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QUASIANALYTICAL ESTIMATES OF INDUCTANCE USING SUBCONDUCTOR METHODS

INDUKTIVITĀTES KVAZIANALĪTISKS APRĒĶINS AR SUBVADĪTĀJU METODI

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Abstract: *Subconductor methods or partial wire methods yield reliable results for conductor parameters such as inductance, capacitance, conductance and resistance and can be used to estimate these parameters for conductor configurations involving complex geometries which cannot be handled using analytical methods. The values of inductances obtained using partial wire method are critically compared with values obtained using analytical methods.*

Keywords: *Frequency-dependent inductance values, transmission tine, subconductor method, partial wire technique, power systems, skin effect.*

Introduction

The inductance is a parameter of electrical and electronic components dependent on geometry and material characteristics. The analysis of electrical and electronic circuits is dependent on the values of circuit parameters such as inductance, capacitance and resistance. As the current density at 50 Hz in conductors with radius greater than 1 cm, is very much different from that of a DC current density, due to skin and proximity effects, the inductance is frequency dependent. In this presentation, only the skin effect is taken into consideration in the evaluation of inductances.

Very often the estimation of inductance is complicated, especially for conductors with complex geometries. Generally the length of such cables is very much greater than the dimensions of the cross section and hence the problem can be viewed as two dimensional.

The skin effect is very often a complicated problem that can be solved analytically only for very few cases involving conductors with simpler geometries. The most commonly presented examples found in many text books are infinitely wide flat sheets and long straight cylindrical lines. The former case leads to the concept of the depth of penetration while the latter involves Bessel functions in its analytical closed-form solutions. While the cylindrical conductor model is practically applicable to cases like electric power transmission lines, it cannot be used to estimate the impedances of conductors with complicated cross sections (e.g., segment conductors, bus-bar, etc.) under the influence of skin effect.

The inductance can be estimated with Finite Element Methods (FEM) fairly straight forward. Such programs are easy to handle, mostly they use a friendly Graphical User Interface (GUI). They are leading to results which are very illustrative, colourful and which help to interpret the results physically with the help of good tables and field plots.

For a practising engineer with access to measurement facilities, such an estimation of cable parameters involving FEM methods can be very useful as the values obtained can be used to optimise the design and production of cables. When the method is established and verified for

given geometries using measurements and quasianalytical methods, the process can lead to reliable estimates, which can effectively be used in design and production stages.
Such approximate methods have been discussed by many authors, such as Ametani and Fuse [2]. The method is essentially a quasianalytical method for estimating inductance, resistance/conductance and capacitance. Very often such estimates are sufficiently accurate for a practising engineer.

1. The subconductor method

A. Basics

An alternative to solve the skin effect problem with FEM is a network model based on coupled-inductance theories [4] in which the conductor cross section is divided into m subconductors. The current density in each (thin) subconductor will be constant. The current direction of the subconductor is the direction of the current density.
By assuming uniform current distribution for each partial wire, we obtain m magnetically coupled R-L circuits parallel. Therefore, the actual current distribution over the conductor cross section is approximated by the currents flowing in the R-L branches. As long as m is sufficiently large, accurate results can be obtained. This method has been widely used for finding impedances of cables.
The coupled-inductance model is basically a circuit-oriented model which, though not as powerful as the FEM, is familiar to all electrical engineers and easy to code with a high level computer language.

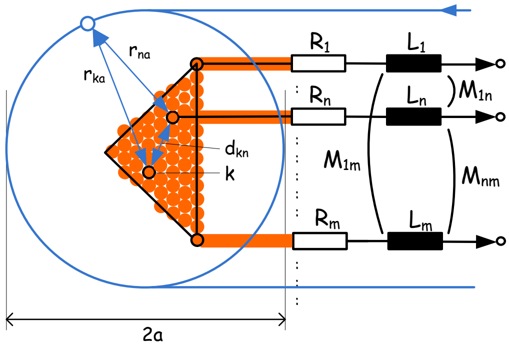

Fig. 1. **Solid triangle conductor with sub conductors and return conductor**

To begin with, the resistance and inductance of a conductor is estimated for non-uniform current density. The bulk conductor is considered as a collection of m cylindrical subconductors each with resistance R_n and self-inductance L_n with mutual inductances M_{nk} to all the other conductors. Such a network is excited with a voltage U_0. Using the distribution of the total current I_0 in all the partial wires, the current density G_0 and the impedance Z_0 can be estimated.
A return conductor of radius *a* is included in the network thus guaranteeing zero as the total sum of current in the system. As such the return conductor has no influence on the magnetic field and hence no effect on the calculation of inductance values.

