most of these works have no detailed information on the interbank market. To circumvent this lack of data, some contributions have therefore focused on a segment of the market for which bilateral interbank exposures were available at individual bank level. For instance, Furfine (2003) investigated a small fraction of US interbank exposures related to the Federal Reserve's large-value transfer system. Similarly, Degryse and Nguyen (2007) investigated how the structure of the interbank market influences contagion risk, making use of panel data on large exposures to banks in Belgium. In this way, they identified, over time, the pattern of contagion risk due to interbank defaults. They performed a sort of stress test to evaluate how the failure of an individual bank, caused by a sudden and idiosyncratic shock, could cause a systemic crisis in the Belgian financial system, explaining the time-series behaviour of contagion. They found that moving from a complete structure to a multiple-money-centre structure (i.e. a situation dominated by higher concentration on the banking market) decreases both the risk and the impact of domestic contagion.

Other contributions have covered the whole interbank market and had to make some assumptions about the structure of the matrix. For instance, Upper and Worms (2004) used aggregate interbank assets and liabilities from banks' balance sheets to estimate the matrix of interbank relationships by maximising the entropy of claims. In this hypothesis, each bank lends to all the others, so that the market is complete in the sense of Allen and Gale (2000). Wells (2004) and van Lelyveld and Liedorp (2006) have actual data on large bilateral exposures and used maximum entropy techniques just to estimate the rest of the interbank matrix. Moreover, van Lelyveld and Liedorp (2006) made a valuable contribution by comparing the results based on the maximum entropy proxy with survey data on exposures in the Netherlands. They showed that the approximation does not seem to introduce a bias in the estimate of the actual linkages between banks.

With regard to the Italian interbank market, Mistrulli (2005 and 2010) carried out a survey to evaluate contagion in the banking system comparing the hypothesis of the 'complete' structure maximising the entropy of interbank linkages with the 'multiple money centre' structure observed in Italy. To this end, a single dataset including actual bilateral exposures was used. The results indicate that the maximum entropy approximation tends to provide a biased estimate of the extent of financial contagion. In particular, the estimated matrix overrates the vulnerability to contagion, but this does not hold true in general, depending on different elements such as the size of the interbank linkages, the recovery rates of interbank exposures and banks' capitalisation.

About shocks modelling, two main approaches are used in these papers. The first and most used approach relies on the artificial failure of single banks that (possibly) causes subsequent collapses in

the banking environment considered (e.g. van Lelyveld and Liedorp, 2006; Mistrulli, 2005), while the second is based on common shocks for the whole system (as in Elsinger, 2006).

Based on this second approach, the study reported in this paper aims to assess how an approximation of interbank linkages (that can possibly be very different from the actual linkages) affects the evaluation of systemic risk in a financial environment. It simulates the behaviour of the banking system in the presence of contagion, under the maximum entropy assumption. The starting point for this paper is a Monte Carlo simulation as in De Lisa et al. (2011) that make it possible to obtain directly scenarios with multiple defaults. Banks' assets are considered to be correlated. Therefore, in bad economic cycles multiple banks are exposed to potential failure. In particular, this paper estimates the distribution of aggregated excess losses (i.e. the losses of a bank that exceed the capital buffer) in the banking system, assuming that the default of one bank can trigger the default of others, which are linked to the failed bank via the interbank market matrix.

In order to test the soundness of the maximum entropy assumption, this hypothesis is relaxed to see if variations in the structure of the interbank market lead to significantly different systemic excess losses. Changes in the interbank matrix aim to relax the hypothesis of complete markets (as defined in Allen and Gale (2000)): for each bank analysed, a certain proportion of exposures are set to zero and the related contagion effects are compared with those obtained in maximum entropy conditions.

3. Methodology

3.1 Interbank matrix structure

This analysis assesses the uncertainty in results of simulations, due to approximation of the interbank matrix. Available data for each bank cover only total credits and debts to other banks, but information on bilateral exposures between banks is not publicly available. For this reason, the interbank matrix must be inferred by making assumptions on how interbank debts and credits are spread over the system.

Following Upper and Worms (2004), the first step is to approximate the interbank matrix with the maximum entropy one, i.e. assume that banks maximise the dispersion of their interbank credits and debts. This maximum entropy matrix is taken as the reference base in the numerical experiment presented in the next section.

Considering a banking system made up of J banks, the interbank exposures can be represented as a $J \times J$ matrix $IB = \{x_{jk}\}, j, k = 1, \dots, J$.

$$IB_{J \times J} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1J} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2J} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3J} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{J1} & x_{J2} & x_{J3} & \cdots & x_{JJ} \end{pmatrix}$$

where x_{jk} represents the exposure (debt) of bank j to bank k .

Diagonal elements $\{x_{jj}\}$, $j = 1, \dots, J$, representing self-exposures, are set at zero, so the interbank matrix of bilateral exposures becomes:

$$IB_{J \times J} = \begin{pmatrix} 0 & x_{12} & x_{13} & \cdots & x_{1J} \\ x_{21} & 0 & x_{23} & \cdots & x_{2J} \\ x_{31} & x_{32} & 0 & \cdots & x_{3J} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{J1} & x_{J2} & x_{J3} & \cdots & 0 \end{pmatrix}.$$

Only the total amount of interbank credits and interbank debts are known, i.e. $x_k = \sum_j x_{kj}$ and $x_j = \sum_k x_{kj}$ respectively. Moreover, as the samples considered do not cover the whole system, typically the values for total credits and total debits differ, i.e. $\sum_k x_k \neq \sum_j x_j$.

