02 Jun 2016

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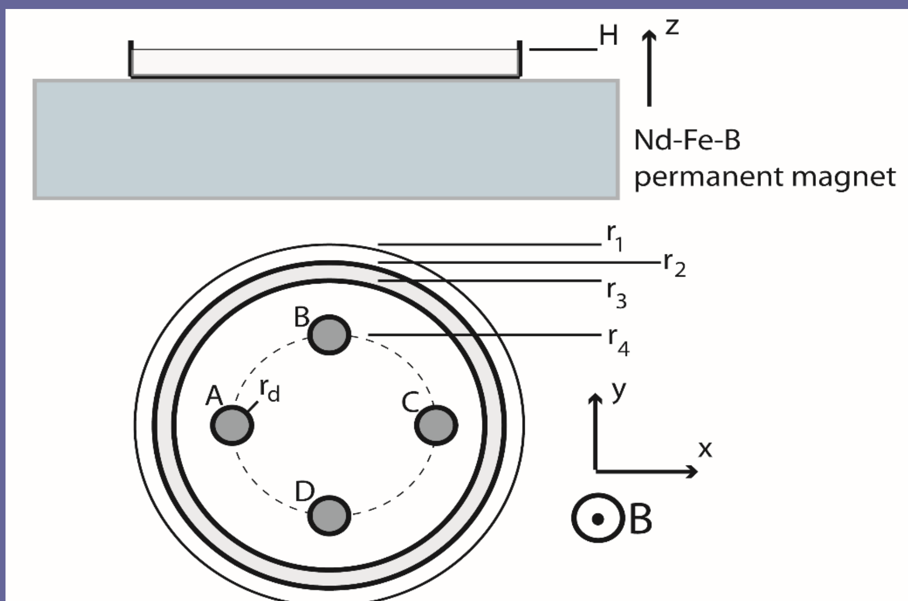
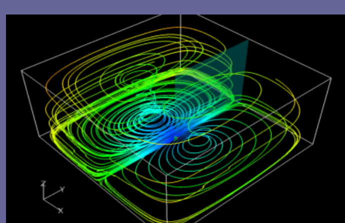
229th ECS Meeting, San Diego, May 29-Jun 2, 2016

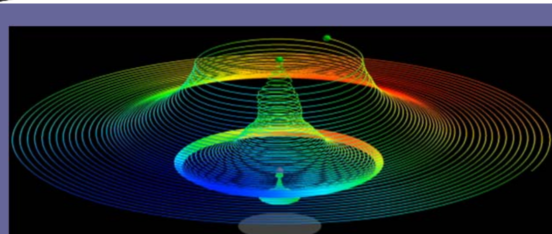
Acknowledgements

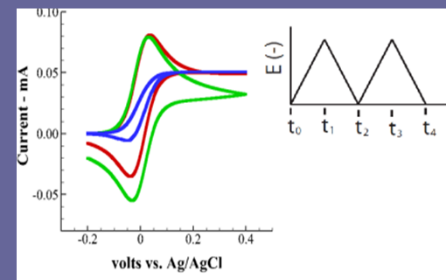
National Science Foundation Grants

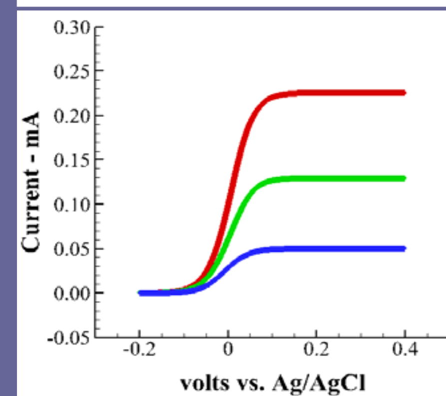
Award number CBET-1336722
Award number CHE-0719097





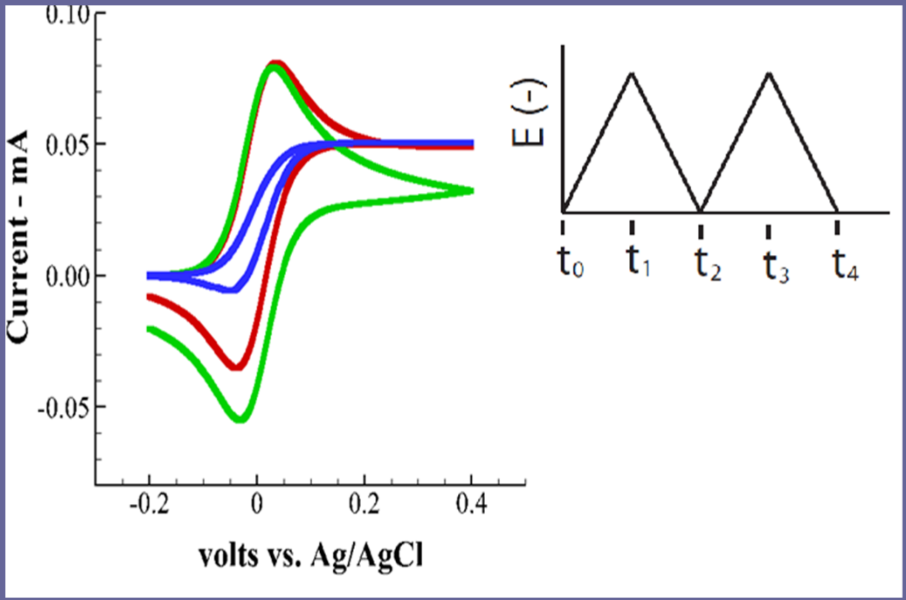






Outline

- Motivation
- Problem Description
- Technical Highlights
- Results
- Discussion
- Conclusions
- Ongoing and Future Work