B. Determination of Network Parameters

The resistance of one conductor is

$$R_i = m\frac{l}{\kappa A_0} \tag{1}$$

with conductivity κ and cross-sectional area $A_0 = \pi r_0^2$ for each subconductor.
In the estimation of the inductance values, three conductors are taken into account as shown in Figure 2: conductor n, conductor k and the return conductor of radius a. The inductance

values can be estimated using [3, chapter 5.23, 6.5] through the calculation of the magnetic fields.

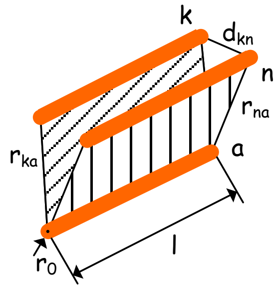

Fig. 2. **For the calculation of self and mutual inductances using three sample conductors n, k and a**

L_n is the self inductance of the conductor loop n,a with the length l, resulting from the n-th and the corresponding return conductor *a* surrounded. M_{kn} is the mutal inductance of the loops with the conductors n,a and k,a.

$$L_n = \frac{\mu l}{\pi} \ln \frac{r_{na}}{r_0} \tag{2}$$

$$M_{kn} = \frac{\mu l}{2\pi} \ln \frac{r_{ka} r_{na}}{r_{kn} r_0} \tag{3}$$

The network consisting of these partial wires are then connected to a voltage source as depicted in Figure 3.

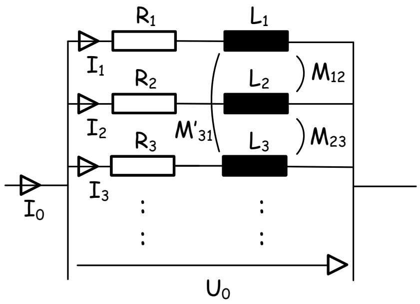

Fig. 3. **Ladder network of a transmission line involving discrete components**

C. Determination of individual currents in each of the conductors in the subconductor bundle

The equivalent circuit shown in Figure 3 is described by the system of equations, given by (4):

$$U_n = (R_n + j\omega L_n)\underline{I}_n + \sum_{\substack{k=1 \\ k \neq n}}^{m} M_{nk} \underline{I}_k \tag{4}$$

With $n = 1 \ldots m$, U_n the applied voltage on conductor n, resistance and inductance of conductor n being R_n and L_n, and the mutual inductances between conductor n and k being $M_{nk} = M_{kn}$, with $k,n = 1,2,\ldots m$.

For each conductor, an equation of the form given in (4) will be used and the whole set of equations can be written as a matrix equation as given in (5).

$$\underline{U} = \underline{R}\,\underline{I} + j\omega \underline{M}_0 \underline{I} = \underline{Z}\,\underline{I} \tag{5}$$

The alphabets with underscore indicate the different matrices as indicated below:

$\underline{U} = \begin{pmatrix} U_1 \\ U_2 \\ \vdots \end{pmatrix}$ representing the voltages

$\underline{R} = \begin{pmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$ representing the resistances of individual conductors

$\underline{M}_0 = \begin{pmatrix} L_1 & M_{12} & M_{13} & \cdots \\ M_{21} & L_2 & M_{23} & \cdots \\ M_{31} & M_{32} & L_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$ representing the different self-inductances (diagonal elements) and mutual inductances.

Then the currents can be expressed as

$$\underline{I} = \underline{Z}^{-1}\underline{U} \tag{6}$$

The total current density will be the sum of the individual currents obtained from (6).

2. Verification for a cylindrical bulk conductor

For the purposes of verification, a bulk conductor with current $I(t) = \hat{I}\cos\omega t$ is selected. Figure 4 shows how the sub-conductors are selected for the given bulk conductor.

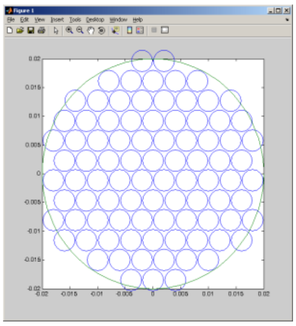

Fig. 4. **Bulk conductor seen as a bunch of wires**

As mentioned before the return conductor is a pipe of radius a.

The system will be studied using equations (4) and (5) for estimating the currents.