To take this into account, a row and a column must be added to the matrix, representing the net positions with regard to the ‘rest of the world’. The interbank matrix is extended to a $(J+1) \times (J+1)$ matrix such that $\sum_k x_k = \sum_j x_j$. This difference can be attributed to banks in proportion to their total exposure, as follows:

$$\text{If } \sum_k x_k > \sum_j x_j \text{ the last column will contain: } \left\{ x_{j, J+1} = \frac{x_j}{\sum_j x_j} \left(\sum_j x_j - \sum_k x_k \right) \right\}, j = 1, \dots, J$$

$$IB_{N+1 \times N+1} = \begin{pmatrix} 0 & x_{12} & x_{13} & \cdots & x_{1J} & \frac{x_1}{\sum_j x_j} \left(\sum_j x_j - \sum_k x_k \right) \\ x_{21} & 0 & x_{23} & \cdots & x_{2J} & \frac{x_2}{\sum_j x_j} \left(\sum_j x_j - \sum_k x_k \right) \\ x_{31} & x_{32} & 0 & \cdots & x_{3J} & \frac{x_3}{\sum_j x_j} \left(\sum_j x_j - \sum_k x_k \right) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{J1} & x_{J2} & x_{J3} & \cdots & 0 & \frac{x_N}{\sum_j x_j} \left(\sum_j x_j - \sum_k x_k \right) \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Symmetrically, if $\sum_k x_k < \sum_j x_j$ the last row will contain:

$$\left\{ x_{J+1k} = \frac{x_k}{\sum_k x_k} \left(\sum_k x_k - \sum_j x_j \right) \right\}, k = 1, \dots, J.$$

Keeping these constraints and assuming that the individual interbank exposures in the sample display maximum dispersion, so that each bank lends to each of the others in proportion to its share of the total interbank credit. All the other values can be calculated. In this way the largest lender will be the largest creditor for all other banks, and banks with no debts will evidently result in a column of zeros.

The corresponding matrix is obtained numerically via the ENTROP algorithm (see Blien and Graef (1997)).

In order to test the robustness of the maximum entropy assumption, variations were introduced in the interbank matrix to evaluate if these changes induce a significant variation in results.

Variations in the matrix of bilateral interbank exposures obtained via the ENTROP algorithm were introduced with a procedure that preserves the totals but introduces one zero more at each step. In this way an incomplete matrix is obtained that concentrates interbank activities into a limited pre-set number of non-zero values.

The procedure develops as follows:

Considering, for example, a 5×5 IB matrix:

$$IB_{5 \times 5} = \begin{pmatrix} 0 & x_{12} & x_{13} & x_{14} & x_{15} \\ x_{21} & 0 & x_{23} & x_{24} & x_{25} \\ x_{31} & x_{32} & 0 & x_{34} & x_{35} \\ x_{41} & x_{42} & x_{43} & 0 & x_{45} \\ x_{51} & x_{52} & x_{53} & x_{54} & 0 \end{pmatrix}$$

- (1) Select, randomly, two different rows and two different columns, which identify four different elements of the matrix (e.g. rows 1 and 2 and columns 3 and 4 identify x_{13} , x_{14} , x_{23} and x_{24}). Provided all four values are different from zero, these elements are going to be changed in order to obtain a new matrix with one additional zero.

$$IB_{5 \times 5} = \begin{pmatrix} 0 & x_{12} & 5 & 8 & x_{15} \\ x_{21} & 0 & 6 & 12 & x_{25} \\ x_{31} & x_{32} & 0 & x_{34} & x_{35} \\ x_{41} & x_{42} & x_{43} & 0 & x_{45} \\ x_{51} & x_{52} & x_{53} & x_{54} & 0 \end{pmatrix}$$

- (2) Evaluate which of the four elements has the lowest value (in this example this is $x_{13} = 5$).
- (3) The lowest value is subtracted from itself and also from the element in the other row and other column $x'_{13} = x_{13} - 5 = 0$; $x'_{24} = x_{24} - 5$ and added to the element in the same row but different column and in the same column but different row $x'_{23} = x_{23} + 5$; $x'_{14} = x_{14} + 5$.

The new matrix IB' will be:

$$IB'_{5 \times 5} = \begin{pmatrix} 0 & x_{12} & 5-5 & 8+5 & x_{15} \\ x_{21} & 0 & 6+5 & 12-5 & x_{25} \\ x_{31} & x_{32} & 0 & x_{34} & x_{35} \\ x_{41} & x_{42} & x_{43} & 0 & x_{45} \\ x_{51} & x_{52} & x_{53} & x_{54} & 0 \end{pmatrix} \Rightarrow IB'_{5 \times 5} = \begin{pmatrix} 0 & x_{12} & 0 & 13 & x_{15} \\ x_{21} & 0 & 11 & 7 & x_{25} \\ x_{31} & x_{32} & 0 & x_{34} & x_{35} \\ x_{41} & x_{42} & x_{43} & 0 & x_{45} \\ x_{51} & x_{52} & x_{53} & x_{54} & 0 \end{pmatrix}$$

In this way row and column totals are maintained, but a zero is introduced where the lowest value was originally placed.

This procedure is then iterated, up to the pre-set number of zeros.

For each country, the process starts with the maximum entropy matrix and adjusts it as described in the previous section in order to produce 20 series of interbank matrices with 20%, 35%, 50%, 65% and 80% more elements set to zero (other than the diagonal elements or elements already set at zero). To

perform a ‘*ceteris paribus*’ analysis, for each simulation the variation in the interbank matrix is set randomly, whereas the internal losses suffered by each bank (see next section) are always the same. In this way different results for the same country can only be due to variations in the interbank matrix.

3.2 Generating scenarios

In order to verify the effectiveness of contagion, the authors consider it fundamental to generate market scenarios as close as possible to the real market situation. To do this, a Monte Carlo simulation coherent with a Basel II framework and based on balance-sheet data (see De Lisa et al. (2011)) was performed with banks’ correlated assets. The correlation between banks’ assets is fundamental, as in this way market scenarios often include cases where one or a few bank defaults are rounded by some other cases of near-to-default banks, which are situations that are more likely to start financial contagion.

Table 1: Number of primary defaults (before contagion)

Number of primary defaults	BE	IE	IT	PT
1	8 663	8 931	6 696	8 855
2	959	806	1 493	840
3	252	183	696	197
4	73	51	330	69
5	29	16	185	21
> 5	24	13	600	18
Total	10 000	10 000	10 000	10 000

Simulations are based on the following three steps:

- (1) Estimate, based on assets, the average assets probability to default [PD] of each bank j \hat{PD}_j calculated as the PD that allows the actual value of the capital requirement for that specific bank K_j (extracted from balance-sheet data) to be equal to its numerically calculated value, setting the other variables, i.e. loss given default (LGD), maturity (M) and size (S), to their standard values:

$$\hat{PD}_j : K(\hat{PD}_j | LGD = 0.45 \ M = 2.5 \ S = 50) = K_j$$

where:

$$K_j(PD_{ij}, LGD_{ij}, M_{ij}, S_{ij}) = \sum_j C_{ij}(PD_{ij}, LGD_{ij}, M_{ij}, S_{ij}) \times A_{ij} \quad i = 1, \dots, I$$

is the sum of the capital allocation parameter (C_{ij}) of each exposure i of bank j multiplied by its amount A_{ij} .