Motivation

- Speed of chemical reactions is dictated by mixing efficiency
- Low Reynolds number (~1) makes mixing a challenge
- Introducing Turbulent-like features can enhance mixing
- Chaotic advection provides foundation for analysis and control
- Simulations based on Navier-Stokes equations provide many insights
- Powerful post-processing tools help understand underlying mixing phenomena
- Simulation results can help develop new mixing and flow control strategies

Problem Description

- Stirring vs. mixing: An important distinction
- Stirring increases the mean value of gradients
- Mixing decrease the mean value of gradients
- Stirring leads to stretching and folding of the interface
- Chaotic advection is an efficient way of stirring
- Mixing is complete when the gradients disappear

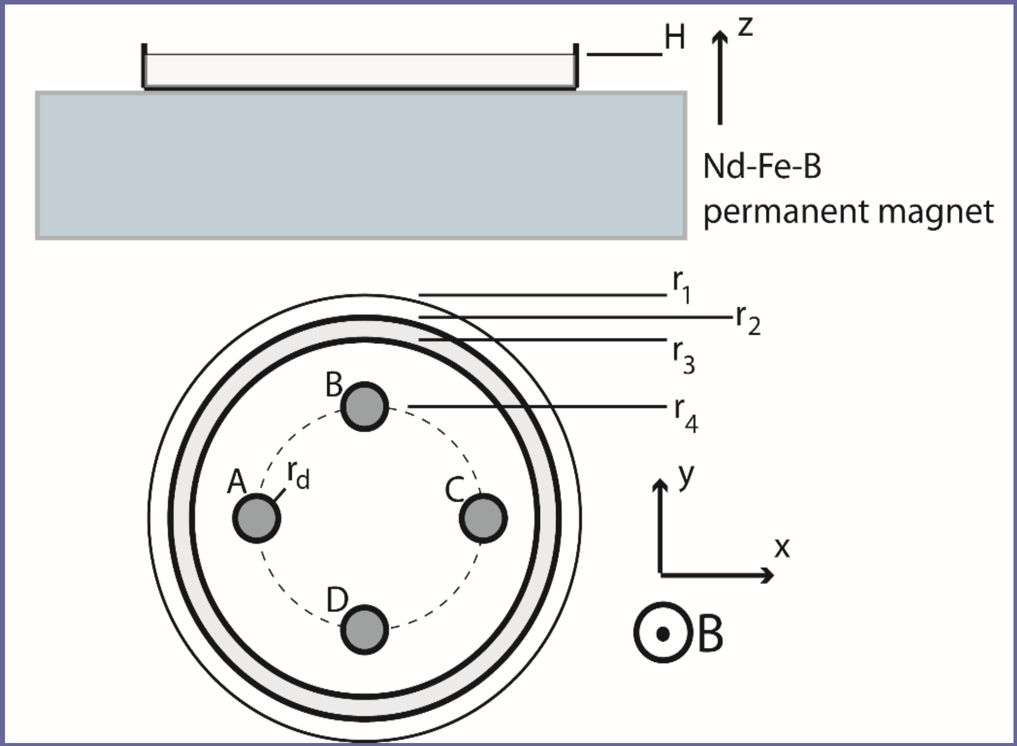
Problem Description

- MHD can introduce a non-intrusive driving force
- Lorentz force $\bar{j} \times \bar{B}$ for MHD is due to ionic current-magnetic field cross product
- Flow field and species concentration fields are obtained by solving unsteady, 3D Navier-Stokes equations
- Extensive post-processing is performed to visualize the results and obtain mixing quality
- Passive numerical particles are tracked by integrating the advection equation $\frac{d\bar{X}}{dt} = \bar{V}(\bar{X}, t)$

Problem Description

Cell geometry

Shallow open cell
r_1 = 3mm
r_2 = 2.4 mm
r_3 = 2mm
r_d = 0.16 mm
H = 0.5 mm



NaCl solution: ρ = 1000 kg/m^3, μ = 0.001 kg/(m.s), C^* = 0.1M,
ϕ = 0.04 V, σ = 1.29 S/m.
Magnetic field: B_z = 0.36 Tesla