The inductance is the sum of self- and external inductances $L = L_o + L_i$. The external inductance is dependent on the magnetic flux in the surrounding medium of air and hence independent of frequency but dependent on the radius of the conductor. The self-inductance does not depend on the radius of the conductor but on frequency.

The self-inductance is given by analytical equations as given in [1, chapter 8.8.1.2]. Due to the existing rotational symmetry is the value for self-inductance not dependent on angle.

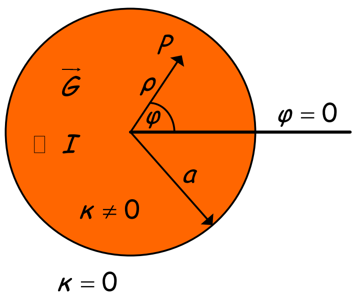

Fig. 5. **Bulk conductor with AC current I(t)**

The current density is given by [1, chapter 8.8.1.2].

$$G\,\rho = \frac{1}{2}\frac{\alpha a I_0\ \alpha\rho}{I_1\ \alpha a}G_0 \text{ with } G_0 = \frac{\hat{I}}{\pi a^2};\ \alpha = j\omega\kappa\mu \tag{7}$$

Using the current density, the impedance now can be evaluated as given in (8).

$$Z_0 = R_0 + j\omega L_0 = \frac{l}{\pi\kappa a^2}\ 1+j\ \frac{a}{\delta}\frac{I_0\left[\ 1+j\ a/\delta\right]}{2I_1\left[\ 1+j\ a/\delta\right]} \quad \text{with } \delta = \sqrt{\frac{2}{\omega\kappa\mu}} \tag{8}$$

Figure 6 shows the estimated values using equivalent subconductor bundles and compares these with the values obtained using analytical methods. The analytical methods using close form expressions and the sub-conductor method gives results which tally fairly well with each other as can be seen for values plotted for R_{subc} & R_{ana} and ωL_{subc} & ωL_{ana} (subconductor & analytical).

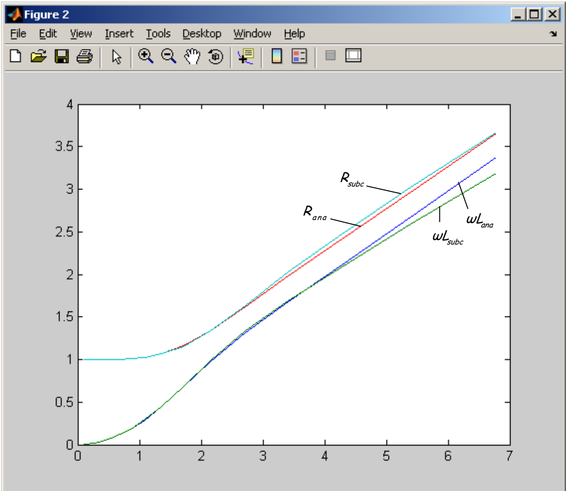

Fig. 6. **Results R und ωL for solid conductors each normalised w.r.t DC resistance**

3. Segmented Conductor

With the power cables connected to consumers in remote areas, these can serve as possible substitutes for ADSL or for any other electrical signals, thus making it essential to know the values
R', L', C', G', as the transmission characteristics is dependent on these cable parameters. As a last example in this paper, a segmented conductor as shown in Figure 7 will be studied.

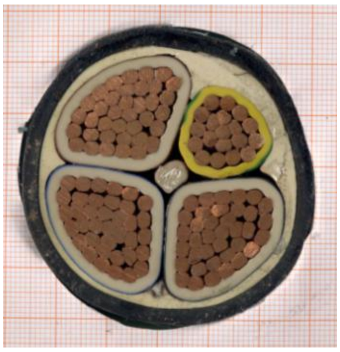

Fig. 7. **Segmented conductor used for testing the methodology used in determining the cable parameters**

Figure 8 shows the segmented conductor of Figure 7 in a form better suitable for estimation purposes.

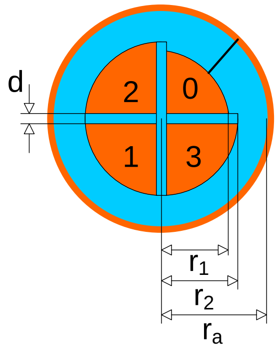

Fig. 8. **The cross-sectional area of conductor studied**

Transmission line models are normally based on the transmission line theory very often based on the ladder network with discrete components as shown in Figure 9. This will be used for determining the transmission functions for the network shown in Figure 9.