$$C_{ij}(PD_{ij}, LGD_{ij}, M_{ij}, S_{ij}) = \left[LGD_{ij} \times \left[\sqrt{\frac{1}{1-R(PD_{ij}, S_{ij})}} N^{-1}(PD_{ij}) + \sqrt{\frac{R(PD_{ij}, S_{ij})}{1-R(PD_{ij}, S_{ij})}} N^{-1}(0.999) \right] - PD_{ij} \times LGD_{ij} \right] \times \left[1 + (M_{ij} - 2.5) B(PD_{ij}) \right] \times \left(1 - 1.5 \times B(PD_{ij}) \right)^{-1} \times 1.06$$

where:

$$B_{ij}(PD_{ij}) = \left[0.11852 - 0.05478 \ln(PD_{ij}) \right]^2$$

$$R_{ij}(PD_{ij}, S_{ij}) = 0.12 \frac{1 - e^{-50PD_{ij}}}{1 - e^{-50}} + 0.24 \left[1 - \frac{1 - e^{-50PD_{ij}}}{1 - e^{-50}} \right] - 0.04 \left[\frac{S_{ij} - 5}{45} \right]$$

(2) For each simulation n , calculate bank j 's losses L_{nj} performing a Monte Carlo simulation based on:

$$L_{nj}(z_{nj}, \hat{PD}_j) = \left[0.45 N \left[\sqrt{\frac{1}{1-R(\hat{PD}_j, 50)}} N^{-1}(\hat{PD}_j) + \sqrt{\frac{R(\hat{PD}_j, 50)}{1-R(\hat{PD}_j, 50)}} N^{-1}(z_{nj}) \right] - 0.45 \hat{PD}_j \right] \times \left(1 - 1.5 B(\hat{PD}_j) \right)^{-1} \times 1.06$$

where

$n = 1, \dots, N$ simulations

$z_{nj} \sim N(0,1) \forall j, n$

$\text{cov}(z_{nj}, z_{nk}) = 0.5 \forall j \neq k$

These primary banks' simulated losses are then compared with banks' capital. Whenever in each bank j

$$L_{nj}(z_{nj}, \hat{PD}_j) \geq CAP_j$$

losses net of provisions exceed total capital and bank j is considered to default in simulation n .

These net losses $L_{nj}(z_{nj}, \hat{PD}_j) - CAP_j$ are recorded (when at least one bank defaults) as 'no contagion losses'.

This produces a wealth of synthetic market scenarios, distributed as implicitly defined by the Basel II Regulation, correlated between banks, and based on proxies of assets PD and actual values of the total capital of each bank considered. This is the starting point for testing contagion effects.

(3) Following James (1991), it was assumed that, whenever a bank defaults, 40% of the amount of its interbank debts are passed onto creditor banks and distributed between them, so that:

$$L_{nj}^c(z_{nj}, \hat{PD}_j, IB) = L_{nj}(z_{nj}, \hat{PD}_j) + \sum_k D_k x_{kj} \text{ where } D_k = 1 \text{ if bank } k \text{ defaulted, and zero otherwise.}$$

Considering this, bank j defaults when

$$L_{nj}^c(z_{nj}, \hat{PD}_j, IB) \geq CAP_j.$$

Contagion is looped up to the cycle where no more banks default.

Finally, net losses $L_{nj}^c(z_{nj}, \hat{PD}_j, IB) - CAP_j$ are recorded (when at least one bank defaults).

Simulations were performed in order to have 10,000 significant values for each country considered and for each interbank matrix. Setting the same starting seed in a random number generator assures that differences in contagion results are due to the interbank matrix variation.

4. Data

Some authors point out that different features of banking systems could lead to different effects of the maximum entropy hypothesis. For this reason, the analysis was conducted on four banking systems: Belgium (BE), Italy (IT), Ireland (IE) and Portugal (PT). These banking systems show different distributions of banks' concentration ratio and business models. This makes it possible to evaluate if the impact of changes in the hypotheses over the interbank matrix is related to countries' specific characteristics. Data are based on the *Bankscope* dataset, as of December 2009, integrated with ECB and central banks' values.

Table 2 contains aggregate information about the data considered for each country.

Table 2: Description of the samples used for simulations

	Number of banks	Sample % population	Capital (m€)	Total assets (m€)	Interbank debts (m€)	Interbank credits (m€)
BE	23	82.26 %	48 401	878 336	97 493	84 727
IE	24	101.91 %	65 392	1 221 181	276 738	148 729
IT	473	81.81 %	270 876	2 827 051	188 375	195 958
PT	14	66.49 %	26 341	323 762	43 561	34 504

	Capital/total assets	Interbank debts/total assets	Interbank credits/total assets	Herfindhal index (over total assets)	Herfindhal index (over interbank debts)	Herfindhal index (over interbank credits)
BE	0.055	0.111	0.096	0.293	0.304	0.256
IE	0.054	0.227	0.122	0.154	0.177	0.214
IT	0.096	0.067	0.069	0.054	0.092	0.117
PT	0.081	0.135	0.107	0.259	0.228	0.345

The sample of banks covered in each country ('sample population') is calculated with reference to the amount of total assets reported by the ECB¹.

¹ Source: European Central Bank (2010), EU banking structures: <http://www.ecb.int/pub/pdf/other/eubankingstructure201009en.pdf>.

Capitalisation levels, measured by the capital/total assets ratio, first approximate the extent to which banks are resilient to defaults of their own assets. That also depends on the riskiness of the assets, which is taken into account in the scenario-generating process.

Columns containing interbank volumes represent the size of interbank debts and credits over total assets. Herfindhal indices monitor concentration in the banking system relative to total asset and interbank exposures. The index is generally calculated as:

$$H = \sum_{k=1}^N s_k^2,$$

where s_k is the market share of firm k in the market with respect to the variable considered (total assets, interbank debts and credits).

Looking at the tables above, Belgium has a small number of banks and, according to its Herfindhal indices, a highly concentrated banking system in terms of total assets and interbank exposures. The Irish banking system is not highly concentrated and is made up of a small number of banks highly exposed in the interbank market.