Problem Description

Governing Equations

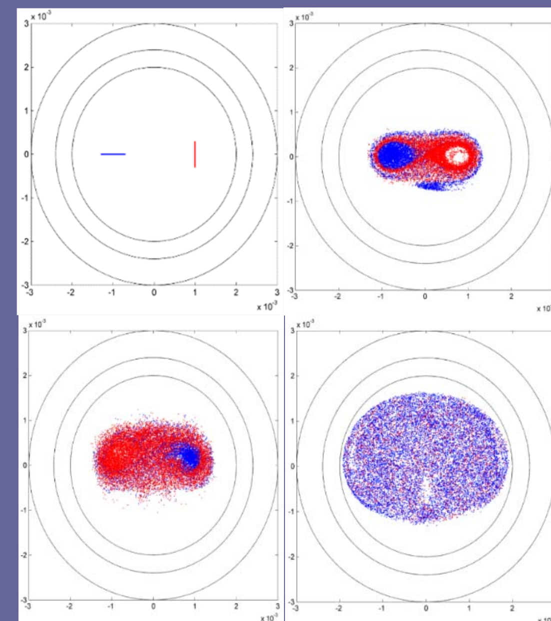
Continuity: $\nabla \cdot \mathbf{V} = 0$

Momentum: $\rho \dfrac{D\mathbf{V}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{V} + \mathbf{F}_\mathrm{L}$

Lorenz Force: $\mathbf{F}_\mathrm{L} = -\sigma \nabla \phi \times \mathbf{B}$

Electric Field Outside Double Layer:

$$\nabla^2 \phi = 0$$



Auxiliary Equations

Passive Particle Trajectory: $\dfrac{d\mathbf{X}(t)}{dt} = \mathbf{V}(\mathbf{X}, t)$

MixingPerformance: $\alpha(t) = 1 - \dfrac{\delta^2(t)}{\delta^2(0)}$

$$\delta^2(t) = \iiint_V \left[C(x,y,z,t) - \bar{C} \right]^2 dxdydz$$

2nd order, implicit

$$\frac{3\phi^{n+1} - 4\phi^n + \phi^{n-1}}{2\Delta t} = F(\phi^{n+1})$$

Problem Description

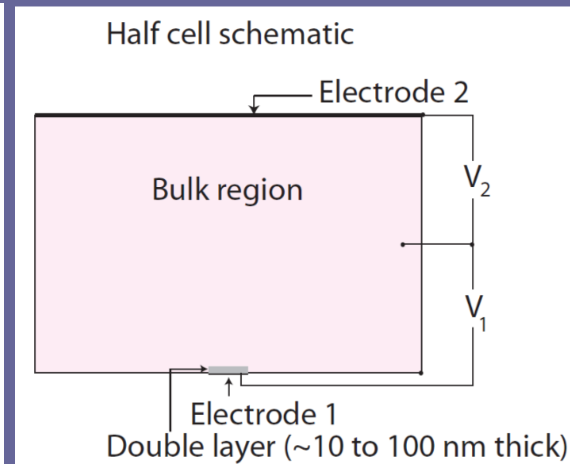
Gouy-Chapman Model for Double Layer

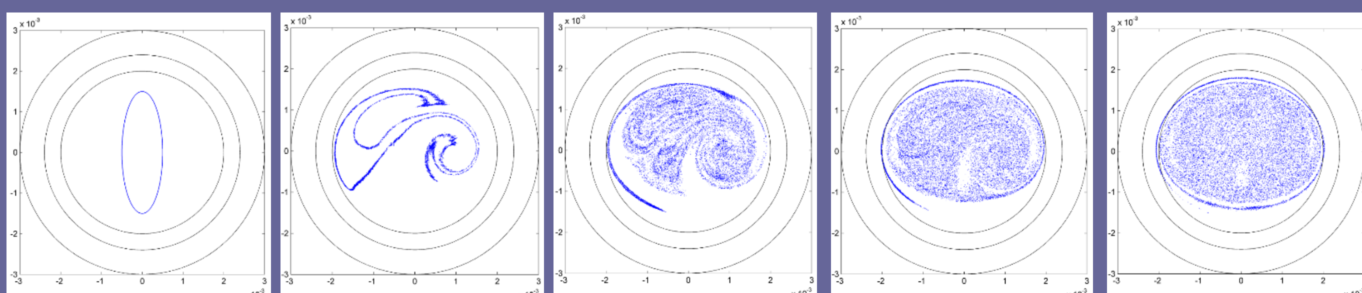
Boltzmann distribution: $$\mathrm{C}_i = C_i^* \exp\left(-\frac{z_i F\phi}{RT}\right)$$

Electric field: $$\nabla^2\phi = -\frac{\rho_e}{\varepsilon} = -\frac{F}{\varepsilon}\sum_i z_i C_i^* \exp\left(-\frac{z_i F\phi}{RT}\right)$$

1D solution for DL: $$\frac{\tanh(K\phi)}{\tanh(K\phi_0)} = \exp(-\kappa x)$$

$$K = Fz/(4RT), \qquad \kappa = \left(2F^2/\varepsilon RT\right)^{1/2} z\sqrt{C^*}$$