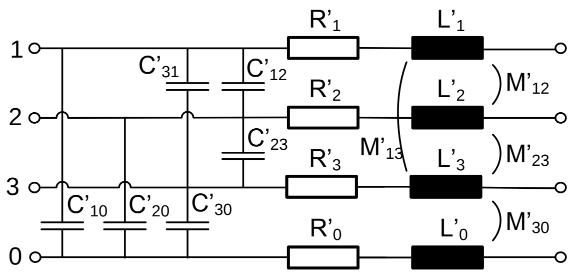

Fig. 9. **Ladder network of a transmission line involving discrete components**

The „dashes“ " ' " are used to imply parameter values per unit length, for example $R' = R / l$. The four conductors in the network shown in Figure 9 correspond to the four bulk conductors of Figure 7 or Figure 8. Using different combinations of bulk conductors for feeding and return conductors and the system equations, the current in each wire can be measured.
As an example the cable segments 2 and 3 are supplied with currents. The equivalent circuit is shown in Figure 10.

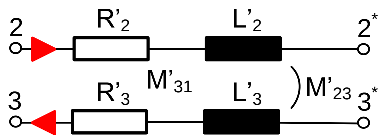

Fig. 10. **Equivalent circuit for the case involving current supply to conductors 1 and 2**

Each conductor in such a set of clustered conductors has its own resistance and inductance and in addition mutual inductances to each and every other conductor in the cluster. The verification is now limited to conductor segments 2 and 3 only.

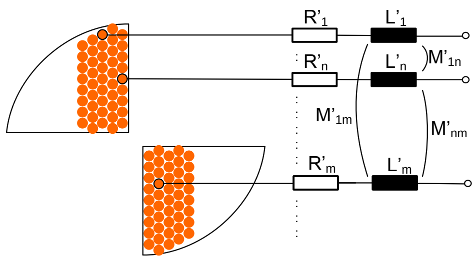

Fig. 11. **Bulk of conductor viewed as a cluster of snugly lying set of single conductors in line with partial wire techniques**

The bulk conductor is divided into a set of conductors as shown in Figure 11 and is described using the equivalent electrical circuit by (4) and (5). Figure 12 shows the locations of all the conductors for a sub-condcutor system consisting of 2 x 93 conductors. A system of equations with 186 equations will then be solved.

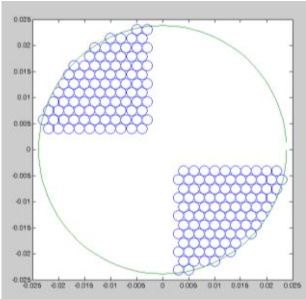

Fig. 12. **Distribution of the cluster of wires for segments 2 and 3 of cable shown in Fig. 8**

The current distribution at $f = 50\,Hz$ is given in Figure 13. For the sake of comparison, the conductivity of segment 0 and 1 is set to zero.

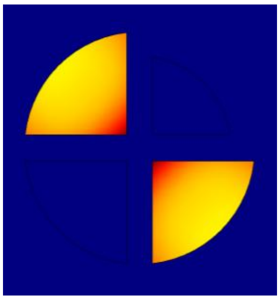

Fig. 13. **Current distribution when only conductor segments 2 and 3 are fed**

Figure 14 shows the resistance and inductance values of the segments based on partial wire method and the values obtained with Comsol Multiphysics for the actual cable geometry (shown by the blue circles).

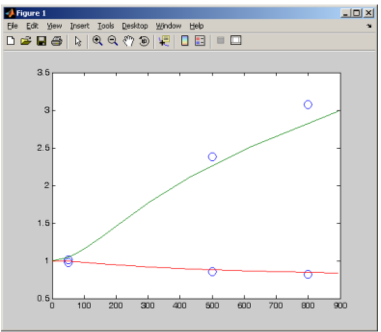

Fig. 14. **Inductance and resistance values estimated (continuous lines) using partial wire method compared to estimates using Comsol Multiphysics (circles)**

The difference in the resistance values can be attributed to non-conducting areas (filling factor of the circles with respect to the bulk conductor). This can be improved by improving the model.

Conclusions

This paper shows the numerical advantage of partial wire method in the estimation of cable parameters. Partial wire network can easily represent the skin-effect using a network of these conductors. The methodology developed was tested for some simple cable geometries which can be studied using close form solutions. Building upon this, the method was also applied for massive bulk conductors placed in a magnetic field.

For planar problems, the method is basically a two dimensional problem with dedicated GUI making the evaluation straight forward and simple.

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