Italy has the largest number of banks, high capitalisation, low interbank exposures and low Herfindhal indices. Portugal has the smallest number of banks, a high capitalisation level and, together with Belgium, the highest level of concentration in terms of both total assets and interbank exposures.

5. Results

5.1 Effects on contagion

The consequences of assuming ‘maximum entropy’ for interbank exposures are not evident *a priori*. On the one hand, the maximum entropy assumption could lead to underestimation of contagion risk: the consequences of a default are actually spread across all the other banks, limiting the effects on each single bank. On the other, this assumption reflects the connectedness between all banks, even where no real interbank links exist, thus possibly creating fictitious ways of propagating contagion. For this reason, the influence of variations in the interbank matrix is verified for the whole probability distribution of estimated losses.

As expected, concentration in the interbank matrix does affect variability. In particular, the higher the concentration in interbank connections (number of zeros in the interbank matrix), the higher the variability in results. As can also be seen, higher interbank values (Ireland) result in higher variability, while a higher number of banks (Italy) possibly induces more stability.

In this regard, Table 3 reports the average ratios constructed as standard error over average. Remember that simulations are run in order to have 10000 scenarios with at least one default in each country. For

each scenario 20 different interbank matrices were constructed for each concentration level, so that in the end contagion in each country can be monitored by five matrices (one for each concentration level) with dimensions 10000 x 20, for both losses and defaults. Variability in a single banking system is thus evaluated with the average value of the standard error/average ratio calculated for each row of the five matrices.

Table 3: Variability — average value in standard error of contagion simulations results

	+ 20 % zeros	+ 35 % zeros	+ 50 % zeros	+ 65 % zeros	+ 80 % zeros
BE	0.6 %	0.7 %	2.1 %	3.3 %	5.7 %
IE	11 %	26 %	34 %	56 %	78 %
IT	0.004 %	0.008 %	0.013 %	0.023 %	0.045 %
PT	2 %	4 %	4 %	6 %	7 %

The general trend is an increase in variability as the simulation moves up from a situation with 20 % of zeros added in the interbank matrices to 80 %. This trend is confirmed in all four countries considered, even if differences between them can be seen from the differences in the magnitude of variability (see, for example, the comparison between Ireland and Italy).

The authors also investigated if changes in the interbank matrix produce an effect on losses aggregated on the basis of the magnitude of contagion. Tables 4.1 to 4.4 and 5.1 to 5.4 report three possible levels, individualised by the amount of losses originated in the cases of maximum entropy and of no contagion. In this regard, it must be remembered that simulations were run with and without contagion, in order to evaluate the effects of linkages between banks.

In detail, the ‘Overall’ column in Tables 4.1 to 4.4 represents the average number of defaults, over the 10000 simulated scenarios, while Tables 5.1 to 5.4 report the average excess losses calculated per country. The other three columns show the same losses split on the basis of the magnitude of contagion. More specifically:

- NO CONTAGION contains cases where: $ExcessLoss(NoContagion) = ExcessLoss(BaseScenario)$
- SMALL CONTAGION contains cases where:
 $ExcessLoss(BaseScenario) - ExcessLoss(NoContagion) \leq ExcessLoss(NoContagion)$
- LARGE CONTAGION contains cases where:
 $ExcessLoss(BaseScenario) - ExcessLoss(NoContagion) > ExcessLoss(NoContagion)$

The ‘BASE’ row refers to the maximum entropy situation, whereas +20 %, +35 %, +50 %, +65 % and +80 % indicate subsequent changes in the interbank matrix.

Table 4.1: Number of defaults by contagion magnitude — Belgium

BE	CONTAGION			
	Overall (10 000 cases)	NO (6 747 cases)	SMALL (2 275 cases)	LARGE (978 cases)
BASE	1.84	1.00	2.67	5.75
+20 %	1.85	1.00	2.68	5.76
+35 %	1.87	1.00	2.72	5.82
+50 %	1.90	1.00	2.80	5.97
+65 %	1.96	1.04	2.95	6.00
+80 %	2.12	1.07	3.37	6.48

Table 4.2: Number of defaults by contagion magnitude — Ireland

IE	CONTAGION			
	Overall (10 000 cases)	NO (6 174 cases)	SMALL (937 cases)	LARGE (2 889 cases)
BASE	4.41	1.00	2.21	12.40
+20 %	4.46	1.03	2.41	12.45
+35 %	4.51	1.11	2.81	12.35
+50 %	4.65	1.19	3.73	12.34
+65 %	4.82	1.49	4.30	12.10
+80 %	5.51	2.34	6.58	11.94

Table 4.3: Number of defaults by contagion magnitude — Italy

IT	CONTAGION			
	Overall (10 000 cases)	NO (6 694 cases)	SMALL (3 305 cases)	LARGE (0 cases)
BASE	2.15	1.00	4.48	-
+20 %	2.15	1.00	4.48	-
+35 %	2.15	1.00	4.48	-
+50 %	2.15	1.00	4.48	-
+65 %	2.15	1.00	4.48	-
+80 %	2.15	1.00	4.48	-

Table 4.4: Number of defaults by contagion magnitude — Portugal

PT	CONTAGION			
	Overall (10 000 cases)	NO (6 814 cases)	SMALL (2 478 cases)	LARGE (708 cases)
BASE	1.85	1.00	3.19	5.39
+20 %	1.85	1.00	3.18	5.38
+35 %	1.83	1.01	3.07	5.33
+50 %	1.81	1.01	2.99	5.39
+65 %	1.79	1.03	2.88	5.31
+80 %	1.74	1.01	2.76	5.16