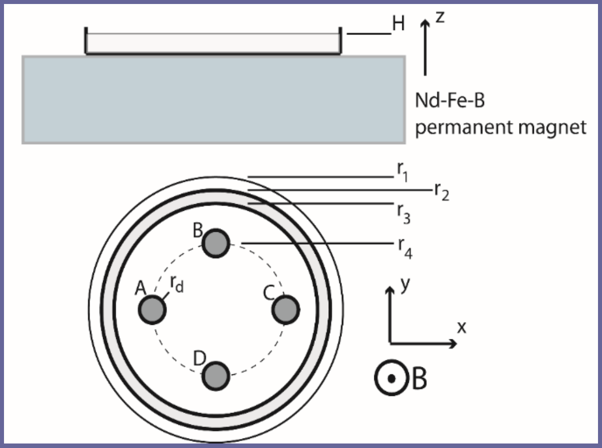
T = 10s
t = 1T, 5T, 10T, 20T

Problem Description

Switching Scheme 1

$$\phi_A = 0.04\text{V} \quad \phi_C = 0 \quad kT < t < kT + \frac{T}{2}$$

$$\phi_A = 0 \quad \phi_C = 0.04\text{V} \quad kT + \frac{T}{2} < t < (k+1)T$$



Switching Scheme 2

$$\phi_A = 0.04\text{V} \quad \phi_C = -0.04\text{V} \quad \nabla\phi_B = \nabla\phi_D = 0 \quad kT < t < kT + \frac{T}{2}$$

$$\nabla\phi_A = \nabla\phi_C = 0 \quad \phi_B = 0.04\text{V} \quad \phi_D = -0.04\text{V} \quad kT + \frac{T}{2} < t < (k+1)T$$

$k = 0, 1, 2, 3, \ldots\ldots\ldots$

Results

- **Scheme 1, Scheme 2**

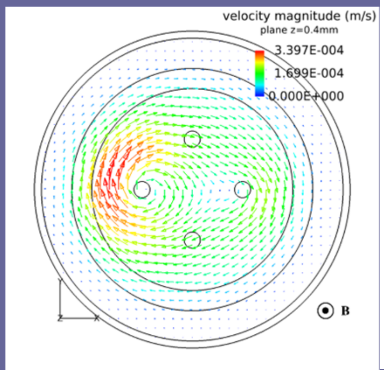
  Velocity vectors
  Velocity profiles
  Potential contours
  Current lines
  Poincaré maps
  Mixing performance

- **Scheme 2**

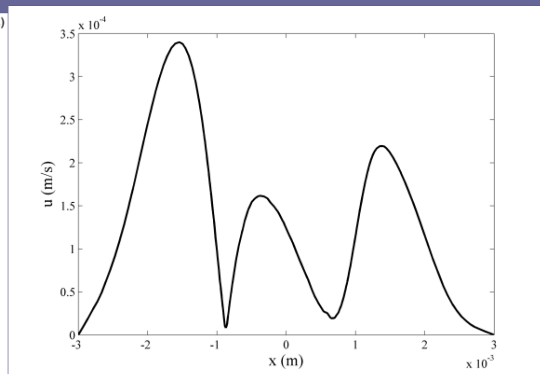
  …+
  Particle concentration maps
  Species concentration contours
  Material line deformation

- **Observations**

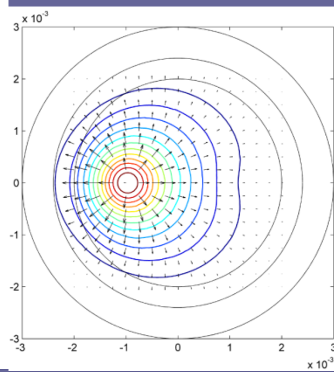
Results



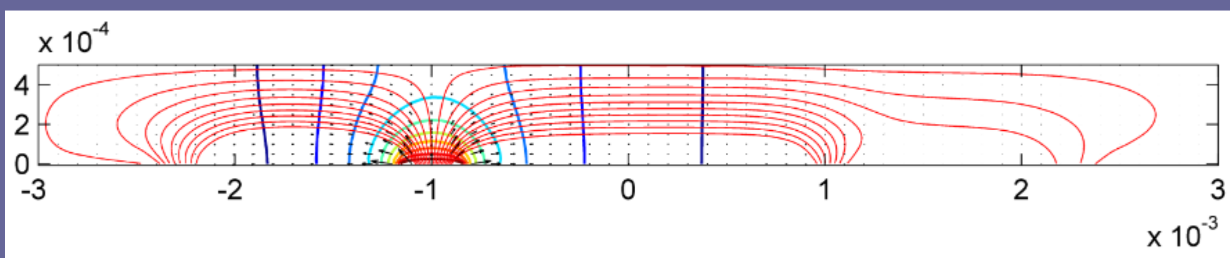
(a)



(b)



(c)