Figure 1: Number of defaults by contagion magnitude — Belgium

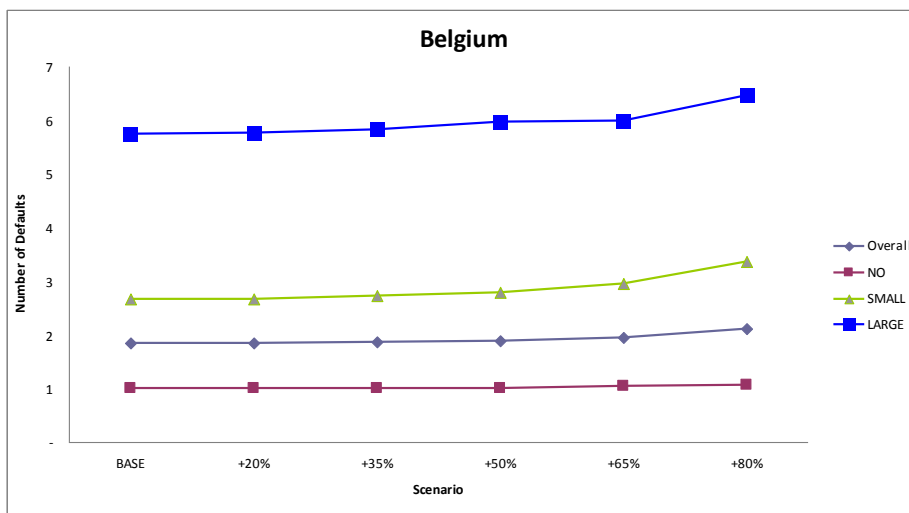


Figure 2: Number of defaults by contagion magnitude — Ireland

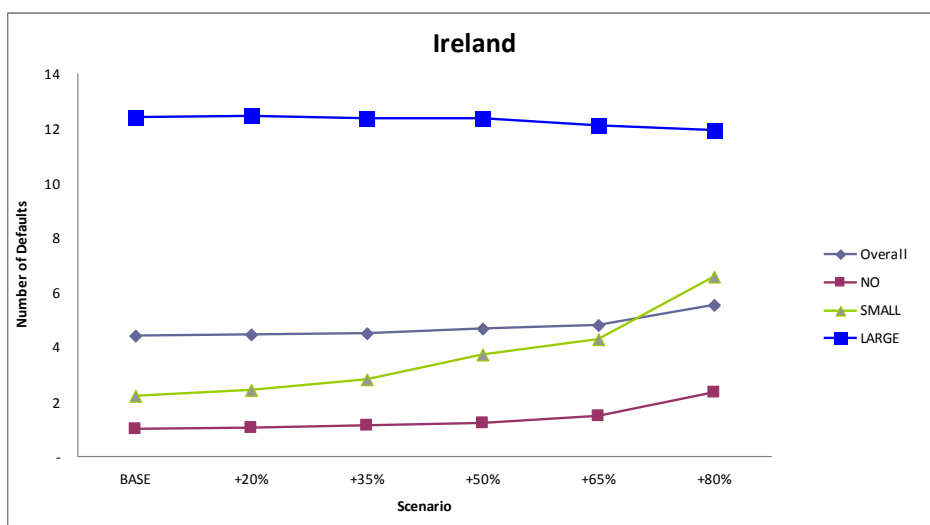


Figure 3: Number of defaults by contagion magnitude — Italy

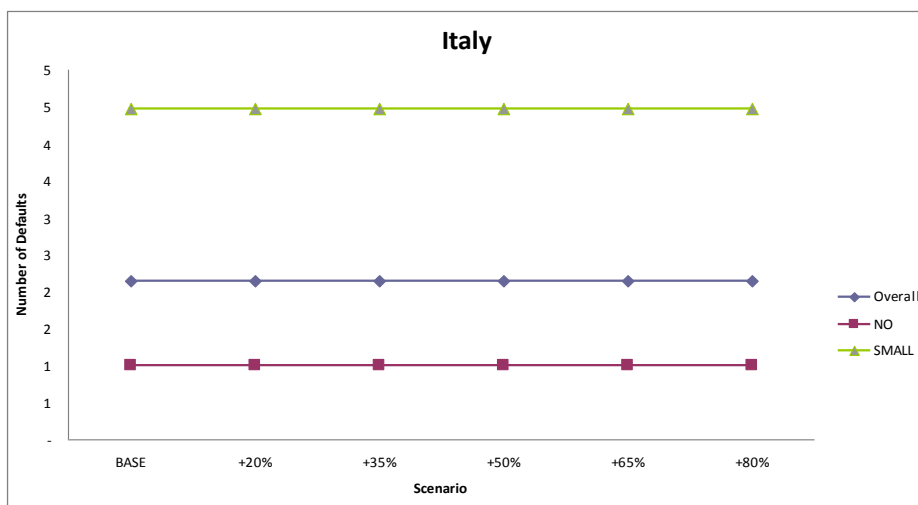


Figure 4: Number of defaults by contagion magnitude — Portugal

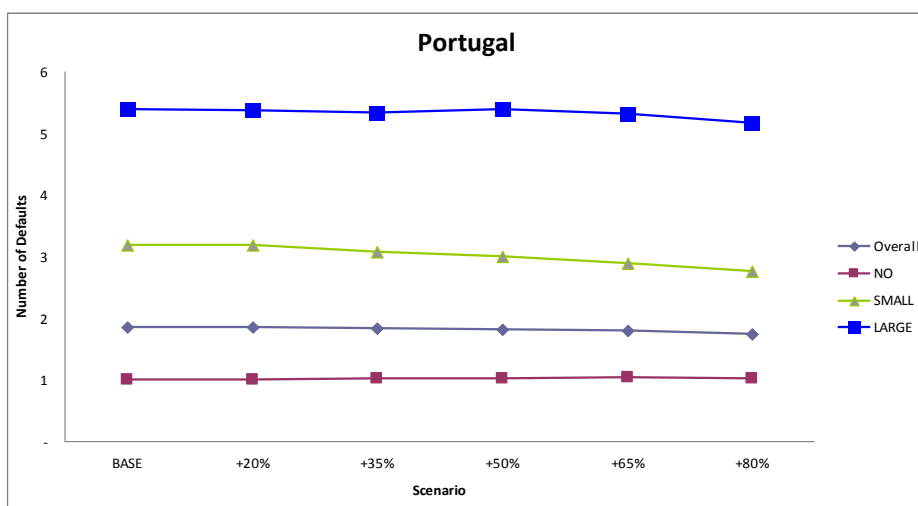


Table 5.1: Average value of losses by contagion magnitude — Belgium

BE	CONTAGION			
	Overall (10 000 cases)	NO (6 747 cases)	SMALL (2 275 cases)	LARGE (978 cases)
BASE	2 696 176	498 464	5 063 463	12 352 041
+20 %	2 693 324	498 724	5 065 719	12 314 761
+35 %	2 693 778	498 956	5 064 582	12 320 454
+50 %	2 703 282	499 291	5 109 217	12 311 490
+65 %	2 710 214	504 140	5 147 584	12 259 669
+80 %	2 761 571	512 407	5 394 238	12 153 998

Table 5.2: Average value of losses by contagion magnitude — Ireland

IE	CONTAGION			
	Overall (10 000 cases)	NO (6 174 cases)	SMALL (937 cases)	LARGE (2 889 cases)
BASE	16 998 231	989 367	2 394 299	55 946 867
+20 %	17 049 103	1 074 374	3 407 988	55 612 516
+35 %	16 968 441	1 291 853	5 529 794	54 180 373
+50 %	17 206 620	1 565 217	9 522 706	53 125 572
+65 %	17 321 954	2 517 032	12 076 994	50 662 249
+80 %	19 941 129	6 014 341	22 203 313	48 969 972

Table 5.3: Average value of losses by contagion magnitude — Italy

IT	CONTAGION			
	Overall (10 000 cases)	NO (6 694 cases)	SMALL (3 305 cases)	LARGE (0 cases)
BASE	171 048	47 199	421 817	-
+20 %	171 046	47 199	421 812	-
+35 %	171 045	47 199	421 807	-
+50 %	171 052	47 199	421 828	-
+65 %	171 047	47 199	421 815	-
+80 %	171 042	47 200	421 800	-

Table 5.4: Average value of losses by contagion magnitude — Portugal

PT	CONTAGION			
	Overall (10 000 cases)	NO (6 814 cases)	SMALL (2 478 cases)	LARGE (708 cases)
BASE	881 506	68 602	1 984 053	4 846 226
+20 %	881 388	68 664	1 990 277	4 822 161
+35 %	879 098	68 842	1 989 835	4 789 649
+50 %	884 939	68 910	2 007 381	4 810 091
+65 %	887 535	69 586	2 037 210	4 735 842
+80 %	898 572	70 164	2 093 690	4 688 501