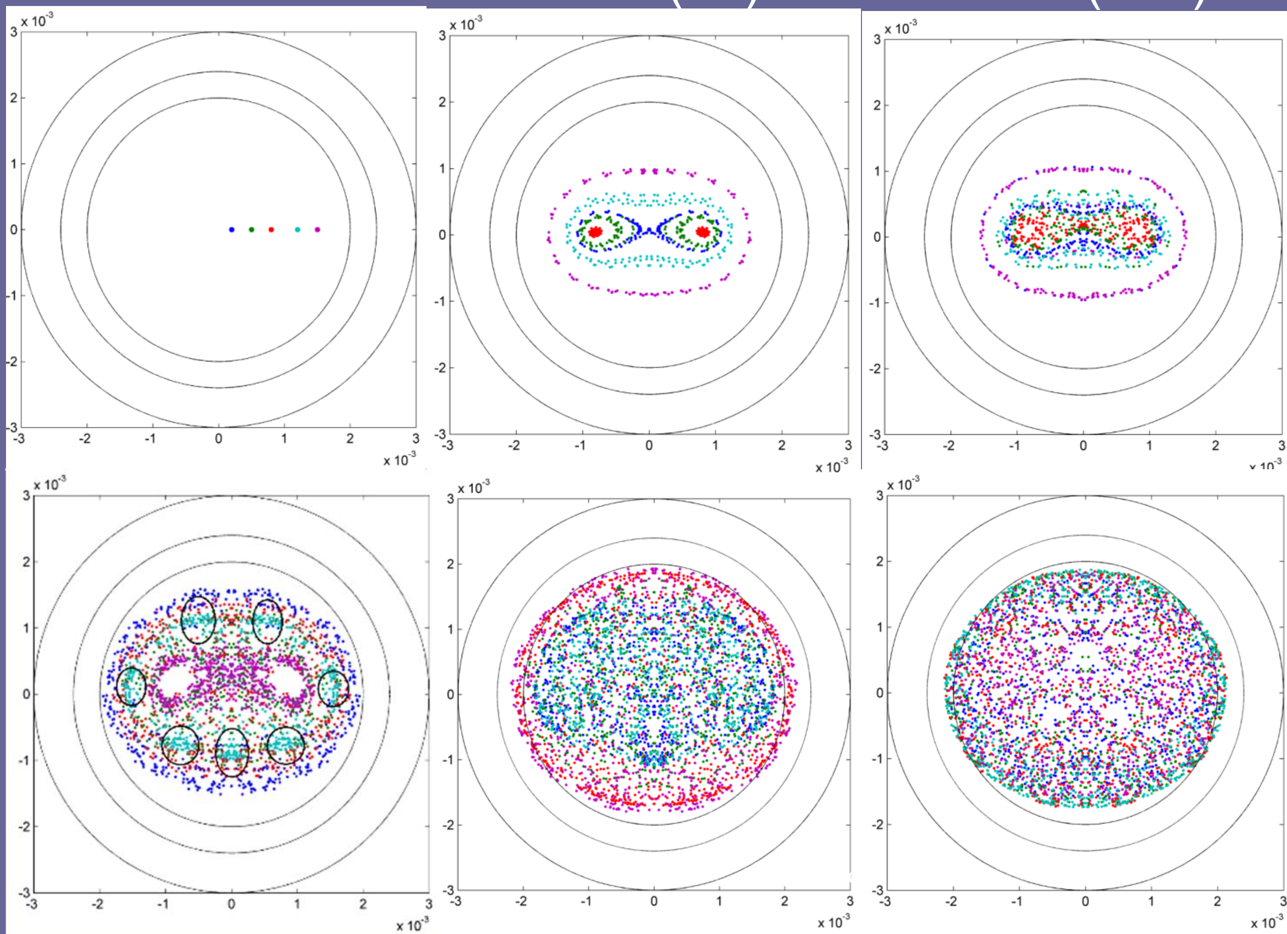
(d)

Scheme 1

(a) Velocity vectors
(b) Velocity profile
(c), (d) Potential contours and current flux lines
(c: z = 0.4 mm plane, d: y = 0 plane)

Results

t = 0 | T = 1s (75) | T = 2s (100)