Figure 5: Average value of losses by contagion magnitude — Belgium

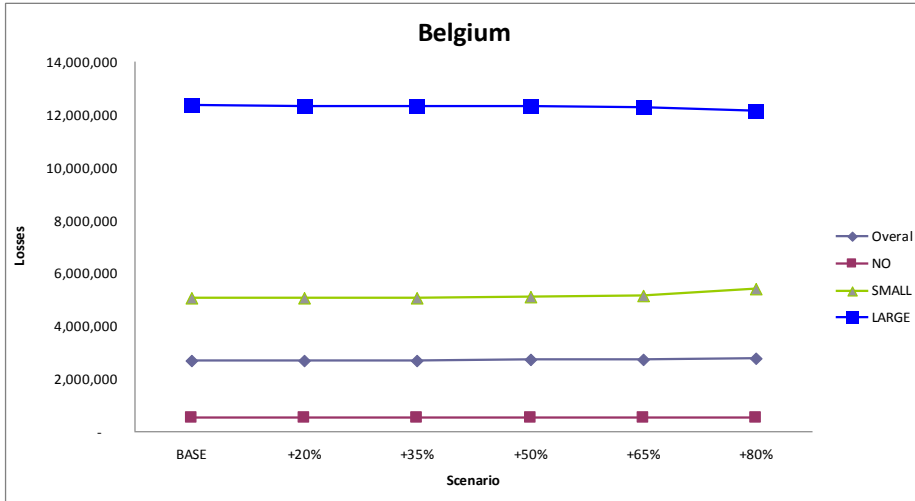


Figure 6: Average value of losses by contagion magnitude — Ireland

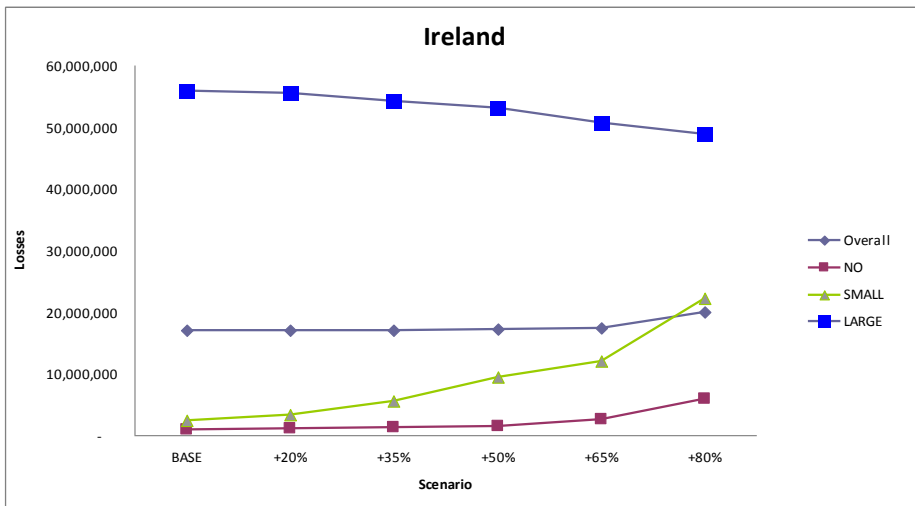


Figure 7: Average value of losses by contagion magnitude — Italy

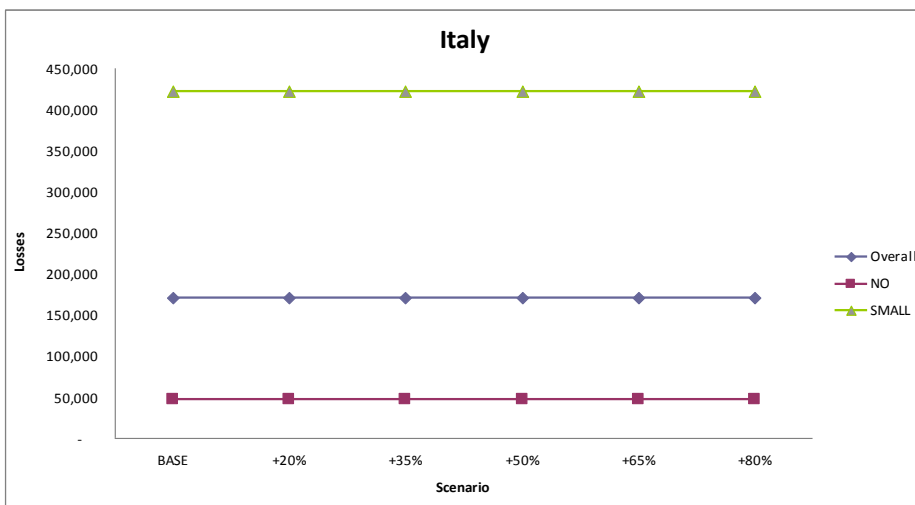
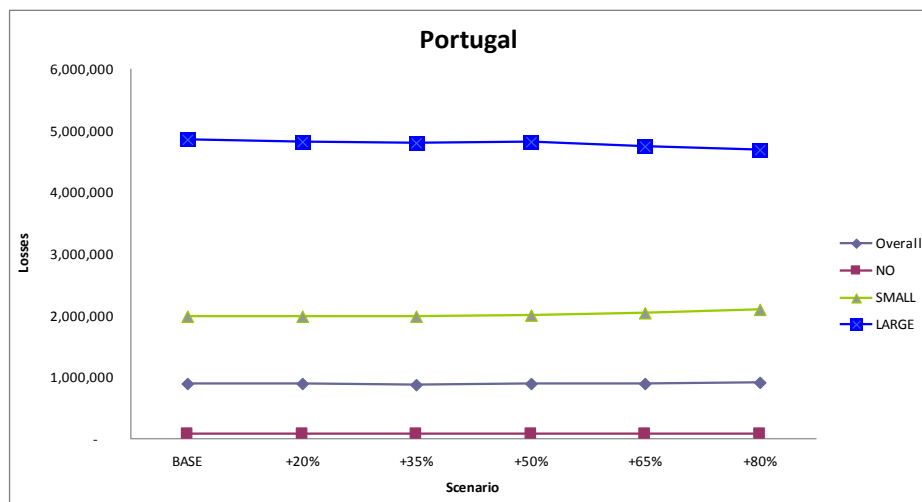


Figure 8: Average value of losses by contagion magnitude — Portugal



The average results (‘Overall’ column) clearly indicate that, considering all 10 000 scenarios, changes in the interbank matrix (zeros added) do not significantly influence the amount of excess losses found in the base case of maximum entropy. The only exception is Ireland +80 %, where the amount of losses jumps when the extreme concentration level is reached. Nevertheless, some differences originate when the results are split into groups selected by the size of contagion. In the small contagion case, the maximum entropy hypothesis (base values) tends to lead to underestimation of excess losses, whereas in big crises (large contagion) maximum entropy seems to overestimate contagion effects (as found by Mistrulli, 2010).

Looking at differences between countries, banking systems with a large number of banks (Italy) tend to have more stability in results, producing almost the same estimates for excess losses despite the hypothesis over the interbank matrix.

Countries with a smaller number of banks experience more significant changes in the amount of losses. In particular, contagion is more vulnerable to changes in the interbank structure in countries that have more sizable interbank exposures (and lower capitalisation) and are therefore more exposed to financial contagion (Ireland). In these situations the no contagion and small contagion simulations are highly underestimated, whereas large contagion cases are slightly overestimated.

5.2 Effects on losses probability distribution

Different evaluations can be found when considering the probability distribution of financial crises by crisis of final effects values (instead of contagion effects).

Tables 6.1 to 6.4 report, for each country, the distribution of the 10 000 simulated scenarios. In each table, column 1 (no contagion) shows the magnitude of systemic excess losses without contagion effects, column 2 (base scenario) shows results for the baseline scenario (i.e. under the maximum entropy assumption) and columns 3 to 8 contain results for the matrices with 20 %, 35 %, 50 %, 65 % or 80 % of matrix elements set to zero.

Comparison between columns 1 and 2 could be useful to address the effects of contagion. For instance, Table 6.1 indicates that contagion in the Belgian banking system has no effect on systemic excess losses under the 40th percentile of the distribution.

For each of these five classes of variation (20 %, 35 %, 50 %, 65 % and 80 %), the authors estimated 20 probability distributions obtained via 20 different interbank matrices. The averages and standard errors are reported in the tables below.

The results show that variations in the structure of the interbank matrix do not really affect the probability distribution of banking crisis estimates. This is probably (and almost partially) due to the fact that when the interbank matrix is incomplete, contagion affects some banks more and does not affect others, thus inducing different results, but, when considering the whole system, larger effects on some banks and lower on others balance out and the final distribution (which is re-ordered by crisis size) is not deeply affected.

Table 6.1: Estimated losses probability distribution — Belgium

BE	No contagion	Base scenario	+20 % zeros		+35 % zeros		+50 % zeros		+65 % zeros		+80 % zeros	
			Average	Standard error %	Average	Standard error %	Average	Standard error %	Average	Standard error %	Average	Standard error %
90 %	4963902	8076588	8067476	1%	8075254	1%	8083277	1%	8123097	2%	8325802	5%
80 %	2124436	3165878	3169533	0%	3176077	1%	3223853	3%	3263529	4%	3559248	8%
70 %	696941	1260621	1267342	1%	1274006	1%	1318665	6%	1346789	7%	1489448	13%
60 %	238197	269098	269019	0%	269248	0%	268486	1%	272527	2%	273569	4%
50 %	113708	119845	119820	0%	119809	0%	119648	0%	120114	1%	120170	1%
40 %	60175	61529	61526	0%	61513	0%	61499	0%	61702	1%	61801	1%
30 %	31970	32241	32239	0%	32243	0%	32237	0%	32270	0%	32271	0%
20 %	16151	16249	16255	0%	16256	0%	16259	0%	16266	0%	16267	0%
10 %	6500	6517	6517	0%	6517	0%	6518	0%	6522	0%	6523	0%

Table 6.2: Estimated losses probability distribution — Ireland

IE	No contagion	Base scenario	+20 % zeros		+35 % zeros		+50 % zeros		+65 % zeros		+80 % zeros	
			Average	Standard error %	Average	Standard error %	Average	Standard error %	Average	Standard error %	Average	Standard error %
90 %	4985377	62698542	62758442	0%	62717637	1%	63004224	1%	63004066	2%	62939981	3%
80 %	2657548	51321600	51355471	2%	50964341	4%	50558473	7%	49386188	8%	53126401	7%
70 %	1610770	6867626	6830817	2%	8176396	91%	11430821	103%	12758165	87%	35753569	37%
60 %	987499	2094352	2128881	2%	2210744	12%	2414177	13%	2760687	22%	7317439	111%
50 %	591401	943087	968819	4%	999867	7%	1082851	10%	1192853	15%	1997922	34%
40 %	338844	470510	475435	4%	489601	8%	518941	12%	566961	19%	759291	40%
30 %	182764	225856	227429	3%	231543	6%	241571	8%	257633	13%	289531	16%
20 %	87586	98216	98576	2%	99634	3%	102256	4%	106534	7%	112791	10%
10 %	33409	35908	35957	1%	36246	2%	36883	3%	37968	5%	39415	9%

Table 6.3: Estimated losses probability distribution — Italy

IT	No contagion	Base scenario	+20 % zeros		+35 % zeros		+50 % zeros		+65 % zeros		+80 % zeros	
			Average	Standard error %	Average	Standard error %	Average	Standard error %	Average	Standard error %	Average	Standard error %
90 %	314 591	316 000	315 998	0%	315 992	0%	315 954	0%	315 892	0%	315 887	0%
80 %	119 961	120 194	120 204	0%	120 184	0%	120 188	0%	120 174	0%	120 158	0%
70 %	65 722	65 722	65 722	0%	65 728	0%	65 732	0%	65 739	0%	65 739	0%
60 %	40 647	40 647	40 647	0%	40 647	0%	40 647	0%	40 647	0%	40 647	0%
50 %	26 612	26 612	26 612	0%	26 612	0%	26 612	0%	26 612	0%	26 614	0%
40 %	16 822	16 822	16 822	0%	16 822	0%	16 822	0%	16 822	0%	16 822	0%
30 %	10 481	10 481	10 481	0%	10 481	0%	10 481	0%	10 481	0%	10 481	0%
20 %	5 828	5 828	5 828	0%	5 828	0%	5 828	0%	5 828	0%	5 828	0%
10 %	2 368	2 368	2 368	0%	2 368	0%	2 368	0%	2 368	0%	2 368	0%

Table 6.4: Estimated losses probability distribution — Portugal

PT	No contagion	Base scenario	+20 % zeros		+35 % zeros		+50 % zeros		+65 % zeros		+80 % zeros	
			Average	Standard error %	Average	Standard error %	Average	Standard error %	Average	Standard error %	Average	Standard error %
90 %	1 731 395	2 465 932	2 469 978	0%	2 466 611	1%	2 496 896	1%	2 519 816	2%	2 593 774	5%
80 %	567 998	866 993	862 903	1%	856 110	3%	877 474	2%	895 202	4%	964 488	13%
70 %	196 915	274 012	272 333	1%	268 626	4%	270 659	3%	268 213	3%	268 228	5%
60 %	85 532	96 355	96 196	0%	96 170	1%	95 781	1%	95 759	2%	94 690	1%
50 %	42 275	44 584	44 569	0%	44 629	1%	44 438	0%	44 615	2%	44 240	1%
40 %	22 892	23 454	23 463	0%	23 507	1%	23 436	0%	23 518	1%	23 367	0%
30 %	12 956	13 306	13 283	0%	13 263	1%	13 219	0%	13 252	1%	13 160	0%
20 %	6 778	6 833	6 830	0%	6 833	0%	6 825	0%	6 835	0%	6 813	0%
10 %	2 933	2 942	2 942	0%	2 944	0%	2 943	0%	2 948	0%	2 942	0%

6. Conclusions

This paper tested maximum entropy approximation for the interbank matrix in simulating contagion effects in banking systems. In the process, an uncertainty test was performed on the maximum entropy matrix by developing an algorithm that allows more concentrated interbank exposures to be obtained. A Monte Carlo method was applied to generate banking crises scenarios that were used to test contagion effects. Results obtained from the maximum entropy interbank matrix were then compared with the results derived from higher concentration in the matrices.

The probability distribution of losses is rather stable even with 80 % more zeros in the matrix.

Conversely, when considering the magnitude of contagion, it can be seen that excess losses tend to be underestimated when the maximum entropy matrix is used in banking systems with large interbank exposures and in ‘small contagion crises’. Otherwise ‘large contagion crises’ tend to be associated with overestimation of excess losses.

As in Mistrulli (2010), the authors found that underestimation of contagion by maximum entropy is heightened by the specific features of the banking system. More specifically, high levels of capitalisation, low interbank exposure and large samples seem to produce more stable results. On the other hand, low capitalisation, high interbank exposure and a small number of banks result in underestimation of excess losses in ‘small contagion crises’ and overestimation in ‘large contagion crises’.

Therefore, the results for different countries seem to be clearly affected by certain characteristics of the banking system. Precise quantification of their individual effects would go beyond the scope of this paper. Nevertheless it is worth developing this approach further and performing a sensitivity analysis in order to quantify better the effects of banking systems’ characteristics on financial contagion.

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Abstract

The main lesson learned from the recent financial crisis is the crucial role of interconnectedness between banks as a factor that can push the effects of bank defaults to extreme levels.

One bank in distress can compromise the ability to repay obligations of its creditor banks, thereby inducing a more general crisis that spreads from the banking system towards the real economy. Several empirical and theoretical studies have focused on the role of the interbank market in causing contagion in financial crises.

In this regard, one frequent problem encountered in dealing with contagion risk in the banking system is that only data on interbank credits and debts aggregated at bank level are publicly available, whereas the whole matrix of interbank linkages would be needed in order to estimate systemic risk correctly.

One common solution is to assume that banks maximise the dispersion of their interbank credits and debts, so that the interbank matrix can be approximated by its maximum entropy.

This paper tests the influence of this hypothesis on simulations by verifying if variations in the structure of the interbank matrix lead to significant changes in the magnitude of contagion.

In order to do this, an algorithm was developed that generates interbank matrices with higher concentration.

Then a Monte Carlo simulation was run by making use of the SYMBOL model (SYstemic Model of Banking Originated Losses) jointly developed by the JRC, DG MARKT, and experts of banking regulation (see De Lisa et al., 2010). We then compared results obtained using the maximum entropy approximated matrix with those obtained from more concentrated matrices.

Numerical experiments, performed on samples of banks from four European countries, highlight that concentration in interbank loans does affect results but that, when considering the probability distribution of losses, even significant changes in the interbank matrix do not deeply affect results.

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