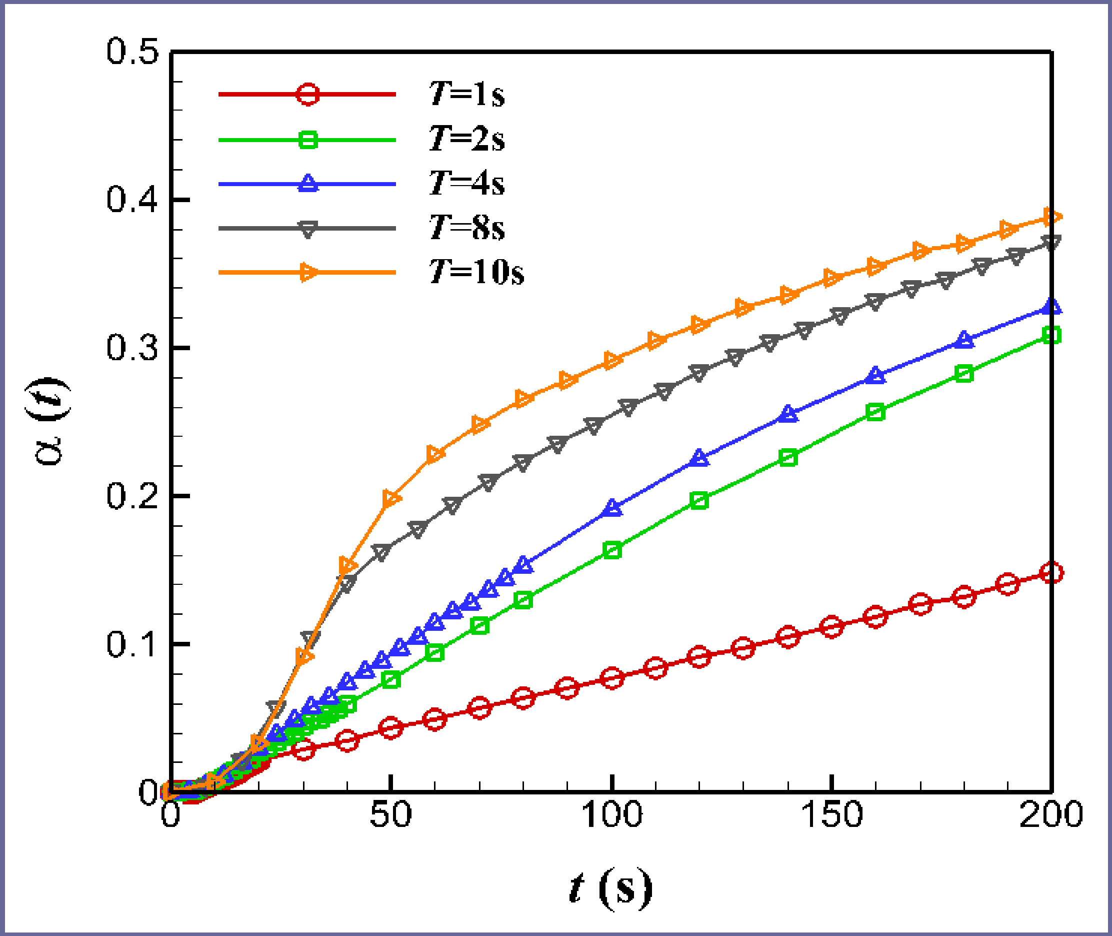
T = 4s (300) | T = 8s (400) | T = 10s (400)

Scheme 1

Poincaré maps
z = 0.4 mm

Number of periods are shown in parentheses

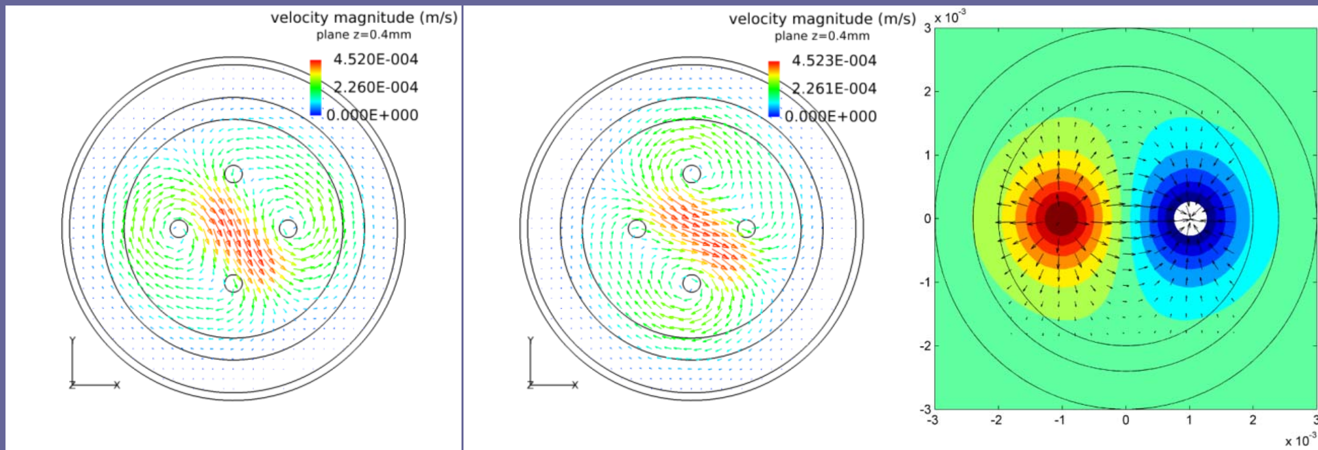
Results



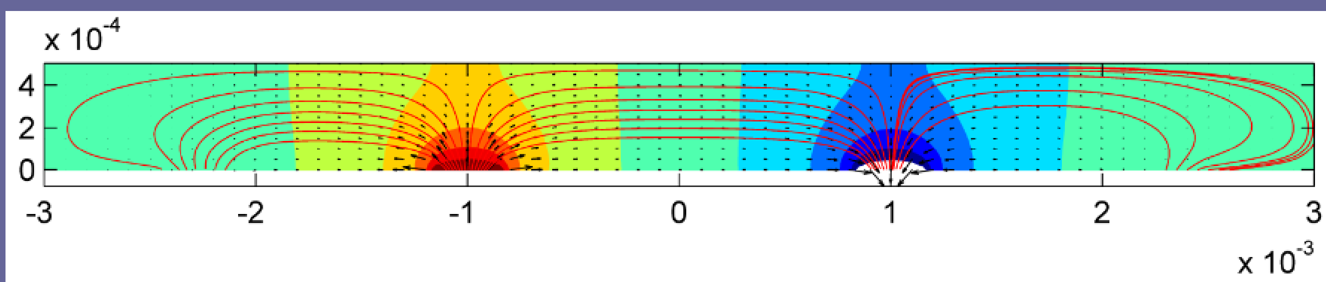
Scheme 1

Evolution of mixing quality vs. time *t* for 5 values of T in plane z = 0.4 mm

Results



(a) (b) (c)

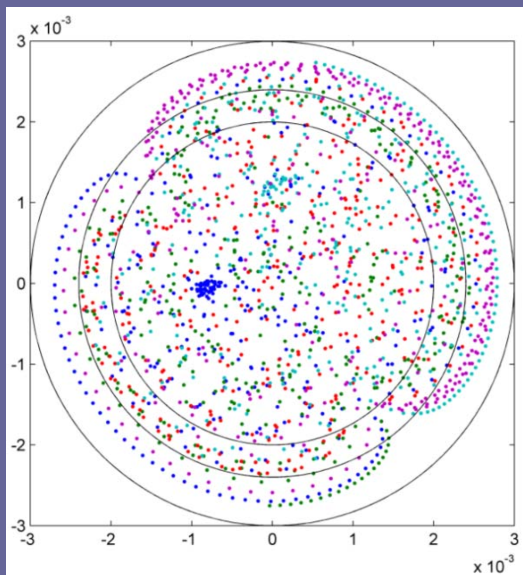


(d)

Scheme 2

(a) Velocity vectors
(b) Velocity vectors
(c), (d) Potential contours and current flux vectors and lines
(c: z = 0.4 mm plane, d: y = 0 plane)

Results



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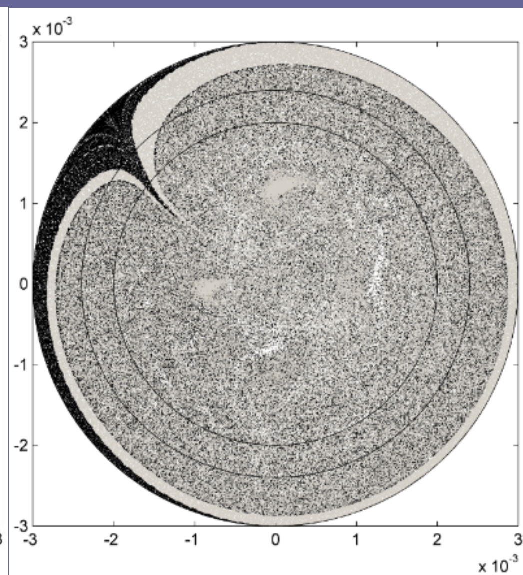

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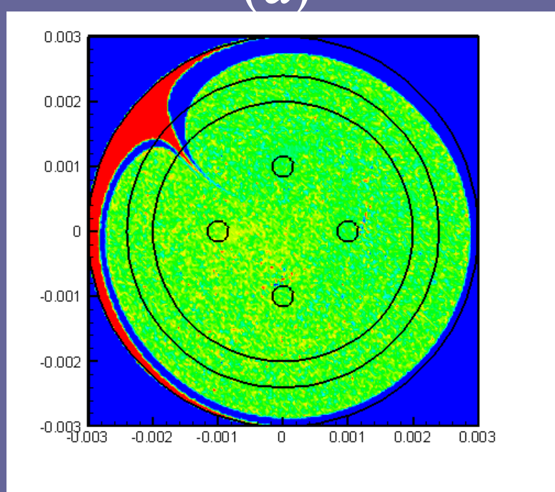
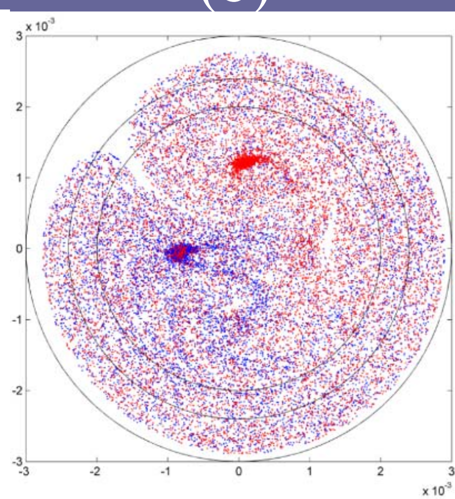


(c)



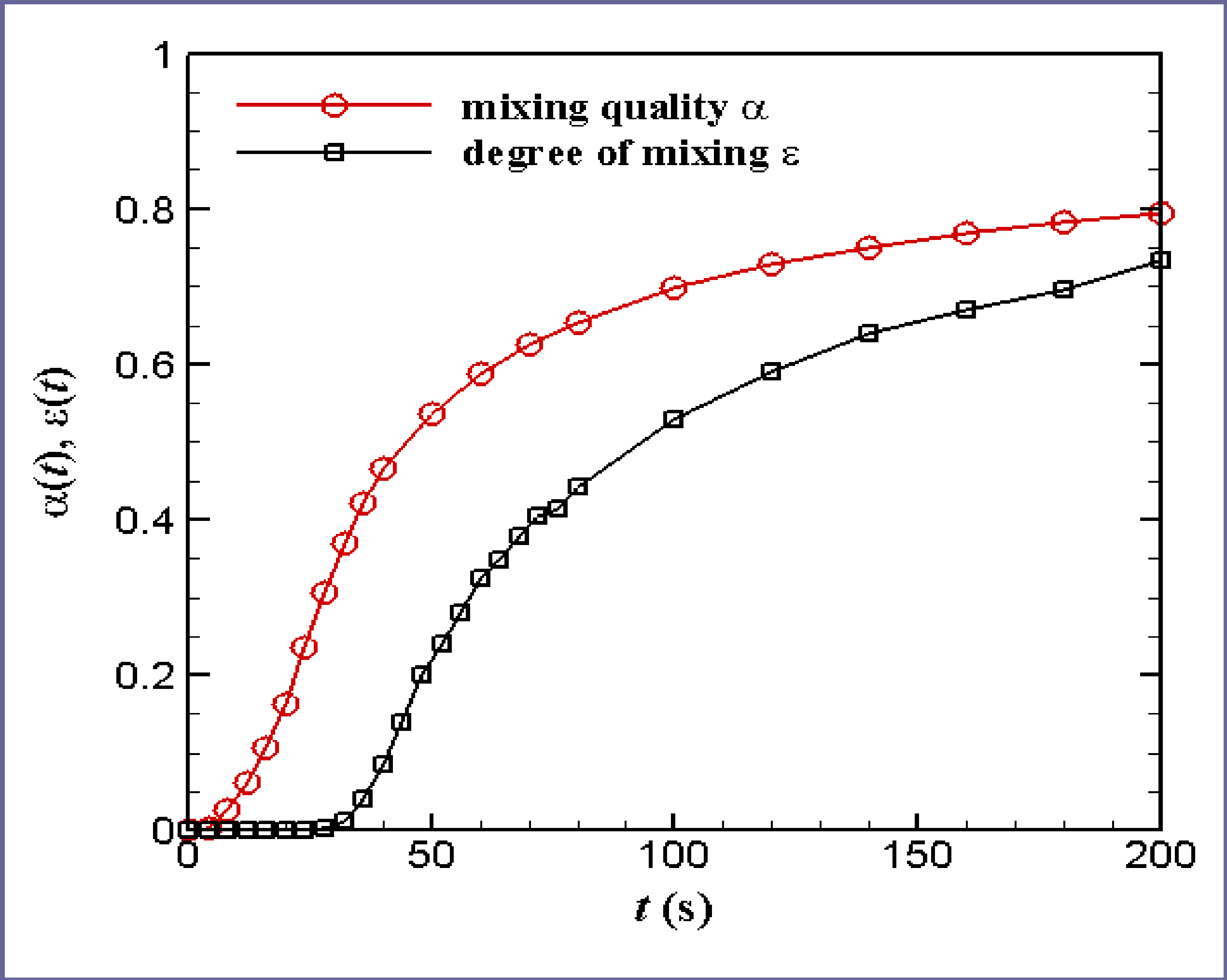
(d)

Scheme 2.

T = 4s, *t* = 200s z = 0.4 mm plane.

(a) Poincaré map
(b) particle concentration map
(c) species concentration contours
(d) Stretching and deformation of two material lines.

Results



Scheme 2

Temporal evolution of mixing quality α and degree of mixing ε. T = 4s.

Results

- Switching Scheme1 is a practical way to create the blinking vortex model
- Scheme 1 results are similar to those from the Stokes flow model
- Results from particles and material lines show that α increases with period T
- Switching Scheme 2 improves α over Switching Scheme 1
- The islands disappear faster in Scheme 2 compared to Scheme 1
- Each pair of disks is insulated during part of the cycle. *No current on the disk, no vortical flow around the disk to isolate it*

Conclusions and Future Work

- Even better mixing may be possible with more number of electrodes
- Optimal geometric placement and judicious choice of switching schemes may further improve mixing.
- Islands disappear faster with the electrodes insulated during part of the cycle
- The Navier-Stokes simulations show differences compared to the Stokes flow model for $Re_H >> 1$
- Good post-processing tools are necessary for analysis

Conclusions and Future Work

- Chaotic advection increases with increase with the dimension of the problem (no chaotic advection in 2D, and increases in the order: unsteady 2D and steady 3D, unsteady 3D
- Length scales and time scales have an influence on chaotic advection
- CFD simulations advance the modeling over potential flow and stokes flow
- Work on Transmission Line Equivalent Circuit (TLEC) model is in progress